Angular momentum sum rule in nuclei

SWADHIN TANEJA
(STONY BROOK UNIVERSITY)

STONY BROOK UNIVERSITY

XVIII International Workshop on Deep-Inelastic Scattering and Related Subjects
Convitto della Calza, Firenze, 19th - 23rd April 2010

Outline

- QCD decomposition of proton spin
- Gauge invariance
- DVCS and Generalized Parton Distributions
- Nucleon GPDs and Spin Sum Rule
- Deuteron GPDs and Spin Sum Rule for spin 1
- Conclusions

QCD decomposition of proton spin

- Energy momentum density tensor, $T^{\mu\nu}$

  T^{00} : Energy density, E

  T^{0i} : Momentum Density, P^i

- Angular momentum,

$$J_{q,g}(\mu) = \left\langle P\frac{1}{2}\middle| \int d^3x (\vec{x} \times \vec{T}_{q,g})^z \middle| P\frac{1}{2} \right\rangle$$

- Angular momentum density,

$$M^{\mu\nu\alpha} = T^{\mu\alpha}x^\nu - T^{\mu\nu}x^\alpha$$

QCD decomposition of proton spin

- In QCD the energy momentum tensor :

$$T^{\alpha\beta} = T_q^{\alpha\beta} + T_g^{\alpha\beta} = \frac{1}{4}\bar{\psi}\gamma^{(\alpha}i\overleftrightarrow{D}^{\beta)}\psi + \left(\frac{1}{4}g^{\alpha\beta}F^2 - F^{\alpha\mu}F^{\beta}_{\ \mu}\right)$$

- QCD angular momentum (Jaffe):

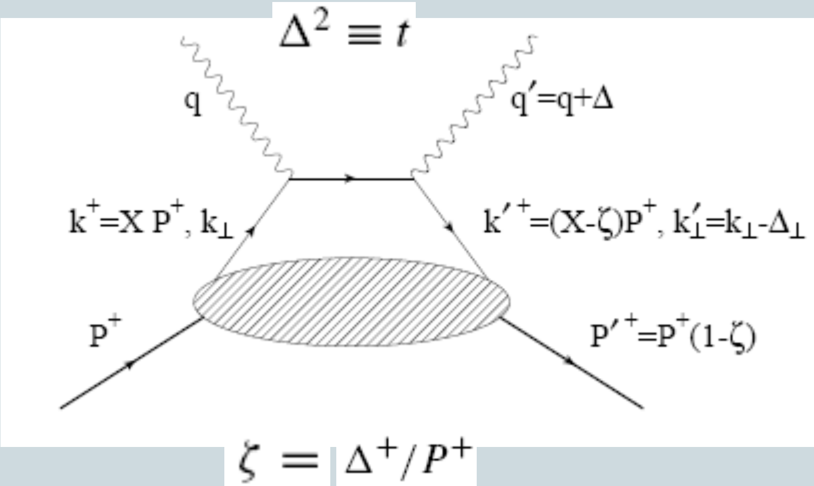
$$\begin{aligned}\vec{J}_{QCD} &= \int d^3x\psi^\dagger\frac{1}{2}\vec{\Sigma}\psi + \int d^3x\psi^\dagger\vec{x}\times\frac{1}{i}\vec{\nabla}\psi \\ &+ \int d^3x\vec{E}^a\times\vec{A}^a + \int d^3x E^{ai}\vec{x}\times\vec{\nabla}A^{ai} \\ &\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g,\end{aligned}$$

Gauge invariance

- By adding a term $$\int d^3x \vec{\nabla} \cdot [\vec{E}^a(\vec{A}^a \times \vec{x})],$$
- Gauge invariant decomposition (used by X. Ji)

$$\begin{aligned}
\vec{J}_{QCD} &= \int d^3x \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \psi^\dagger \vec{x} \times \frac{1}{i}\vec{\mathbb{D}}\psi \\
&+ \int d^3x \vec{x} \times (\vec{E}^a \times \vec{B}^a) \\
&\equiv \vec{S}_q + \vec{L}'_q + \vec{J}'_g.
\end{aligned}$$

DVCS and Nucleon Generalized Parton Distributions



$$\bar{P}^+ \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P', S' | \bar{\psi}(\frac{-z}{2}) \gamma^+ \psi(\frac{z}{2}) | P, S \rangle$$

$$= \bar{u}(P', S') \left\{ \gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\nu}\Delta_\nu}{2M} E(x,\xi,t) \right\} u(P,S)$$

- GPDs are hybrids of PDFs and FFs: describes simultaneously x and t dependences !!

- GPDs give access to spatial d.o.f of partons !

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}.$$

- GPDs give access to orbital angular momentum of partons !

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \left\langle P{+}\Delta,\uparrow \left| \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| P,\downarrow \right\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

Nucleon – Angular momentum Sum rule

- Energy momentum tensor from symmetries

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\Big[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(t)\Delta^{(\mu}\Delta^{\nu)}/M\Big]U(P)$$

- Angular momentum

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$$

- Spin sum rule nucleon (X. Ji)

$$\int dx x\left[H_q(x,\zeta,t=0) + E_q(x,\zeta,t=0)\right] = 2J_q \qquad \text{X. Ji}$$

Deuteron Generalized Parton Distributions

$$\frac{1}{2}\int \frac{d\lambda}{2\pi}\, e^{ix(Pz)} \langle p' |\, \bar{q}(-\tfrac{1}{2}z)\, \not{n}_-\, q(\tfrac{1}{2}z)\, |p\rangle \Big|_{z=\lambda n_-} = -(\epsilon'^*\epsilon)\, H_1$$
$$+ \frac{(\epsilon n_-)(\epsilon'^* P) + (\epsilon'^* n_-)(\epsilon P)}{P n_-} H_2 - \frac{2(\epsilon P)(\epsilon'^* P)}{m^2} H_3$$
$$+ \frac{(\epsilon n_-)(\epsilon'^* P) - (\epsilon'^* n_-)(\epsilon P)}{P n_-} H_4$$
$$+ \left[m^2 \frac{(\epsilon n_-)(\epsilon'^* n_-)}{(P n_-)^2} + \frac{1}{3}(\epsilon'^*\epsilon) \right] H_5 \,,$$

- Off forward matrix element for spin 1 target is written in terms of five unpolarized GPDs.

Berger, Cano, Diehl and Pire (2001)

- Form Factors from GPDs H_1 , H_2 and H_3 are related to charge, magnetic moment and quadrupole moment.

Form Factors $\int_{-1}^{1} dx\, H_i(x,\xi,t) = G_i(t) \qquad (i = 1,2,3).$

$$G_C = G_1 + \frac{2}{3}\eta\, G_Q,$$
$$G_Q = G_1 - G_2 + (1+\eta) G_3,$$
$$G_M = G_2$$

Energy momentum tensor – Spin 1

- Energy momentum tensor from symmetries

$$\begin{aligned}\langle p'|\theta^{\mu\nu}|p\rangle = & - \frac{1}{2}\left[P^\mu P^\nu\right](\epsilon'^*\epsilon)G_{1,2}(t) - \frac{1}{4}\left[P^\mu P^\nu\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}G_{2,2}(t) \qquad (1)\\ & - \frac{1}{2}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right](\epsilon'^*\epsilon)G_{3,2}(t) - \frac{1}{4}\left[\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right]\frac{(\epsilon P)(\epsilon'^* P)}{M^2}G_{4,2}(t)\\ & + \frac{1}{4}\left[\left(\epsilon'^{*\mu}(\epsilon P) + \epsilon^\mu(\epsilon'^* P)\right)P^\nu + \mu\leftrightarrow\nu\right]G_{5,2}(t)\\ & + \left[\left(\epsilon'^{*\mu}(\epsilon P) - \epsilon^\mu(\epsilon'^* P)\right)\Delta^\nu + \mu\leftrightarrow\nu + 2g^{\mu\nu}(\epsilon P)(\epsilon'^* P) - \left(\epsilon'^{*\mu}\epsilon^\nu + \epsilon'^{*\nu}\epsilon^\mu\right)\Delta^2\right]G_{6,2}(t)\end{aligned}$$

- Relations with deuteron GPDs

$$\begin{aligned}\int dx\, x H_1(x,\xi,t) - \frac{1}{3}\int dx\, x H_5(x,\xi,t) &= G_{1,2}(t) + \xi^2 G_{3,2}(t)\\ \int dx\, x H_2(x,\xi,t) &= G_{5,2}(t)\\ \int dx\, x H_3(x,\xi,t) &= G_{2,2}(t) + \xi^2 G_{4,2}(t)\\ \frac{1}{4\xi}\int dx\, x H_4(x,\xi,t) &= \frac{M^2}{-t}\int dx\, x H_5(x,\xi,t) = G_{6,2}(t)\end{aligned}$$

Deuteron – Angular momentum sum rule

.. inserting the Energy Momentum tensor in $\langle p'| \int d^3x (\vec{x} \times \vec{T}^{0i}_{q,g})_z |p\rangle$

$$J_q = \frac{1}{2} \int dX\, X H_2^q(X,0,0) \equiv \frac{1}{2} G_{5,2}(0)$$

S. Liuti, S. Taneja

Quick note: For spin ½ case ,

$$\int dx\, x\, [H_q(x,\zeta,t=0) + E_q(x,\zeta,t=0)] = 2 J_q$$

$G_M(t=0)$

Conclusions

- Understanding the richness of the nucleon and the nuclear structure continues to be our fundamental goal.
- Generalized Parton Distributions offer a new insight not only into the spatial structure, but also the angular momentum composition of nucleons and nuclei.
- We derived a spin sum rule for spin 1 system in terms of GPDs. We realized that the contribution to the spin comes from the same GPD that contributes to the magnetic moment of the system.