Correlations in impact-parameter space in saturation models

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ÉCOLE
POLYTECHNIQUE

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dépasser les frontières

High-energy/high-density QCD

We want to understand dense systems of partons (color glass condensate)

- Evolution equations have been written (B-JIMWLK, BK...)
- Some universal features of dense parton systems are known
- Phenomenological models have been built

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But many points still need to be understood

- Mechanism for saturation, i.e. establish evolution equations *systematically*
- Solutions at non-asymptotic energies
- Dynamics in impact-parameter space

The impact parameter in phenomenological models

Simplest model for the dipole-hadron elastic scattering amplitude:

$$T(r,y,b)=1-\exp\left(-\frac{r^2 Q_s^2(y,b)}{4}\right)$$ Golec-Biernat, Wüsthoff (1999)

$$Q_s^2(y,b)=Q_0^2\exp(\lambda y)\,S(b)$$

Profile in impact-parameter

$$S(b)=\theta(R-b)$$ Golec-Biernat, Wüsthoff (1999)

$$S(b)=\frac{1}{2\pi B}\exp\left(-\frac{b^2}{2B}\right)$$ Kowalski, Motyka, Watt

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Correlations/fluctuations between different impact parameters are neglected.

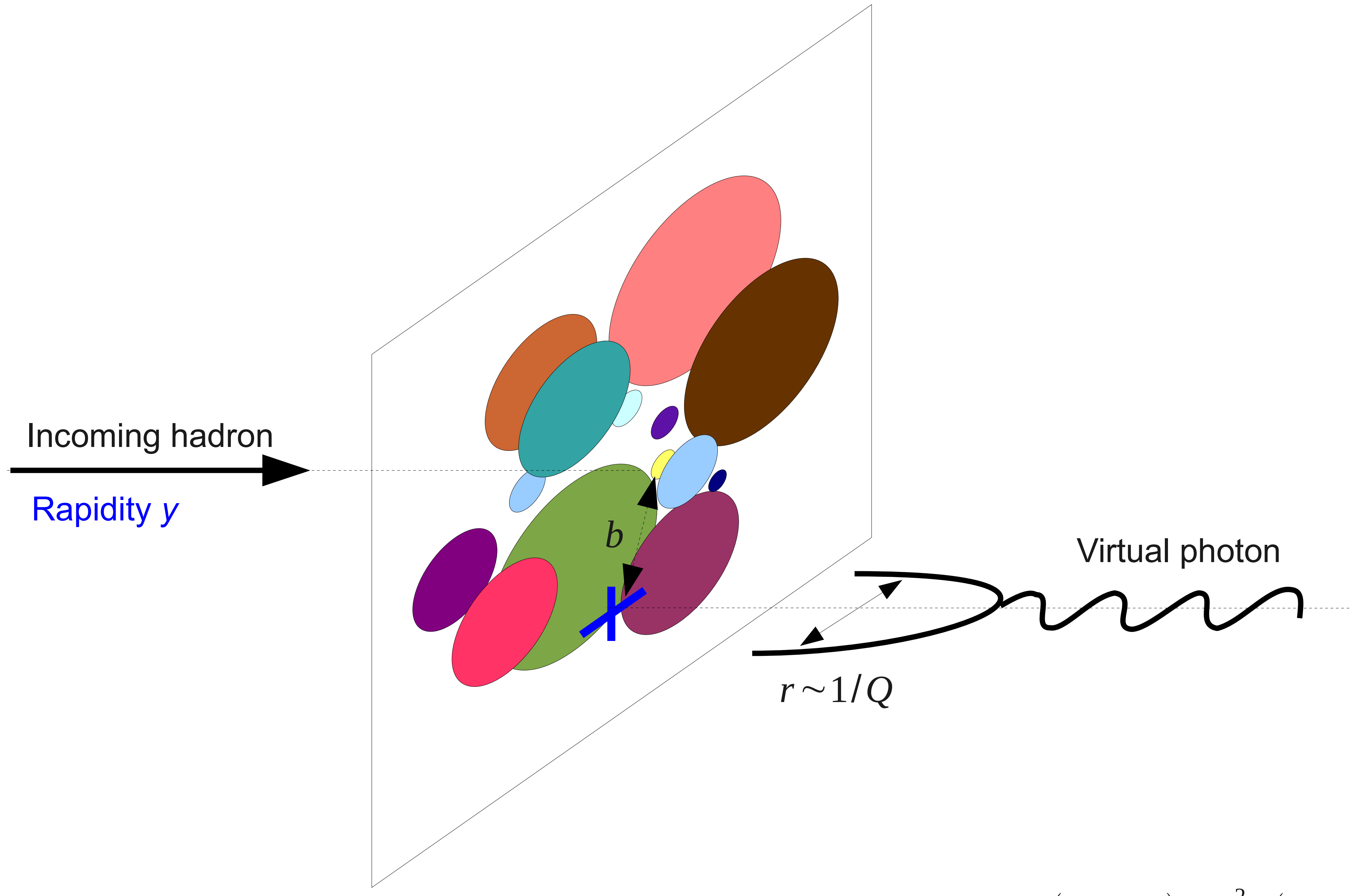
- OK for phenomenology, since total and elastic DIS cross sections are not directly sensitive to fluctuations in impact-parameter space

- But we want to understand better the picture from QCD

Outline

- Microscopic picture of a hadron at high energy
- How correlations may occur between different impact parameters
- Quantitative results for the correlations

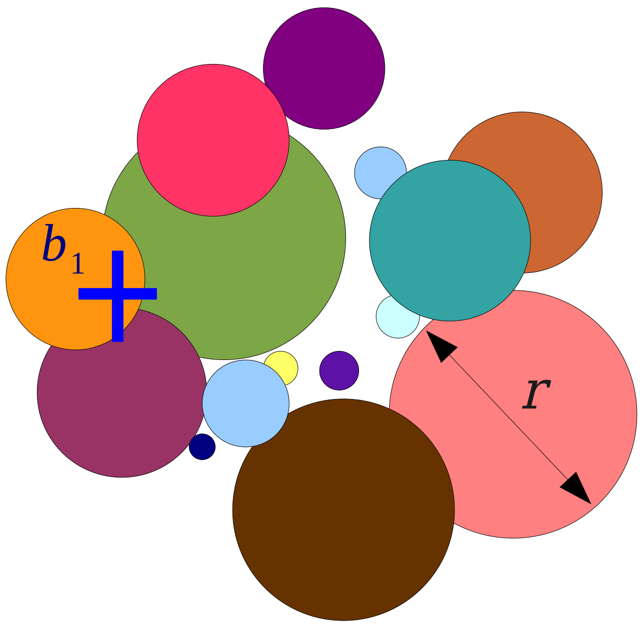
Microscopic picture of a hadron at high energy



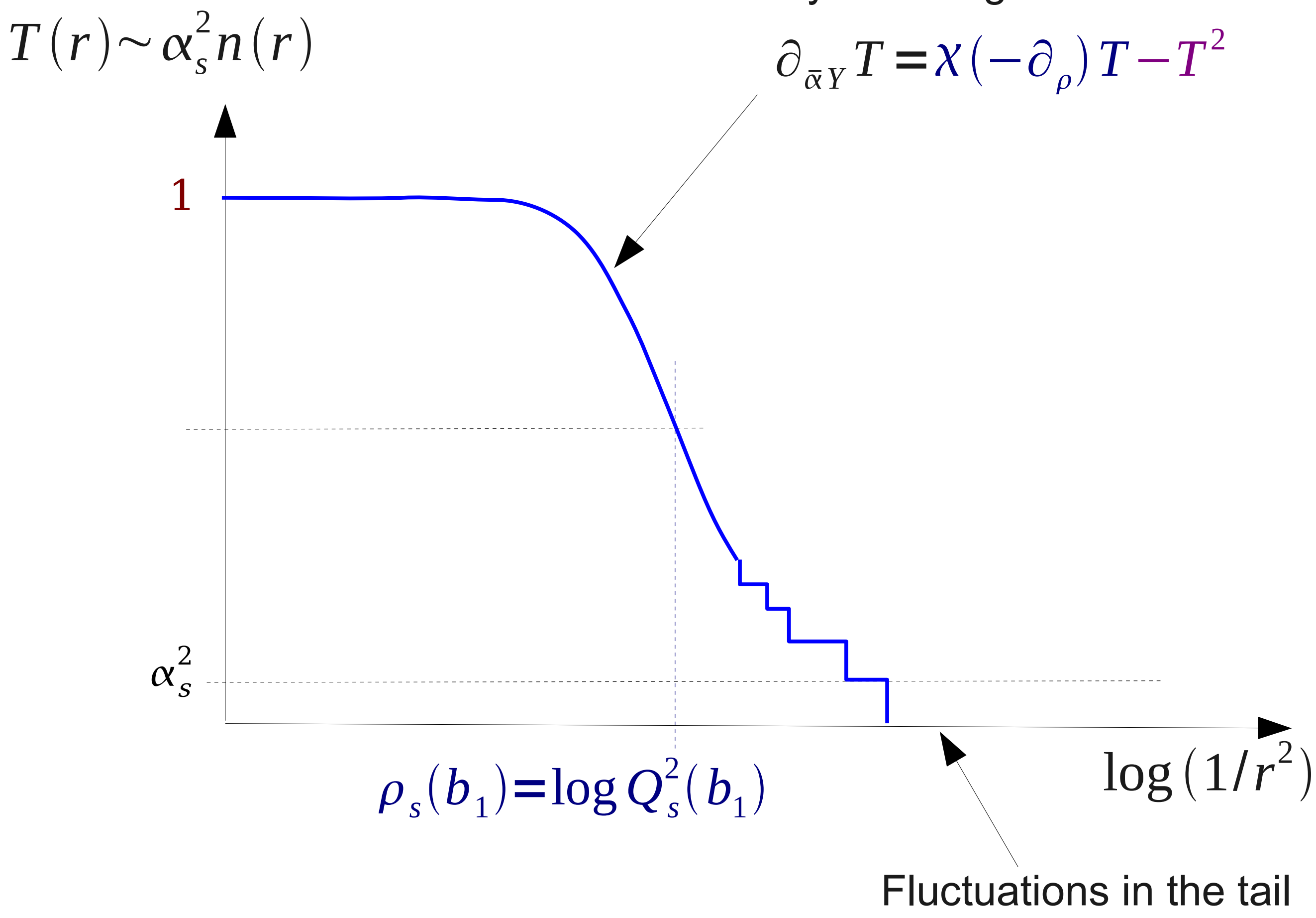
Cross section ~ interaction probability = $T(r, y, b) \sim \alpha_s^2 n(r, y, b)$

Microscopic picture of a hadron at high energy

Rapidity y

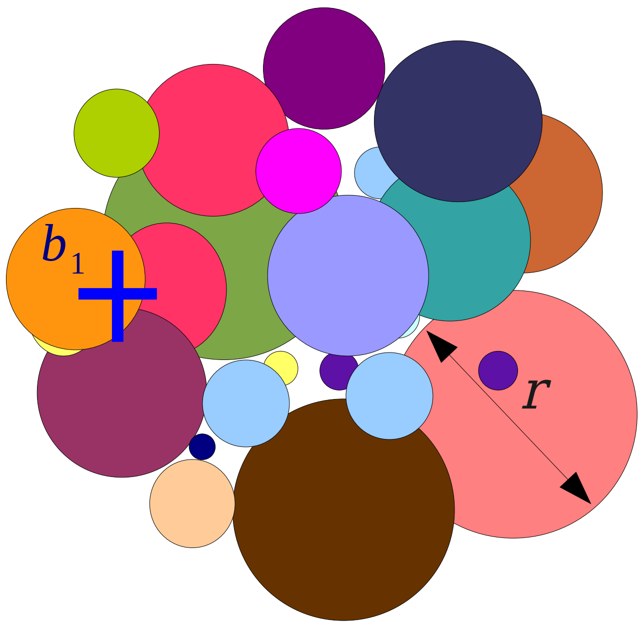


Universal shape
results from the Balitsky-Kovchegov evolution



Microscopic picture of a hadron at high energy

Extra evolution over Δy units of rapidity



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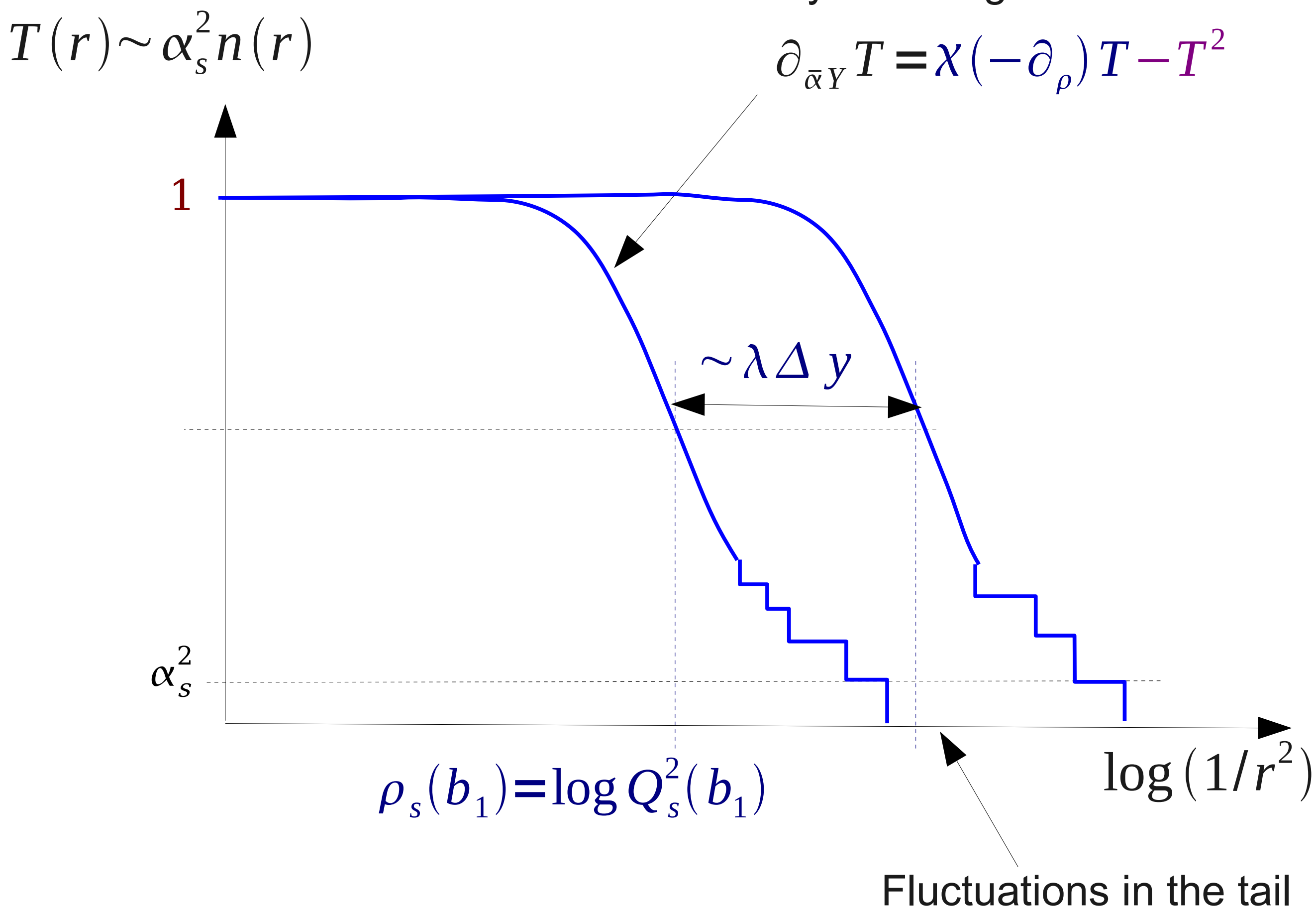

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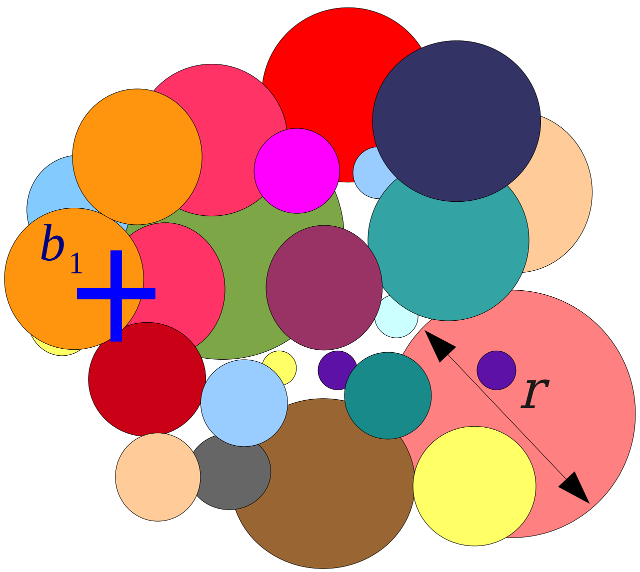

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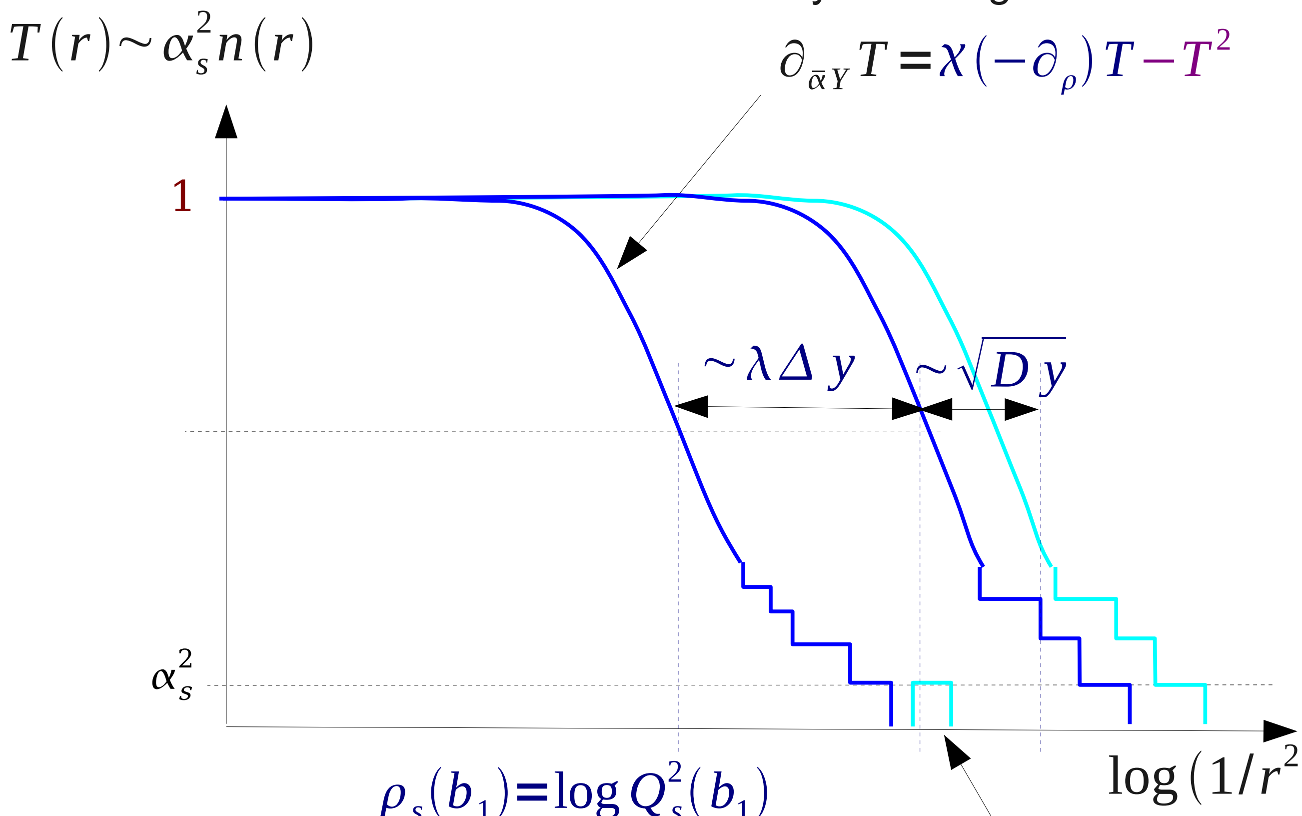
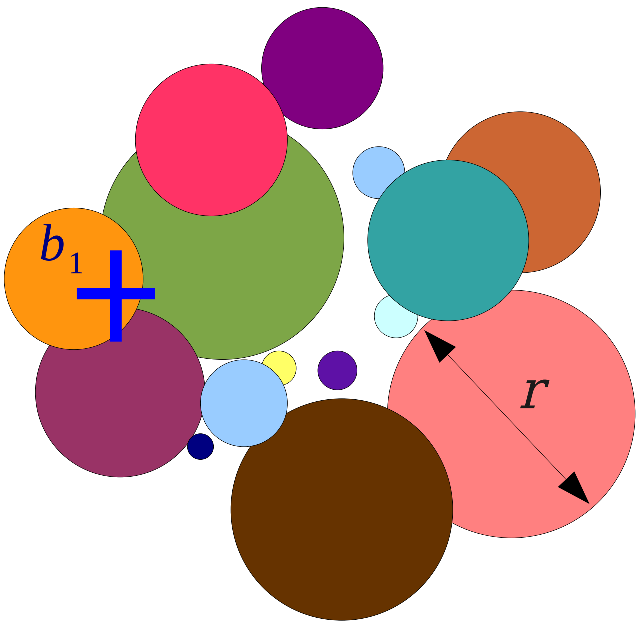


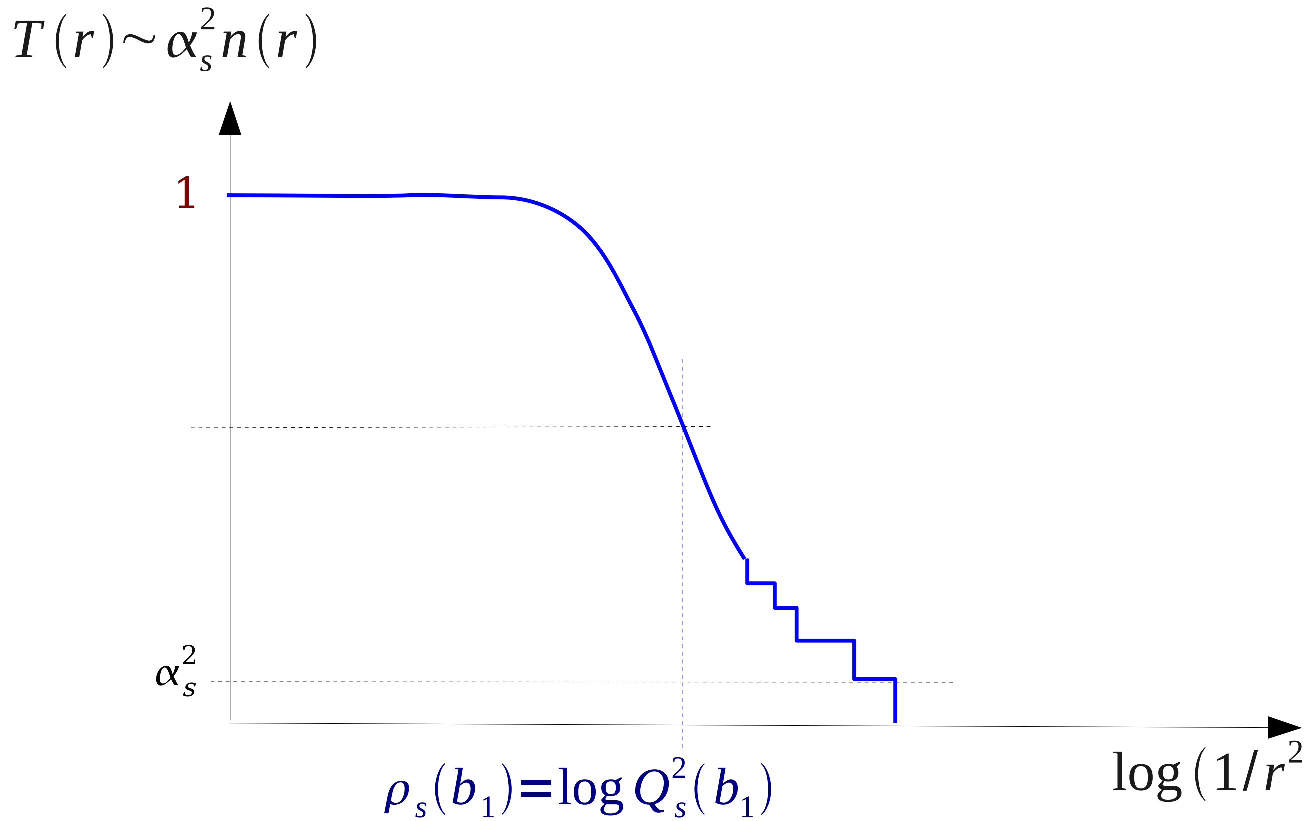
Fluctuations in the tail
Generate random shifts in the saturation scale after $\Delta y \sim \log^2(1/\alpha_s^2)$

$$\langle \rho_s^2(y) \rangle \sim D \times y$$

Microscopic picture of a hadron at high energy

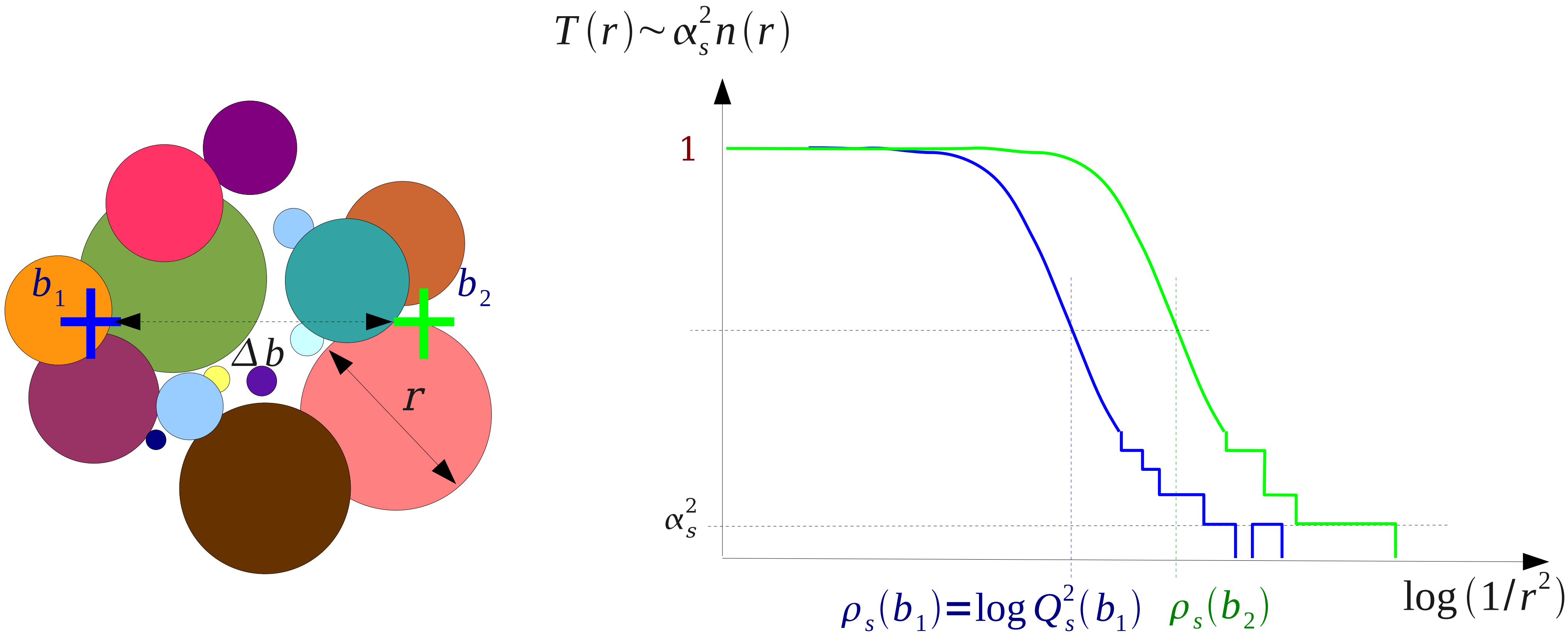
Rapidity *y*





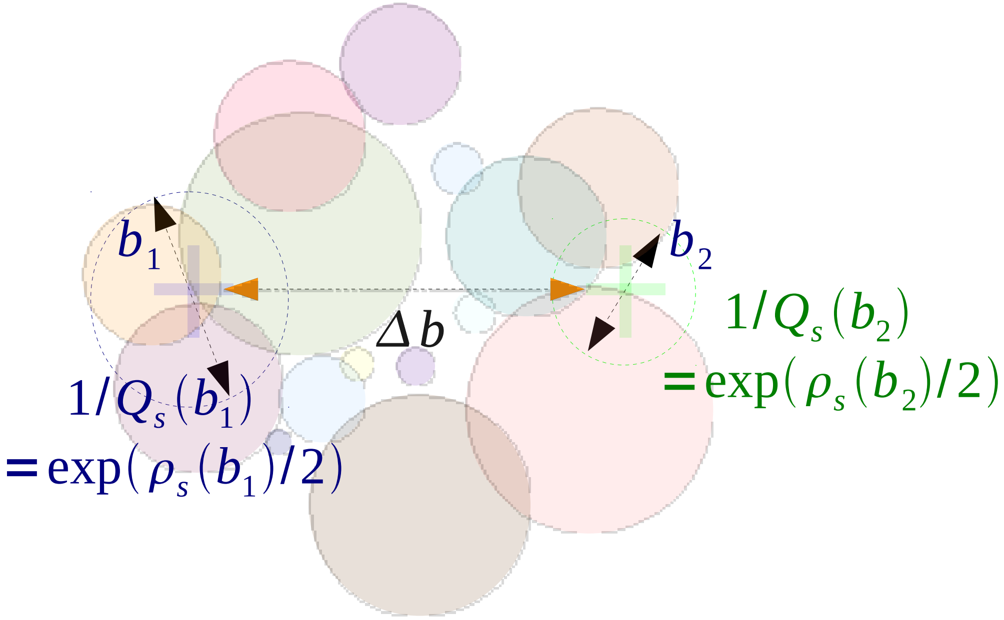
Microscopic picture of a hadron at high energy

Rapidity y



What is the relationship between $\rho_s(b_1)$ and $\rho_s(b_2)$?

How correlations may occur



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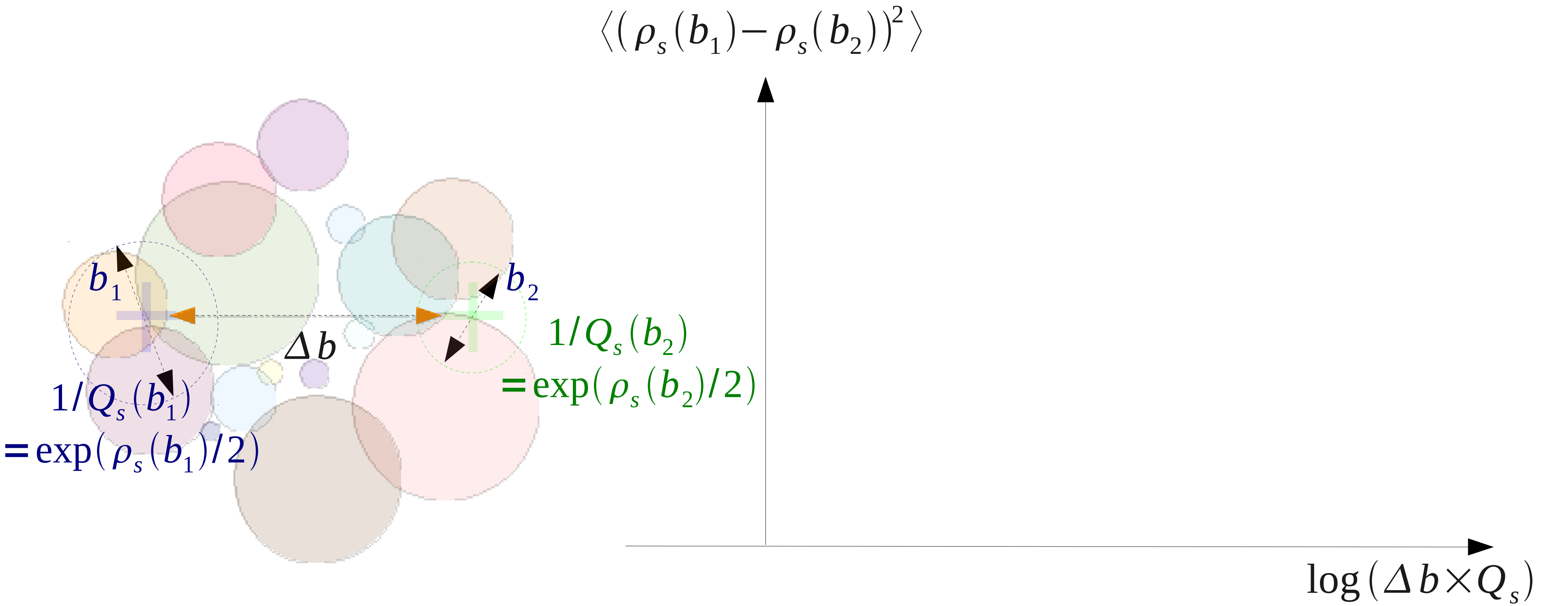

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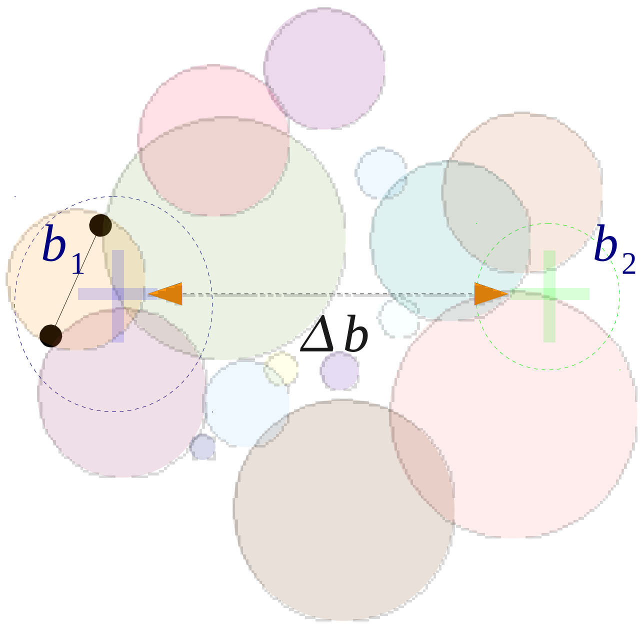

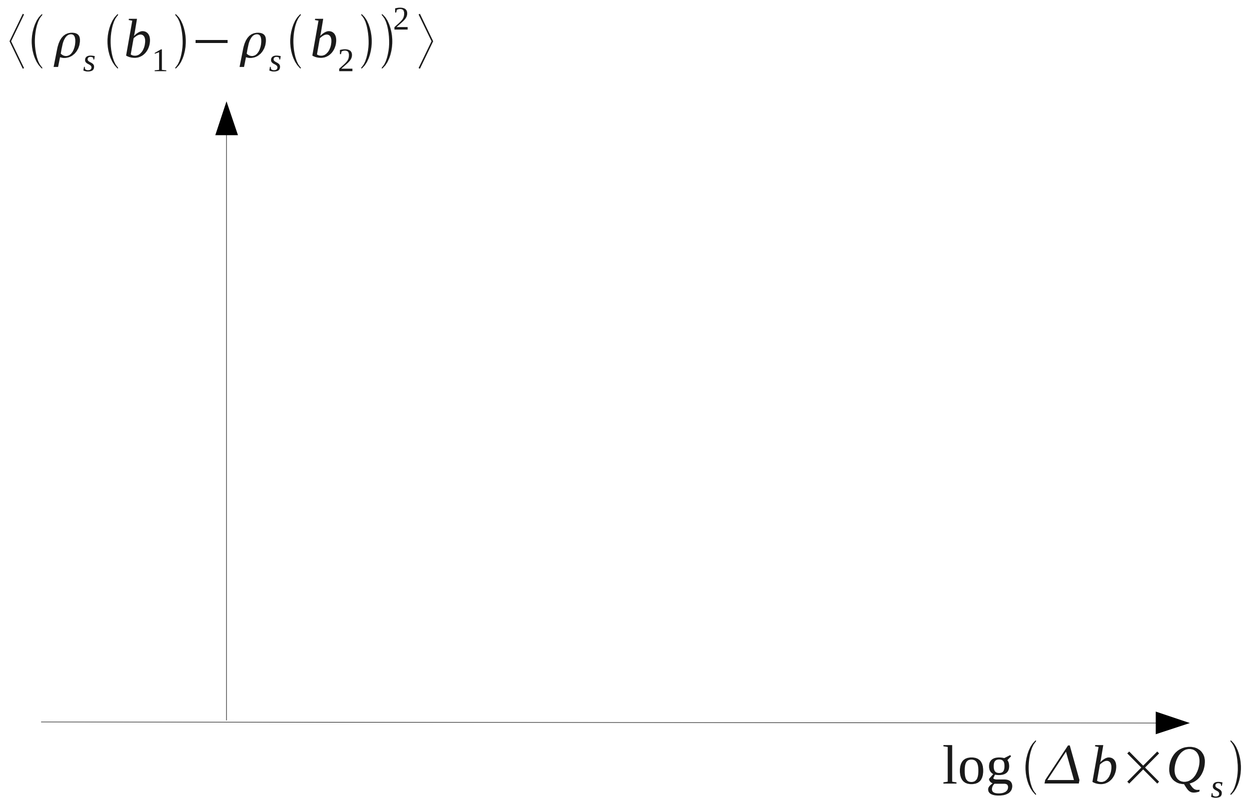

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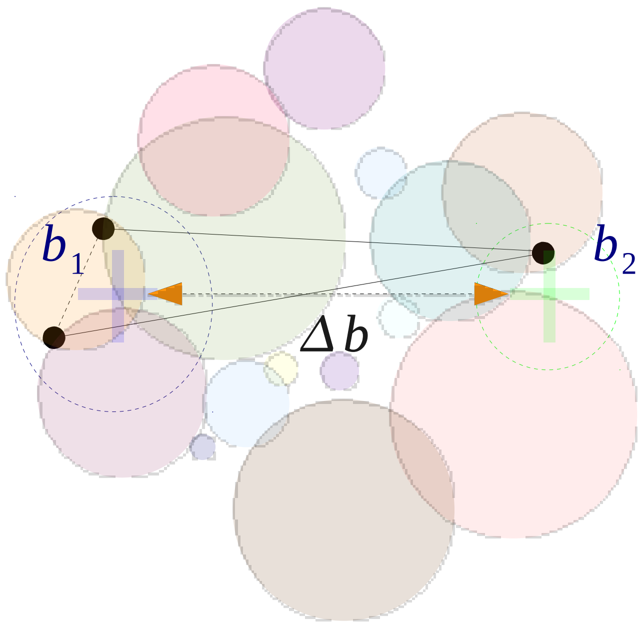

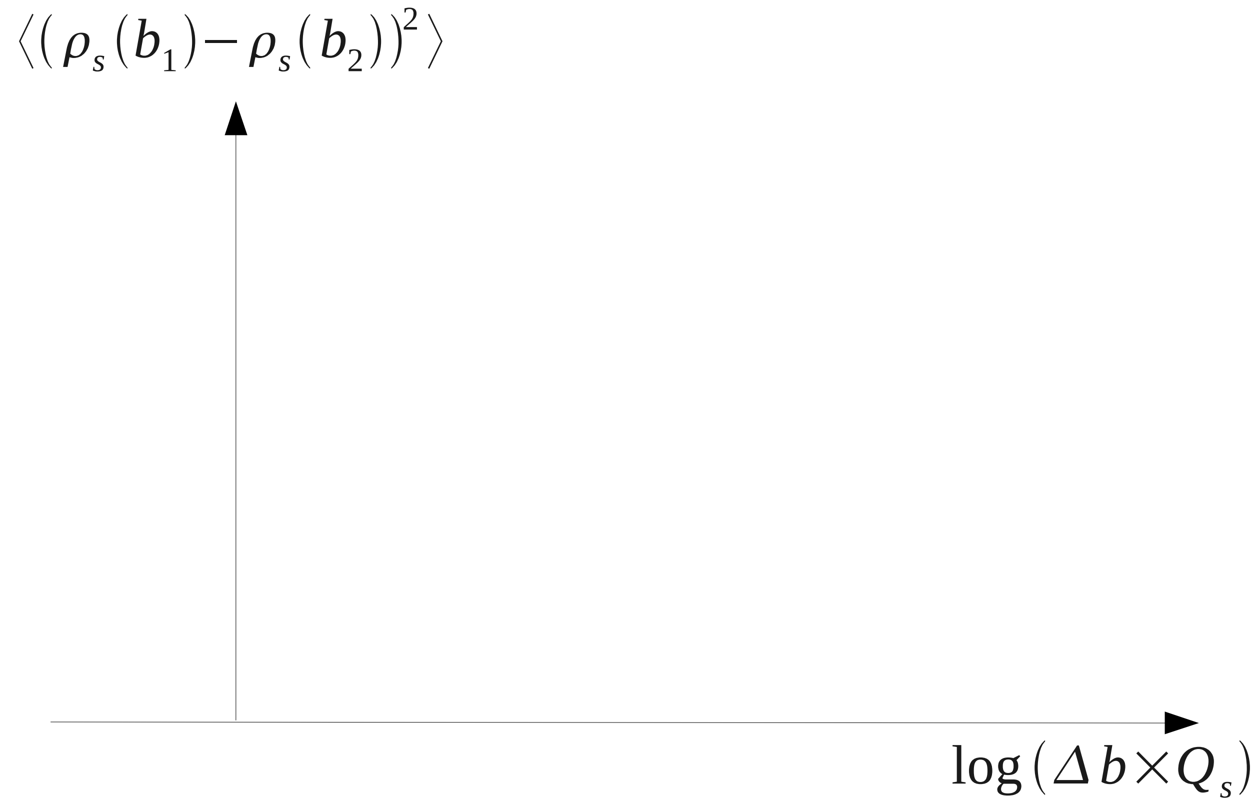

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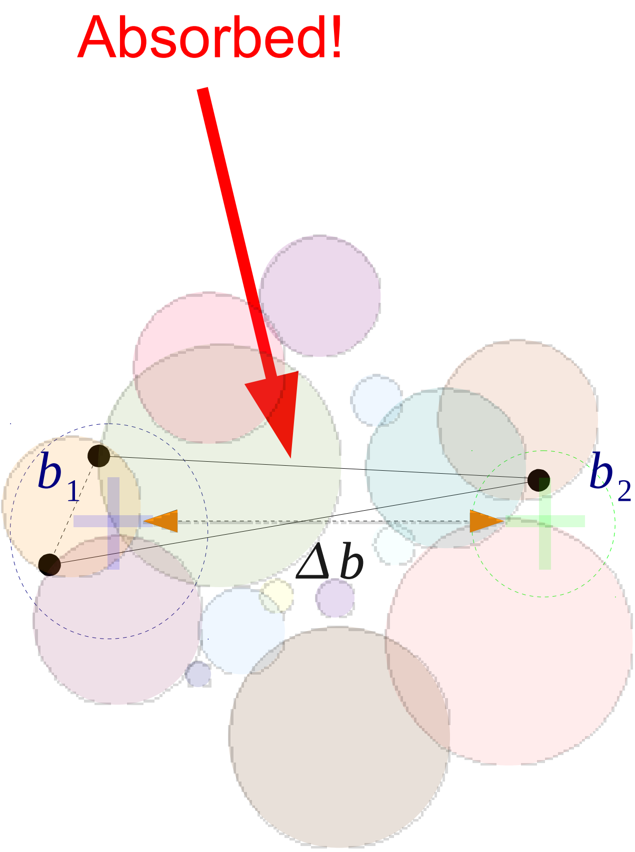
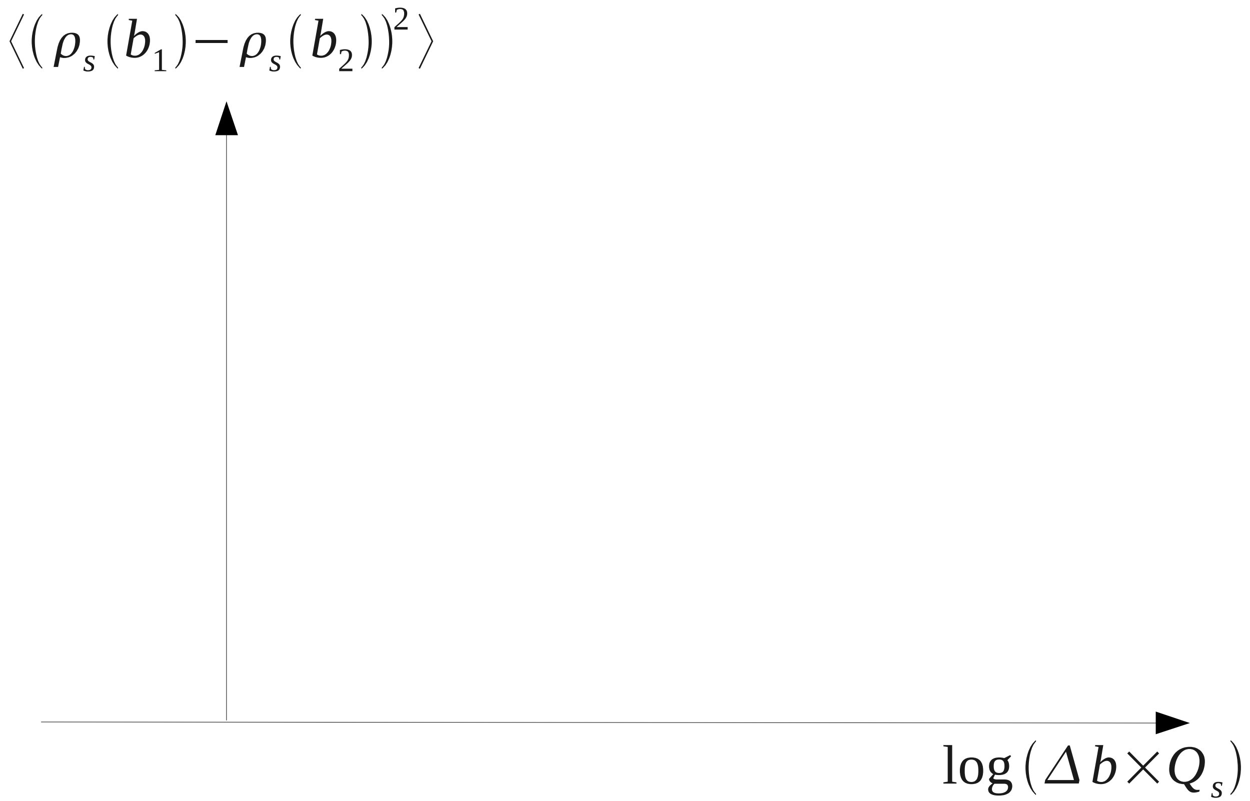
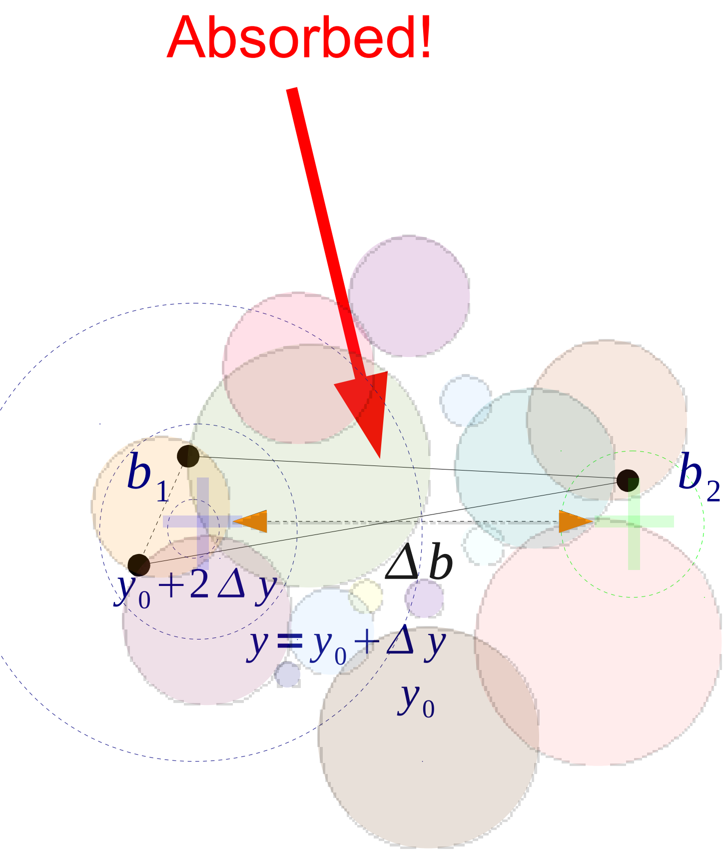
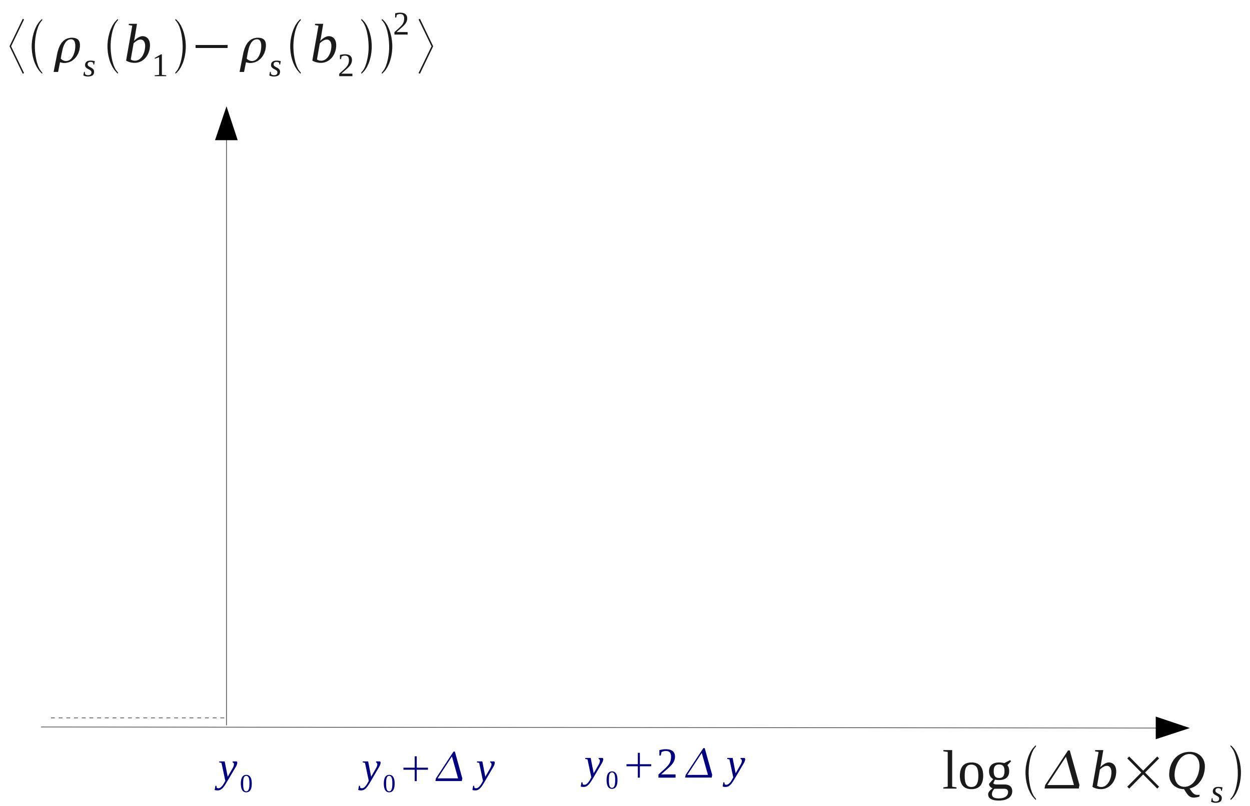




Expect decoupling as soon as $\Delta b > 1/Q_s$

How correlations may occur





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Assume it occurs at rapidity y_0
i.e. up to rapidity y_0 , same ρ_s at b_1 and b_2

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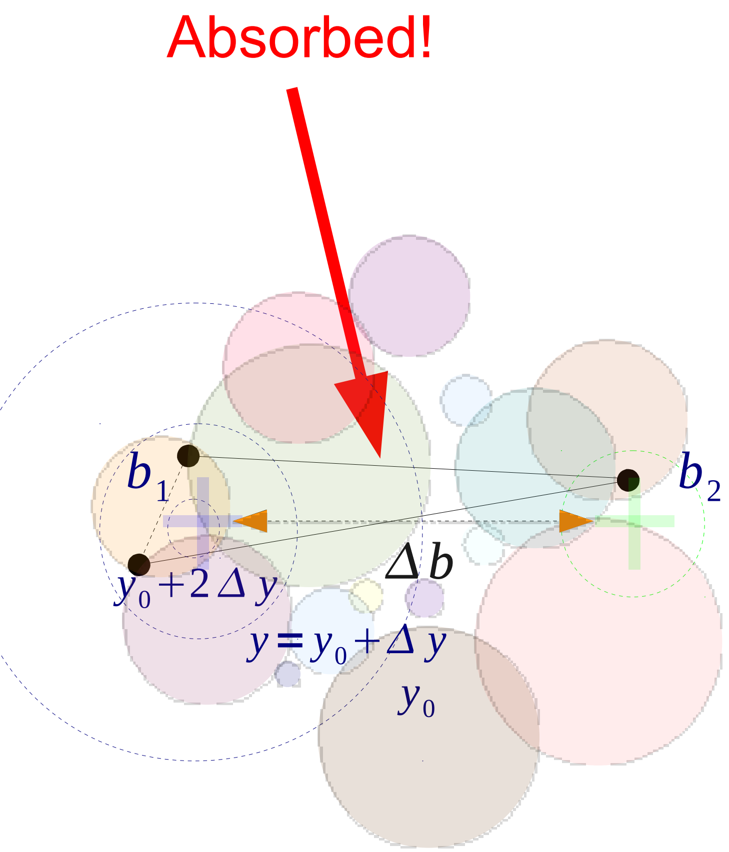

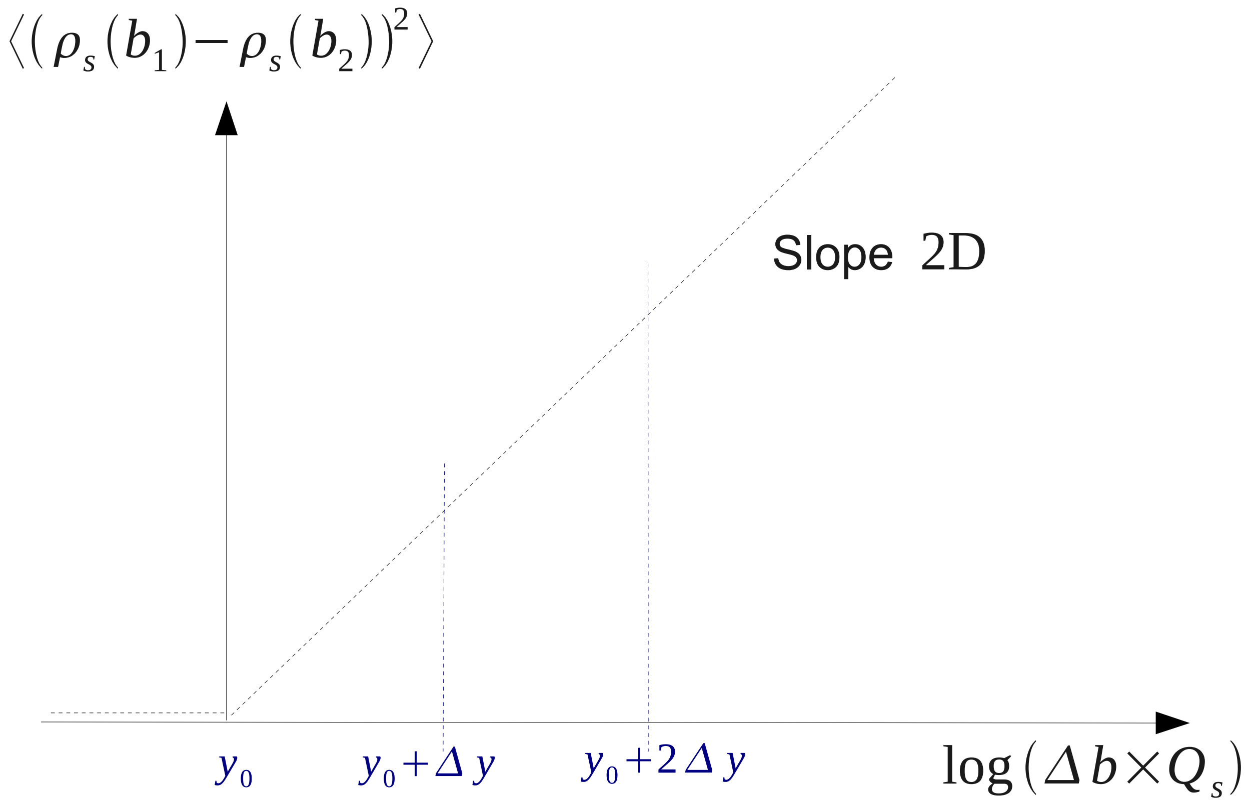

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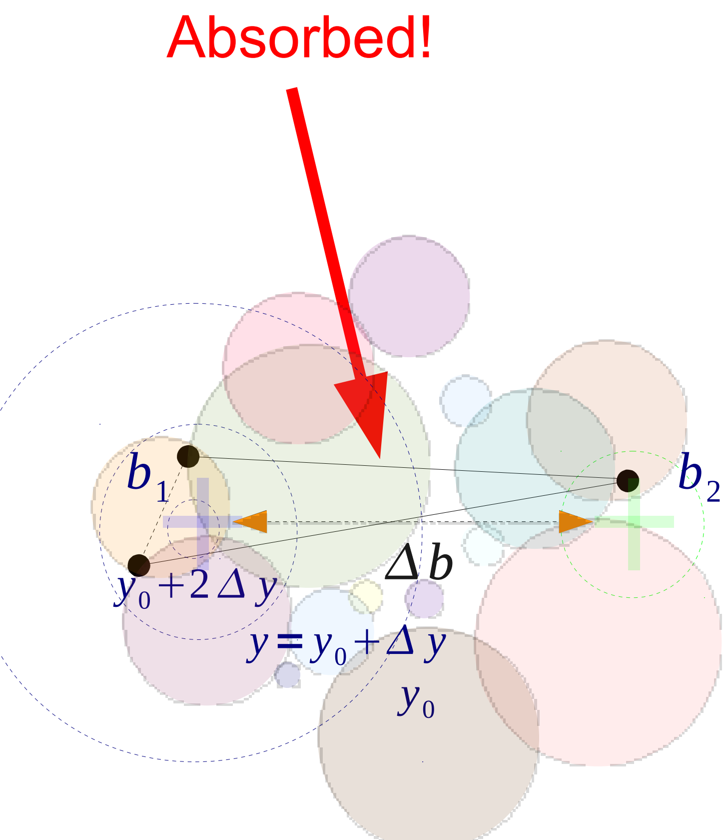
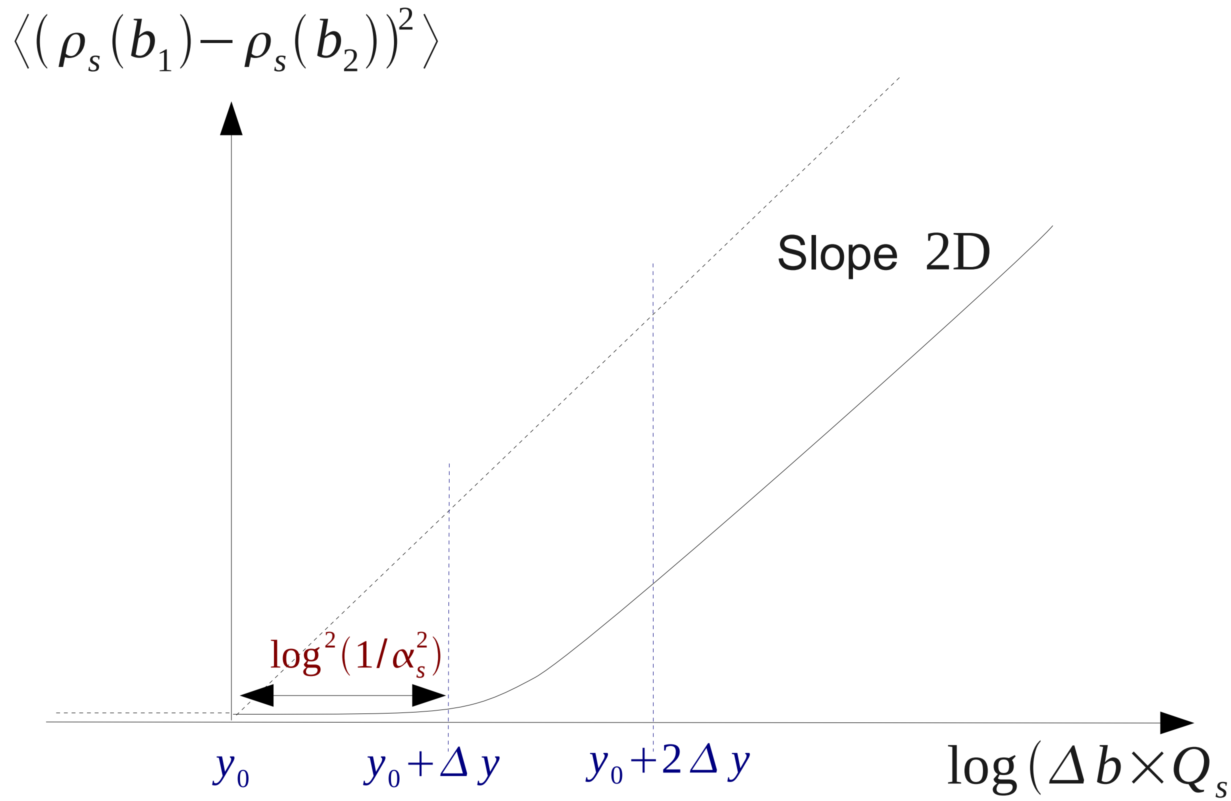
Since the saturation radius decreases exponentially with the rapidity,
the variance of the difference $\rho_s(b_1)-\rho_s(b_2)$ should grow linearly with $\log(\Delta b\times Q_s)$

How correlations may occur

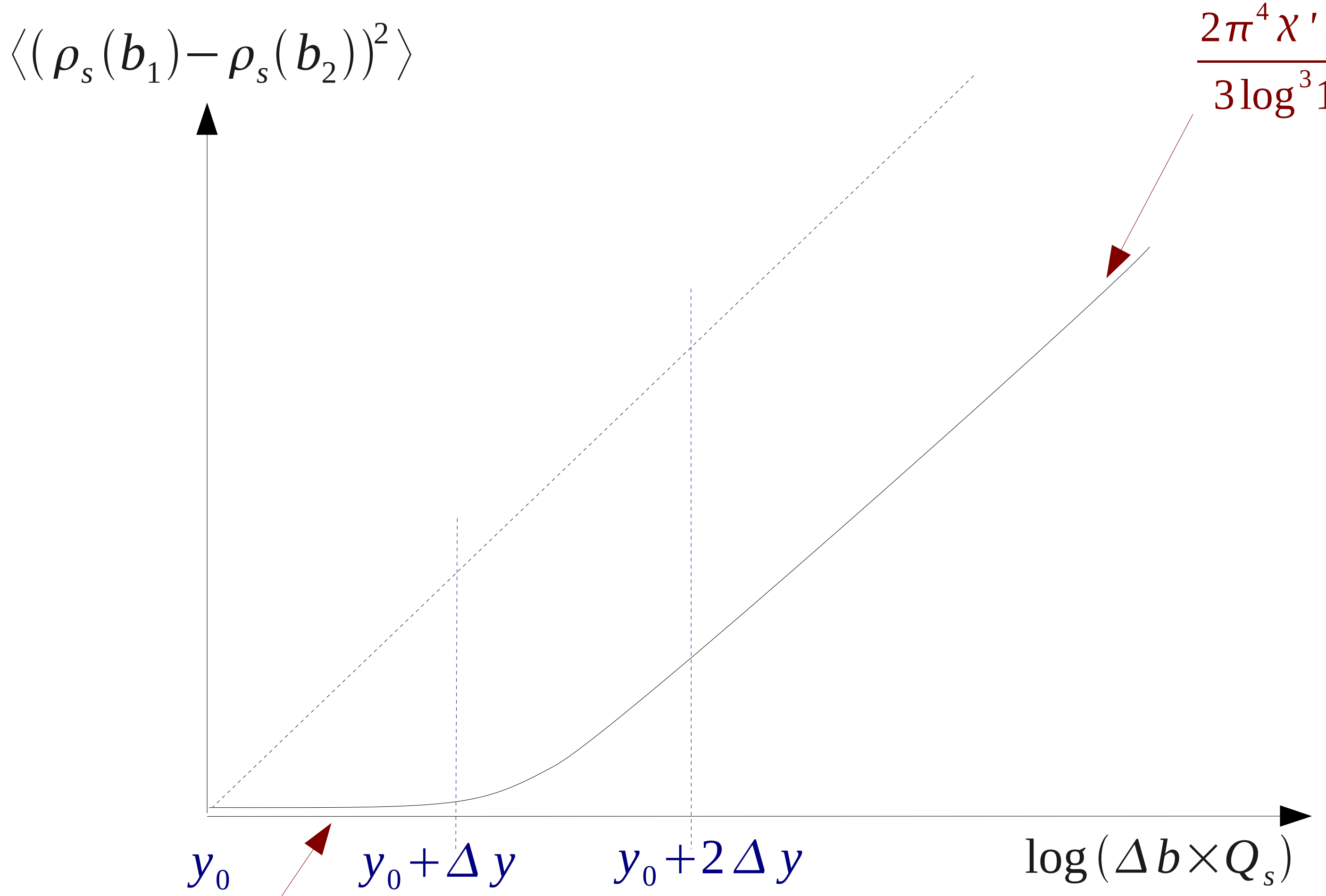




But when decoupling occurs, the fluctuations need $\Delta y \sim \log^2(1/\alpha_s^2)$ units of rapidity to affect the saturation scale

Thus correlations effectively persist for all $\Delta b < \exp\left[\log^2(1/\alpha_s^2)\right]/Q_s$

Quantitative results

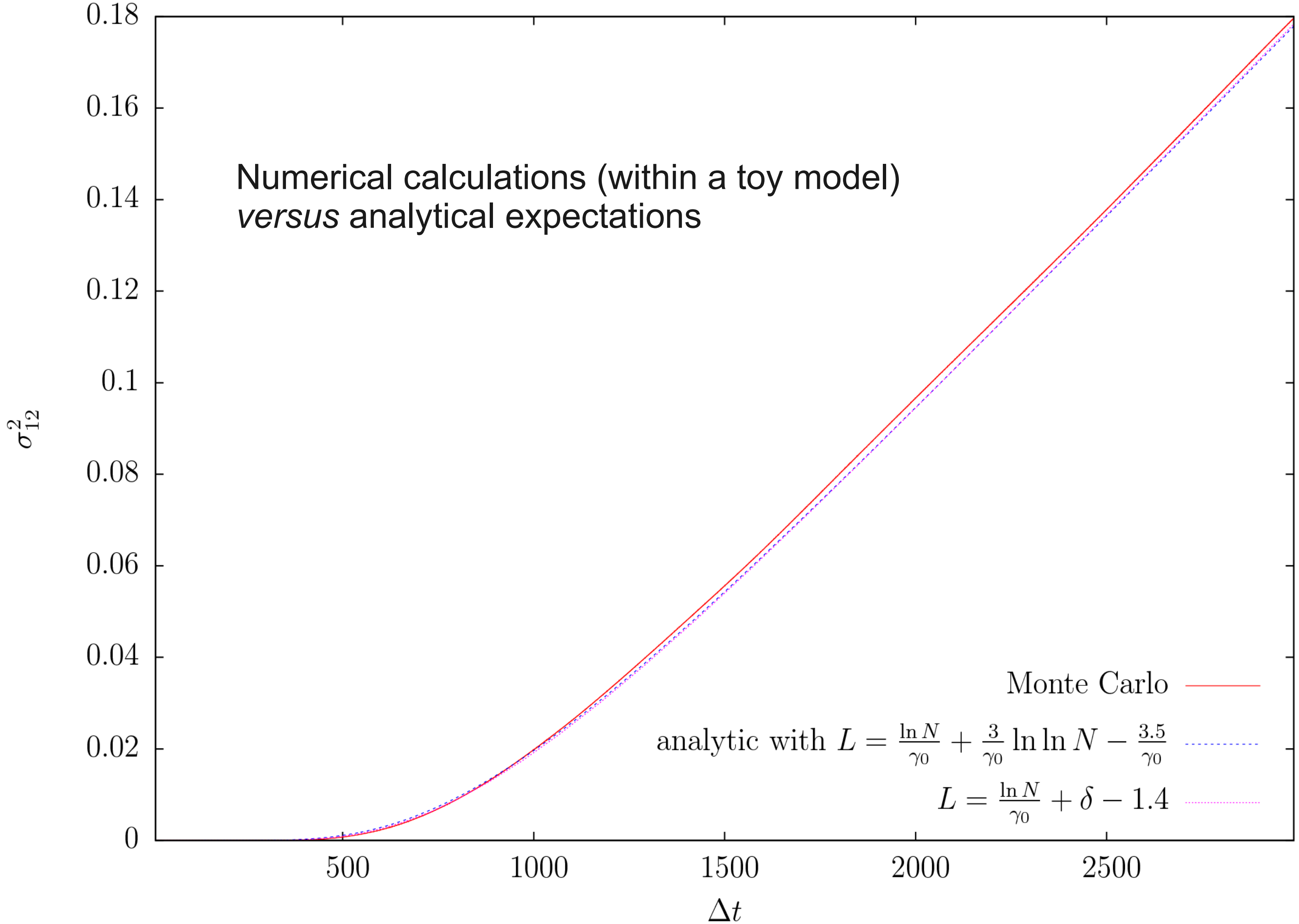


$$\frac{2\pi^4 \chi''(\gamma_0)}{3 \log^3 1/\alpha_s^2} \log(\Delta b Q_s)$$

$$\frac{4}{3\gamma_0^3} \sqrt{\frac{2\pi^3}{\chi''(\gamma_0) \log(\Delta b Q_s)}} \exp\left(-\frac{\log^2 1/\alpha_s^2}{2\gamma_0 \chi''(\gamma_0) \log(\Delta b Q_s)}\right)$$

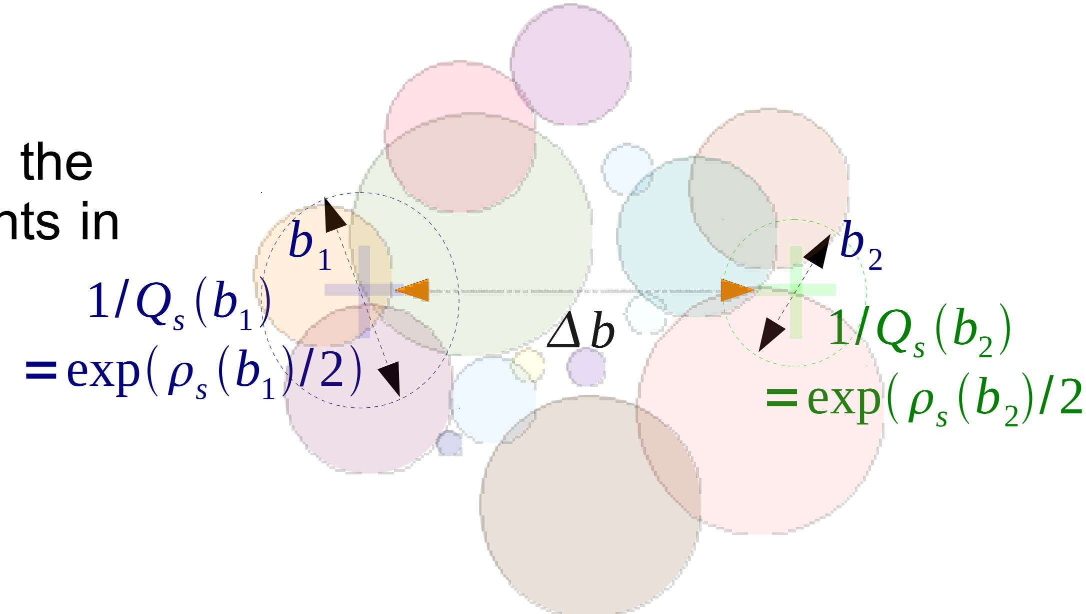
Quantitative results

$N = 10^{50}$ (Very small strong coupling)



Conclusions and outlook

We have studied the correlations of the saturation scale between different points in impact-parameter space



A naïve estimate would be that the correlations vanish over the length $\Delta b \sim 1/Q_s$

We have found that correlations effectively persist for all $\Delta b < \exp\left[\log^2(1/\alpha_s^2)\right]/Q_s$
which means that the density of partons is homogenous over quite large distances

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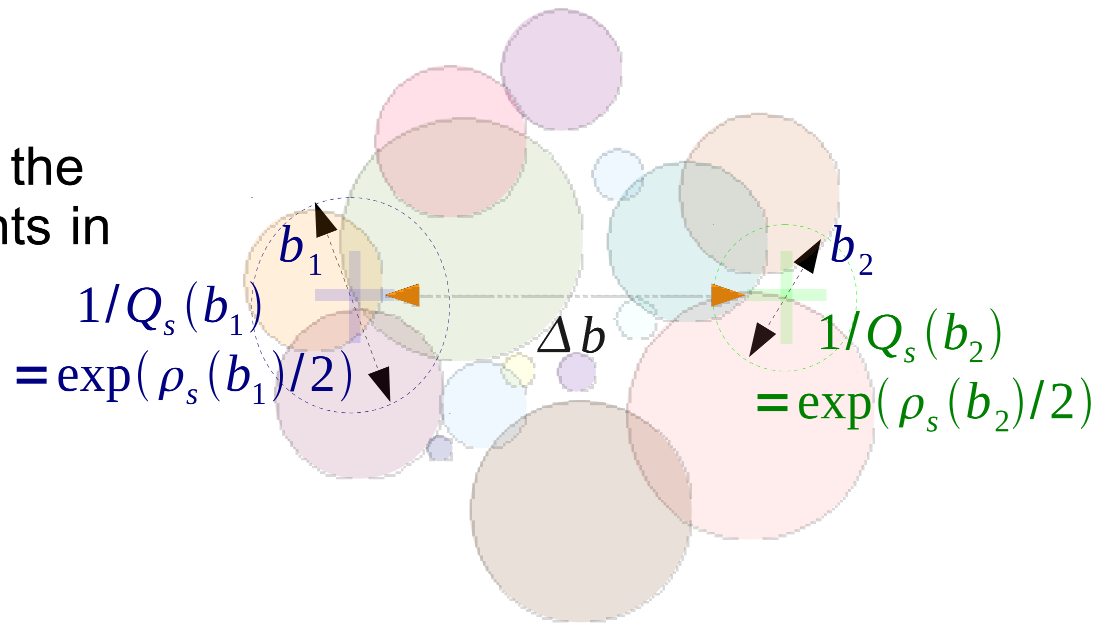

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Analytical results for smaller values of the coupling constant?

How would this new scale manifest itself in the phenomenology?
(diffraction in DIS, heavy-ion collision)