DIS 2010, 21 April, Firenze, Italy

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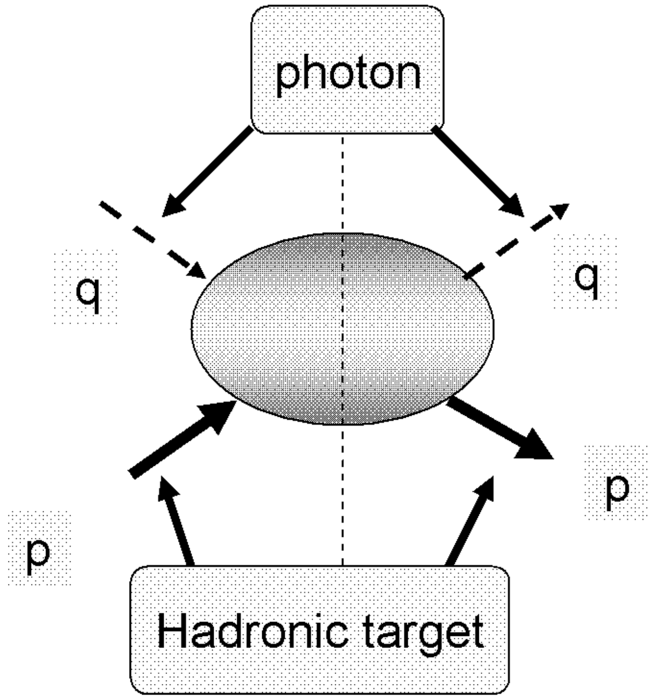
Requirements for initial parton densities following from factorization

talk based on results obtained in collaboration with M. Greco and S.I. Troyan

$$\mathrm{W}_{\mu\nu} =$$



$$\mathrm{W}_{\mu\nu} = \sum P^{a}_{\mu\nu} f_a(x, Q^2) \qquad \mathrm{x} = \mathrm{Q}^2/2pq$$

Projection operators

Structure funcrions

For example:

$$\mathrm{W}^{unpol}_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1 + \left(P_\mu - q_\mu \frac{Pq}{q^2}\right)\left(P_\nu - q_\nu \frac{Pq}{q^2}\right)(1/Pq)F_2$$

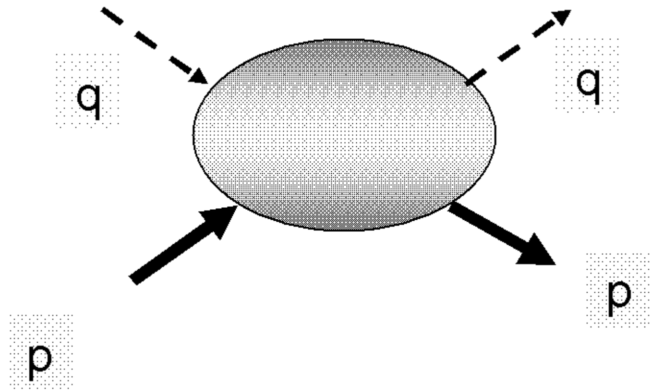
Optical Theorem:

forward Compton scattering amplitude

$$\mathrm{W}_{\mu\nu} = \frac{1}{\pi}\Im A_{\mu\nu} \quad \longrightarrow \quad \mathrm{A}_{\mu\nu} = \sum P^{r}_{\mu\nu} A_r$$

Invariant amplitudes

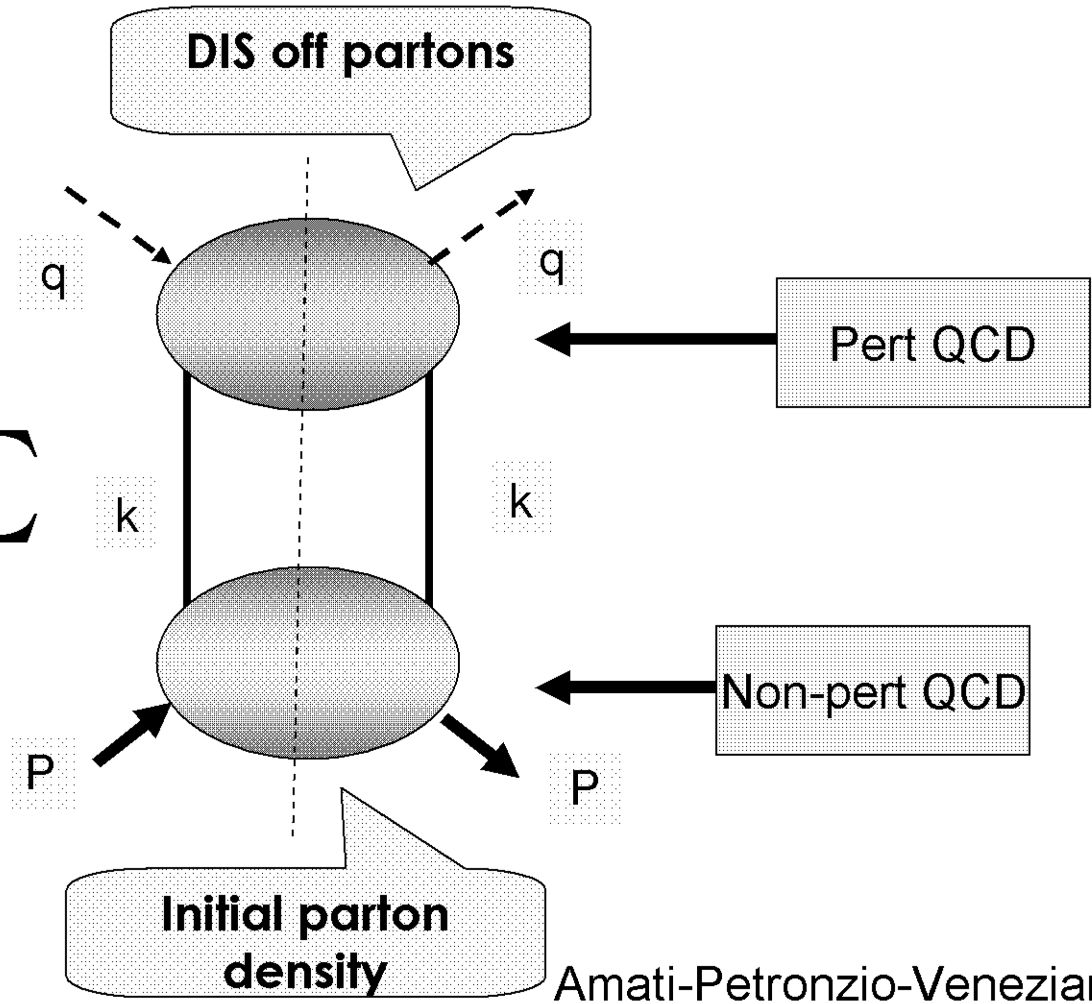
$$\mathrm{W}_{\mu\nu} = \frac{1}{\pi}\Im$$



$$\mathrm{f}_r = \frac{1}{\pi}\Im A_r$$

Factorization:

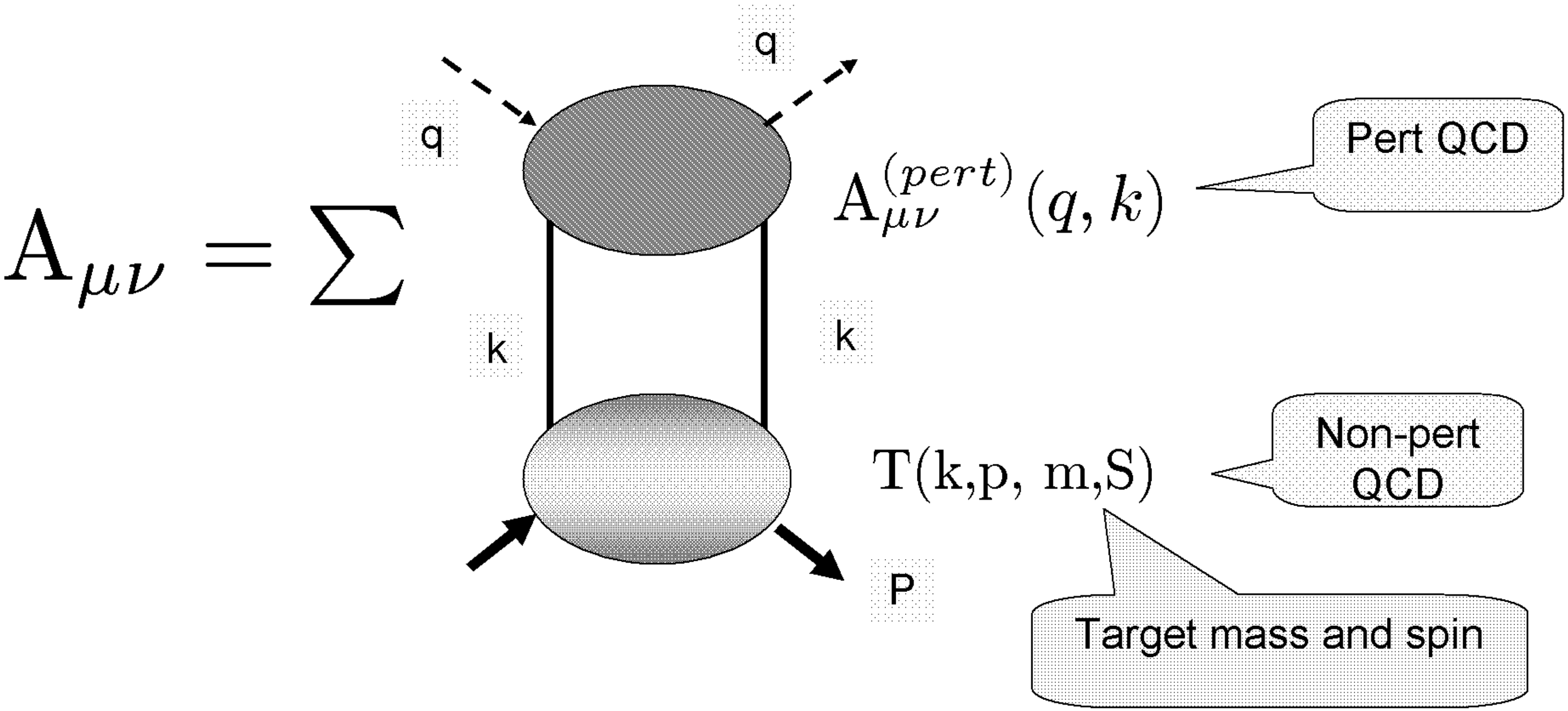
$$W_{\mu\nu} = \sum$$



Collinear factorization (for DGLAP) — Amati-Petronzio-Veneziano, Efremov-Radyushkin, Libby-Sterman, Brodsky-Lepage,..

k_T –factorization (when BFKL is used) — Catani-Ciafaloni-Hautmann

Amplitude of forward Compton scattering



$$A_{\mu\nu}(q,p) = \int \frac{d^4k}{(2\pi)^4} A^{(pert)}_{\mu\nu}(q,k)\frac{1}{(k^2)^2}T(k,p)$$

K acts as IR cut-off for IR-sensitive contributions

$$A\ (q,p) = \int \frac{d^4k}{(2\pi)^4} A^{(pert)}(q,k) \frac{1}{(k^2)^2} T(k,p)$$

any of the invariant amplitudes

Pert contribution

Non-Pert term

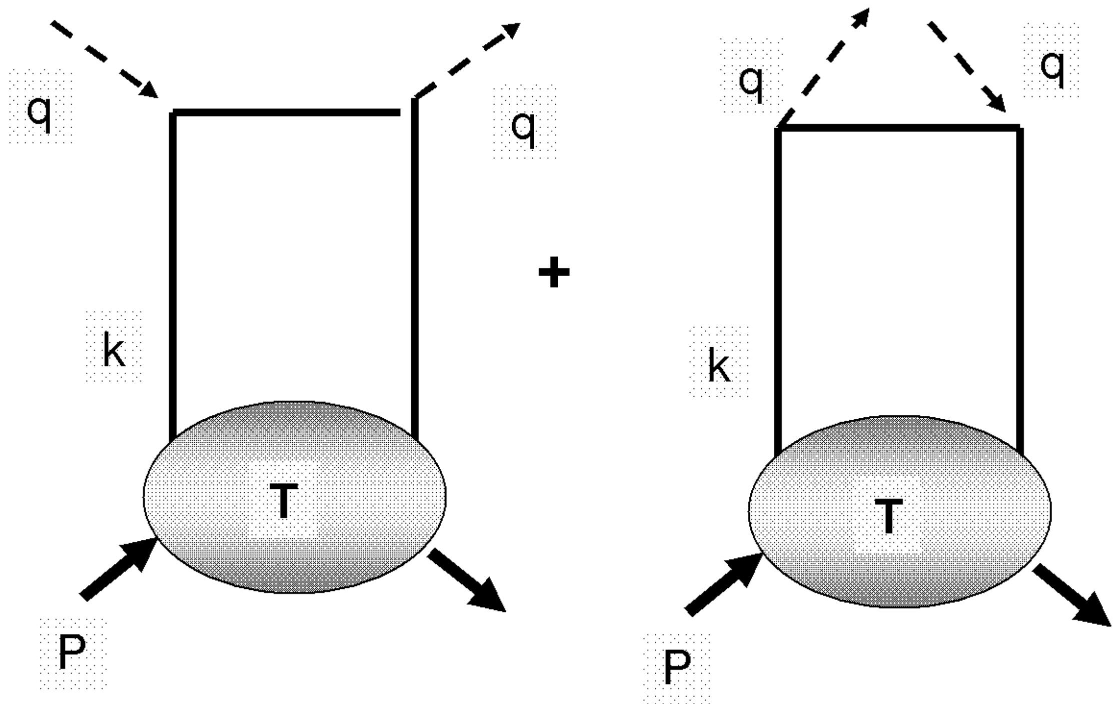
Calculated either in fixed orders or with Evolution Equations

Unknown from theoretical grounds

integration over **k** should be free of UV and IR divergences

requirements for **T**

Born approximation



UV behavior: **At Euclidean k** $\mathrm{d}^4k \sim dk k^3$

So, at large k $\mathrm{A} \sim \int dk \frac{k^3}{k^3} T(k)$ ➡ $\mathrm{T} \sim k^{-1-h}$

In Pert QCD T is gluon propagator: $\mathrm{T} = 1/\mathrm{k}^2$

In Minkowsky space:

Sudakov parameterization

$$\mathrm{k} = \text{-}\ \alpha p + \beta(q + xp) + k_{\perp}$$

So that

$$\mathrm{k}^2 = -\alpha\beta w - k_{\perp}^2, 2pk = \alpha w, 2qk = (\beta + x\alpha)w$$

$$\mathrm{w} = 2\mathrm{pq}$$

$$\mathrm{A}_B^{(pert)} = \frac{\gamma_\mu(\hat{q}+\hat{k})\gamma_\nu}{(q+k)^2} + \frac{\gamma_\nu(\hat{k}-\hat{q})\gamma_\mu}{(q-k)^2}$$

At large α

$$\int_{-\infty}^{\infty} d\alpha\ \hat{k} A_B^{(pert)}(q,k)\hat{k}\ \frac{1}{(k^2)^2}\ T(k,p) \sim \int d\alpha \frac{\alpha^3}{\alpha^3} T(k,p)$$

$$\mathrm{T} = \mathrm{T}((\mathrm{p}+\mathrm{k})^2, k^2) = T(w\alpha, (w\alpha\beta + k_\perp^2))$$

$$\mathrm{T} \sim \alpha^{-1-h}$$

h >0

Beyond the Born approximation

$$\mathrm{A}\ (\mathrm{q,p}) = \int \frac{d^4k}{(2\pi)^4} A^{(pert)}(q,k) \frac{B}{(k^2)^2} T(k,p)$$

where

$$\mathrm{B} \approx w(\alpha^2 + \beta^2) + k_\perp^2$$

There are two different situations beyond Born approximation:

$$\Im A \sim F_1^{singlet}$$ ← **vacuum numbers in t-channel**

Refer as singlet A_S

$$\Im A \sim F_1^{NS}, g_1^{NS}, g_1^S$$ ← **non-vacuum numbers in t-channel**

Refer as non-singlet A_{NS}

singlet and non-singlet have different perturbative contributions:

$$\mathrm{A}_S^{(pert)} = \frac{1}{\beta}\left(\frac{w\beta}{k^2}\right) M\left(\ln(w\beta/k^2),\ \ln(Q^2/k^2)\right)$$

$$\mathrm{A}_{NS}^{(pert)} = \frac{1}{\beta} M\left(\ln(w\beta/k^2),\ \ln(Q^2/k^2)\right)$$

Amplitudes M are different for different amplitudes and in different approaches but their arguments are always the same

$$\mathrm{M} = \sum \ln^j(w\beta/k^2)\ln^l(Q^2/k^2)$$ + non-logarithmic contributions

Do not involve powers of α

$$A_S = \int dk_\perp^2 \frac{d\beta}{\beta} d\alpha \left(\frac{w\beta}{k^2}\right) M_S \left(\ln(w\beta/k^2), \ln(Q^2/k^2)\right) \frac{B}{(k^2)^2} T_S(w\alpha, k^2)$$

$$A_{NS} = \int dk_\perp^2 \frac{d\beta}{\beta} d\alpha M_{NS} \left(\ln(w\beta/k^2), \ln(Q^2/k^2)\right) \frac{B}{(k^2)^2} T_{NS}(w\alpha, k^2)$$

Now let us integrate over α neglecting α -dependence in logs

$$A_S = \int dk_\perp^2 \frac{d\beta}{\beta} (w\beta) M_S \int d\alpha \frac{B}{(k^2)^3} T_S(w\alpha, k^2) \qquad \int d\alpha \frac{\alpha^2}{\alpha^3} T_S(\alpha)$$

$$A_{NS} = \int dk_\perp^2 \frac{d\beta}{\beta} M_{NS} \int d\alpha \frac{B}{(k^2)^2} T_{NS}(w\alpha, k^2) \qquad \int d\alpha \frac{\alpha^2}{\alpha^2} T_{NS}(\alpha)$$

to arrive at

Obligatory for integrability

$$T_S \sim \alpha^{-h} \qquad T_{NS} \sim \alpha^{-1-h}$$

Application to DIS structure functions

$$\mathrm{f}_S = \int dk_\perp^2 \frac{d\beta}{\beta} (w\beta)\, f_S^{(pert)}(w\beta, k^2, Q^2) \int d\alpha \frac{B}{(k^2)^3} \Im T_S(w\alpha, k^2)|$$

stands for singlet F_1 only

$$-\, \mathrm{k}^2 = w\alpha\beta + k_\perp^2$$

$$\mathrm{f}_{NS} = \int dk_\perp^2 \frac{d\beta}{\beta} f_{NS}^{(pert)}(w\beta, k^2, Q^2) \int d\alpha \frac{B}{(k^2)^2} \Im T_{NS}(w\alpha, k^2)$$

any of $\mathrm{F}_1^{NS},\ F_2,\ g_1^S, g_1^{NS},\ g_2$

Factorization of α and β dependence only when $\mathrm{w}\alpha\beta \ll k_\perp^2$

Singlet

$$f_S \approx \int \frac{dk_\perp^2}{k_\perp^2}\frac{d\beta}{\beta}\left(\frac{w\beta}{k_\perp^2}\right) f_S^{(pert)}(Q^2/w\beta, Q^2/k_\perp^2)\Phi_S(\beta, k_\perp^2)$$

$$\Phi_S = \int_{k_\perp^2/w}^{k_\perp^2/w\beta} d\alpha \Im T_S(w\alpha, k_\perp^2)$$

$$\mathrm{Q}^2/w\beta = x/\beta$$

Non-singlet

$$f_{NS} \approx \int \frac{dk_\perp^2}{k_\perp^2}\frac{d\beta}{\beta} f_{NS}^{(pert)}(Q^2/w\beta, Q^2/k_\perp^2)\Phi_{NS}(\beta, k_\perp^2)$$

$$\Phi_{NS} = \int_{k_\perp^2/w}^{k_\perp^2/w\beta} d\alpha \Im T_{NS}(w\alpha, k_\perp^2)$$

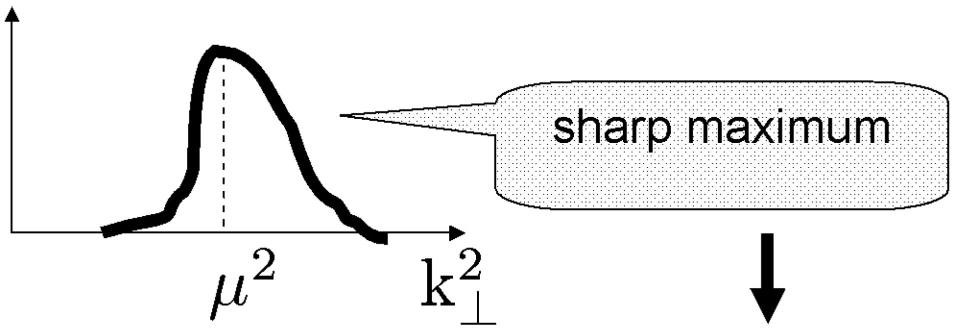
$$T_S \sim \alpha^{-h} \quad \longrightarrow \quad \Phi_S \sim \beta^{-1+h}$$

h>0

$$T_{NS} \sim \alpha^{-1-h} \quad \longrightarrow \quad \Phi_{NS} \sim \beta^{h}$$

Transition to DGLAP: Collinear factorization

$$\Phi = \Phi(\beta, k_\perp^2)$$



For instance: $\Phi \approx \Phi(\beta, k_\perp^2)\delta((k_\perp^2 - \mu^2)$

$$\mathrm{f}_{NS} \approx \int \frac{d\beta}{\beta} f_{NS}^{(pert)}(x/\beta, Q^2/\mu^2)\Phi_{NS}(\beta, \mu^2)$$

Dependence on μ

However, there is no μ dependence in the case of DGLAP because DGLAP collects leading logs of Q^2 only, Sub-leading logs will be μ-dependent

$$\ln^n(Q^2/\mu_1^2) \approx \ln^n(Q^2/\mu_2^2)$$ **+ sub-leading**

$$\Phi_{NS} \sim \beta^h \qquad \Phi_S \sim \beta^{-1+h}$$

Comparison to standard DGLAP fits:

Altarell-Bal-Forte-Ridolfi, Leader-Sidorov-Stamenov, Blumlein-Botcher, Hirai,…

Typical expressions:

parameters a,b,c,d > 0

$$\delta q(y), \delta g(y) \sim y^{-a}(1-y)^b(1+cy^d)$$

$$\sim y^{-a} + \underline{(1+cy^d)\sum(-1)^r C_r^b y^{b-a-r}}$$

singular factor

regular terms

integrability

$$\Phi_S \sim \beta^{-1+h}$$ → singular factor satisfies: a <1 for F_1 singlet

h>0

$$\Phi_{NS} \sim \beta^h$$ → excludes singular factors for all other structure functions

However in practice these requirements are violated

Reason why singular factors are necessary in DGLAP at small x

Most important at small x

DGLAP does not include resummations of $\ln^n(1/x)$, so without singular factors x^{-a} DGLAP expressions grow too slowly to match experiment

Factors x^{-a} bring the appropriate growth at small x They mimic resummation of $\ln^n(1/x)$ and eventually, at $x \longrightarrow 0$ they change the classic DGLAP asymptotics $f \sim e^{\sqrt{\ln(1/x)}}$ for the Regge one $f \sim x^{-a}$

When the resummation is accounted for, they should be dropped, which simplifies fits

CONCLUSION

Integrability of forward Compton amplitudes imposes the following restrictions on DGLAP fits for initial parton densities:

1. Singular factors x^{-a} can be used in fits for singlet F_1 only, providing $a<1$

2. Singular factors should not be used for all other structure functions. Instead, one should use total resummation of $\ln^n(1/x)$

3. Necessity to use singular factors is a good indication that important logs of x are missing from theoretical expressions