

Modelling non-perturbative corrections to heavy-quark fragmentation

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Based on:

U. Aglietti, G. Corcella and G. F., Nucl. Phys. B (2007) [hep-ph/0610035]

G. Corcella and G. F., JHEP (2007) [0706.2357 [hep-ph]]

Outline

- 1 Heavy-quark fragmentation
- 2 Analytic QCD coupling
- 3 Phenomenological analysis: b and c -quark fragmentation
- 4 Conclusions

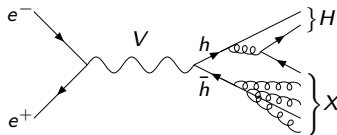


Heavy-quark fragmentation

$$e^+e^- \rightarrow V(Q) \rightarrow H_h + X;$$

$$V = Z^0, \gamma^*, \Upsilon(4s); \quad h = b, c$$

$$m_V, m_h \gg \Lambda_{QCD}$$



Heavy-quark energy distribution factorizes as [Mele,Nason (91),
Cacciari,Catani (01)]:

$$\frac{1}{\sigma} \frac{d\sigma}{dx_h}(x_h; Q^2, m_h^2) = C(x_h; Q^2, \mu_F^2) \otimes E(x_h; \mu_F^2, \mu_{0F}^2) \otimes D^{ini}(x_h; \mu_{0F}^2, m_h^2)$$

$$\text{where } x_h = 2 \frac{E_h}{m_V};$$

Perturbative fixed-order expansion feasible but large perturbative terms (\Rightarrow resum) and non-perturbative corrections (\Rightarrow modelize).

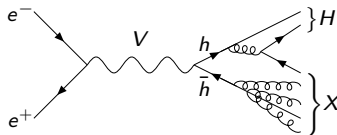


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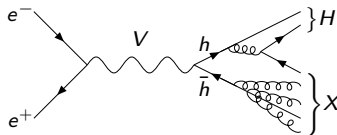


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- Collinear emission of partons with transverse momenta between $m_h^2 \leq k_{\perp}^2 \leq m_V^2$ give rise to large logarithmic terms:

$$\alpha_S^n \ln^k \frac{m_V^2}{m_h^2} \quad (n = 1, 2, \dots, \infty \quad k = 1, 2, \dots, n),$$

which we have resummed using the Altarelli–Parisi (DGLAP) formalism at NLL in $E(x_h; \mu_F^2, \mu_{0F}^2)$.

- In the threshold region ($x_h = 2 \frac{E_h}{m_V} \rightarrow 1$), soft and/or collinear parton emissions give rise to large logarithmic terms:

$$\alpha^n \left[\frac{\ln^k(1-x_h)}{1-x_h} \right]_+ \quad (n = 1, 2, \dots, \infty, k = 0, 1, 2, \dots, 2n-1),$$

which we have resummed at NLO+NNLL in $C(x_h; Q^2, \mu_F^2)$ and $D^{ini}(x_h; \mu_{0F}^2, m_h^2)$ [Sterman ('87), Catani & Trentadue ('89)].



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Perturbative resummation

- In the Mellin space the threshold resummed form factor reads

[Catani & Cacciari ('01)]:

$$\ln C_N = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{\mu_F^2}^{m_Z^2(1-z)} \frac{dk_{\perp}^2}{k_{\perp}^2} A[\alpha_S(k_{\perp}^2)] + B[\alpha_S(m_Z^2(1-z))] \right\}$$

$$\ln D_N^{ini} = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left\{ \int_{m_b^2(1-z)^2}^{\mu_{0F}^2} \frac{dk_{\perp}^2}{k_{\perp}^2} A[\alpha_S(k_{\perp}^2)] + D[\alpha_S(m_b^2(1-z)^2)] \right\}$$

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- When $z \rightarrow 1$ the integration in k_{\perp}^2 involves α_S at the Landau pole: it is necessary a prescription, e. g. the Minimal Prescription [Catani, Mangano, Nason & Trentadue ('96)].
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Model for non-perturbative corrections

- Non perturbative corrections cannot be calculated from first principles. They are usually described using phenomenological hadronization models, with free parameters to be fitted to experimental data.

$$D(x_h) = D_{Pt}(x_h, \mu_{0F}^2, m_h^2) \otimes D_{NP}(x_h, \epsilon_1, \dots, \epsilon_n)$$

$$D_{NP}(x, \epsilon) = \frac{1}{x \left[1 - \frac{1}{x} - \frac{\epsilon}{1-x} \right]^2}, \quad [\textit{Peterson et al. ('83)}]$$

$$D_{NP}(x, \epsilon) = x^\epsilon (1-x), \quad [\textit{Kartvelishvili et al. ('78)}]$$

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Analytic QCD coupling

- Standard QCD coupling: physical cut at $\mu^2 < 0$ and unphysical pole at $\mu^2 = \Lambda_{QCD}^2$:

$$\alpha_S^{lo}(\mu^2) = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda_{QCD}^2}} .$$

- Analytic QCD coupling: same discontinuity along the cut but analytic elsewhere in the complex plane [Shirkov & Solovtsov ('97)]:

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$$\tilde{\alpha}_S(k_\perp^2) = \frac{i}{2\pi} \int_0^{k_\perp^2} ds \text{Disc}_s \frac{\bar{\alpha}_S(-s)}{s}.$$

- At leading order we have:

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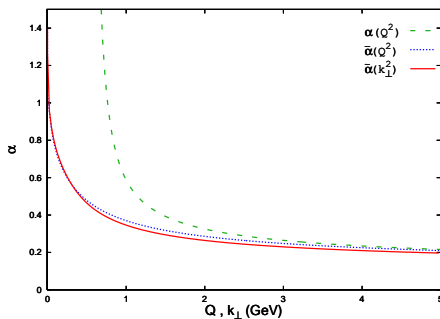


Figure 1: Time-like and space-like analytic couplings compared with the standard one.

We have a modified threshold resummation formula (analogously for C_N and E_N .)

$$\ln D_N^{ini} = \int_0^1 dx \frac{x^{N-1} - 1}{1-x} \left\{ \int_{m_b^2(1-x)^2}^{\mu_{0F}^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \tilde{A} [\tilde{\alpha}_S(k_{\perp}^2)] + \tilde{D} [\tilde{\alpha}_S(m_b^2(1-x)^2)] \right\}$$



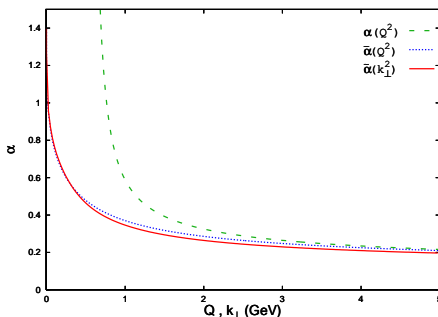


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Phenomenological Analysis: b -quark fragmentation at the Z^0 peak

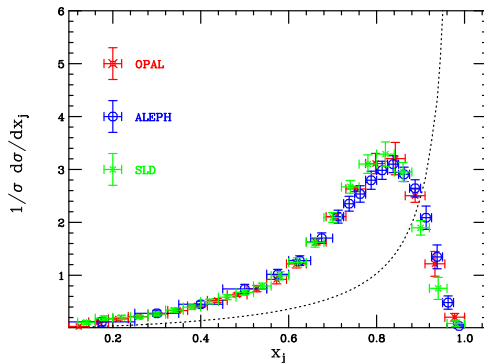


Figure 2: B -hadron spectrum in e^+e^- annihilation at Z^0 peak: prevision of the model compared with experimental data [Alep ('01), Delphi ('02), SLD ('00)].



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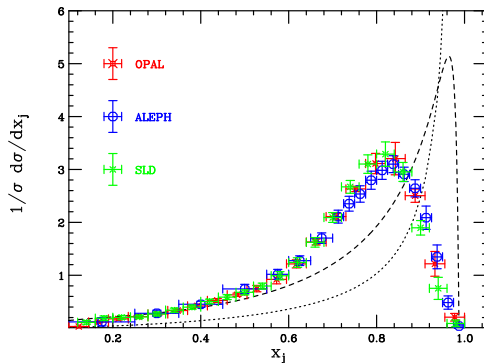


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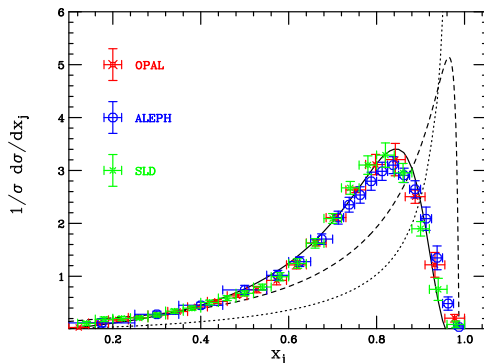


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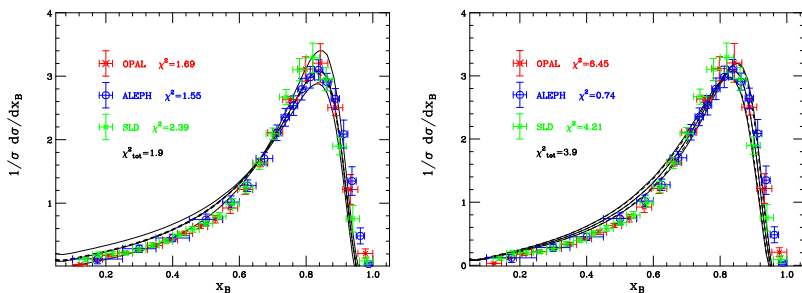


Figure 3: Model dependence on the factorizations scales (left): $\mu_{0F} = m_b/2, m_b, 2m_b$; $\mu_F = m_Z/2, m_Z, 2m_Z$ and on $\alpha_S(m_Z)$ and on m_b (right): $\alpha_S(m_Z) = 0.117, 0.119, 0.121$; $m_b = 4.7, 5.0, 5.3$ GeV.



	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$	$\langle x^4 \rangle$
e^+e^- data σ_N^B	0.7153 ± 0.0052	0.5401 ± 0.0064	0.4236 ± 0.0065	0.3406 ± 0.0064
$[\sigma_N^B]_{\text{th}}$	0.6867 ± 0.0403	0.5019 ± 0.0472	0.3815 ± 0.0465	0.2976 ± 0.0462
$\delta\sigma_N^B(\mu_R)$	0.0014	0.0011	0.0009	0.0007
$\delta\sigma_N^B(\mu_F)$	0.0066	0.0067	0.0059	0.0051
$\delta\sigma_N^B(\mu_{0R})$	0.0022	0.0028	0.0031	0.0033
$\delta\sigma_N^B(\mu_{0F})$	0.0364	0.0414	0.0398	0.0364
$\delta\sigma_N^B(m_b)$	0.0111	0.0145	0.0153	0.0150
$\delta\sigma_N^B(\bar{m}_b)$	0.0004	0.0005	0.0006	0.0006
$\delta\sigma_N^B(\bar{m}_c)$	0.0003	0.0005	0.0006	0.0006
$\delta\sigma_N^B(\bar{m}_s)$	0.0004	0.0007	0.0008	0.0008
$\delta\sigma_N^B(\alpha_S(m_Z^2))$	0.0113	0.0158	0.0173	0.0176
σ_N^b	0.7734 ± 0.0232	0.6333 ± 0.0311	0.5354 ± 0.0345	0.4617 ± 0.0346

Table 1: Moments σ_N^B from [DELPHI ('02)] and moments $[\sigma_N^B]_{\text{th}}$ yielded by our calculation. We quote the uncertainties due to the parameters which enter in the perturbative calculations and compute the theoretical total error.

We also present the moments σ_N^b of the standard NLL parton level result.



Charm-quark fragmentation at Z^0 peak

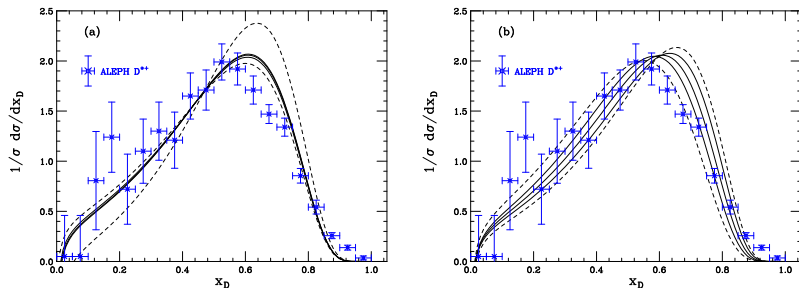


Figure 4: Model dependence on the factorizations scales (left): $\mu_{0F} = m_c/2, m_c, 2m_c$; $\mu_F = m_Z/2, m_Z, 2m_Z$ and on $\alpha_S(m_Z)$ and on m_b (right): $\alpha_S(m_Z) = 0.117, 0.119, 0.121$; $m_b = 1.5, 1.8, 2.1$ GeV.



Charm-quark fragmentation at $\Upsilon(4s)$ peak

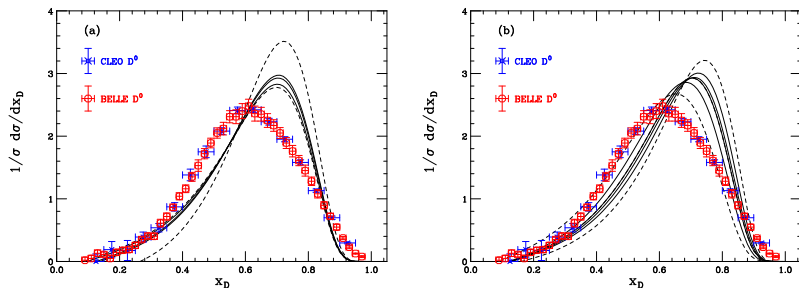


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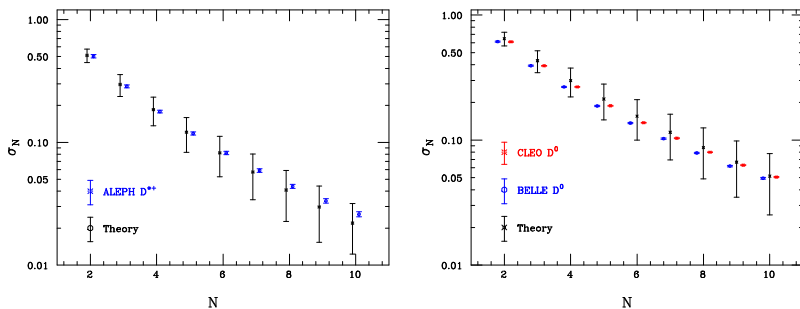


Figure 6: Moments form [Alep, CLEO, and Belle] compared with model predictions.



Conclusions

- We resummed large logarithm correction to heavy flavour fragmentation up to NNLL matching to the NLO fixed order result.
- Using the analytic QCD coupling we develop a model for non-perturbative effects which allow us to describes experimental data in b and c -quark fragmentation without introducing any ad-hoc non-perturbative component. The model works better for processes characterized by an hard scale $Q \sim m_Z$.
The model was also tested in semi-inclusive B decays.
- Possible future perspectives: perform a complete NNLO+NNLL for heavy flavour fragmentation and use the model to study HERA and Tevatron hadron production data and to make prediction for LHC.



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Back-up Slides



Comparison with DMW model

- Since the time-like coupling is regular for any value of k_{\perp} , we can compute the average of the coupling

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which is a free parameter to be determined with a fit to experimental data [Dokshitzer, Marchesini & Webber ('95)].

- Assuming $\alpha_S(m_b) = 0.22$, $n_f = 3$ and $\mu_I = 2 \text{ GeV}$, we obtain at leading order with the time-like coupling:

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