# Scaling properties in deep inelastic scattering

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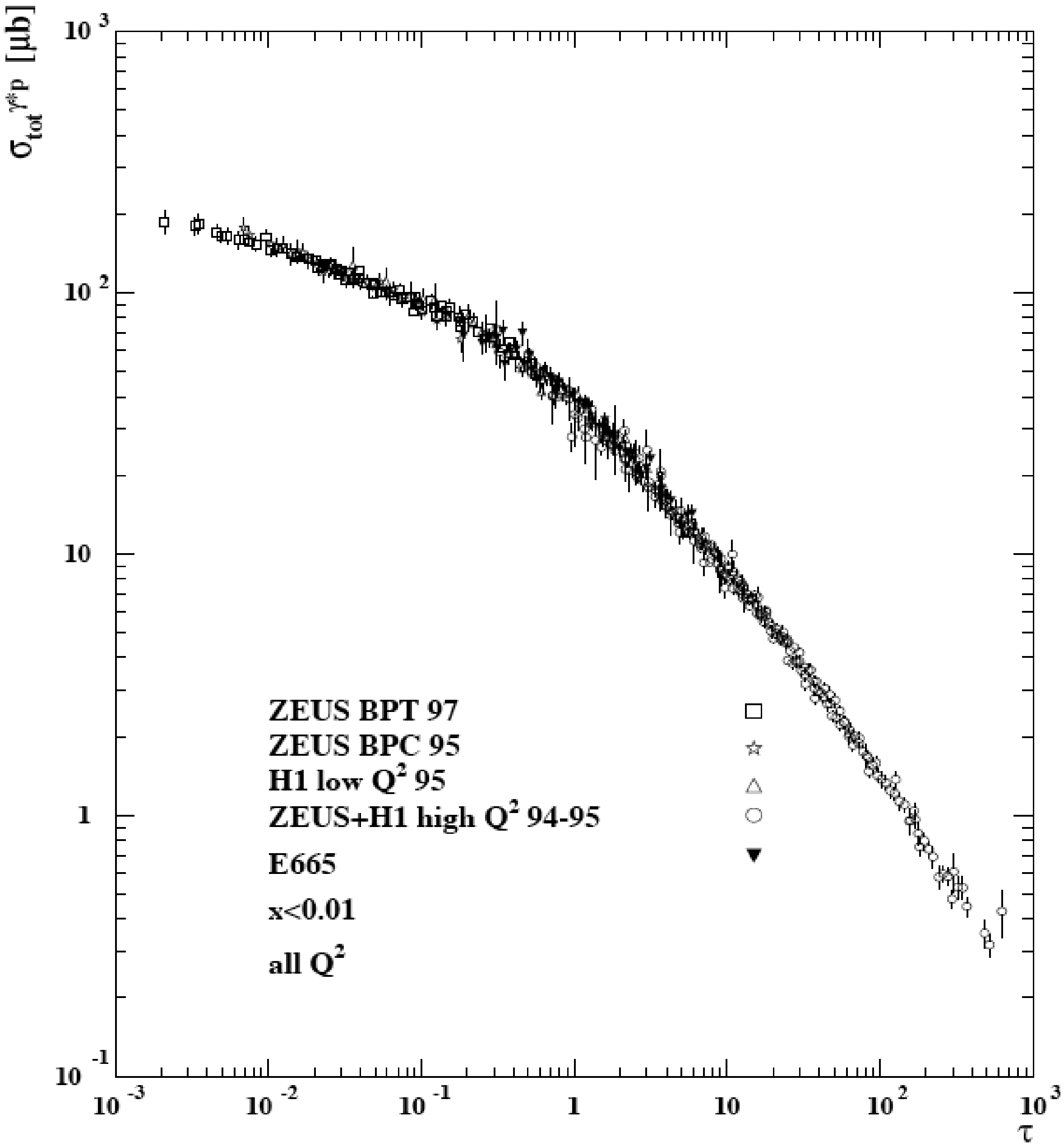
- Scalings in DIS
- Scaling using proton structure function $F_2$
- Fits to $F_2$ data

Work done in collaboration with R. Peschanski

# Original observation: $F_2$ scaling

Original study: plot $\sigma(\gamma^* p)$ as a function of $\tau$: $(\tau = Q^2 \times (x/x_0)^\lambda)$

See: A. M. Stasto, K. Golec-Biernat, and J. Kwiecinski, Phys. Rev. Lett. 86 (2001) 596



# Scalings in DIS

- Stochastic extension of Balitsky-Kovchegov equation for dipole amplitude

$$\frac{\partial T}{\partial Y} = \alpha_S \left[\chi(-\partial_L)T - T^2\right]$$

where $\chi$ is the BFKL kernel, $L = \log Q^2$, $\alpha_S \sim 1/\log Q^2$, $Y = \log 1/x$ (BFKL equation when $\alpha_S$ constant, term in $T^2$ neglected)

- $\alpha_S$ constant:
  - BK equation with $\alpha_S$ constant
  - It is possible to show that the solution of the BK equation does not depend independently on $Y$ and $\log Q^2$ but a combination of both (scaling)
  - Scaling obtained: $\tau = L - \lambda Y$
  - Scaling called "Fixed coupling" in the following

## Scalings in DIS

- Extension of Balitsky-Kovchegov equation

$$\frac{\partial T}{\partial Y} = \alpha_S(Q^2)\left[\chi(-\partial_L)T - T^2\right]$$

- $\alpha_S$ running:
  - $\alpha_S \sim 1/logQ^2$
  - $T^2$ is expected to follow the scaling but it is possible to show that it is impossible that both $L\partial T/\partial Y$ and $\chi(-\partial_L)$ follow the same scaling at the same time
  - Two scalings obtained: $\tau = L - \lambda\sqrt{Y}$, Running coupling I; $\tau = L - \lambda Y/L$ , Running coupling II; See G. Beuf, arXiv:0803.2167

# Scalings in DIS

- Extended Balitsky-Kovchegov equation

$$\frac{\partial T}{\partial Y} = \alpha_S \left[\chi(-\partial_L)T - T^2 + \sqrt{\alpha_S^2 \kappa T}\,\nu(L,Y)\right]$$

- $\alpha_S$ constant, $\kappa$
  - $\nu$ is a gaussian "noise" corresponding to the fluctuation of the number of gluons
  - This term contributes to pomeron loops (gluon splitting)
  - It can be shown that the scaling obtained is: $(L - \lambda Y)/\sqrt{Y}$, called Diffusive scaling in the following
- NB: There can be additional parameters: take $Y - Y_0$, $L = \log Q^2/Q_0^2$
- ATTENTION: $\tau$ MUST BE POSITIVE IN MOST OF THE CONSIDERED PHASE SPACE $Q^2 > 3$ Gev$^2$

## Quality factor

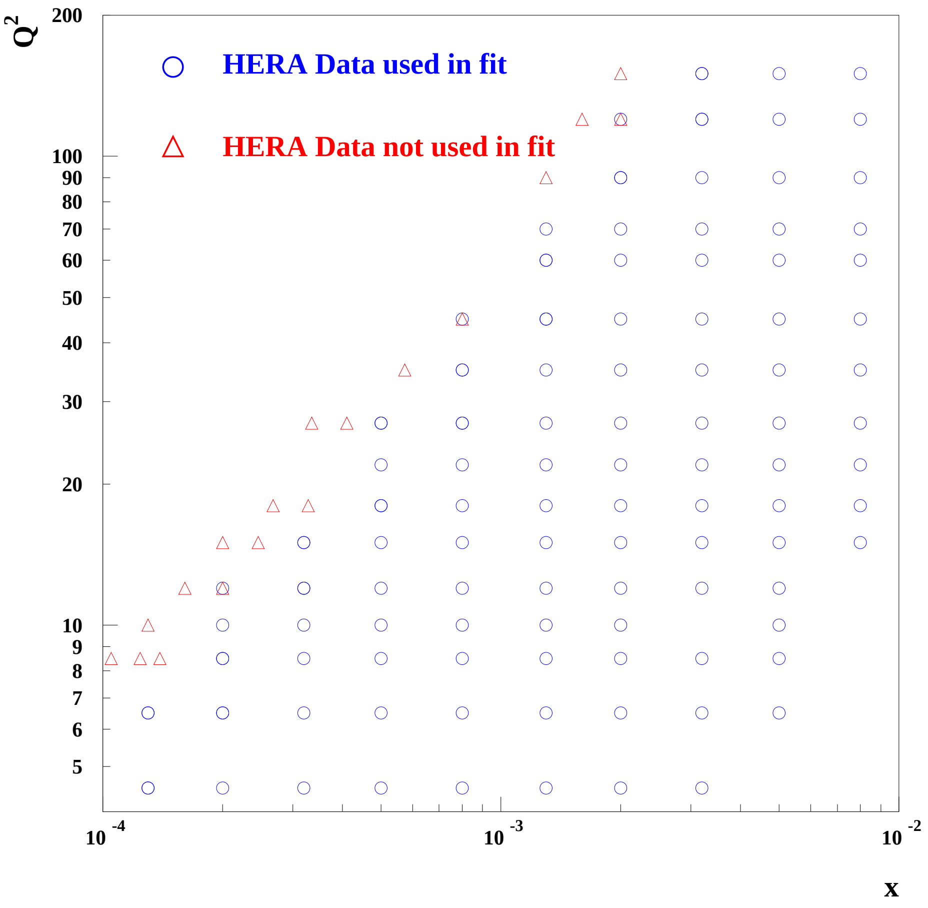
- How to compare the different scalings? DIS cross section $\sim F_2/Q^2$ depends on the $\tau$ variable only or not (for instance fixed coupling $\tau = \log Q^2 - \lambda Y$)
- The form of the $\tau$ dependence is not known and an estimator is needed on how data points ($F_2$ for instance) depend only on $\tau$ or not
- Method:
  - Normalise data sets $v_i = \log(\sigma_i)$, and scalings $u_i = \tau_i(\lambda)$ between 0 and 1 (note that we take the $\log$ of the cross section since the cross section varies by orders of magnitude)
  - Order the scalings in $u_i$
  - Define the quality factor:

$$QF(\lambda) = \left[ \sum_i \frac{(v_i - v_{i-1})^2}{(u_i - u_{i-1})^2 + \epsilon^2} \right]^{-1}$$

    ($\epsilon$ needed in the case that two data points have the same scaling, namely when they have the same $x$ and $Q^2$ for $F_2$, $\epsilon^2$=0.0001)
  - Fit $\lambda$ to maximise QF: when data are close to the scaling law, QF is large

# Scaling tests in DIS using $F_2$

- Combined $F_2$ measurements from H1/ZEUS (small error bars)
- Cuts on data: $4 \leq Q^2 \leq 150$ GeV$^2$, $x \leq 10^{-2}$: stay in perturbative domain, avoid region where valence quark dominates
- Avoid high $y$ region where $F_L$ is large: $y \leq 0.6$
- After all cuts: 117 data points



## Comparison of different scalings

- Value of parameters and QF for $Q^2 \geq 4$ GeV$^2$
- FC, RC1 and RC2 favoured, DS disfavoured

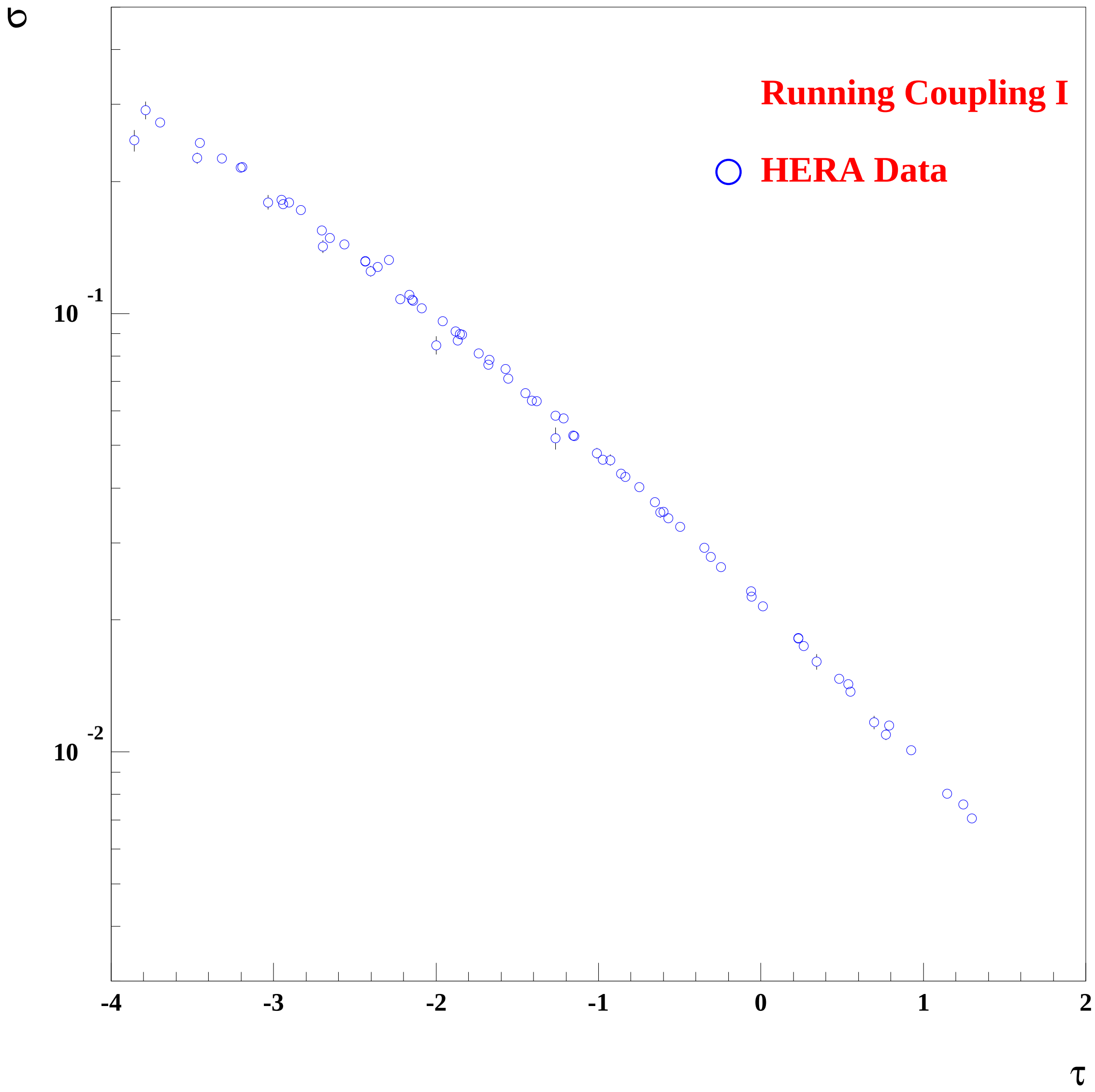
$$
\begin{aligned}
FC &: \quad \tau = \log Q^2 - \lambda \log(\frac{1}{x}) \\
RC1 &: \quad \tau = \log Q^2 - \lambda \sqrt{\log(\frac{1}{x})} \\
RC2 &: \quad \tau = \log(Q^2/0.2^2) - \lambda \frac{\log(\frac{1}{x})}{\log(Q^2/0.2^2)} \\
DS &: \quad \tau = \frac{\log Q^2}{\sqrt{\log \frac{1}{x}}} - \lambda \log(\frac{1}{x})
\end{aligned}
$$

| scaling | parameter | 1/QF |
|---|---|---|
| Fixed Coupling | $\lambda = 0.31$ | 150.2 |
| Running Coupling I | $\lambda = 1.61$ | 137.9 |
| Running Coupling II | $\lambda = 2.76$ | 124.3 |
| Diffusive Scaling | $\lambda = 0.31$ | 210.7 |

NB: Introducing additional parameters ($Q_0$, $Y_0$) does not improve much the quality of scaling

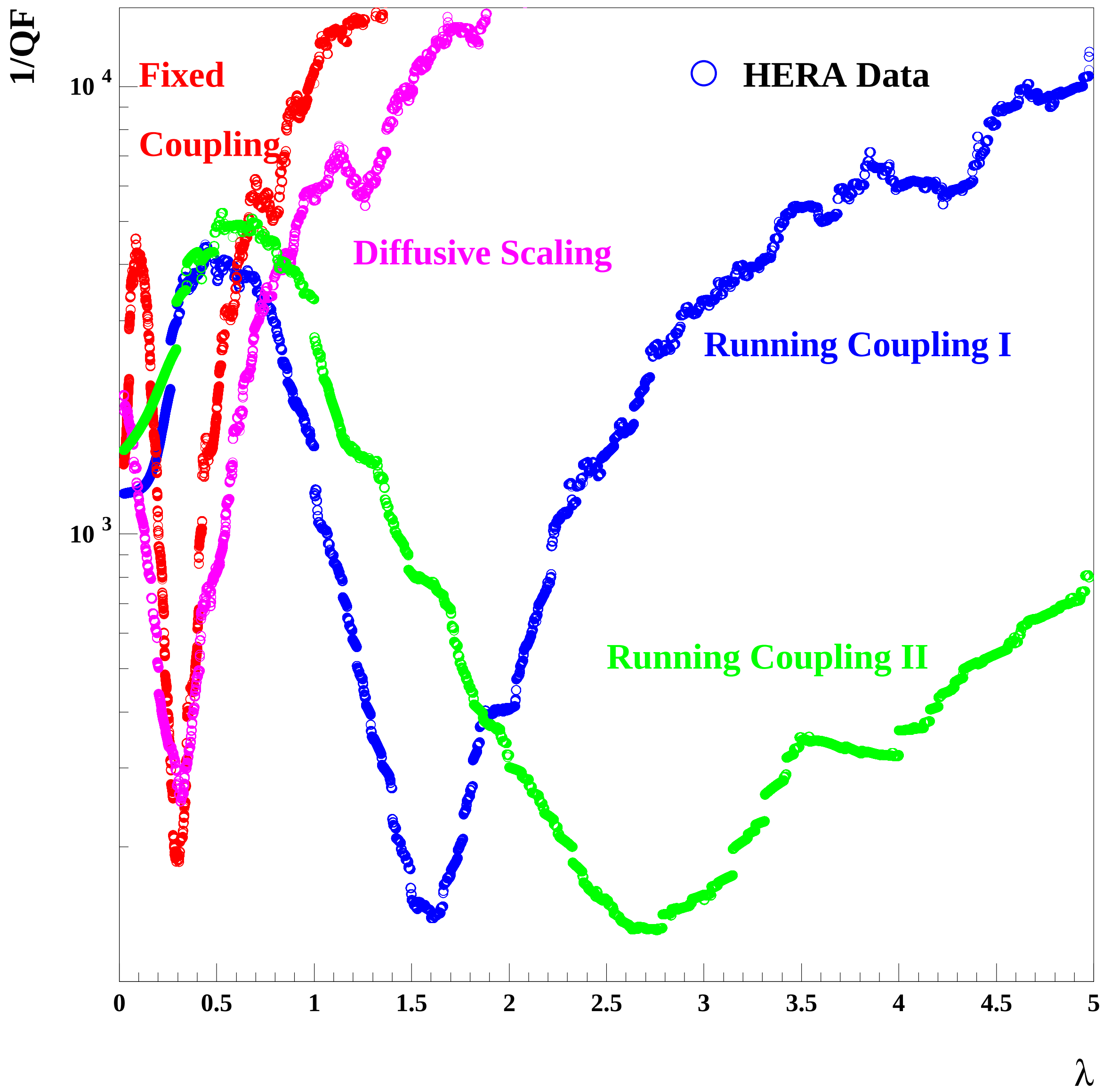
# Scaling plot

As an example, scaling plot for RCI: $\tau > 0$ after definition of $Q_0$ (scaling
$\tau = \log Q^2/Q_0^2 - \lambda \log(\frac{1}{x})$



# Quality factor

Differences in $1/QF$ for FC, RCI, RCII, DS



# Fits to HERA data

- Fit to HERA data inspired by RCI: all data above $Q^2 = 4GeV^2$ should be in the dilute regime (if saturation occurs at HERA, it is at very low $Q^2$)
- Scale $\tau$ and expression for DIS cross section:

$$\begin{aligned} \tau &= \log(\frac{Q^2}{Q_0^2}) - \lambda\sqrt{\log(\frac{1}{x}) - Y_0} \\ \sigma &= N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right) \end{aligned}$$

- Fit formula deduced from the dipole amplitude with saturation (Gregory Soyez) with asymptotic expression of the Airy function which is solution ok Balistsky Kovchegov equation
- Fit using 6 parameters: $\lambda$, $\alpha$, $\beta$, $Q_0$, $Y_0$, $N$
- Explicit moderate scaling violation: $(\log 1/x - Y_0)^{1/4}$ term: fits performed with and without this scaling violation predicted by the dipole model.

# Fit results

- Fit variables:

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda\sqrt{\log(\frac{1}{x}) - Y_0}$$

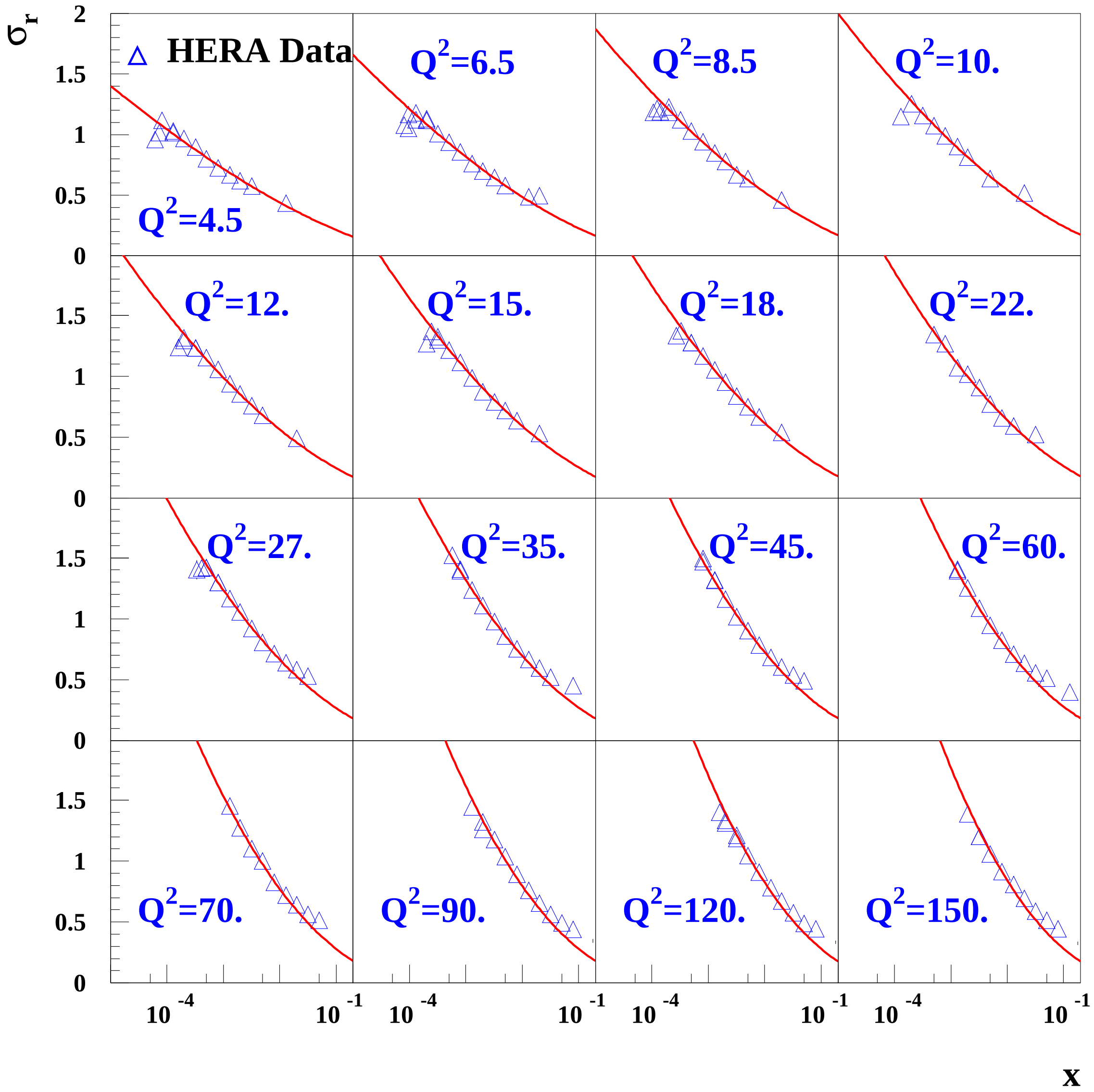
$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)$$

- Fit I: $\chi^2 = 130.1$ for 117 points, $\chi^2/dof = 1.2$
- Fit II without the scaling violation term: $\chi^2 = 119.0$

| Parameter | Fit I | Fit II |
|---|---|---|
| $\lambda$ | 1.54 $\pm$ 0.02 | 1.54 $\pm$ 0.02 |
| $\alpha$ | 0.34 $\pm$ 0.01 | 0.18 $\pm$ 0.01 |
| $\beta$ | 0.24 $\pm$ 0.01 | 0.18 $\pm$ 0.01 |
| $Q_0$ | 0.079 $\pm$ 0.01 | 0.064 $\pm$ 0.01 |
| $Y_0$ | -1.46 $\pm$ 0.02 | 0.50 $\pm$ 0.02 |
| $N$ 0.51 | $\pm$ 0,01 | 0.72 $\pm$ 0,01 |

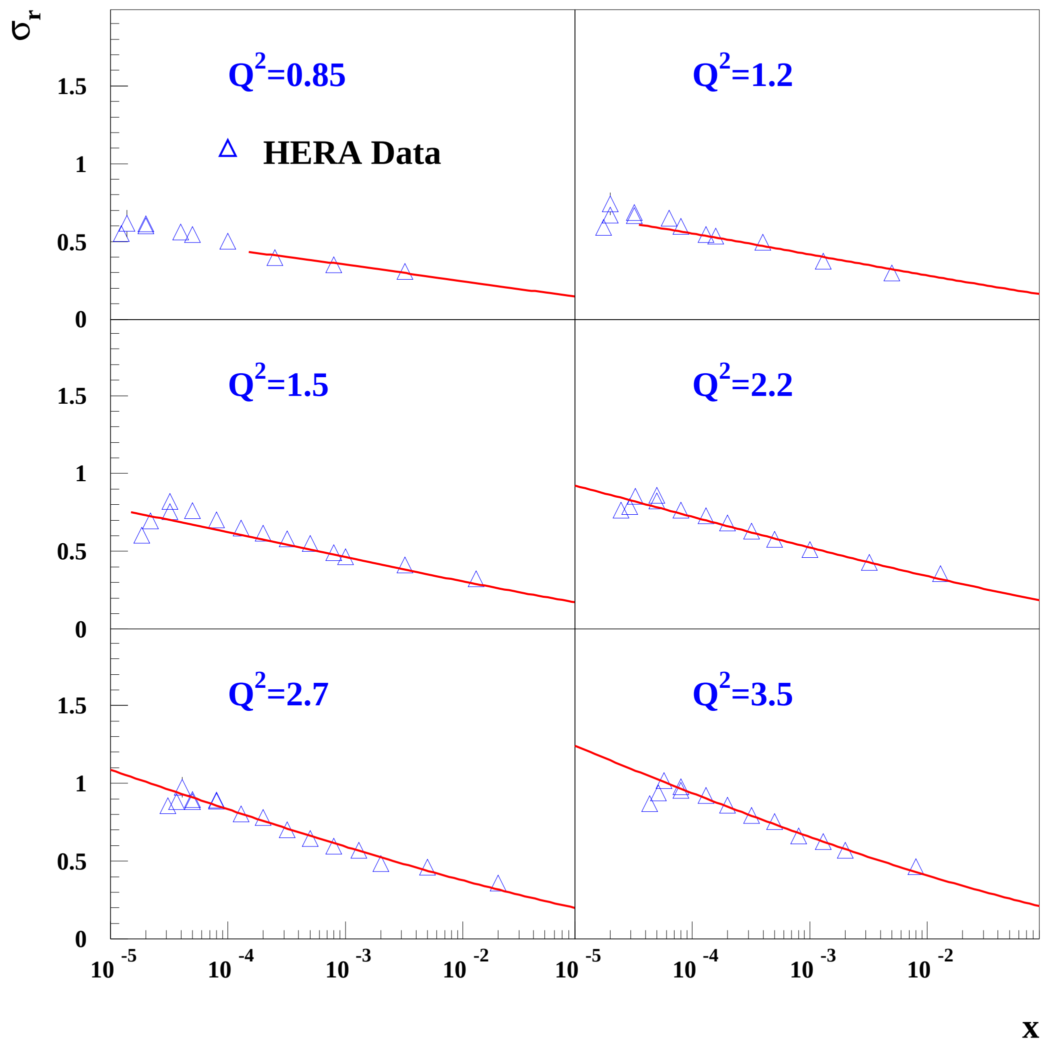
# Fit results

- Good description of HERA low $Q^2$ and low $x$ reduced cross section data
- Fit does not describe the reduction of the reduced cross section at high $y$: $\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$: needs a model of $F_L$ using RCI, in progress



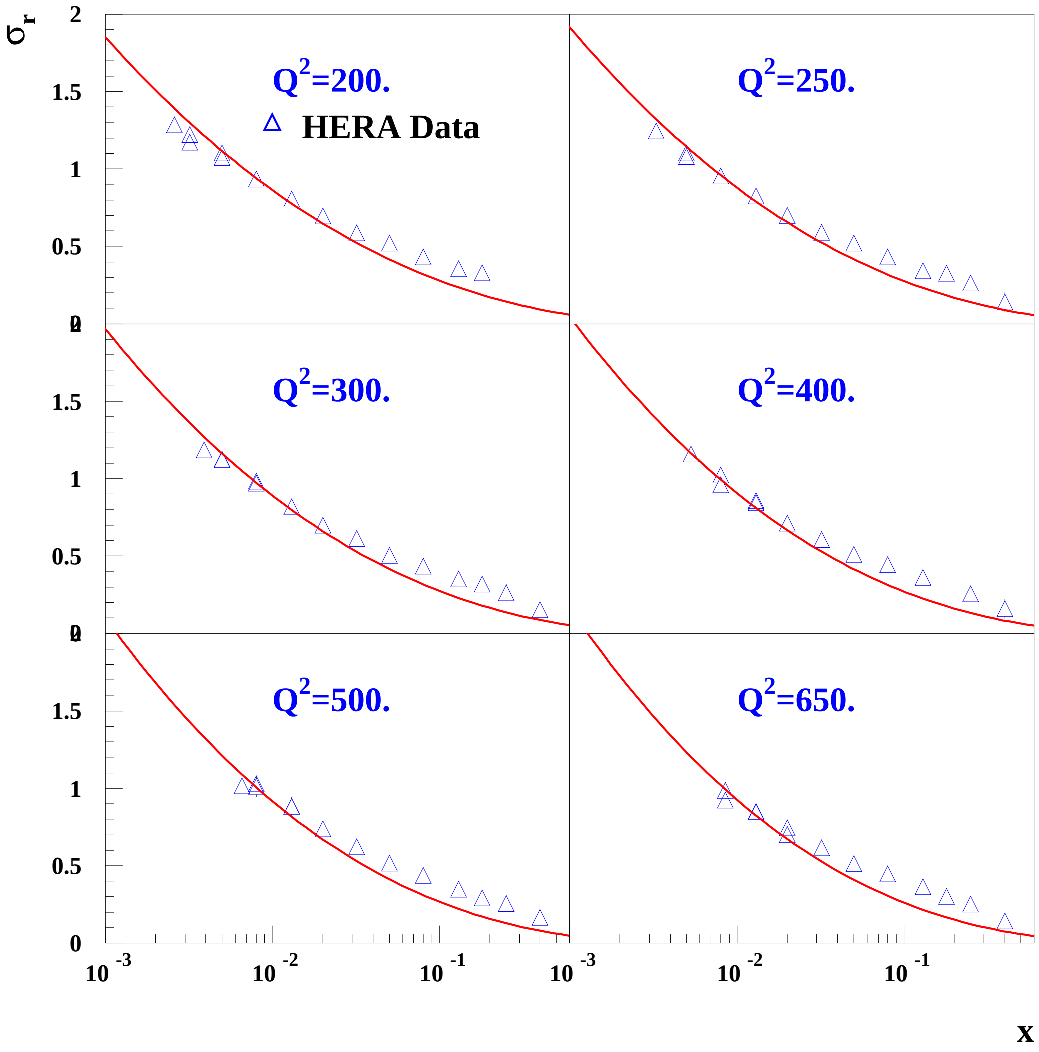
# Fit extrapolation at low $Q^2$

- Leads to a fair description of data at lower $Q^2$
- Need a parameterisation of $F_L$ to describe high $y$ data
- Need a description in the saturated region to describe very low $Q^2$ data: only description in the "dilute" regime so far



# Fit extrapolation at high $Q^2$

Leads to a fair description of data at higher $Q^2$, except at high $x$ (needs valence quark contribution)



# Comparison with other fits

- Same kind of formula for RCII:

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda \frac{\log(\frac{1}{x}) - Y_0}{\log(\frac{Q^2}{Q_0^2})}$$

$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)$$

- $\chi^2 = 190.4$, worse description than for RCI
- Same kind of formula for FC:

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda \log(\frac{1}{x})$$

$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^{2}}{(\log 1/x - Y_0)}\right)$$

- $\chi^2 = 156.4$, worse than RCI, $\chi^2 = 230.5$ without the scaling violation term

# Conclusion

- Different scalings studied in $F_2$ data: fixed coupling, running coupling I and II, diffusive scaling
- Fixed coupling, running coupling I and II lead to a good description of data using the QF formalism
- Diffusive scaling disfavoured
- Fit of $F_2$ data using RCI: parameterised with or without moderate scaling violations, leads to a good description of low $Q^2$, low $x$ $F_2$ data
- Fits disfavour RCII and FC
- Outlook: fits of lower $Q^2$ data in the saturation region; fits of high $y$ data including $F_L$ parameterisation; comparision with numerical solution of BK equation with $\alpha_S$ running