The link between TMDs and PDFs in covariant quark-parton model with orbital motion

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(based on collaboration and discussions with A.Efremov, P.Schweitzer and O.Teryaev)

Outline of talk

- 3D covariant parton model
  - *general remarks*
  - *short overview of previous results*
- Images of quark intrinsic motion
  - *quark OAM*
  - *relations for unpolarized and polarized TMDs*
  - *Numerical predictions on some TMDs*
- Summary & conclusion

3D covariant parton model

- We work with a ‘naive’ 3D parton model, which is based on covariant kinematics and rotational symmetry of parton momenta in nucleon rest frame - for details see published papers
- It appears that main potential is implication of some old and new sum rules and relations among PDF’s and TMDs.
- Illustration…

Sum rules

Obtained structure functions for m→0

obey the known sum rules:

$$g_2(x) = -g_1(x) + \int_x^1 \frac{g_1(y)}{y} dy,$$

which is **Wanzura - Wilczek twist-2 term** for g_2 approximation.

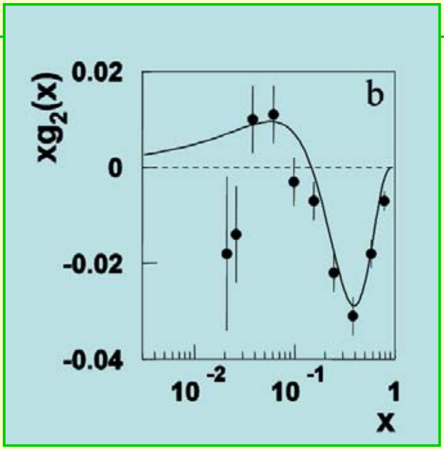
$$\int_0^1 x^\alpha \left[\frac{\alpha}{\alpha+1} g_1(x) + g_2(x) \right] dx = 0$$

For $\alpha = 2, 4, 6, \ldots$ relation corresponds to the **Wanzura - Wilczek sum rules**. Other special cases correspond to the **Burkhardt - Cottingham** ($\alpha = 0$) and the **Efremov - Leader - Teryaev** (ELT, $\alpha = 1$) sum rules.

[P.Z. Phys.Rev.D65, 054040(2002)]

- In this talk m→0 is assumed.

Relations among PDF's



E155 experiment

1. Assuming SU(6), we obtained relations

$$f_1(x) \leftrightarrow g_{1q}(x),\ g_{2q}(x)$$

[P.Z. Phys.Rev. D67, 014019(2003)].

2. **helicity $\leftrightarrow$ transversity**

[A.Efremov, O.Teryaev and P.Z., Phys.Rev.D70, 054018(2004)]

$$\boxed{\delta q(x) = \Delta q(x) + \Delta q_T(x) = \Delta q(x) + \int_x^1 \frac{\Delta q(y)}{y} dy}$$

$$\Rightarrow \quad \boxed{\int_0^1 \delta q(x) dx = 2 \int_0^1 \Delta q(x) dx}$$

3. *helicity ↔ transversity ↔ pretzelosity*

[A.Efremov, P.Schweitzer, O.Teryaev and P.Z.; arXiv:0812.3246]

$$h_{1T}^{\perp(1)}(x) = \Delta q(x) - \delta q(x) = -\Delta q_T(x) = -\int_x^1 \frac{\Delta q(y)}{y} dy$$

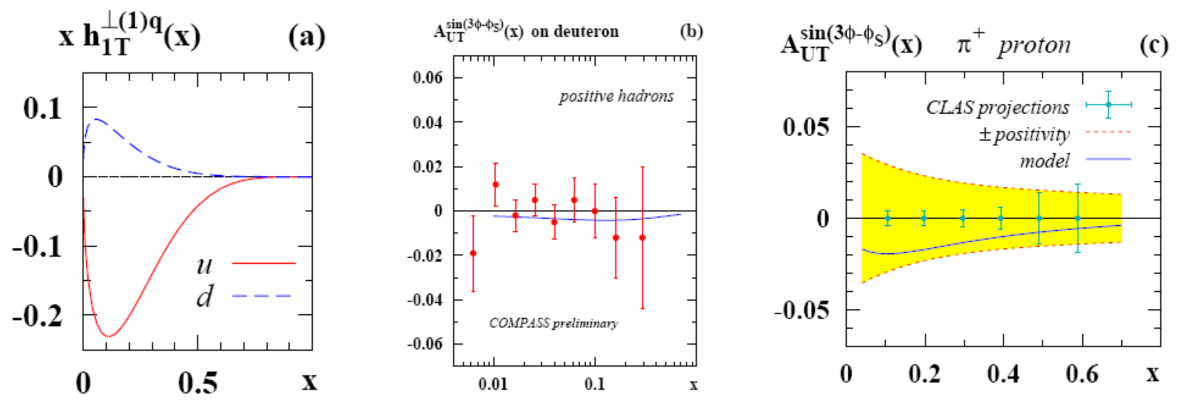


FIGURE 1. (a) $x h_{1T}^{\perp(1)q}(x)$ vs. x from the present approach. (b) $A_{UT}^{\sin(3\phi-\phi_S)}$ as function of x computed with pretzelosity from Fig. 1a for positive hadrons from a deuterium target at COMPASS in comparison to preliminary data [14]. (c) The same as Fig. 1b but for π^+ from proton target at CLAS. The shaded area is the region allowed by positivity [26], the error projections are from [40].

4. *TMDs and relations among them…*

[A.Efremov, P.Schweitzer, O.Teryaev and P.Z. Phys.Rev.D80, 014021(2009)]

…some further implications will be presented in this talk

Images of quark intrinsic motion

Image 1: quark OAM

- Total angular momentum consists of ***j=l+s***.
- In relativistic case ***l,s*** are not conserved separately, only ***j*** is conserved. So, we can have pure states of ***j*** (j^2, j_z) only, which are represented by the bispinor spherical waves:

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{2p_0}} \begin{pmatrix} i^{-l}\sqrt{p_0+m}\,\Omega_{jlj_z}(\omega) \\ i^{-\lambda}\sqrt{p_0-m}\,\Omega_{j\lambda j_z}(\omega) \end{pmatrix},$$

where $\omega = \mathbf{p}/p$, $l = j \pm \frac{1}{2}$, $\lambda = 2j - l$ (l defines the parity) and

$$\Omega_{j,l,j_z}(\omega) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}\, Y_{l,j_z-1/2}(\omega) \\ \sqrt{\frac{j-j_z}{2j}}\, Y_{l,j_z+1/2}(\omega) \end{pmatrix}; \quad l = j - \frac{1}{2},$$

$$\Omega_{j,l,j_z}(\omega) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}\, Y_{l,j_z-1/2}(\omega) \\ \sqrt{\frac{j+j_z+1}{2j+2}}\, Y_{l,j_z+1/2}(\omega) \end{pmatrix}; \quad l = j + \frac{1}{2}.$$

[P.Z. Eur.Phys.J. C52, 121 (2007)]

j=1/2

For $j = j_z = 1/2$ and $l = 0$:

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = i\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{11} = -i\sqrt{\frac{3}{8\pi}}\sin\theta\exp(i\varphi),$$

$$\psi_{kjl j_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{8\pi p_0}}\begin{pmatrix} \sqrt{p_0+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\sqrt{p_0-m}\begin{pmatrix} \cos\theta \\ \sin\theta\exp(i\varphi) \end{pmatrix} \end{pmatrix}.$$

For the superposition

$$\Psi(\mathbf{p}) = \int a_k \psi_{kjl j_z}(\mathbf{p})dk; \qquad \int a_k^{\star} a_k dk = 1$$

the average spin contribution to the total angular momentum is calculated as

$$\langle s \rangle = \int \Psi^{\dagger}(\mathbf{p})\Sigma_z\Psi(\mathbf{p})d^3p; \qquad \Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma_z & \cdot \\ \cdot & \sigma_z \end{pmatrix}.$$

Spin and orbital motion

$$\langle s\rangle = \int a_p^\star a_p \frac{(p_0+m)+(p_0-m)\left(\cos^2\theta-\sin^2\theta\right)}{16\pi p^2 p_0} d^3p$$
$$= \frac{1}{2}\int a_p^\star a_p\left(\frac{1}{3}+\frac{2m}{3p_0}\right)dp.$$

$$\langle l\rangle = \frac{1}{3}\int a_p^\star a_p\left(1-\frac{m}{p_0}\right)dp, \qquad \langle l\rangle + \langle s\rangle = 1/2$$

In chiral limit:

$$m \ll p_0 \Rightarrow \langle s\rangle = 1/6, \qquad \langle l\rangle = 1/3$$

Spin and orbital motion from PDF's

$$s^q = \int g_1^q(x)\,dx$$

$$l^q = -\int h_{1T}^{\perp(1)q}(x)\,dx$$

H. Avakian, A. V. Efremov, P. Schweitzer and F. Yuan
arXiv:1001.5467[hep-ph]

J. She, J. Zhu and B. Q. Ma
Phys. Rev. D 79 (2009) 054008.

Our model:

$$\begin{aligned} \int g_1^q(x)\,dx &= \frac{1}{2}\int \Delta G_q(p_0)\left(\frac{1}{3}+\frac{2m}{3p_0}\right)d^3p \\ -\int h_{1T}^{\perp(1)q}(x)\,dx &= \frac{1}{3}\int \Delta G_q(p_0)\left(1-\frac{m}{p_0}\right)d^3p \end{aligned}$$

Two pictures:

1. wavefunctions (bispinor spherical waves) & operators

$\langle s^q \rangle$	$\langle l^q \rangle$
$\frac{1}{2}\int a_p^* a_p \left(\frac{1}{3}+\frac{2m}{3p_0}\right) dp$	$\frac{1}{3}\int a_p^* a_p \left(1-\frac{m}{p_0}\right) dp$

2. probabilistic distributions & structure functions (in our model)

$\int g_1^q(x)\,dx$	$-\int h_{1T}^{\perp(1)q}(x)\,dx$
$\frac{1}{2}\int \Delta G_q(p_0)\left(\frac{1}{3}+\frac{2m}{3p_0}\right) d^3p$	$\frac{1}{3}\int \Delta G_q(p_0)\left(1-\frac{m}{p_0}\right) d^3p$

$$\boxed{a_p^* a_p \leftrightarrows \Delta G_q(p_0) = G_q^+(p_0) - G_q^-(p_0)}$$

$\Rightarrow$ ***Also in our model OAM can be identified with pretzelosity!***

Application of the model for calculation TMDs

Definition in terms of light-front correlators:

$$\phi(x,\vec{p}_T)_{ij} = \int \frac{\mathrm{d}z^-\mathrm{d}^2\vec{z}_T}{(2\pi)^3}\, e^{ipz}\, \langle N(P,S)|\bar{\psi}_j(0)\,\mathcal{W}(0,z,\mathrm{path})\,\psi_i(z)|N(P,S)\rangle\Big|_{z^+=0,\,p^+=xP^+}$$

$$\frac{1}{2}\,\mathrm{tr}\Big[\gamma^+\,\phi(x,\vec{p}_T)\Big] = f_1 - \frac{\varepsilon^{jk}p_T^j S_T^k}{M_N}\,f_{1T}^{\perp}$$

$$\frac{1}{2}\,\mathrm{tr}\Big[\gamma^+\gamma_5\,\phi(x,\vec{p}_T)\Big] = S_L\,g_1 + \frac{\vec{p}_T\cdot\vec{S}_T}{M_N}\,g_{1T}^{\perp}$$

For details see our quoted TMDs paper and citations therein

In this talk we shall discuss $f_1\,(x,p_T)$ and $g_1\,(x,p_T)$

Image 2: integrated unpolarized distribution function

$$f_1^q(x) = Mx \int G_q(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{dp_1 d^2\mathbf{p}_T}{p_0}$$

Input: probability $G_q(p)$
Nucleon rest frame:
rotational symmetry, one variable
(pP/M in covariant version)

$$p \equiv p_0 = \sqrt{p_1^2 + p_T^2}$$

Integral represents mapping:
(one variable → one variable)

$$G_q \longrightarrow f_1^q$$

Due to special form of integral
there exists inverse mapping:

$$f_1^q \longrightarrow G_q$$

Solution:

$$G_q(p) = -\frac{1}{\pi M^3}\left(\frac{f_1^q(x)}{x}\right)'; \qquad x = \frac{2p}{M}$$

For proof and related results see PZ, Eur.Phys.J. C52, 121 (2007)

⇒

$G_q(p)$, $f_1^q(x)$ *- are equivalent descriptions*

Input for f_1^q:
MRST LO at *4 GeV²*

$$P_q(p) = 4\pi p^2 M G_q(p)$$

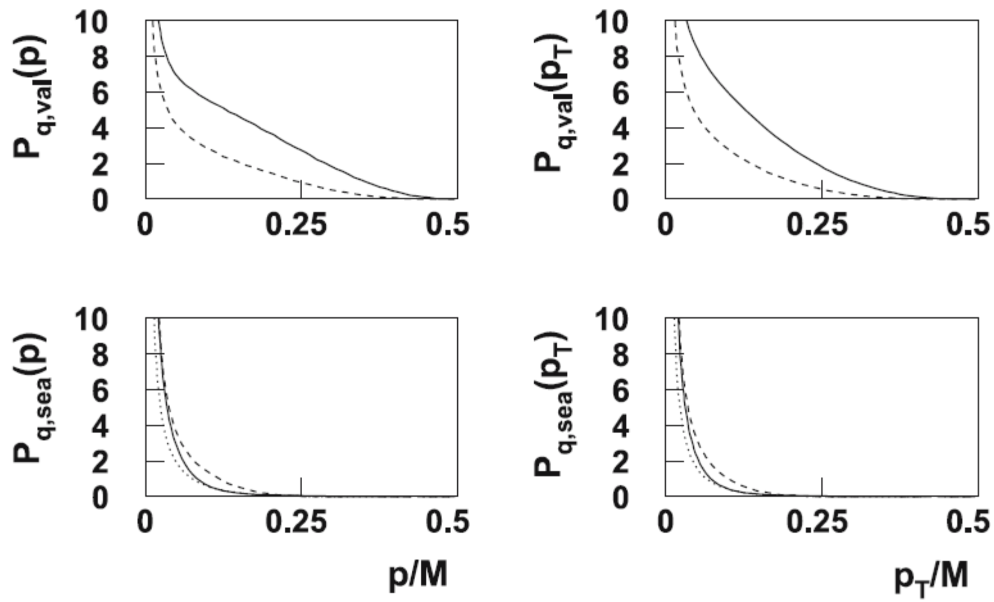


Fig. 1. The quark momentum distributions in the rest frame of the proton: the p and $p_{\rm T}$ distributions for valence quarks $P_{q,\rm val} = P_q - P_{\bar q}$ and sea quarks $P_{\bar q}$ at $Q^2 = 4\,\rm GeV^2$. Notation: $u, \bar u$ is indicated by a *solid line*, $d, \bar d$ by a *dashed line* and $\bar s$ by a *dotted line*

Calculation of $\langle p\rangle_{q,\rm val}$ gives roughly 0.11 GeV/c for u and 0.083 GeV/c for d quarks. Since $G_q(p)$ has rotational symmetry, the average transversal momentum can be calculated to be $\langle p_{\rm T}\rangle = \pi/4 \cdot \langle p\rangle$.

Our results are well compatible with analysis based on some other approaches:

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1. Covariant parton model

Jackson, Ross, Roberts; Phys.Lett. B 226, 159(1979):

$\langle (k_T/M)^2 \rangle \approx 0.025 \quad \Longrightarrow \quad \boldsymbol{\langle k_T/M \rangle \approx 0.15}$

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2. Statistical models

- very good description of structure functions in a broad kinematical region

Bhalerao, Kelkar, Ram; Phys.Lett. B476, 285 (2000):
Temperature is fixed to ***T=0.06 GeV***

Bourrely, Soffer, Buccela; Phys.Lett. B648, 39 (2007):
Model implies ***T ≈ 0.1 GeV***

Remark: Temperature corresponding to transition from hadronic matter to QGP is estimated to ***T ≈0.17 GeV*** – by QCD on lattice

Image 3: unintegrated unpolarized distribution function

starting point:

$$f_1^q(x) = Mx \int G_q(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{dp_1 d^2\mathbf{p}_T}{p_0}$$

$$p \equiv p_0 = \sqrt{p_1^2 + p_T^2}$$

1. step: δ – function allows integration

we need roots:

$$\frac{p_0+p_1}{M} - x = 0$$

set of variables: p_1, p_T, x – only 2 independent, x depends also on p_T.

Solution – just one root for p_1 :

$$\tilde{p}_1 = \frac{Mx}{2}\left(1-\left(\frac{p_T}{Mx}\right)^2\right) \Rightarrow \tilde{p}_0 = \frac{Mx}{2}\left(1+\left(\frac{p_T}{Mx}\right)^2\right)$$

δ – function term is simplified:

$$\delta\left(\frac{p_0+p_1}{M} - x\right)dp_1 = \frac{\delta(p_1-\tilde{p}_1)dp_1}{\left|\frac{d}{dp_1}\left(\frac{p_0+p_1}{M} - x\right)_{p_1=\tilde{p}_1}\right|} = \frac{\delta(p_1-\tilde{p}_1)dp_1}{x/p_0}$$

Integration:

$$f_1^q(x) = M\int d^2\mathbf{p}_T \int G_q(p_0)\delta\left(p_1 - \tilde{p}_1\right)dp_1 = M\int d^2\mathbf{p}_T G_q(\tilde{p}_0)$$

2. step: definition of ***$f_1^q(x,p_T)$*:**

$$f_1^q(x,\mathbf{p}_T) = MG_q(\tilde{p}_0)$$

$$\tilde{p}_0 = \frac{Mx}{2}\left(1 + \left(\frac{p_T}{Mx}\right)^2\right)$$

Remark:

$f_1^q(x,p_T)$ depends on ***x, p_T*** via ***one*** variable. It is due to fact, that one variable in ***$G_q(p)$*** reflects rotational symmetry in the rest frame. In this way **x, p_T** ***are not*** independent variables. Obviously both functions give equivalent description.

3. step: final result $f_1^q\,(x,p_T)$:

Since we know that

$$f_1^q(x,\mathbf{p}_T) = M G_q\left(\frac{M}{2}\xi\right)$$

$$\xi = x\left(1+\left(\frac{p_T}{Mx}\right)^2\right)$$

and

$$G_q\left(\frac{M}{2}\xi\right) = -\frac{1}{\pi M^3}\left(\frac{f_1^q(\xi)}{\xi}\right)'$$

$$\Rightarrow \qquad f_1^q(x,\mathbf{p}_T) = -\frac{1}{\pi M^2}\left(\frac{f_1^q(\xi)}{\xi}\right)'$$

New rule also follows from covariance + rotational symmetry…

…one can check: $f_1^q(x) = \int f_1^q(x,\mathbf{p}_T)\,d^2\mathbf{p}_T$

Is it really possible to get $f_1(x,p_T)$ from $f_1(x)$?

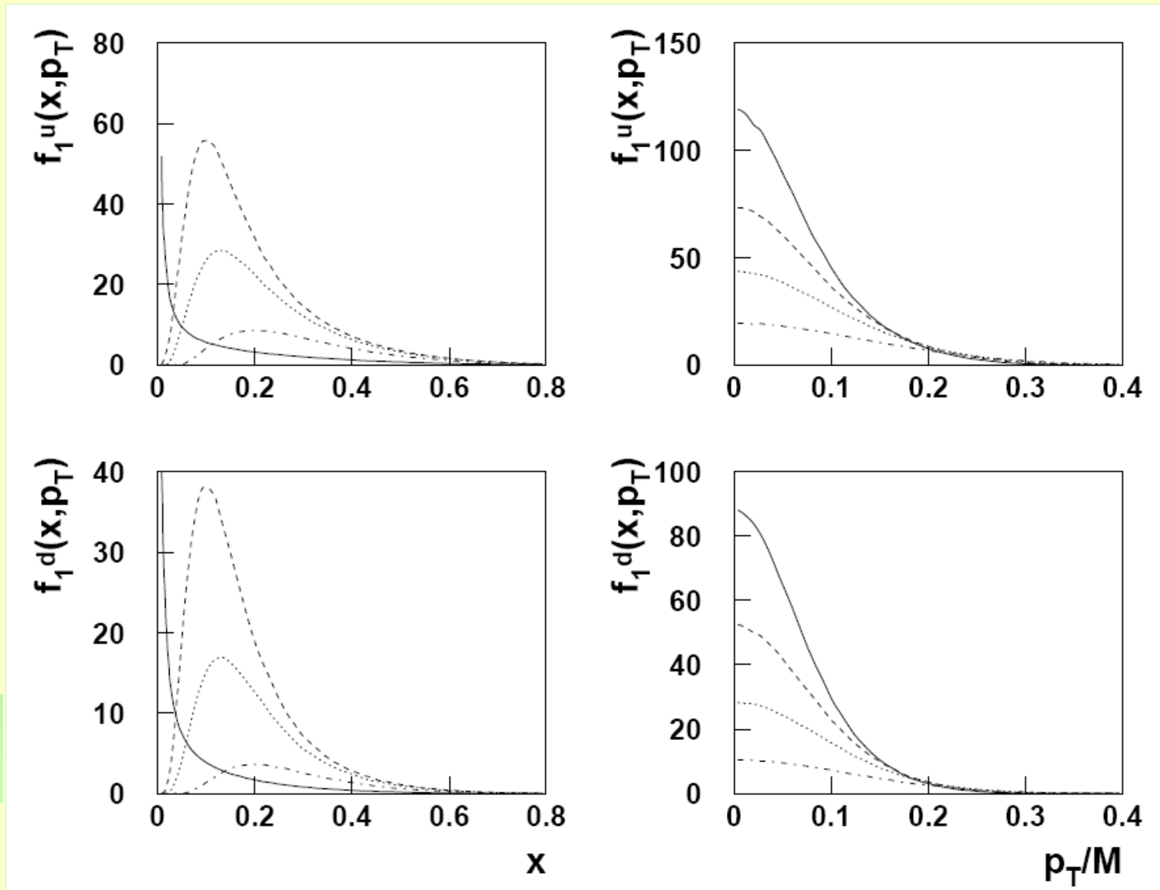
$$f_1^q(x,\mathbf{p}_T) \Leftrightarrow G_q(\mathbf{p}) \Leftrightarrow G_q(p_0) \Leftrightarrow G_q(\frac{pP}{M}) \Leftrightarrow f_1^q(x)$$

Remarks:

- all functions depend also on Q^2. Due to equivalence e.g. evolution $f_1^q(x,p_T,Q^2)$ can be obtained by evolution $f_1^q(x,Q^2)$.
- due to rotational symmetry all functions involve equivalent information
- on very general level, rotational symmetry is connected with the fact, that spin of nucleon $J=1/2$. Deformation of this symmetry would imply, that nucleon spin functions depend not only on x,Q^2 but also on qS.

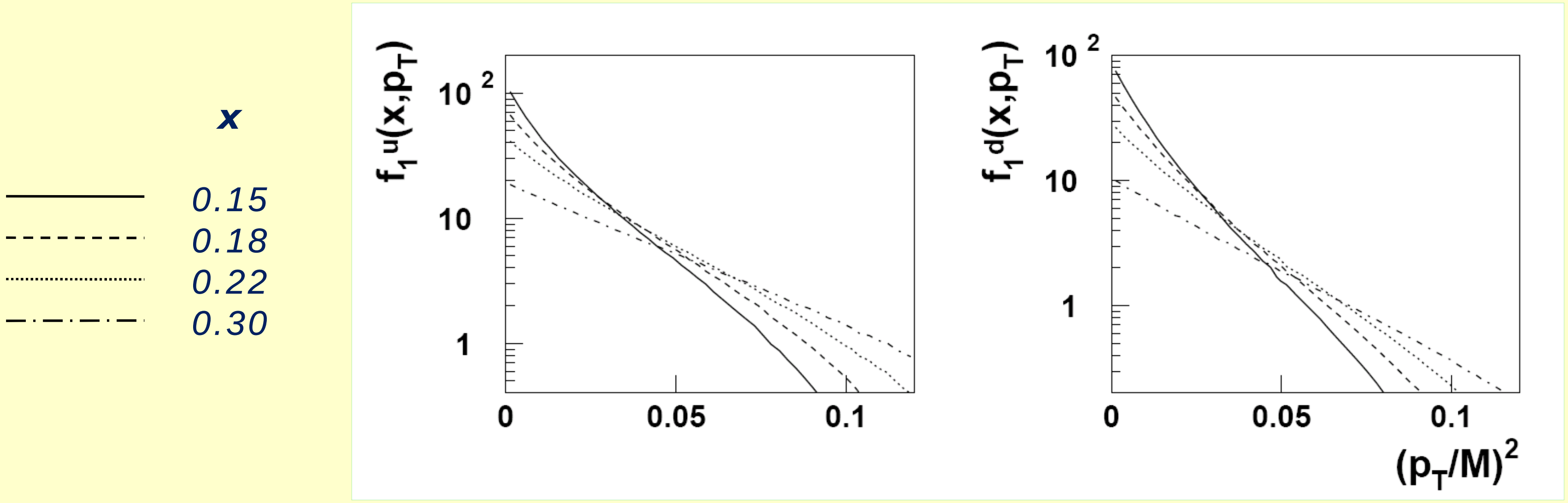
Numerical results: $f_1^q(x,p_T)$

p_T/M	x
$q(x)$	0.15
0.10	0.18
0.13	0.22
0.20	0.30



Input for $f_1(x)$
MRST LO at $4\ GeV^2$

Numerical results – logarithmic scale



- Gaussian shape – is supported by phenomenology
- $\langle p_T^2 \rangle$ depends on x

Image 4: unintegrated polarized distribution function

The procedure is more complicated than for the unpolarized case,
for details see *arXiv:0912.3380[hep-ph] & Phys.Rev.D80, 014021(2009)*

Results:

$$g_1^q(x,p_T) = \frac{2x-\xi}{\pi M^2 \xi^3}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right),$$

$$g_{1T}^{\perp q}(x,p_T) = \frac{2}{\pi M^2 \xi^3}\left(3g_1^q(\xi) + 2\int_\xi^1 \frac{g_1^q(y)}{y}dy - \xi\frac{d}{d\xi}g_1^q(\xi)\right)$$

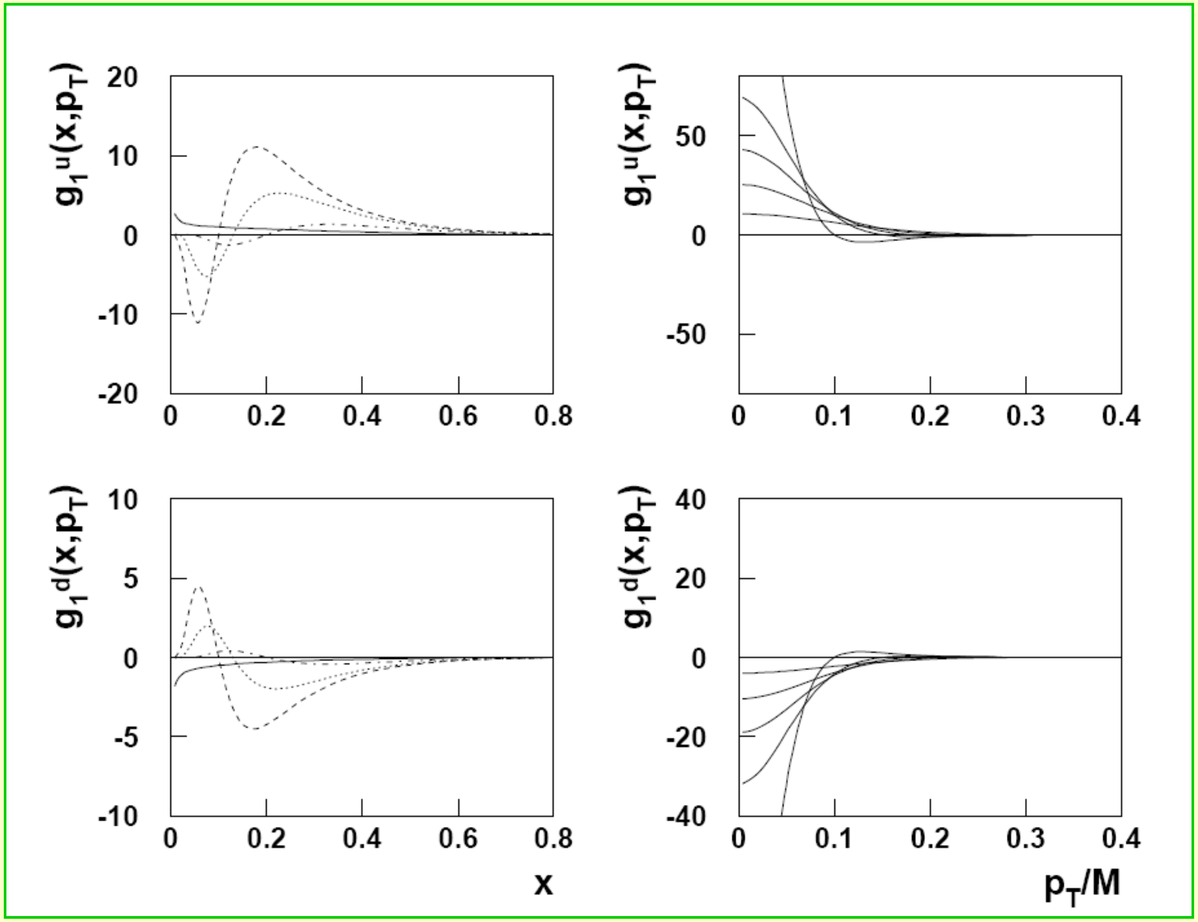
$$\xi = x\left(1+\left(\frac{p_T}{Mx}\right)^2\right)$$

$$\Rightarrow \quad \frac{g_1^q(x,p_T)}{g_{1T}^{\perp q}(x,p_T)} = \frac{x}{2}\left(1-\left(\frac{p_T}{Mx}\right)^2\right)$$

Numerical results: $g_1^q(x,p_T)$

p_T/M

$g_{1q}(x)$ ———
0.10 - - - - -
0.13 ....
0.20 —·—·—



x
0.10
0.15
0.18
0.22
0.30

Input for g_1 :
LSS LO at $4\ GeV^2$

Comment #1

In general $g_{1q}(x,\mathbf{p}_T)$ change sign.

It is due to the factor:

$$2x-\xi = x\left(1-\left(\frac{p_T}{Mx}\right)^2\right) = 2\tilde{p}_1/M$$

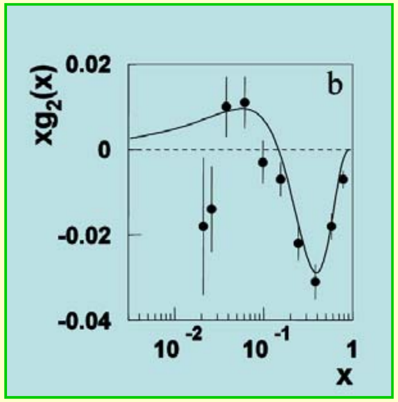
The situation is similar to the $g_2(x)$ case:

$$g_2(x) \;=\; -\frac{1}{2}\int \Delta G(p_0)\left(p_1+\frac{p_1^2-p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{d^3p}{p_0},$$

For both expressions it holds:

- great x is related to positive p_1
- small x is related to negative p_1

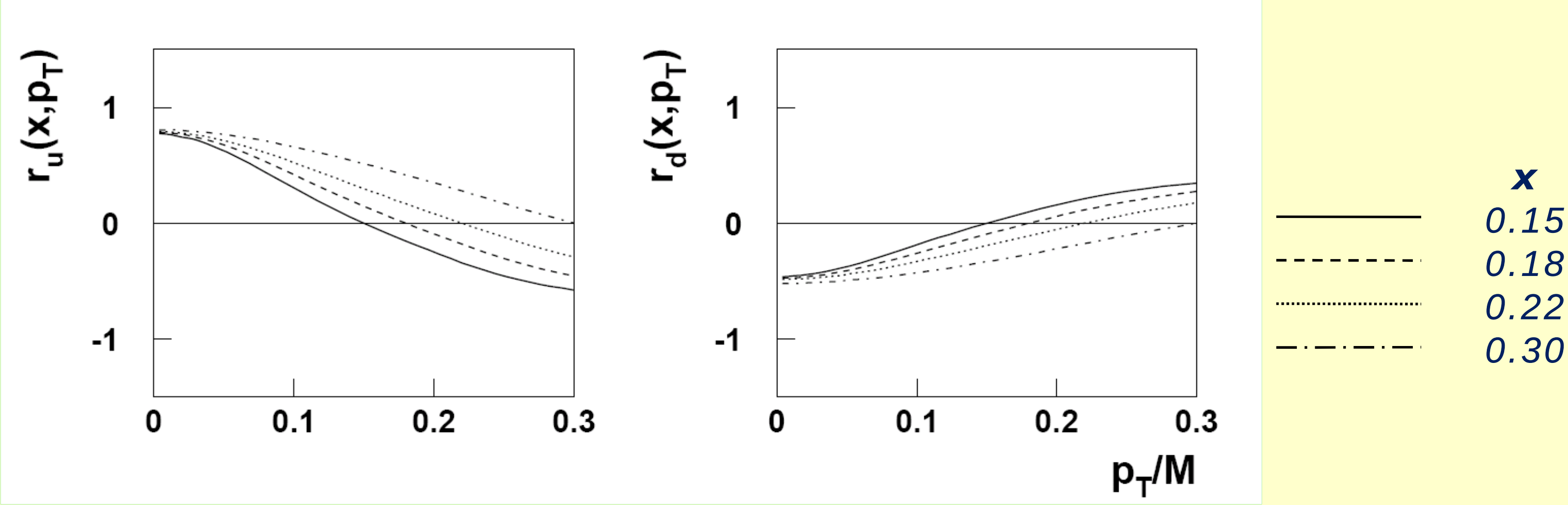
$\Rightarrow$ *both expressions have opposite signs for great and small x*



Comment #2

(inspired by discussion with Jaques Soffer & Claude Bourrely)

$$r_q(x, \mathbf{p}_T) = \frac{g_1^q(x, \mathbf{p}_T)}{f_1^q(x, \mathbf{p}_T)}$$



Input on $f_1(x)$ and $g_1(x)$ from statistical model:

C. Bourrely, J. Soffer and F. Buccella, Eur. Phys. J. C **23**, 487 (2002); Mod. Phys. Lett. A **18**, 771 (2003); Eur. Phys. J. C **41**, 327 (2005); Mod. Phys. Lett. A **21**, 143 (2006); Phys. Lett. B **648**, 39 (2007).

Comment #3

Can we measure all these TMDs curves?

The curves are related to the initial momentum of quarks, but one can measure only products of hadronization (jets). The reconstruction back to quark initial momentum may not be very precise task, one should expect rather some smeared TMDs...?

?

Summary & conclusion

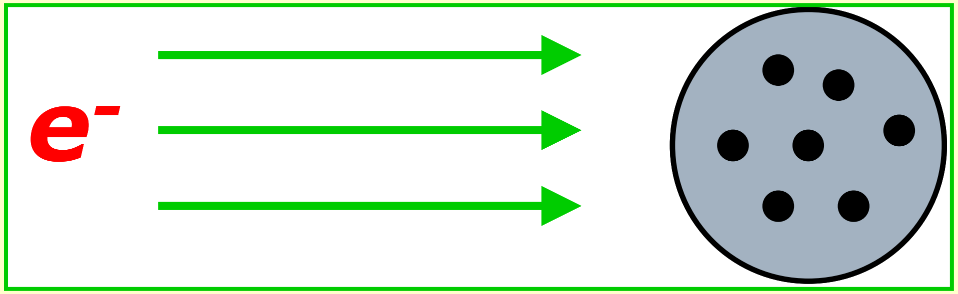
We discussed some aspects of quark motion inside nucleon within the 3D covariant parton model:

- OAM can be identified with the pretzelosity distribution also in this model
- We studied polarized and unpolarized TMDs a showed relations to their unitegrated couterparts
- With the use of these relations we calculated parton distribution functions $f_1(x,p_T)$ and $g_1(x,\mathbf{p}_T)$
- Requirement of **relativistic covariance + rotational symmetry** represents powerful tool for revealing new relations among distribution functions, including (1D)PDFs ↔ (3D)TMDs relations.

Thank you !

3D covariant parton model

- General framework



$$\Delta\sigma(x,Q^2) \sim |A|^2$$

$$|A|^2 = L_{\alpha\beta}W^{\alpha\beta}$$

The quarks are represented by the quasifree fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry

$$G_q^{\pm}(p_0)d^3p; \qquad p_0 = \sqrt{m^2 + \mathbf{p}^2}\,,$$

which are expected to depend effectively on Q^2. These distributions measure the probability to find a quark in the state

$$u(p,\lambda\mathbf{n}) = \frac{1}{\sqrt{N}}\left(\begin{array}{c}\phi_{\lambda\mathbf{n}}\\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}}\end{array}\right); \qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}},$$

where m and p are the quark mass and momentum, $\lambda = \pm 1/2$ and $\mathbf{n}$ coincides with the direction of target polarization $\mathbf{J}$.

$W^{\alpha\beta} \Rightarrow$

$$F_1(x,Q^2)$$
$$F_2(x,Q^2)$$
$$g_1(x,Q^2)$$
$$g_2(x,Q^2)$$

Structure functions

- **Input:**
  3D distribution functions in the proton rest frame (starting representation)

The distributions allow to define the generic functions G and ΔG:

$$G(p_0) = \sum_q e_q^2 G_q(p_0), \quad G_q(p_0) \equiv G_q^+(p_0) + G_q^-(p_0),$$

$$\Delta G(p_0) = \sum_q e_q^2 \Delta G_q(p_0), \quad \Delta G_q(p_0) \equiv G_q^+(p_0) - G_q^-(p_0)$$

from which the structure functions can be obtained.

- **Result:**
  structure functions
  (x=Bjorken x_B !)

If one assumes $Q^2 \gg 4M^2x^2$, then:

$$F_2(x) = Mx^2 \int G(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

$$g_1(x) = \frac{1}{2}\int \Delta G(p_0)\left(m + p_1 + \frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0},$$

$$g_2(x) = -\frac{1}{2}\int \Delta G(p_0)\left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

F_1, F_2 - manifestly covariant form:

$$F_1(x) = \frac{M}{2}\left(\frac{B}{\gamma} - A\right), \qquad F_2(x) = \frac{Pq}{2M\gamma}\left(\frac{3B}{\gamma} - A\right),$$

where

$$A = \frac{1}{Pq}\int G\left(\frac{Pp}{M}\right)[m^2 - pq]\,\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$B = \frac{1}{Pq}\int G\left(\frac{pP}{M}\right)\left[\left(\frac{Pp}{M}\right)^2 + \frac{(Pp)(Pq)}{M^2} - \frac{pq}{2}\right]\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$\gamma = 1 - \left(\frac{Pq}{Mq}\right)^2.$$

g_1, g_2 - manifestly covariant form:

$$g_1 = Pq\left(G_S - \frac{Pq}{qS}G_P\right), \qquad g_2 = \frac{(Pq)^2}{qS}G_P,$$

where

$$G_P = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[\frac{pS}{pP+mM}1 + \frac{1}{mM}\left(pP - \frac{pu}{qu}Pq\right)\right] \times \delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$G_S = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[1 + \frac{pS}{pP+mM}\frac{M}{m}\left(pS - \frac{pu}{qu}qS\right)\right] \times \delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0};$$

$$u = q + (qS)S - \frac{(Pq)}{M^2}P.$$

Comments

- In the limit of usual approach assuming $p = xP$, (i.e. intrinsic motion is completely supressed) one gets known relations between the structure and distribution functions:

$$F_2(x) = x \sum_q e_q^2 q(x) \qquad g_1(x) = \frac{1}{2} \sum_q e_q^2 (q^+(x) - q^-(x))$$

- We work with a ‘naive’ 3D parton model, which is based on covariant kinematics (and not infinite momentum frame). Main potential: implication of some old and new sum rules and relations among PDF’s and TMDs.