Determining α_s at NNLO from Event-Shape Data

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Motivation: Jets in e^+e^--Annihilation

- Very prominent role for phenomenology:
  - discovery of gluon and its properties,
  - precise determination of the QCD coupling constant α_s:

  $\longrightarrow$ NLO value of α_s from LEP data suffers mainly from theoretical scale uncertainty:

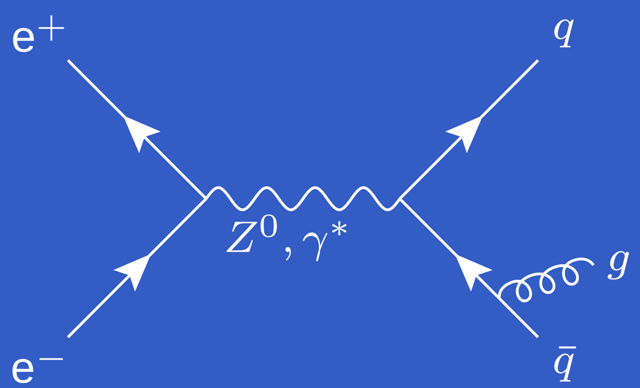
$$\alpha_s\,(M_Z) \;=\; 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \pm 0.0013(\text{had}) \pm 0.0047(\text{scale})$$

[LEPQCDWG]

- Ideal for QCD tests and theoretical developments:
  - no hadronic initial state,
  - computational tools extended to more difficult initial state in a second step.

Jet Observable in e^+e^--Annihilation

- $e^+e^- \rightarrow 3$ jets at leading order:



$$\frac{d\sigma}{dE_g\, d\cos\theta_{\bar{q}g}} \propto \sigma_0 \frac{\alpha_s}{2\pi} \frac{1}{E_g(1-\cos\theta_{\bar{q}g})}$$

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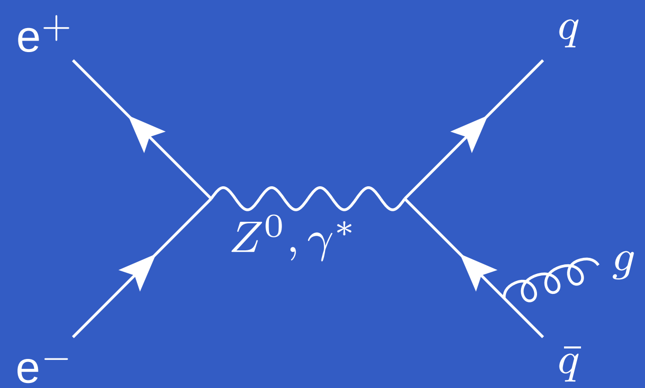

Born cross section for $Z, \gamma \to q\bar{q}$

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Bremsstrahlung:
enhancement for $E_g \to 0$
and for $\theta_{\bar{q}g} \to 0$

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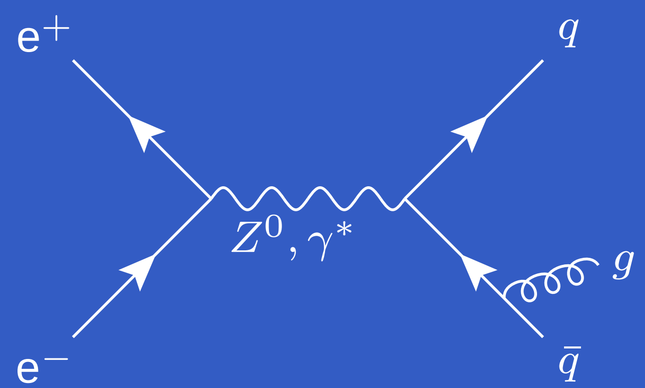

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Bremsstrahlung:
enhancement for $E_g \rightarrow 0$
and for $\theta_{\bar{q}g} \rightarrow 0$

- Experimental observable:
  - $\rightarrow$ jet rates (number of jets),
  - $\rightarrow$ event-shape distributions,
  - $\rightarrow$ event-shape moments.

- Well suited also for theoretical pQCD predictions since many are IR and collinear safe.

Jet Rates

- Definition of a jet relies on a jet algorithm,

$\longrightarrow$ recombination of particles to jets according to distance measure and rec. scheme

- The most widely used is the Durham jet algorithm:

$$y_{ij,D} = \frac{2\min\left(E_i^2, E_j^2\right)(1-\cos\theta_{ij})}{E_{\text{vis}}^2},$$

$\longrightarrow$ new pseudo-particle defined combining 4-momenta: 'E' recombination scheme.

- Combination procedure iterated as long as $y_{ij,D} < y_{\text{cut}}$.

- Experimental studies based on jet rates:

$$R_n = \frac{n\text{−jet cross section}}{\text{total hadronic cross section}}, \qquad n = 2, 3, 4, 5.$$

Jet Rates

- Definition of a
  → recombination of p … ec. scheme
- The most wide
  → new pseudo-partic … scheme.
- Combination p … t.
- Experimental s

$$R_n = \quad , 4, 5.$$

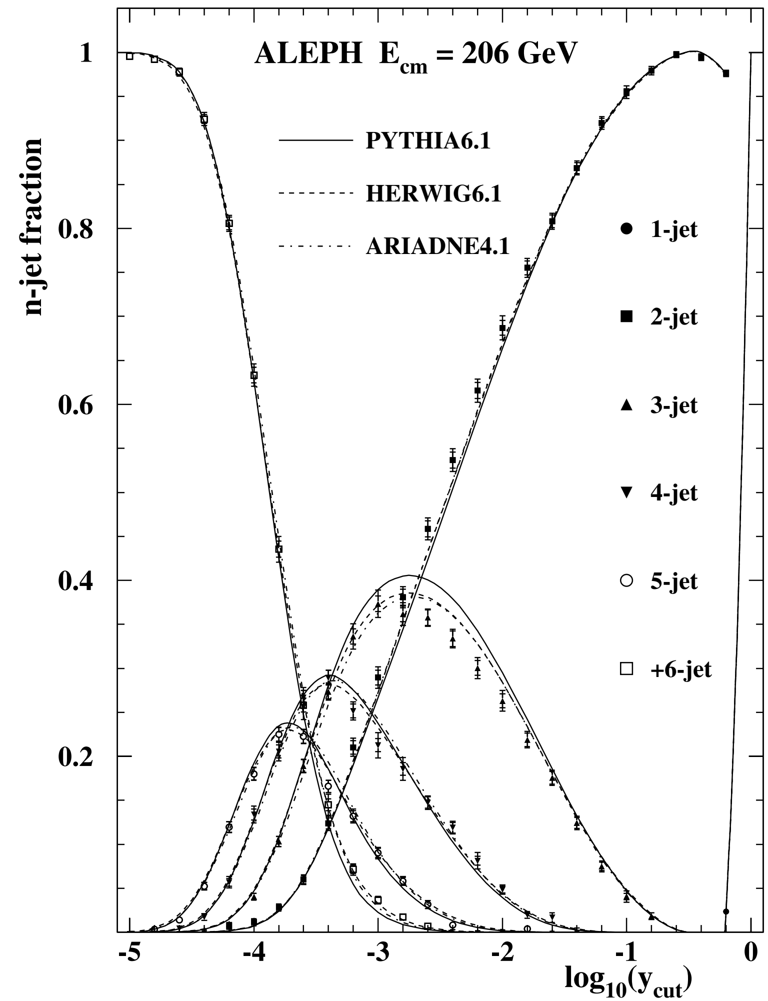
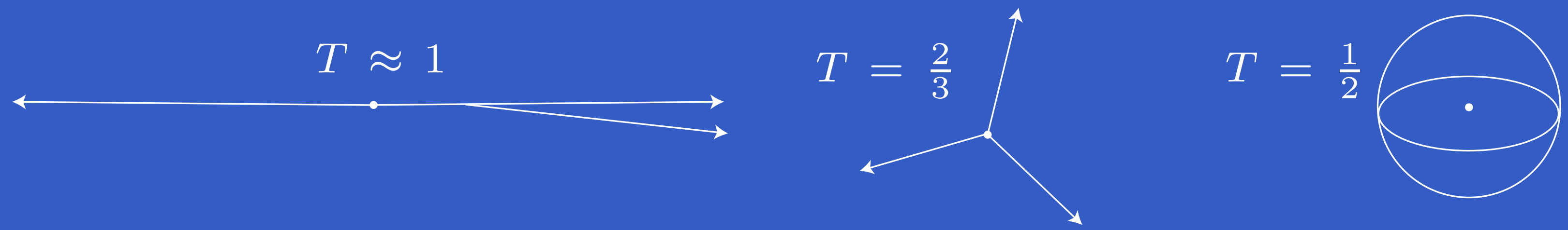


Fig. 7. Measured n-jet fractions for $n = 1, 2, 3, 4, 5$ and $n \geq 6$ and the predictions of Monte Carlo models, at a centre-of-mass energy of 206 GeV [ALEPH Collaboration, 2004]

Event-Shape Observables

- Parametrize geometrical properties of energy-momentum flow,
- canonical example: Thrust

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$



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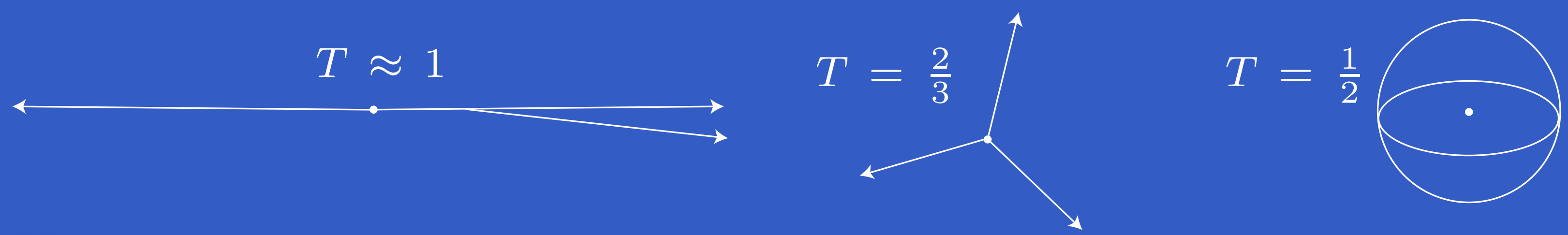

- LEP standard set:
  - Thrust: [Brandt,Farhi]

    $$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

  - Heavy jet mass: [Clavelli,Wyler]

    $$\rho = \max_i \frac{\left(\sum_{n \in H_i} |\vec{p}_n|\right)^2}{E^2_{\text{tot-vis.}}}$$

  - C-parameter: EV of tensor [Parisi]

    $$\Theta^{\alpha\beta} = \frac{1}{\sum_i |\vec{p}_i|} \sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}$$

  - Jet Broadenings: [Rakow,Webber]

    $$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}$$

    $$B_T = B_1 + B_2$$

    $$B_W = \max(B_1, B_2)$$

  - Durham $2 \to 3$ jet parameter: Y_3

    [Catani,Dokshitzer,Olsson,Turnock,Webber]

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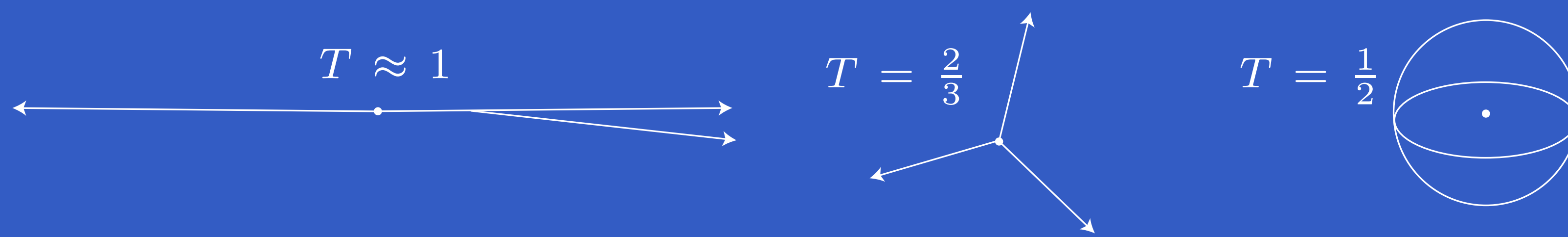

- LEP standard set:
  - Thrust: [Brandt,F
    $T = \max_{\vec{n}}$
  - s: [Rakow,Webber]
    $\vec{p}_k \times \vec{n}_T|$
    $|\vec{p}_j|$
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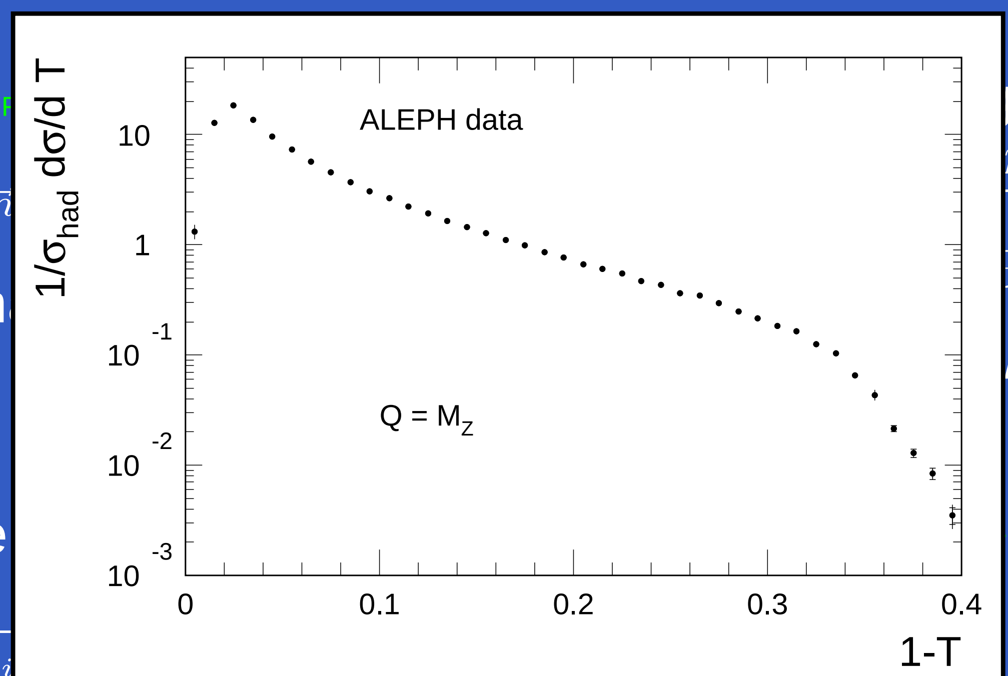
    $\Theta^{\alpha\beta} = \overline{\sum_i}$



Event-Shape Observables

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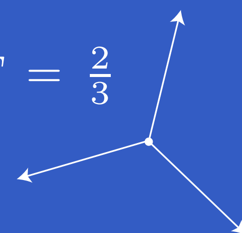
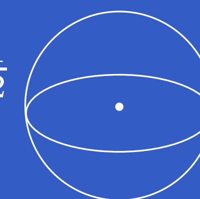
$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

$T \approx 1$

$T = \frac{2}{3}$



$T = \frac{1}{2}$



Need resummation

- LEP standard set:
  - Thrust: [Brandt,F...

    $T = \max_{\vec{n}}$ ...

  - Heavy jet m...

    $\rho = \max_i$ ...

  - C-paramete...

    $\Theta^{\alpha\beta} = \frac{\ldots}{\sum_i \ldots}$

...s: [Rakow,Webber]

$\ldots_k \times \vec{n}_T| \; / \; \ldots \vec{p}_j|$

...2

...$B_1, B_2)$

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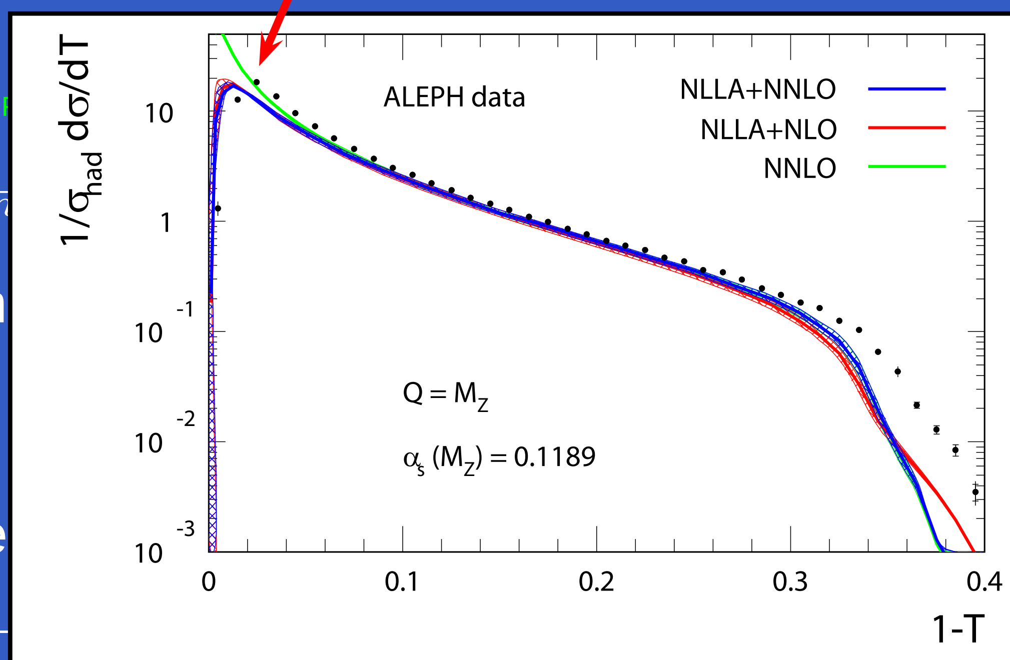

Moments of Event-Shape Observables

- n-th moment of event-shape observable y defined by:

$$\langle y^n \rangle = \frac{1}{\sigma_{\rm had}} \int_0^{y_{\rm max}} y^n \frac{{\rm d}\sigma}{{\rm d}y} {\rm d}y$$

- Higher moments more sensible to multijet region.
- Complementary to distributions $\rightarrow$ fully inclusive in phase space,
  - experimentally determined by summing over events:

$$\langle y^n \rangle_{\rm exp} = \sum_{i=1}^{N} y_i^n .$$

- Hadronization corrections expected to be additive,
  - divide perturbative and non-perturbative contributions:

$$\langle y^n \rangle = \langle y^n \rangle_{\rm pt} + \langle y^n \rangle_{\rm np} ,$$

Moments of Event-Shape Observables

- n-th m
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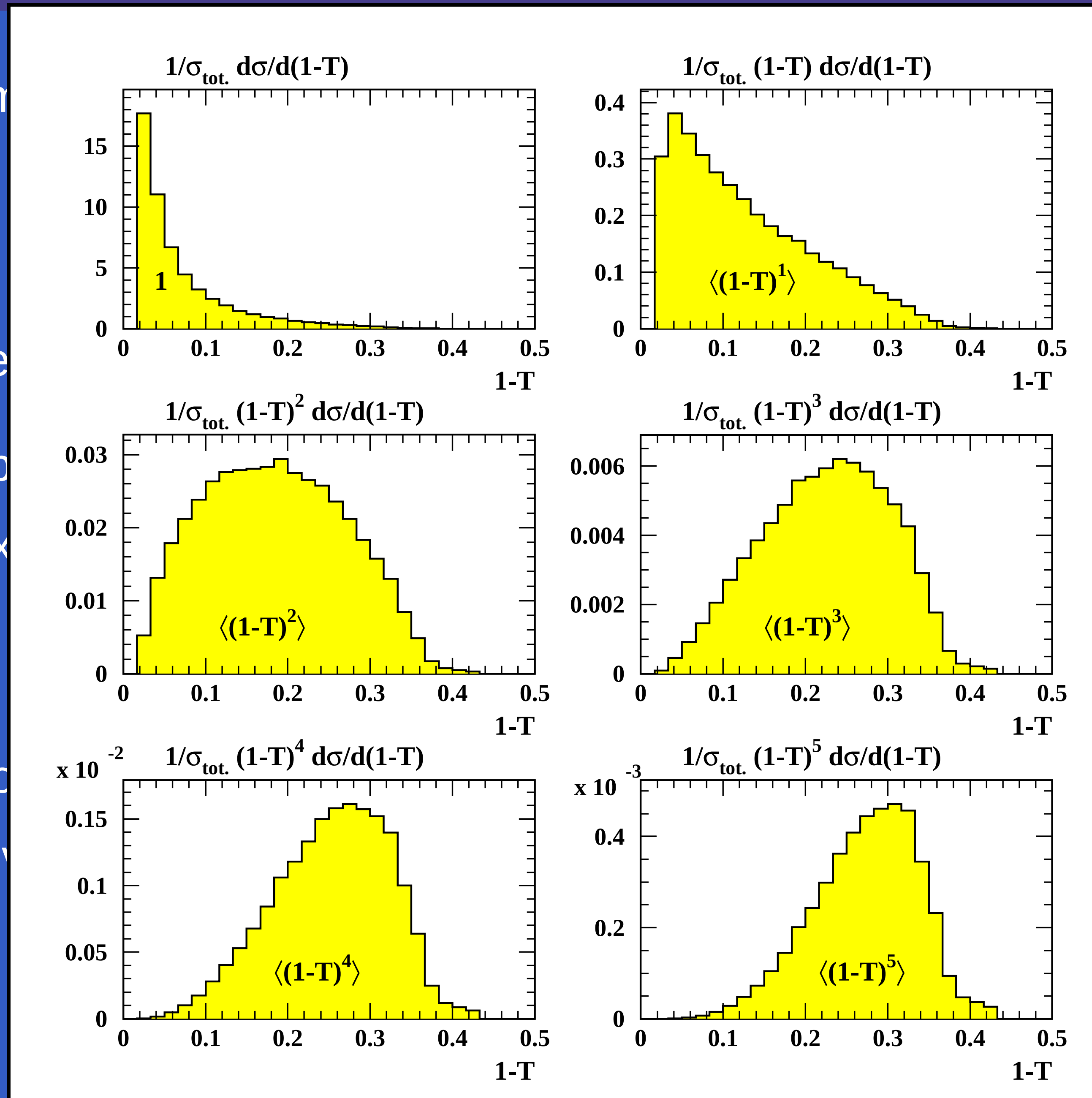

[C. Pahl, 2007]

Recent Theoretical Progress

- State-of-the-art up to recently:
  - fixed NLO calculations, [Ellis, Ross, Terrano; Kunszt, Nason; Giele, Glover; Catani, Seymour]
  - NLL resummation, [Catani, Trentadue, Turnock, Webber; Banfi, Salam, Zanderighi].
- Very important progress in the last three years:
  - for all event-shape observables (in particular LEP standard set):
    - fixed NNLO computation of jet rates and event-shape observables [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich; Weinzierl]
    - matching of NNLO+NLLA [Gehrmann, Stenzel, G.L.]
    - non-perturbative corrections to moments at NNLO [Gehrmann, Jaquier, G.L.]
  - only for thrust:
    - N^3LL resummation in SCET and matching with NNLO, [Schwartz; Becher, Schwartz]
    - non-perturbative corrections to NNLO+NLLA distribution, [Davison, Webber]

Determination of α_S

- Recent works:
  - α_S fit from NNLO and NNLO+NLLA event-shape distributions and ALEPH data, [Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, G. L., Stenzel.]
  - α_S fit with NNLO+NLLA distribution and non-perturbative power corrections for T, [Davison, Webber]
  - α_S fit with N^3LLA predictions for T and ALEPH data, [Becher, Schwartz]
  - α_S fit from NNLO and NNLO+NLLA event-shape distributions and JADE data, [Bethke, Kluth, Pahl, Schieck and JADE Collaboration.]
  - α_S fit using NNLO predictions for moments of event-shape and JADE and OPAL data, [Gehrmann, Jaquier, G. L.]
  - α_S fit using NNLO predictions for jet rates and ALEPH data, [Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Stenzel.]

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Uncertainties in α_S from distributions

- Experimental uncertainties:
  - track reconstr., event selection, detector corrections: cut variations or MC
  - background and ISR (LEP2),

  $\sim 1\%$

- Hadronization uncertainties:
  - difference between various models for hadronization: $\sim 0.7 - 1.5\%$

    Pythia (String frag.), Herwig (Cluster frag.), Ariadne (Dipole + String frag.),

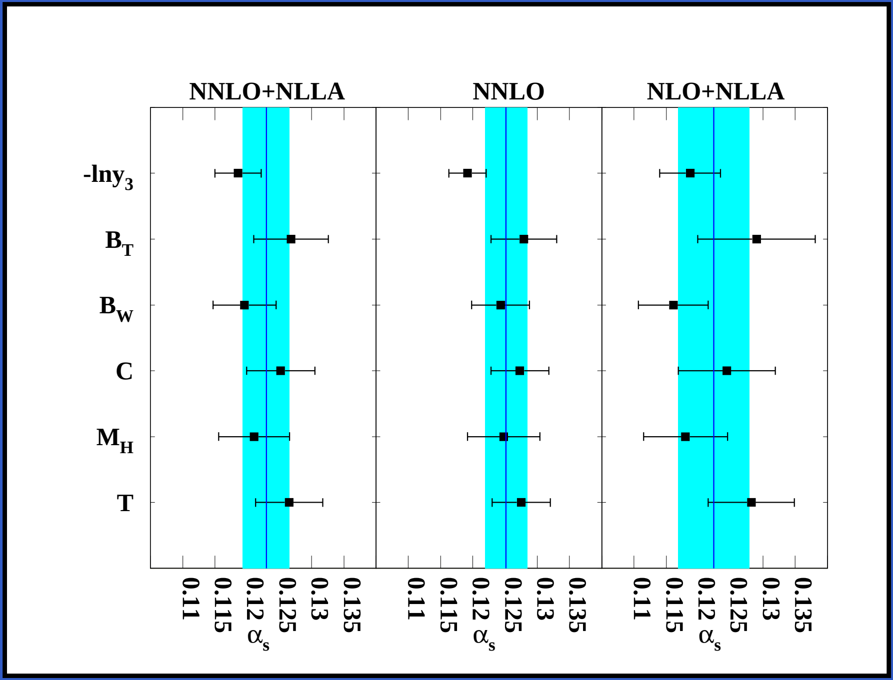
- Theoretical uncertainties (pQCD and resummation):
  - variation of theoretical parameters: $x_\mu, \ldots$
  - uncertainty for b-quark mass correction.

  $\sim 3.5 - 5\%$

- Uncertainty band method to estimate missing higher orders

  [Ford, Jones, Salam, Stenzel, Wicke.]

Extraction of α_s: NNLO+NLLA



- reduced scatter among variables at NNLO
- reduced scale uncertainty compared to NLO and NLO+NLLA
- scale uncertainty increase from NNLO to NNLO+NLLA $\rightarrow$ two-loop running not compensated in resummation

$$\alpha_s\,(M_Z) = 0.1224 \pm 0.0009(\text{stat.}) \pm 0.0009(\text{exp.}) \pm 0.0012(\text{had.}) \pm 0.0035(\text{theo.})$$

- Two class of observables:
  - T, C-par., B_{tot}: higher fit result $\rightarrow$ sizeable missing higher order
  - $-\ln y_3$, B_w, M_H: lower fit result, $\rightarrow$ good convergence of pert. expansion

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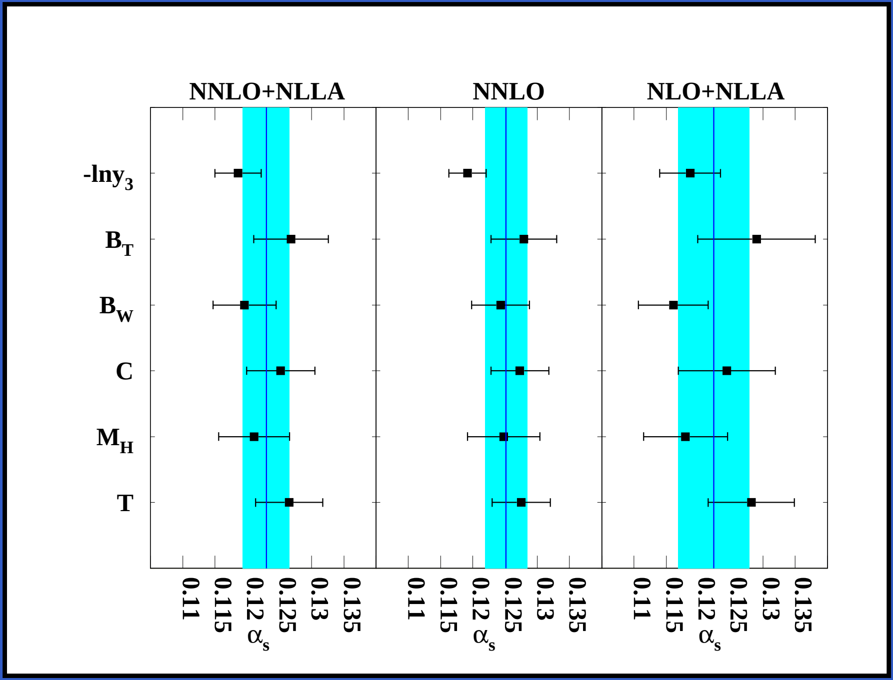

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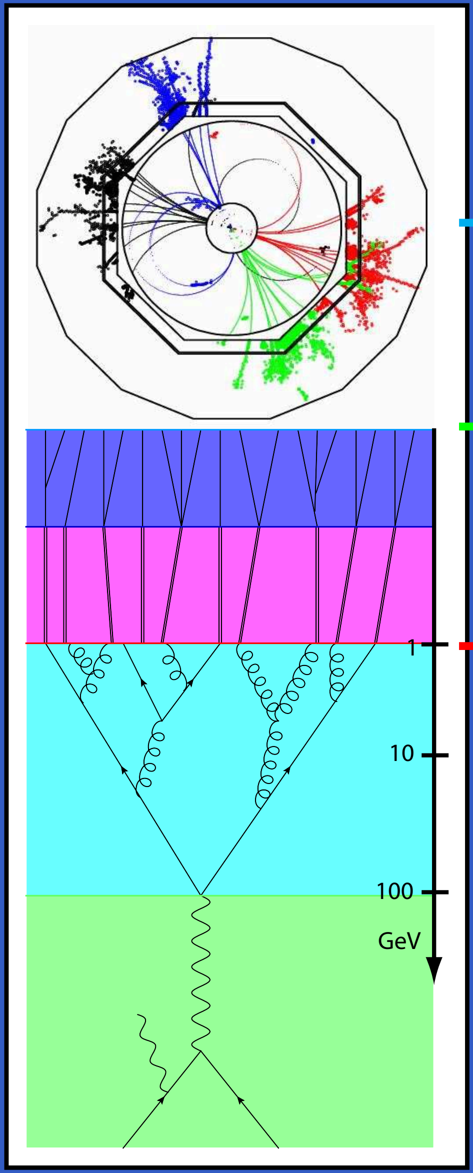
What about hadronization corrections?

Determination of α_s: Hadronization

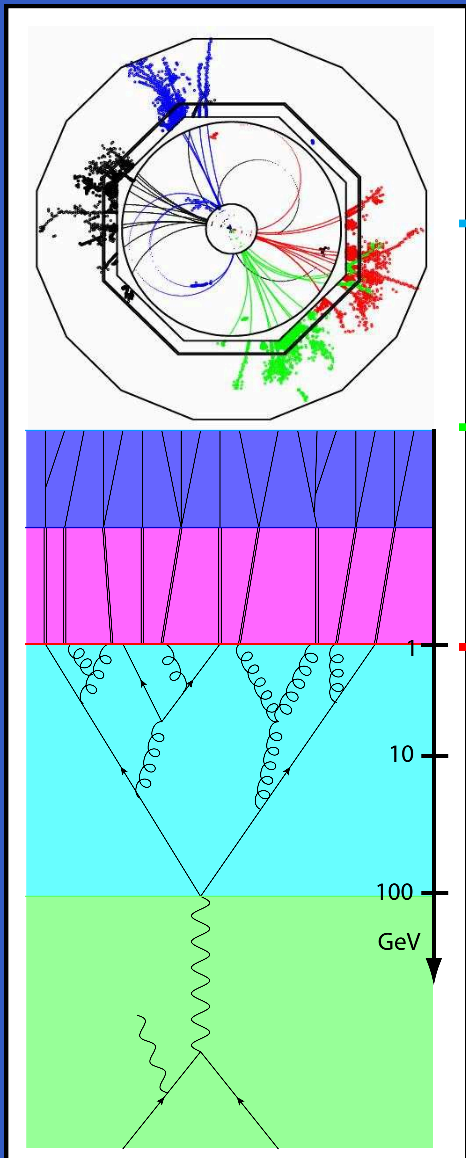


detector level

hadron level

parton level

Determination of α_s: Hadronization



detector level

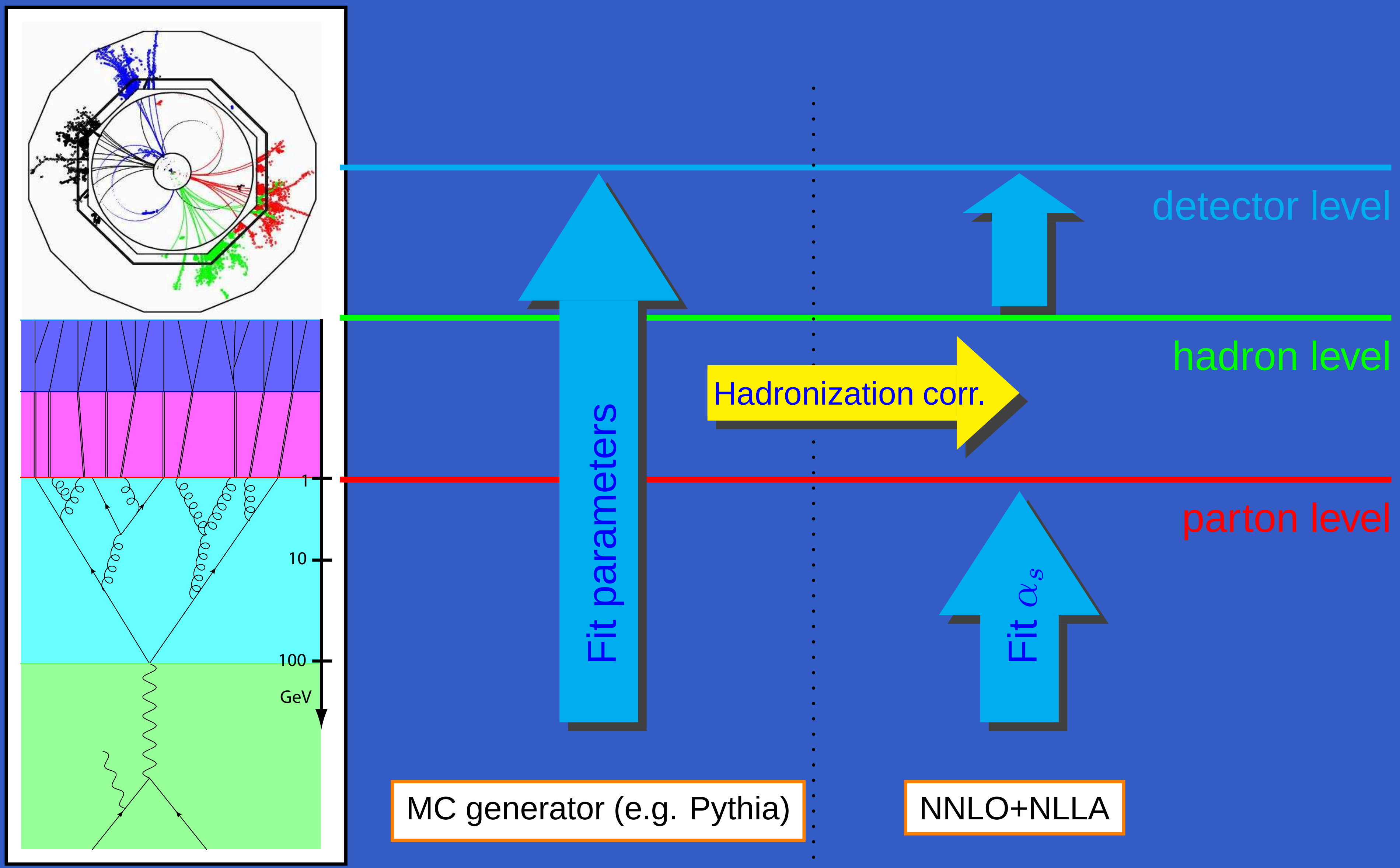
hadron level

parton level

Fit parameters

MC generator (e.g. Pythia)

Determination of α_s: Hadronization



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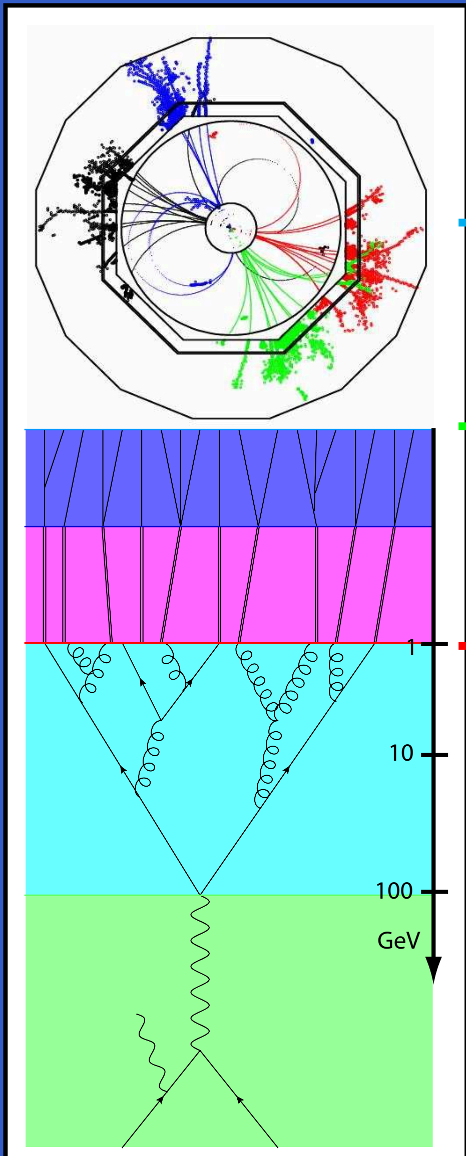
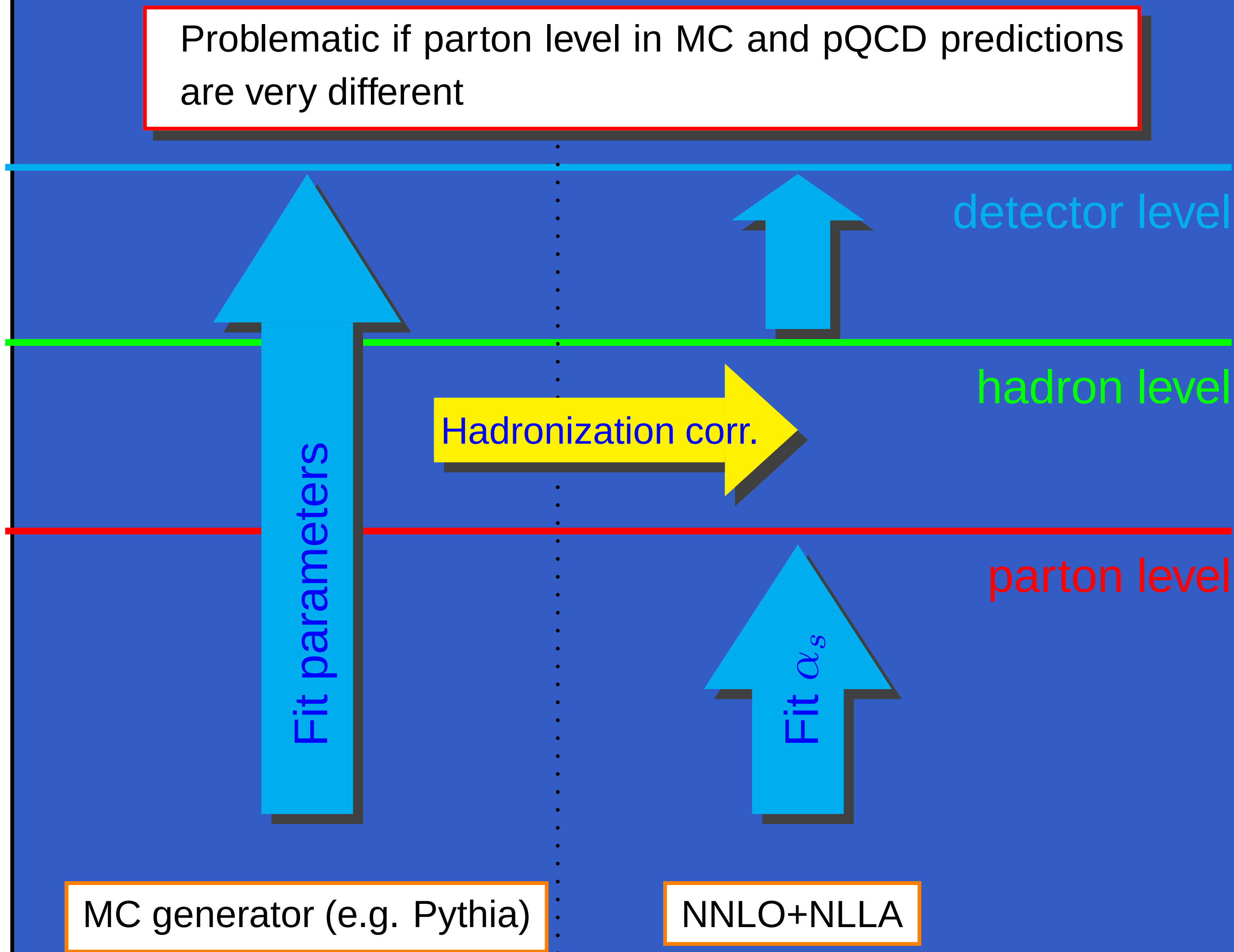
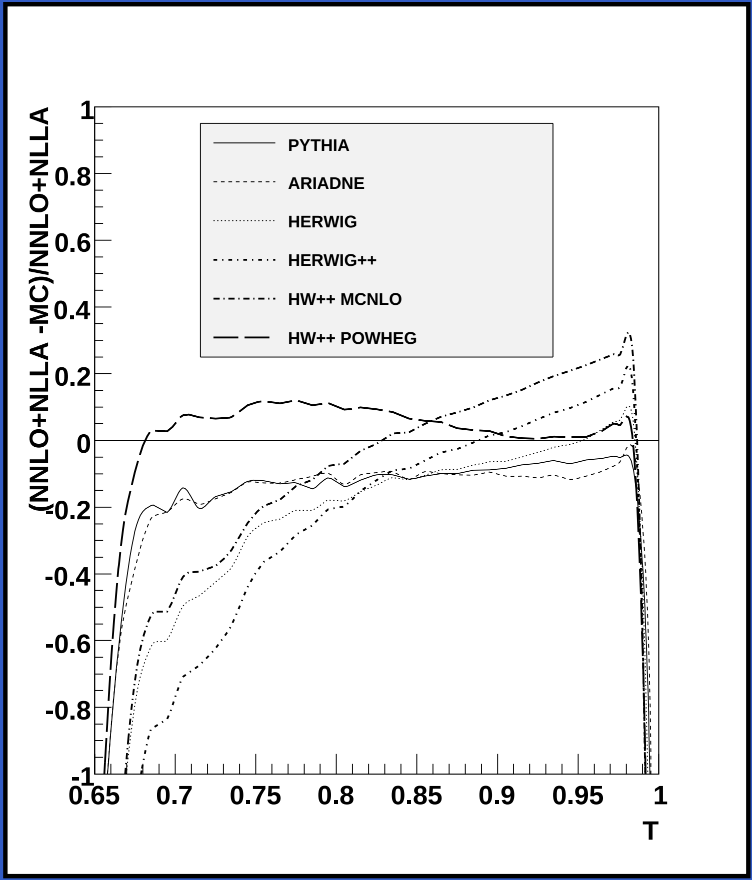
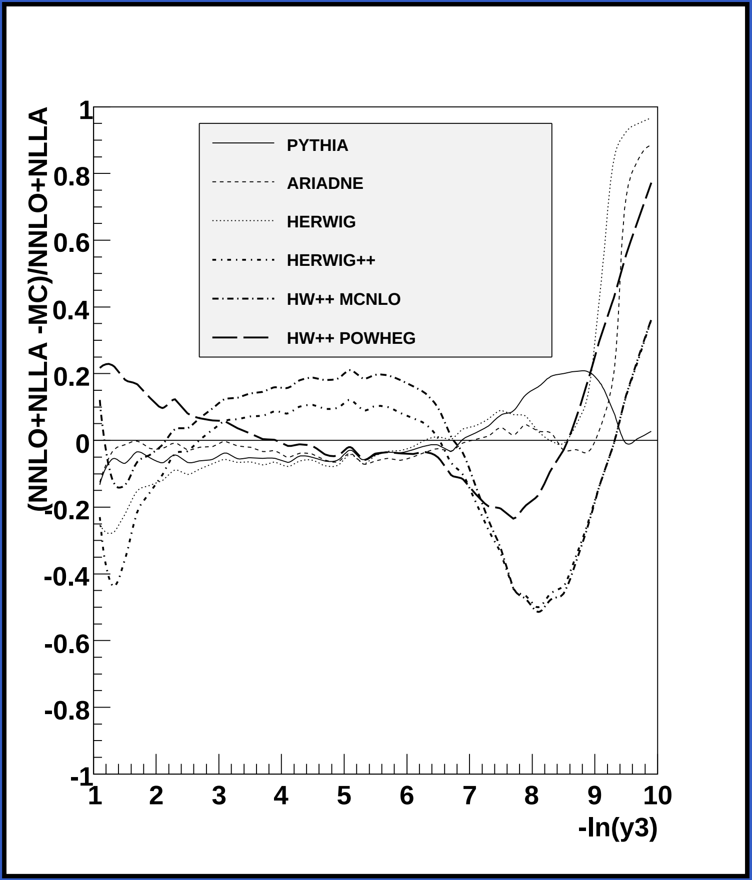




Determination of α_s: Hadronization

- Comparison with modern MC generators:





$\alpha_s\,(M_z)$	T	C	M_H	B_W	B_T	$-\ln y_3$
PYTHIA	0.1266	0.1252	0.1211	0.1196	0.1268	0.1186
χ^2/N_{dof}	0.16	0.47	4.4	4.4	0.84	1.89
HW++ POWHEG	0.1189	0.1179	0.1236	0.1169	0.1224	0.1142
χ^2/N_{dof}	1.46	2.55	3.8	3.9	1.54	0.56

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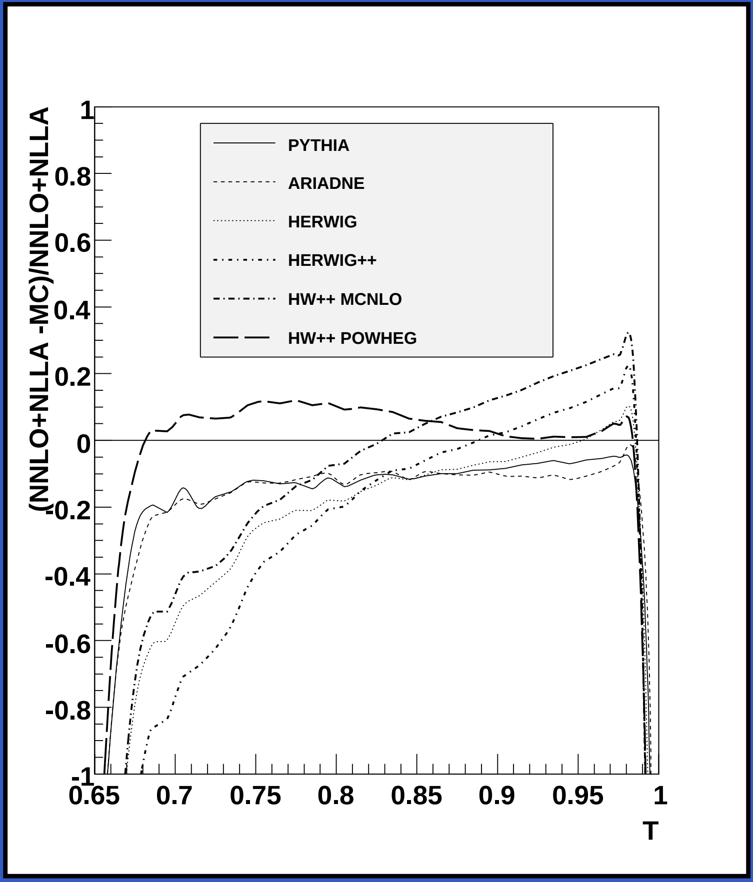
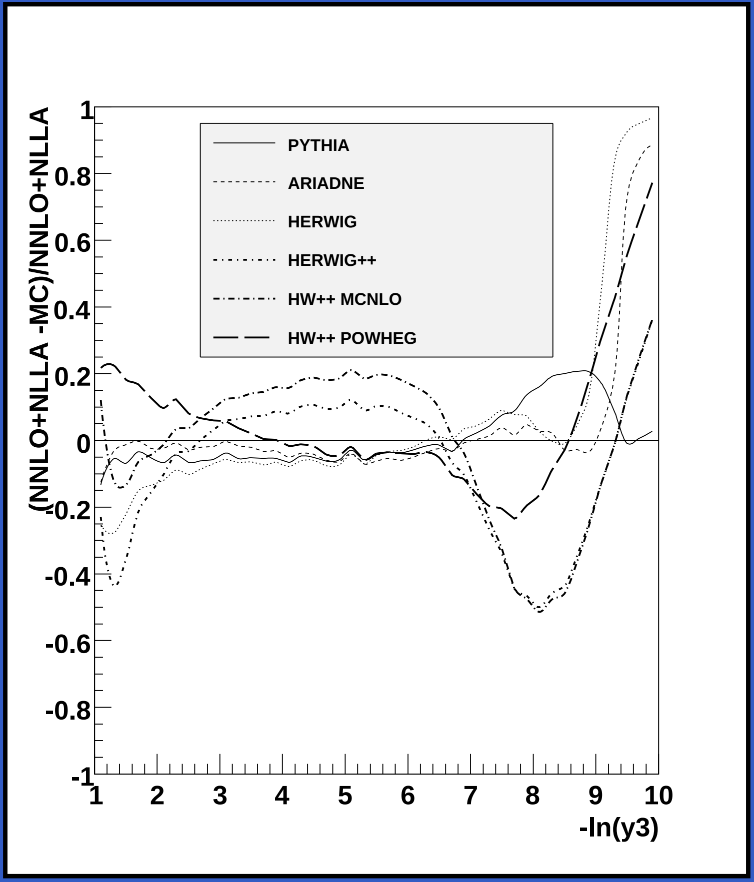




- Thrust: MC parton level prediction larger than in NNLO+NLLA

→ Pythia parameters tuned such that missing HO terms are (over-)compensated and hadronization corrections are effectively too small

Determination of α_S using moments

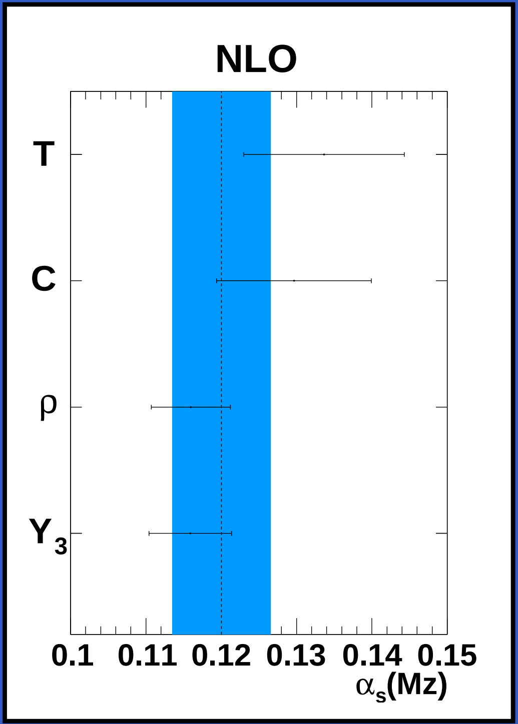
- Combine NNLO results with non-perturbative power corrections from dispersive model [Dokshitzer, Marchesini,Webber.]
  $\rightarrow$ replace α_s below $\mu_I \approx 2$ GeV by average coupling α_0.
- Non-perturbative corrections result in a shift of the distribution:

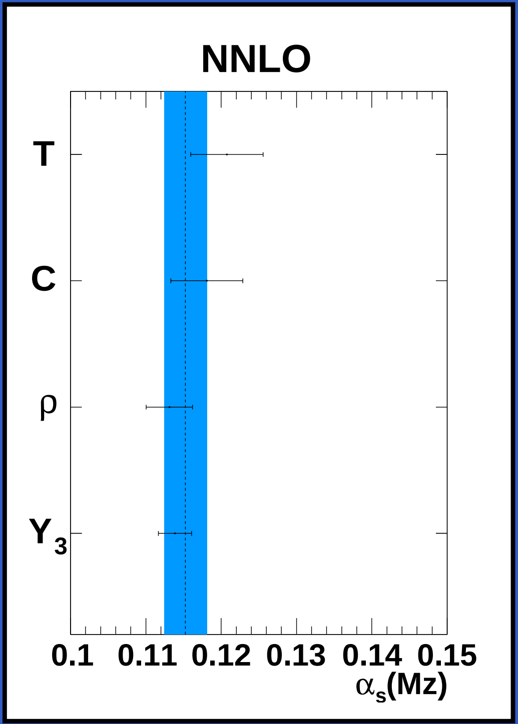
$$\langle y^n \rangle = \int_0^{y_{\max}} \mathrm{d}y \, (y + a_y P)^n \, \frac{1}{\sigma_{\mathrm{tot}}} \frac{\mathrm{d}\sigma_{\mathrm{pt}}}{\mathrm{d}y}(y) \, .$$

- Analytical power correction $P = P(\mu_R)$ extended to NNLO, [Gehrmann, Jaquier, G.L.]
- new fit of α_s and α_0 to JADE and OPAL data for $n = 1, \ldots, 5$:
  - total experimental error used in χ^2,
  - theoretical uncertainty determined by varying μ_R, μ_I and $\mathcal{M}$.

Determination of α_S using moments

Result from analytical power corrections:





Combined result:

$$\alpha_s(M_Z) = 0.1153 \pm 0.0017(\mathrm{exp}) \pm 0.0023(\mathrm{th}),$$
$$\alpha_0 = 0.5132 \pm 0.0115(\mathrm{exp}) \pm 0.0381(\mathrm{th}),$$

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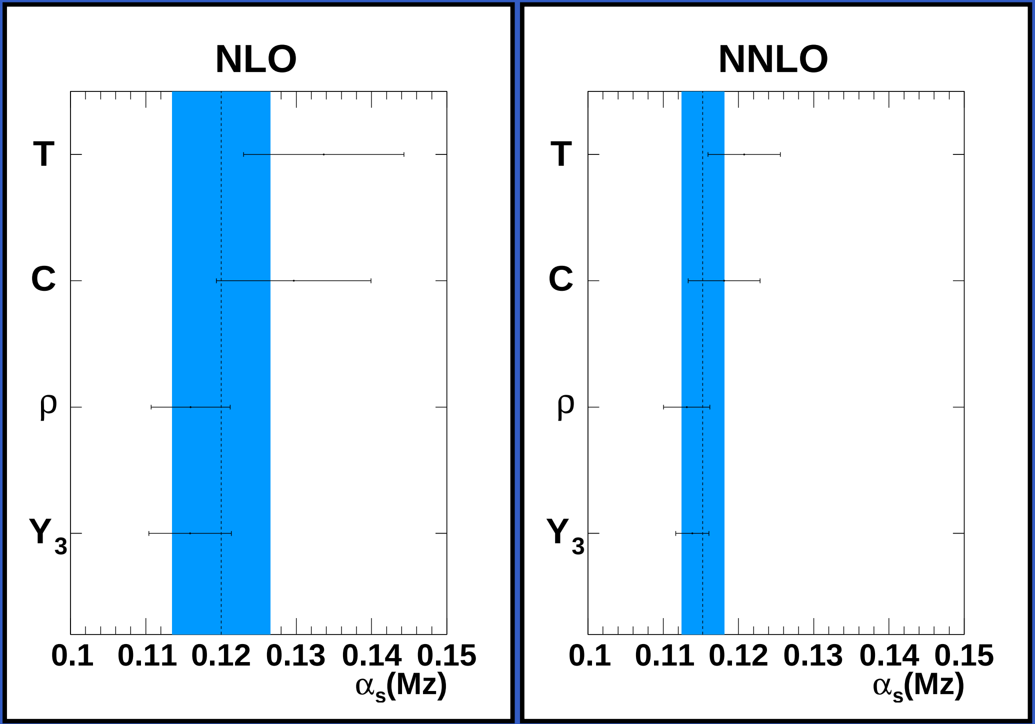
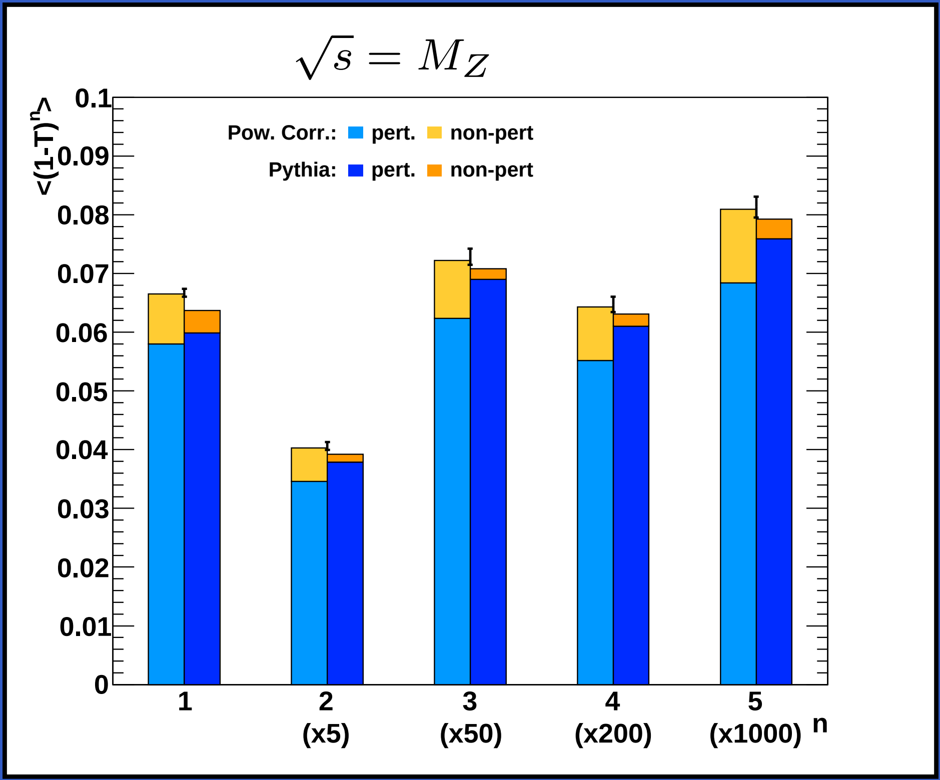


Comparison with Monte Carlo:



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- With Pythia:
  - $\longrightarrow$ smaller hadronization correction
  - $\longrightarrow$ higher partonic predictions

Determination of α_s using jet rates

- In region $10^{-1} > y_{\rm cut} > 10^{-2}$ only very small hadronization corrections $\rightarrow$ motivates a dedicated extraction of α_s

- Separated fits for $-1.3 > \ln(y_{\rm cut}) > -5.1$,
- stability up to $\ln(y_{\rm cut}) = -4.5$,

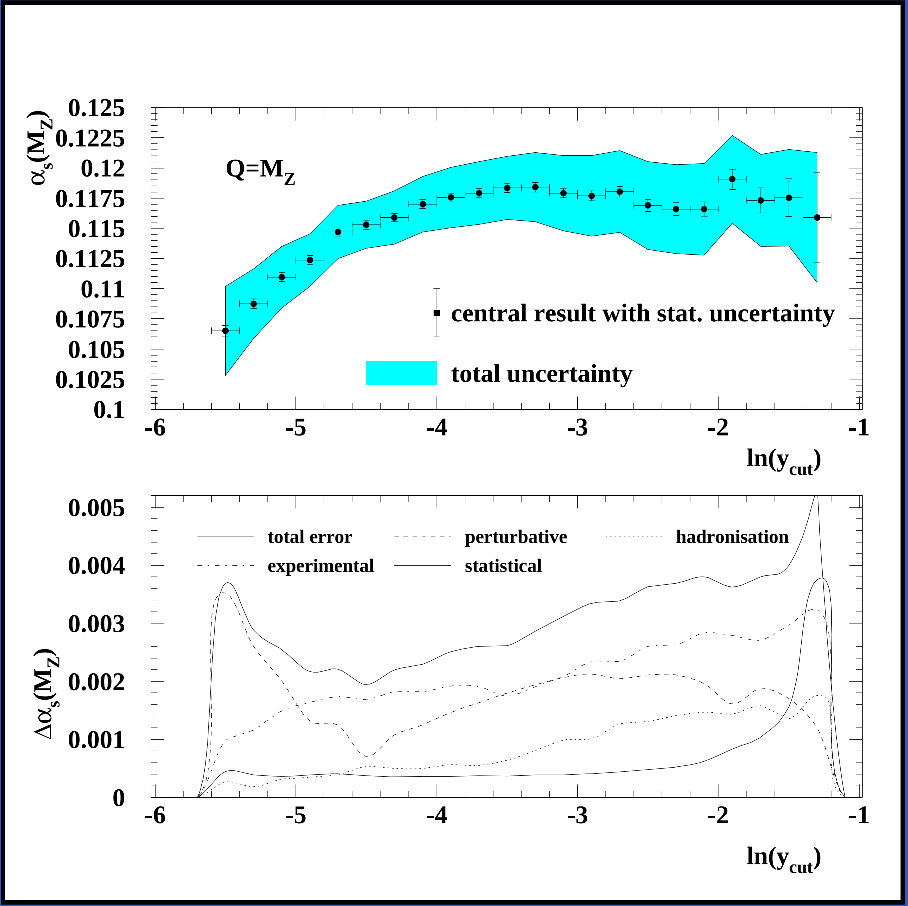
  (onset of large logarithms beyond),

Result at $y_{\rm cut} = 0.02$:

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(\exp) \pm 0.0015(\text{th})$$

- more precise then extractions from

  event-shape distributions.



Conclusions and Outlook

- New NNLO result on jet observables together with high precision data allow improved extraction of α_s:
  - from NLLA+NNLO event-shape distributions,:

    $$\alpha_s\left(M_Z\right) = 0.1224 \pm 0.0009(\text{stat.}) \pm 0.0009(\text{exp.}) \pm 0.0012(\text{had.}) \pm 0.0035(\text{theo.})$$

  - from NNLO event-shape moments with analytical power corrections:

    $$\alpha_s\left(M_Z\right) = 0.1153 \pm 0.0017(\mathrm{exp}) \pm 0.0023(\mathrm{th})$$

  - from NNLO three-jet rate:

    $$\alpha_s\left(M_Z\right) = 0.1175 \pm 0.0020(\mathrm{exp}) \pm 0.0015(\mathrm{th})$$

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  - from NNLO event-shape moments with analytical power corrections:

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  - from NNLO three-jet rate:

    $$\alpha_s\,(M_Z) = 0.1175 \pm 0.0020(\mathrm{exp}) \pm 0.0015(\mathrm{th})$$
- From event-shape analysis some further important observations:
  - combination of NNLO results with hadronization from LO MC not reliable,
  - in LO MC hadronization corrections might be underestimated,
  - further studies in this direction are needed in view of the precision needed at LHC.

Backup Slides

Fixed Order Calculations

- NLO and NNLO calculations: [Gehrmann, Gehrmann-De-Ridder, Glover, Heinrich]
  - careful subtraction of real and virtual divergencies using antenna method:

$$
\begin{aligned}
d\sigma_{\mathrm{NLO}} &= \int_{d\Phi_{m+1}} \left(d\sigma^{\mathrm{R}}_{\mathrm{NLO}} - d\sigma^{\mathrm{S}}_{\mathrm{NLO}} \right) + \left[\int_{d\Phi_{m+1}} d\sigma^{\mathrm{S}}_{\mathrm{NLO}} + \int_{d\Phi_m} d\sigma^{V}_{\mathrm{NLO}} \right] \\
d\sigma_{\mathrm{NNLO}} &= \int_{d\Phi_{m+2}} \left(d\sigma^{\mathrm{R}}_{\mathrm{NNLO}} - d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \right) + \int_{d\Phi_{m+2}} d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \\
&+ \int_{d\Phi_{m+1}} \left(d\sigma^{\mathrm{V},1}_{\mathrm{NNLO}} - d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \right) + \int_{d\Phi_{m+1}} d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \\
&+ \int_{d\Phi_m} d\sigma^{\mathrm{V},2}_{\mathrm{NNLO}} .
\end{aligned}
$$

  - Implemented in the EERAD3 integration programme.

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Fixed Order Calculations

- Theoretical NNLO prediction $\left(\bar{\alpha}_s = \frac{\alpha_s}{2\pi}\,, x_\mu = \frac{\mu}{Q}\right)$:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \bar{\alpha}_s(\mu)\frac{\mathrm{d}A}{\mathrm{d}y}(y) + \bar{\alpha}_s^2(\mu)\frac{\mathrm{d}B}{\mathrm{d}y}(y,x_\mu) + \bar{\alpha}_s^3(\mu)\frac{\mathrm{d}C}{\mathrm{d}y}(y,x_\mu) + \mathcal{O}(\bar{\alpha}_s^4)\,.$$

However:

$$\frac{1}{\sigma_{\mathrm{had}}}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \frac{\sigma_0}{\sigma_{\mathrm{had}}(Q,\mu)}\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu)$$

Measured

Theory prediction

- use simple expansion in α_s, or "exact" ratio up to calculated order
- issues:
  - mass effects
  - EWK effects (factorization)
  - $\longrightarrow$ Effects below per-cent range

Fixed Order Calculations

- For an observable y the differential cross section at NNLO is given by $\left(\bar{\alpha}_s = \frac{\alpha_s}{2\pi}, x_\mu = \frac{\mu}{Q}\right)$:

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \bar{\alpha}_s(\mu)\frac{\mathrm{d}A}{\mathrm{d}y}(y) + \bar{\alpha}_s^2(\mu)\frac{\mathrm{d}B}{\mathrm{d}y}(y,x_\mu) + \bar{\alpha}_s^3(\mu)\frac{\mathrm{d}C}{\mathrm{d}y}(y,x_\mu) + \mathcal{O}(\bar{\alpha}_s^4)\,.$$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}y}(y,Q,\mu) = \underbrace{\bar{\alpha}_s(\mu)\frac{\mathrm{d}A}{\mathrm{d}y}(y)}_{LO} + \underbrace{\bar{\alpha}_s^2(\mu)\frac{\mathrm{d}B}{\mathrm{d}y}(y,x_\mu)}_{NLO} + \underbrace{\bar{\alpha}_s^3(\mu)\frac{\mathrm{d}C}{\mathrm{d}y}(y,x_\mu)}_{NNLO} + \mathcal{O}(\bar{\alpha}_s^4)\,.$$

LO	$\gamma^* \to q\bar{q}g$	tree level	NNLO	$\gamma^* \to q\bar{q}g$	two loop
				$\gamma^* \to q\bar{q}gg$	one loop
NLO	$\gamma^* \to q\bar{q}g$	one loop		$\gamma^* \to q\bar{q}q\bar{q}$	one loop
	$\gamma^* \to q\bar{q}gg$	tree level		$\gamma^* \to q\bar{q}q\bar{q}g$	tree level
	$\gamma^* \to q\bar{q}q\bar{q}$	tree level		$\gamma^* \to q\bar{q}ggg$	tree level

- Coefficient functions $\frac{dA}{dy}$, $\frac{dB}{dy}$, $\frac{dC}{dy}$ are functions of $L \equiv \log\frac{1}{y}$,

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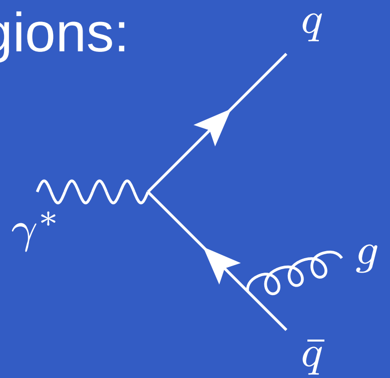
Fixed Order Calculations

- Logarithms are originated from integration over soft and collinear regions:



$$\propto \sigma_0 \frac{\alpha_s}{2\pi} \frac{1}{E_g(1-\cos\theta_{\bar{q}g})}$$

- Integrating over the phase space:

$$\frac{d\sigma}{dy} \propto \int \frac{dE_g}{E_g} \frac{d\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}} \delta\left(y - y\left(E_g, \theta_{\bar{q}g}\right)\right)$$
$$\propto \frac{1}{y} \log\left(\frac{1}{y}\right)$$

- They describe the enhancement due to soft and collinear emissions.

Fixed Order Calculations

- Consider cumulative cross section $R\left(y,Q,\mu\right) \equiv \frac{1}{\sigma_{\rm had}} \int_0^y \frac{d\sigma(x,Q,\mu)}{dx} dx,$

$$R\left(y,Q,\mu\right) = 1 + \mathcal{A}\left(y\right)\bar{\alpha}_s\left(\mu\right) + \mathcal{B}\left(y,x_\mu\right)\bar{\alpha}_s^2\left(\mu\right) + \mathcal{C}\left(y,x_\mu\right)\bar{\alpha}_s^3\left(\mu\right).$$

$\bar{\alpha}_s\mathcal{A}\left(y\right)$	$\bar{\alpha}_s L$	$\bar{\alpha}_s L^2$				
$\bar{\alpha}_s^2\mathcal{B}\left(y,x_\mu\right)$	$\bar{\alpha}_s^2 L$	$\bar{\alpha}_s^2 L^2$	$\bar{\alpha}_s^2 L^3$	$\bar{\alpha}_s^2 L^4$		
$\bar{\alpha}_s^3\mathcal{C}\left(y,x_\mu\right)$	$\bar{\alpha}_s^3 L$	$\bar{\alpha}_s^3 L^2$	$\bar{\alpha}_s^3 L^3$	$\bar{\alpha}_s^3 L^4$	$\bar{\alpha}_s^3 L^5$	$\bar{\alpha}_s^3 L^6$

Contribution becomes smaller ↓

- If L is NOT large, contributions become smaller line-by-line.
- In phase space region where $y \to 0$, $L \to \infty$:

$\longrightarrow$ coefficient functions become large spoiling the convergence of the series expansion.

- Main contribution comes from highest power of the logarithms.

Fixed Order Calculations

- Consider cumulative cross section $R(y,Q,\mu) \equiv \frac{1}{\sigma_{\text{had}}} \int_0^y \frac{d\sigma(x,Q,\mu)}{dx} dx,$

$$R(y,Q,\mu) = 1 + \mathcal{A}(y)\,\bar{\alpha}_s(\mu) + \mathcal{B}(y,x_\mu)\,\bar{\alpha}_s^2(\mu) + \mathcal{C}(y,x_\mu)\,\bar{\alpha}_s^3(\mu).$$

$\bar{\alpha}_s \mathcal{A}(y)$	$\bar{\alpha}_s L$	$\bar{\alpha}_s L^2$				
$\bar{\alpha}_s^2 \mathcal{B}(y,x_\mu)$	$\bar{\alpha}_s^2 L$	$\bar{\alpha}_s^2 L^2$	[illegible]	$\bar{\alpha}_s^2 L^4$		
$\bar{\alpha}_s^3 \mathcal{C}(y,x_\mu)$	$\bar{\alpha}_s^3 L$	$\bar{\alpha}_s^3 L^2$	[illegible] L^3	$\bar{\alpha}_s^3 L^4$	$\bar{\alpha}_s^3 L^5$	$\bar{\alpha}_s^3 L^6$

Contribution becomes smaller ↓

Need RESUMMATION!

- If L is NOT large, contributions [illegible]e smaller line-by-line.
- In phase space region wh[illegible] 0, $L \to \infty$:

$\longrightarrow$ coefficient functions [illegible] large spoiling the convergence of the series expansion.

- Main contribu[illegible]omes from highest power of the logarithms.

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Resummed Calculations

- Idea: resum the highest powers of the logarithms to all orders in perturbation theory

$\bar{\alpha}_s \mathcal{A}(y)$	$\bar{\alpha}_s L$	$\bar{\alpha}_s L^2$				
$\bar{\alpha}_s^2 \mathcal{B}(y, x_\mu)$	$\bar{\alpha}_s^2 L$	$\bar{\alpha}_s^2 L^2$	$\bar{\alpha}_s^2 L^3$	$\bar{\alpha}_s^2 L^4$		
$\bar{\alpha}_s^3 \mathcal{C}(y, x_\mu)$	$\bar{\alpha}_s^3 L$	$\bar{\alpha}_s^3 L^2$	$\bar{\alpha}_s^3 L^3$	$\bar{\alpha}_s^3 L^4$	$\bar{\alpha}_s^3 L^5$	$\bar{\alpha}_s^3 L^6$

Resummed Calculations

- Idea: resum the highest powers of the logarithms to all orders in perturbation theory

$\bar{\alpha}_s \mathcal{A}(y)$	$\bar{\alpha}_s L$	$\bar{\alpha}_s L^2$				
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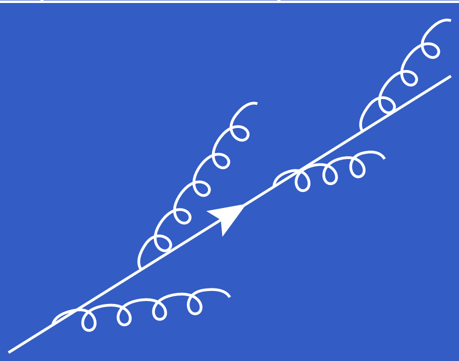
$\bar{\alpha}_s^3 \mathcal{C}(y, x_\mu)$	$\bar{\alpha}_s^3 L$	$\bar{\alpha}_s^3 L^2$	$\bar{\alpha}_s^3 L^3$	$\bar{\alpha}_s^3 L^4$	$\bar{\alpha}_s^3 L^5$	$\bar{\alpha}_s^3 L^6$

- Leading logarithms
- Next-to-Leading logarithms
- From trivial exponentiation



Resummed Calculations

- For suitable observables, resummation of logarithms leads to exponentiation

$$\Sigma(y) = e^{L\,g_1(\alpha_s L) + g_2(\alpha_s L) + \ldots}$$

with

$$L\,g_1(\alpha_s L) = G_{12} L^2 \bar{\alpha}_s + G_{23} L^3 \bar{\alpha}_s^2 + G_{34} L^4 \bar{\alpha}_s^3 + \ldots \text{ (LL)}$$

$$g_2(\alpha_s L) = G_{11} L \bar{\alpha}_s + G_{22} L^2 \bar{\alpha}_s^2 + G_{33} L^3 \bar{\alpha}_s^3 + \ldots \text{ (NLL)}$$

- Integrated cross section at NLLA to be matched with NNLO:

$$R(y) = \left(1 + C_1 \alpha_s + C_2 \alpha_s^2 + C_3 \alpha_s^3\right) \times e^{L\,g_1(\alpha_s L) + g_2(\alpha_s L) + \bar{\alpha}_s^2 G_{21} L + \bar{\alpha}_s^3 G_{32} L^2 + \bar{\alpha}_s^3 G_{31} L} + D(y)$$

$$\Rightarrow R(y) = \underbrace{C(\alpha_s)\,\Sigma(y)}_{\text{logarithmic part}} + \underbrace{D(y)}_{\text{remainder function: } \to 0 \text{ as } y \to 0}$$

C_1, C_2, C_3, G_{21}, G_{32}, G_{31}, $D(y)$: to be determined by matching with fixed order.

Matching

- Different matching schemes
  - R-matching scheme:
    - Two predictions for $R\,(y)$ are matched and double-counting terms are subtracted.
    - Unknown matching coefficients C_1, C_2, C_3, G_{21}, G_{32}, G_{31} numerically determined from fixed order result.
  - Log(R)-matching scheme:
    - Logarithm of $R\,(y)$ is matched and double-counting terms are subtracted.
    - All matching coefficients from expansion of resummed result.

Log-R matching scheme

- To NLLA + NNLO the integrated cross section in the Log-R matching scheme is given by

$$\begin{aligned}\ln\left(R\left(y,\alpha_S\right)\right) &= L\,g_1\left(\alpha_s L\right) + g_2\left(\alpha_s L\right)\\ &+\bar{\alpha}_S\left(\mathcal{A}\left(y\right) - G_{11}L - G_{12}L^2\right) + \\ &+\bar{\alpha}_S^2\left(\mathcal{B}\left(y\right) - \frac{1}{2}\mathcal{A}^2\left(y\right) - G_{22}L^2 - G_{23}L^3\right)\\ &+\bar{\alpha}_S^3\left(\mathcal{C}\left(y\right) - \mathcal{A}\left(y\right)\mathcal{B}\left(y\right) + \frac{1}{3}\mathcal{A}^3\left(y\right) - G_{33}L^3 - G_{34}L^4\right)\quad .\end{aligned}$$

• fixed order • resummation

- To ensure the vanishing of the matched expression at the kinematical boundary $y_{\max}$

$$L \longrightarrow \tilde{L} = \frac{1}{p}\ln\left(\left(\frac{y_0}{y\,x_L}\right)^p - \left(\frac{y_0}{y_{\max}\,x_L}\right)^p + 1\right),$$

with $y_0 = 6$ for $y = C$ and $y_0 = 1$ otherwise, ($x_L = p = 1$).

[Ford, Jones, Salam, Stenzel, Wicke.]

Renormalization scale dependence

- The full renormalization scale dependence is given by making the following replacements,

$$\alpha_s \;\rightarrow\; \alpha_s(\mu)\ ,$$

$$\mathcal{B}(y) \;\rightarrow\; \mathcal{B}(y,\mu) = 2\,\beta_0\;\ln x_\mu\,\mathcal{A}(y) + \mathcal{B}(y)\ ,$$

$$\mathcal{C}(y) \;\rightarrow\; \mathcal{C}(y,\mu) = (2\,\beta_0\;\ln x_\mu)^2\,\mathcal{A}(y) + 2\;\ln x_\mu\;[2\,\beta_0\mathcal{B}(y) + 2\,\beta_1\,\mathcal{A}(y)] + \mathcal{C}(y)\ ,$$

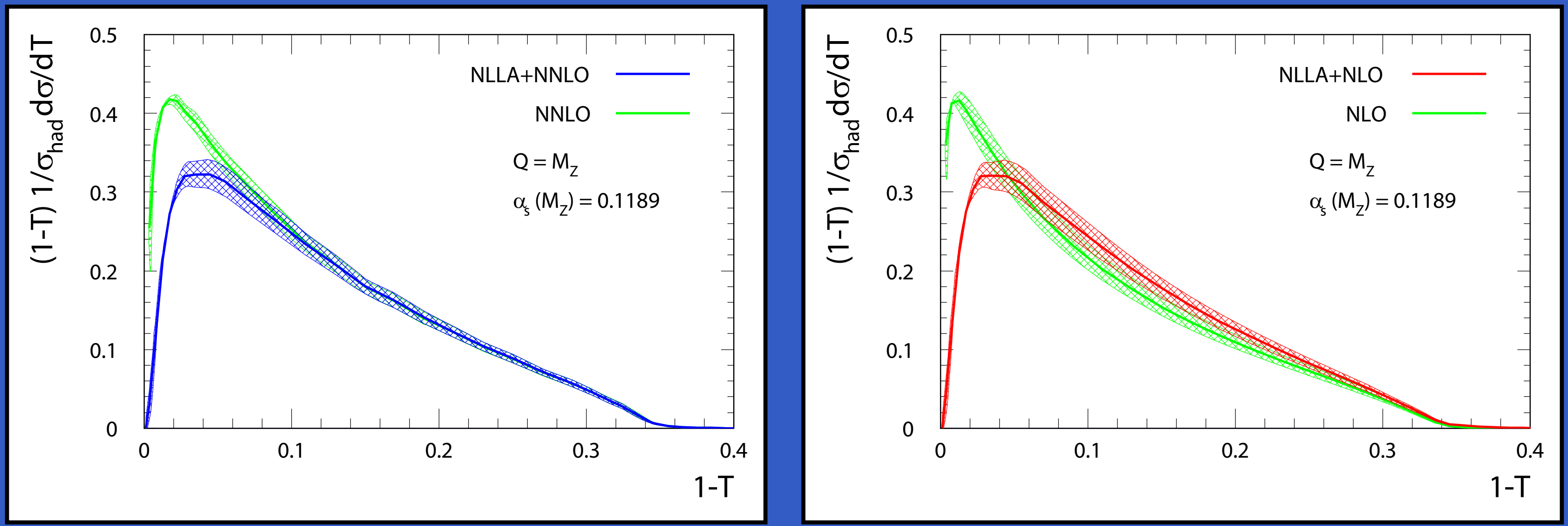
$$g_2(\alpha_S L) \;\rightarrow\; g_2\left(\alpha_S L,\mu^2\right) = g_2(\alpha_S L) + \frac{\beta_0}{\pi}(\alpha_S L)^2\;g_1'(\alpha_S L)\;\ln x_\mu\ ,$$

$$G_{22} \;\rightarrow\; G_{22}(\mu) = G_{22} + 2\beta_0 G_{12}\ln x_\mu\ ,$$

$$G_{33} \;\rightarrow\; G_{33}(\mu) = G_{33} + 4\beta_0 G_{23}\ln x_\mu\ .$$

Results: renormalization scale dependence

- Thrust T: consider $\tau = 1 - T$

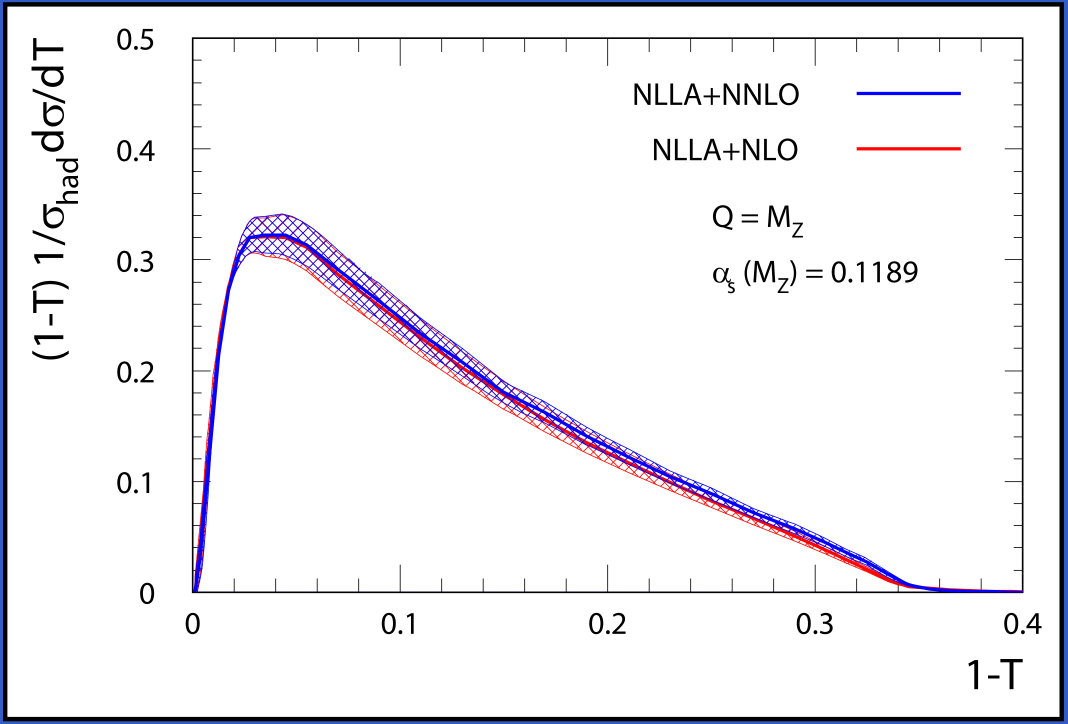


- Difference between NLLA+NNLO and NNLO restricted to the two-jet region,

  whereas NLLA+NLO differ in normalisation throughout the full kinematical range.

Results: renormalization scale dependence

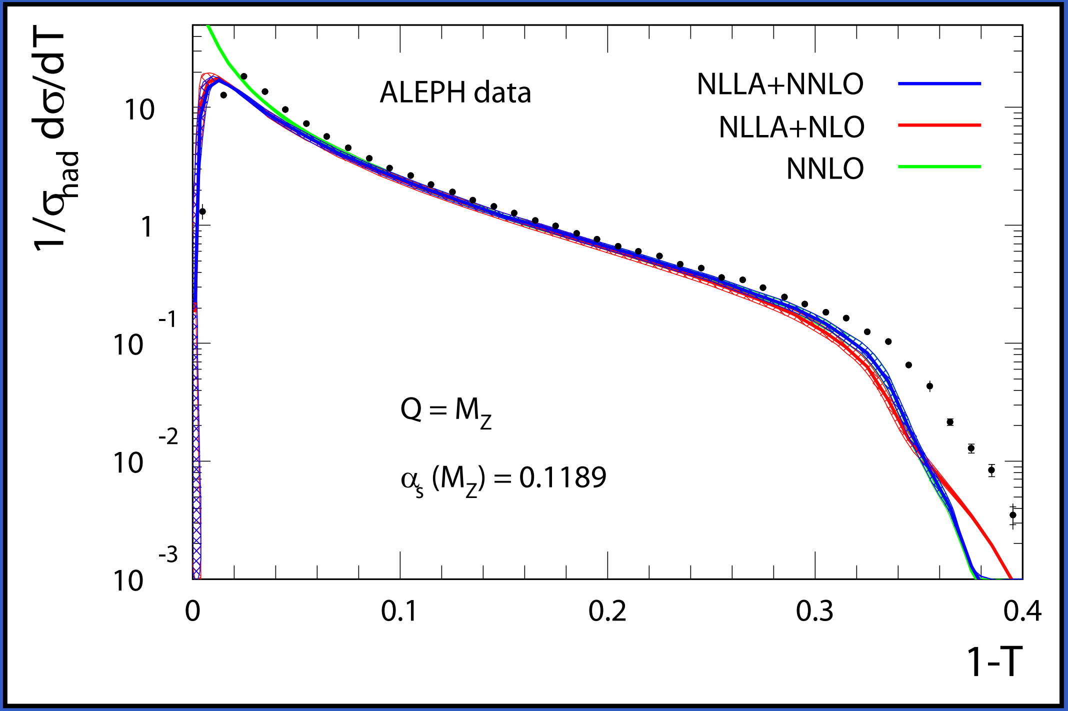
- Thrust T: consider $\tau = 1 - T$



- Difference between NLLA+NNLO and NLLA+NLO moderate in the three-jet region.
- Renormalization scale dependence reduced in three-jet region.

Results: renormalization scale dependence

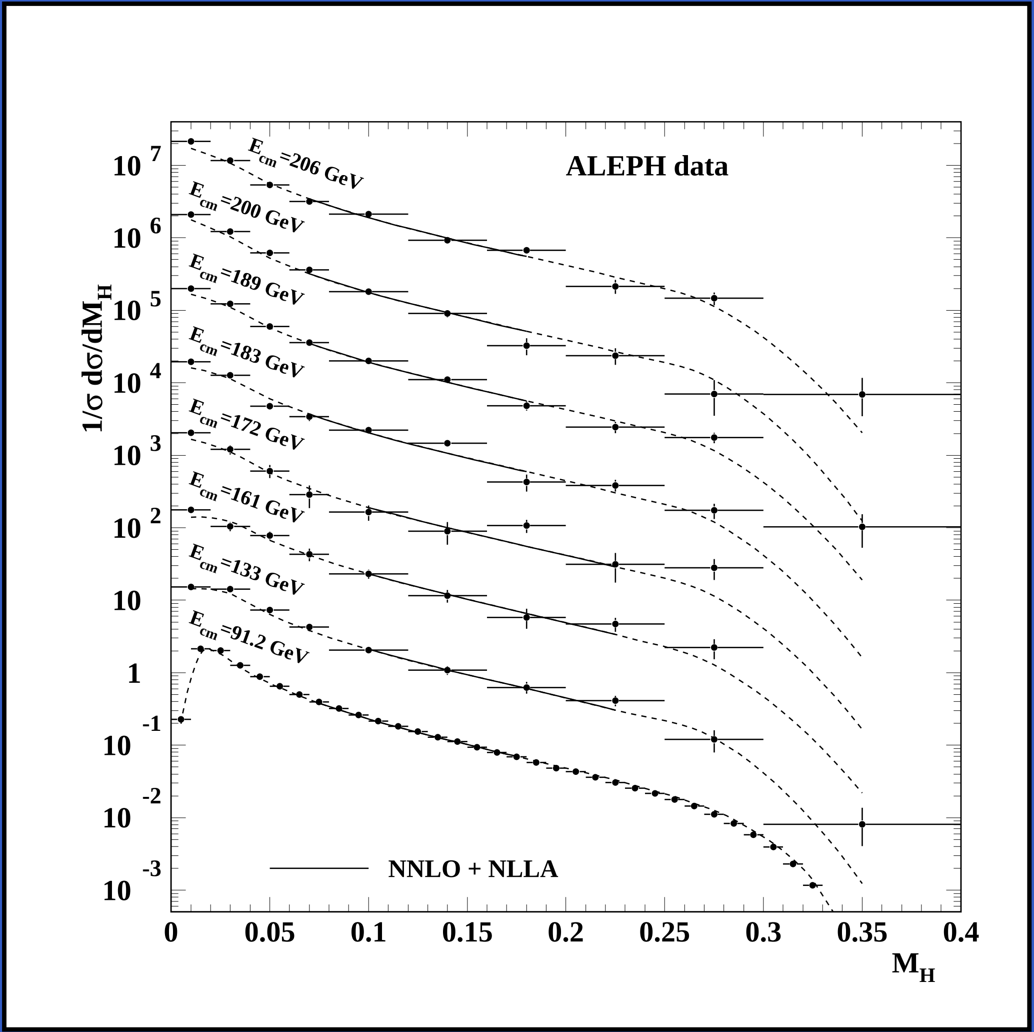
- Thrust T: consider $\tau = 1 - T$



- Description of the hadron-level data improves between parton-level NLLA+NLO and parton-level NLLA+NNLO, especially in the three-jet region.

Determination of α_S: NLLA+NNLO fits

- data are fit in the central part of the event-shape distribution,
- only statistical uncertainties are included in the χ^2.
- good fit quality (but statistical uncertainties due to NNLO coefficient)



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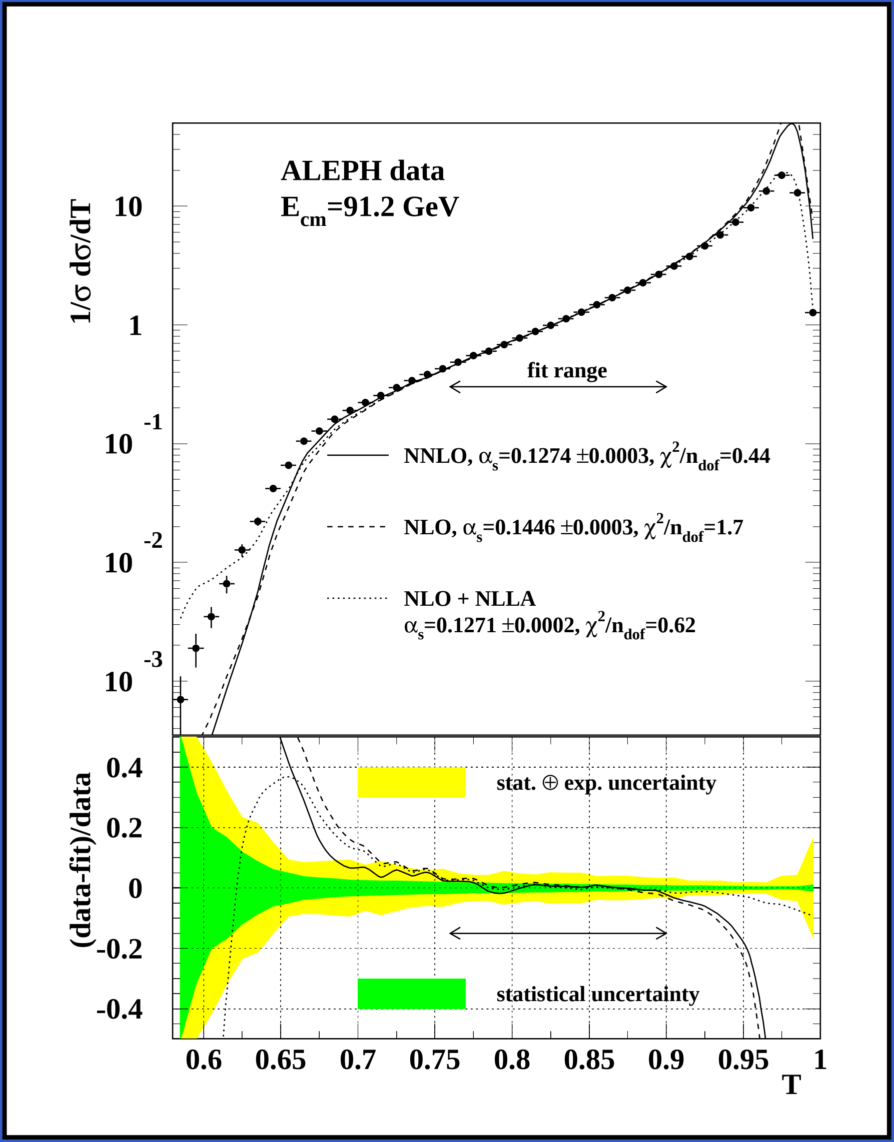
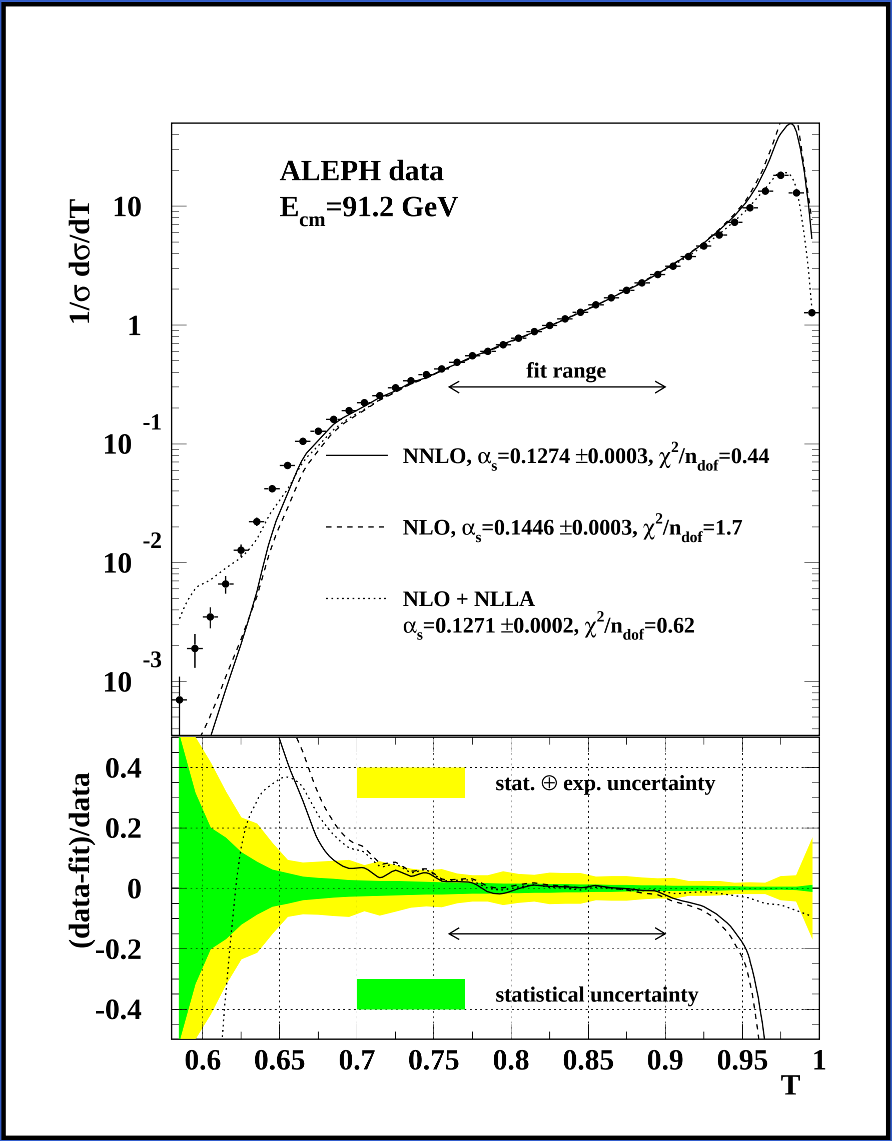
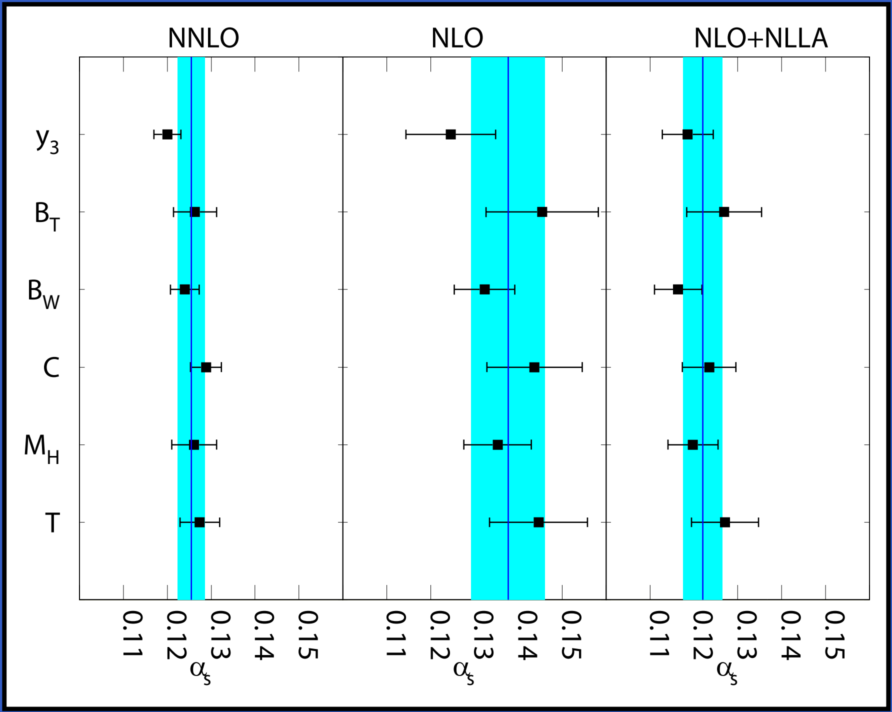
Determination of α_S: NNLO fits



- fit to fixed order calculations gives higher values for α_S,
- tendency to decrease from NLO to NNLO.

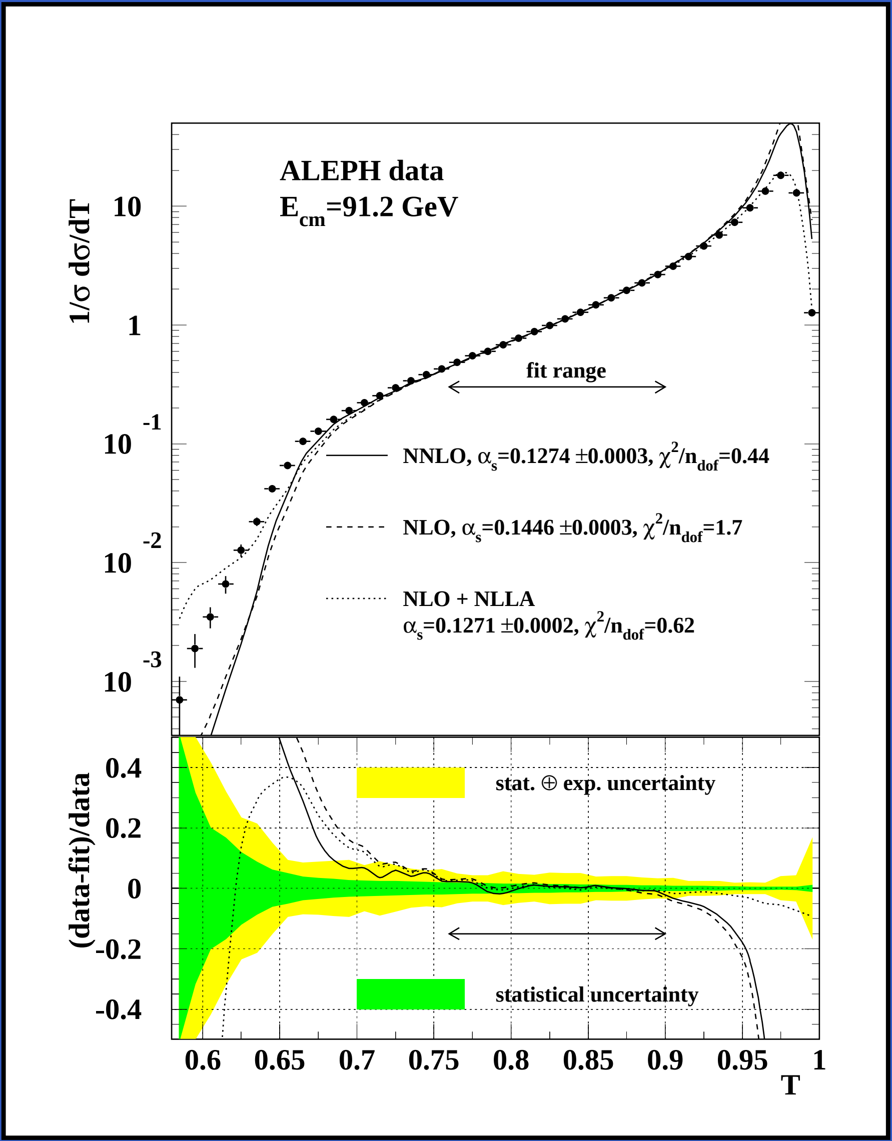
Determination of α_S: NNLO fits

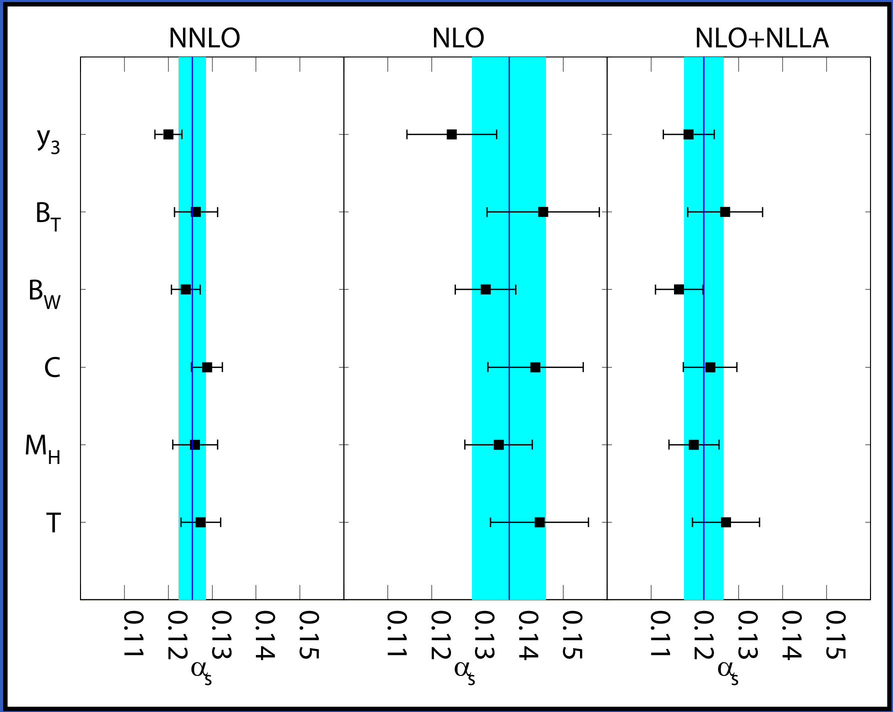




- much less scatter at NNLO
- reduced perturbative uncertainty: 0.003

Determination of α_S: NNLO fits





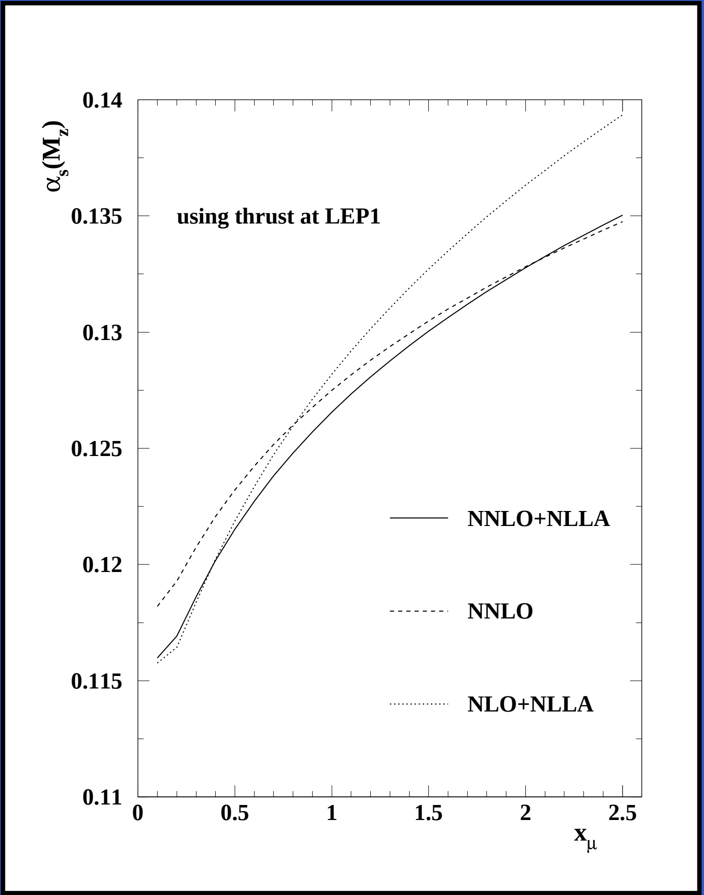
- much less scatter at NNLO
- reduced perturbative uncertainty: 0.003

$$\alpha_s(M_Z) = 0.1240 \pm 0.0008(\text{stat.}) \pm 0.0010(\text{exp.}) \pm 0.0011(\text{had.}) \pm 0.0029(\text{theo.})$$

Determination of α_S: NNLO+NLLA fits

- Beware: consistent matching would require full NNLLA results (at present known only for T).
- a slight increase of the scale uncertainty is observed: two loop running terms not compensated in NLLA.

data set	LEP1 + LEP2	LEP2
$\alpha_s\,(M_Z)$	0.1224	0.1224
stat. error	0.0009	0.0011
exp. error	0.0009	0.0010
pert. error	0.0035	0.0034
hadr. error	0.0012	0.0011
total error	0.0039	0.0039

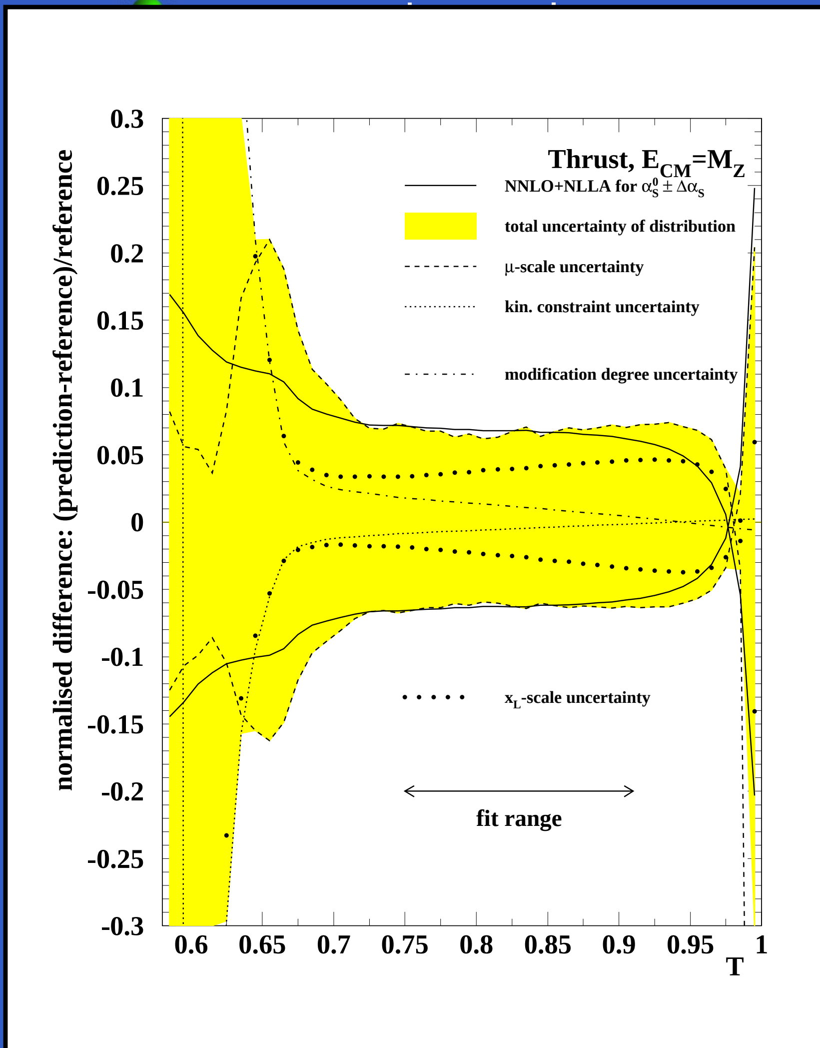


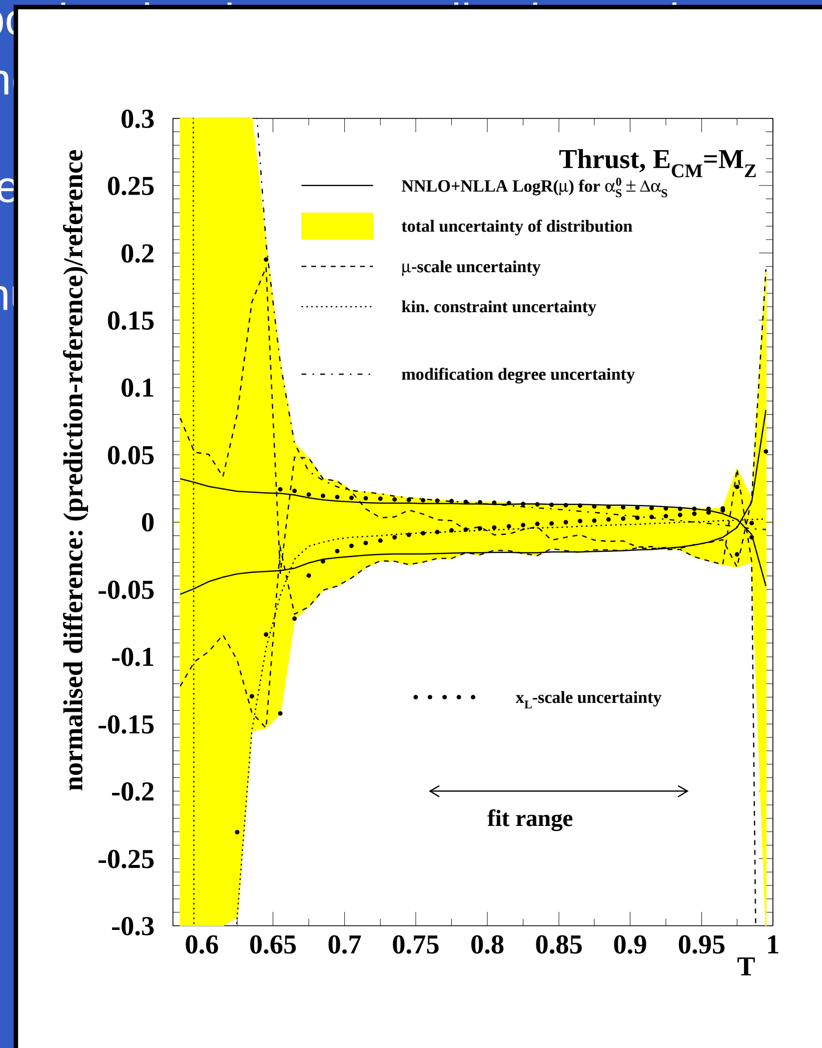
Determination of α_S: perturbative uncertainty

- The $\log$ R(μ)- matching scheme:
  - compute the two-loop terms proportional to the renormalization scale in resummation and matching functions,
  - central values of individual fits are not affected...
  - ... but theoretical uncertainty is much reduced.

Determination of α_S: perturbative uncertainty

- The $\log$ R(μ)- matching scheme:





Determination of α_S: perturbative uncertainty

- The $\log$ R(μ)- matching scheme:
  - compute the two-loop terms proportional to the renormalization scale in resummation and matching functions,
  - central values of individual fits are not affected...
  - ... but theoretical uncertainty is much reduced.

$\log$ R

data set	LEP1 + LEP2	LEP2
$\alpha_s\,(M_Z)$	0.1224	0.1224
stat. error	0.0009	0.0011
exp. error	0.0009	0.0010
pert. error	0.0035	0.0034
hadr. error	0.0012	0.0011
total error	0.0039	0.0039

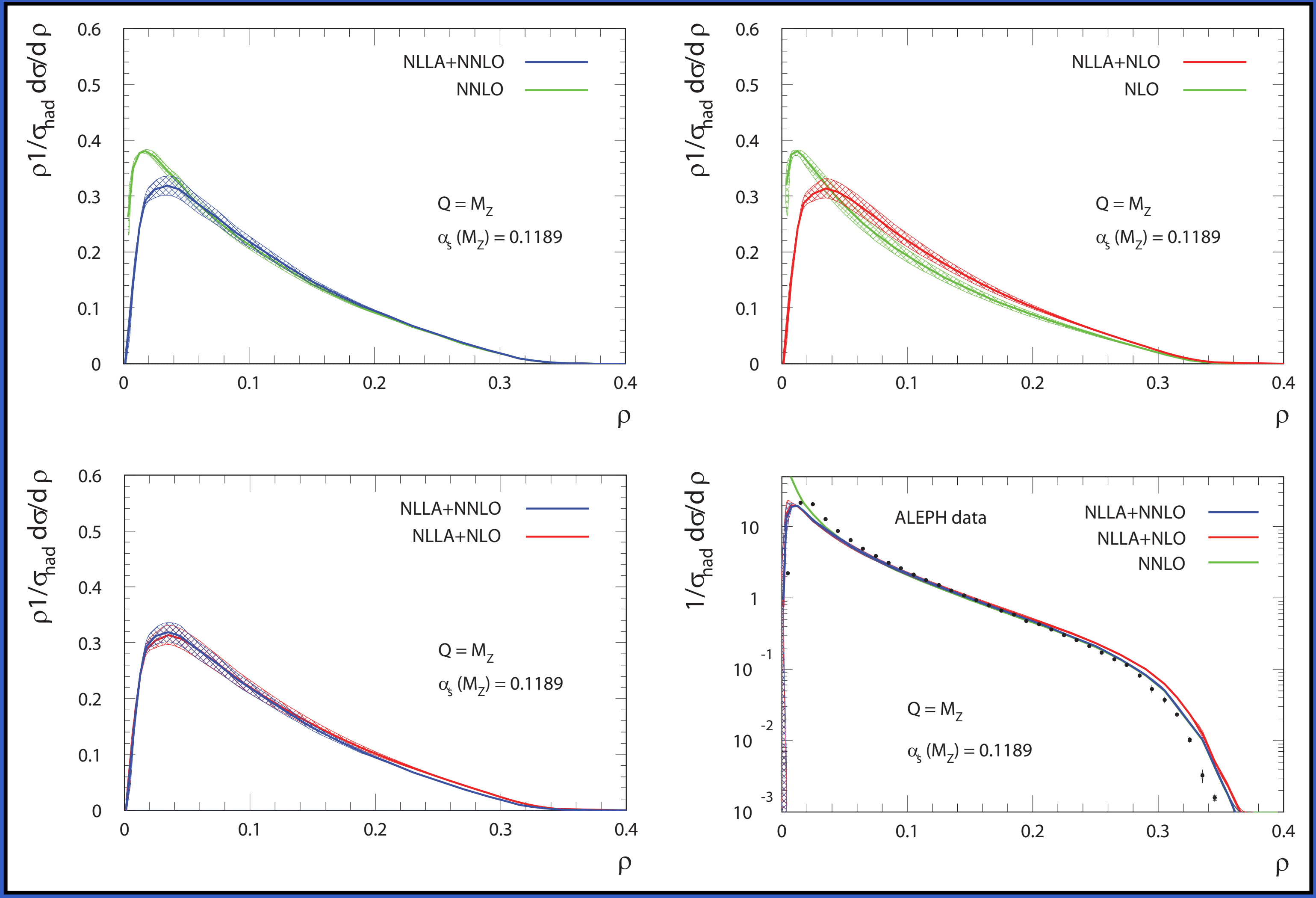
$\log$ R(μ)

data set	LEP1 + LEP2	LEP2
$\alpha_s\,(M_Z)$	0.1227	0.1226
stat. error	0.0008	0.0010
exp. error	0.0009	0.0010
pert. error	0.0022	0.0021
hadr. error	0.0012	0.0011
total error	0.0028	0.0028

- cancelations are probably overestimated,
- conservative result is more reliable.

Results

- Heavy Jet Mass ρ:



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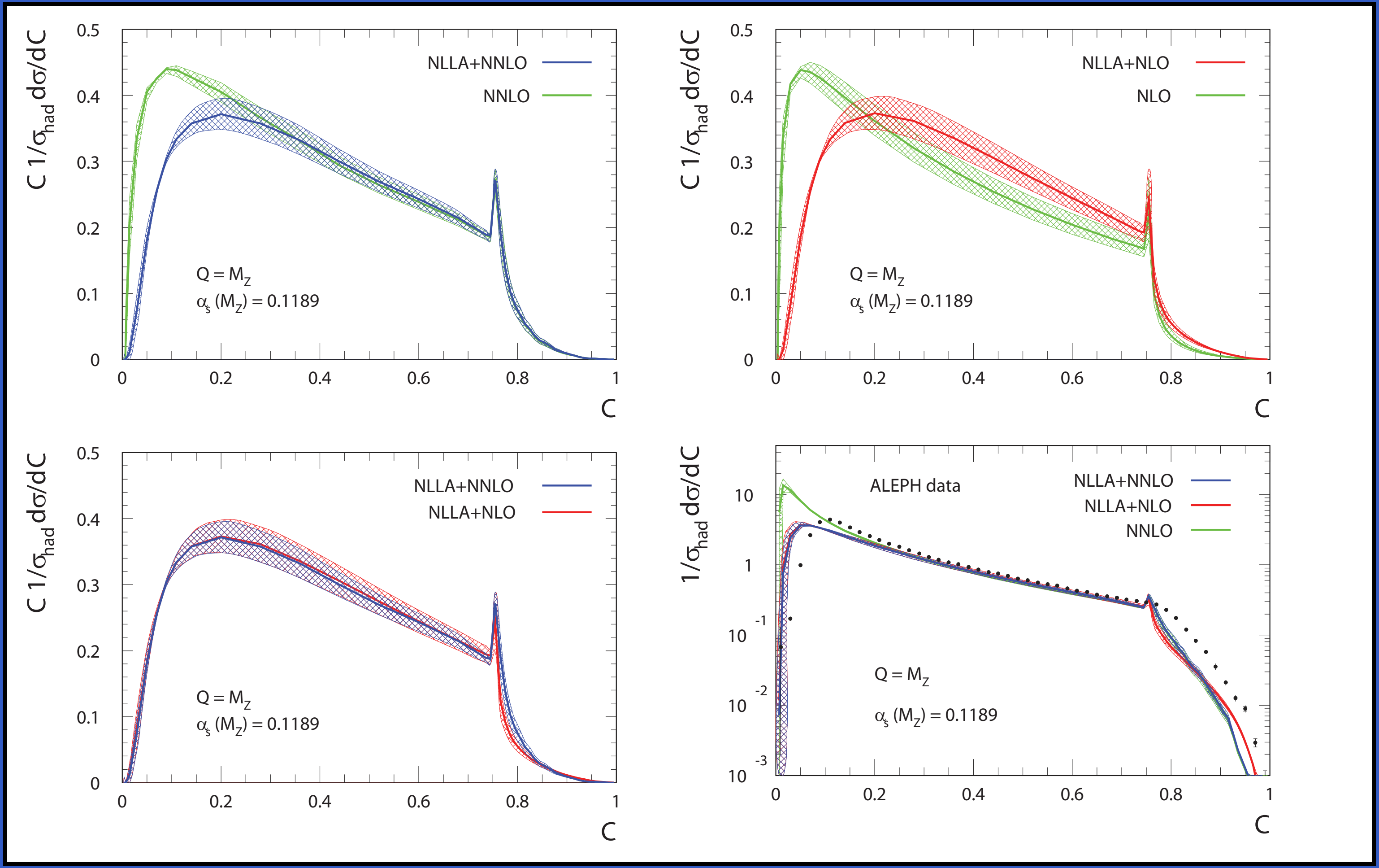
Results

- C-parameter C:



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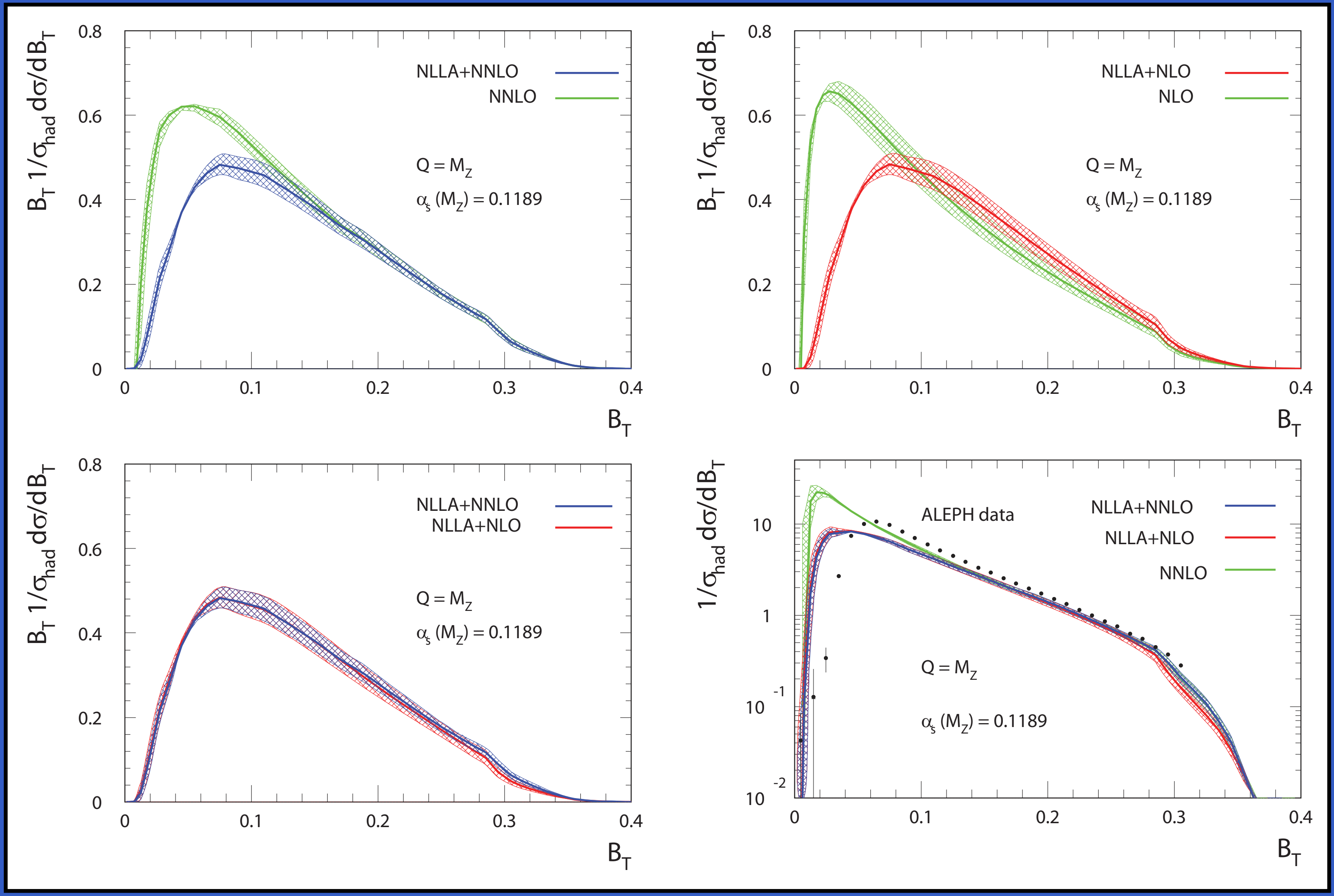
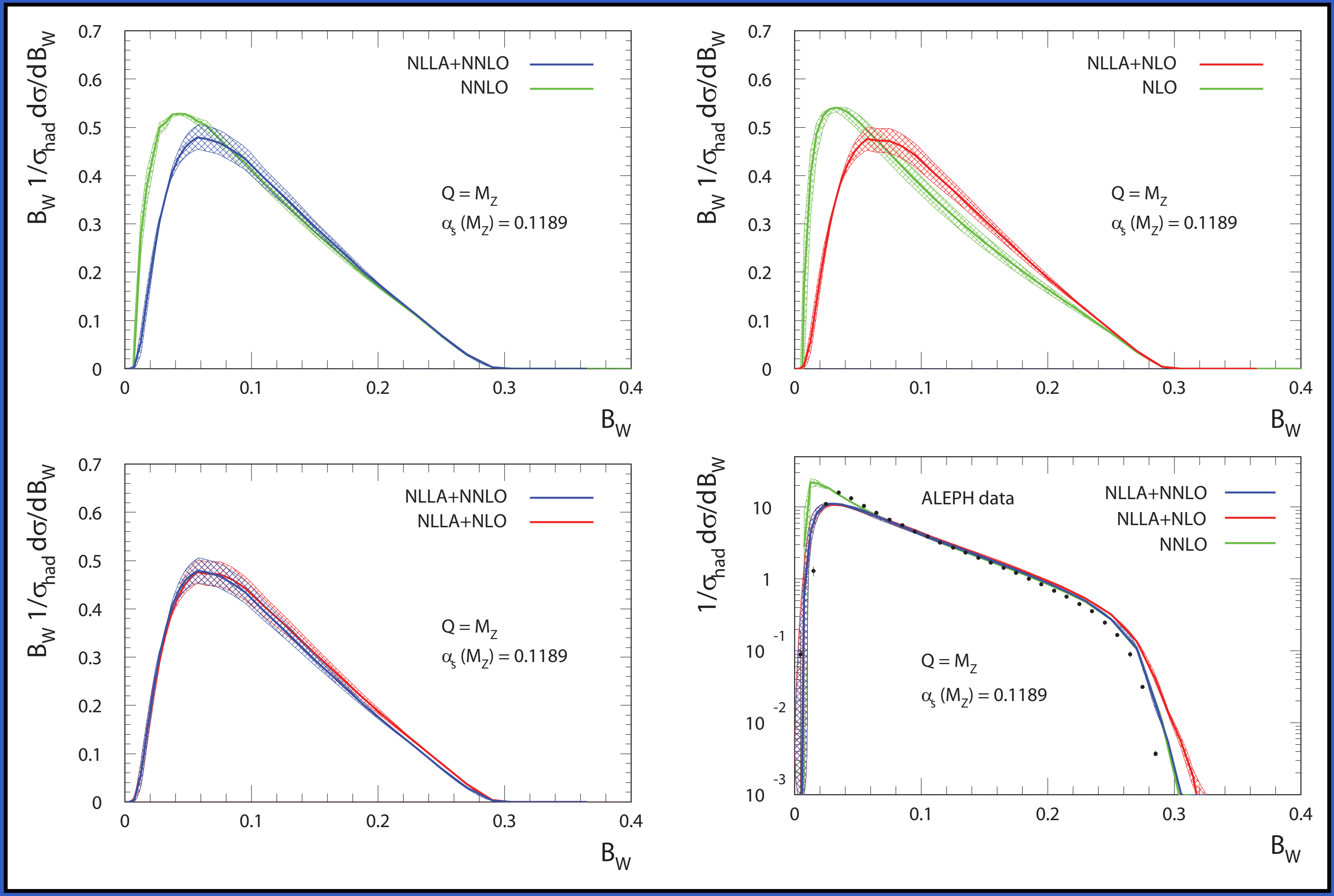
Results

- Total jet broadening B_T:



Results

- Wide jet broadening B_W:



Results

- Two-to-three jet parameter for Durham algorithm Y_3: