Saturation and linear transport equation

Krzysztof Kutak

Universiteit Antwerpen

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MOTIVATIONS

Saturation – way to fulfill unitarity requirements in high energy limit of QCD

QCD evolution equations: BK (Balitsky, Kovchegov), JIMWLK (Jalian-Marian,
McLerran,Weigert,Iancu,Leonidov,Kovner),

but also:
Models: GBW (Golec-Biernat Wusthoff), ITC(Iancu Itakura McLearran)
McLerran, Venugopalan,...

GBW first saturation model, successful in description of data
a lot of further extensions and refinements

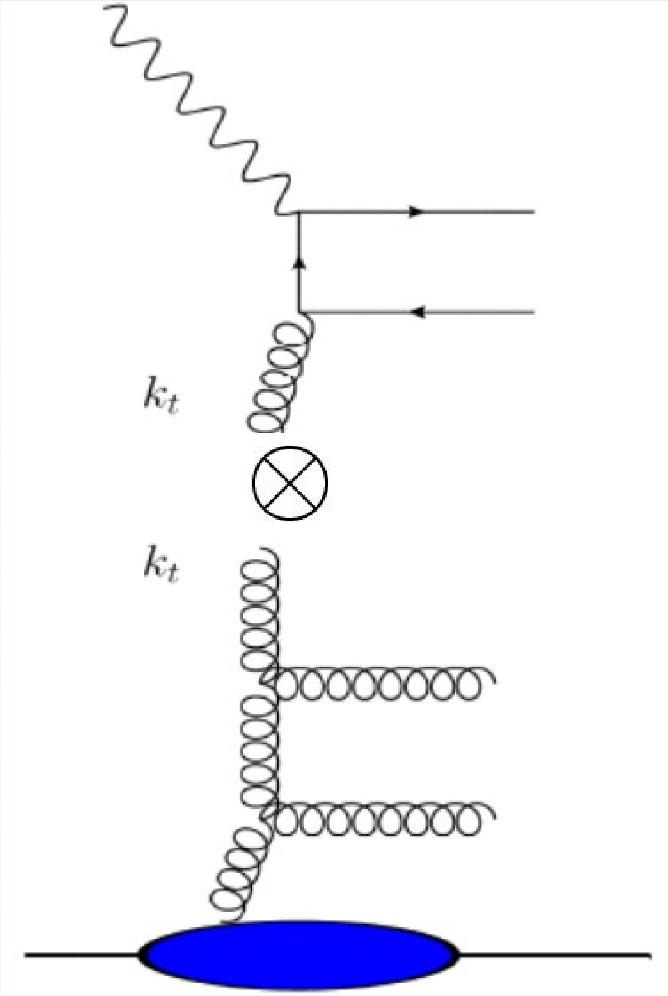
Is there any equation to which GBW is a solution?
Is the any link between GBW and BK?

High energy limit of QCD

Observable ~ ME $\otimes$ parton density

Depend not only on longitudinal momentum fraction but also on its transversal component

Consistent resumation both logs of rapidity and logs of hard scale



High energy limit of QCD

$f(x,k_t)$ - sum up terms - $$\sum_n \sum_m \alpha_s^m ln^n (s/\mu)$$

Lipatov ,Fadin, Kuraev "77

Ciafaloni '89, Catani, Fiorani, Marchesini,

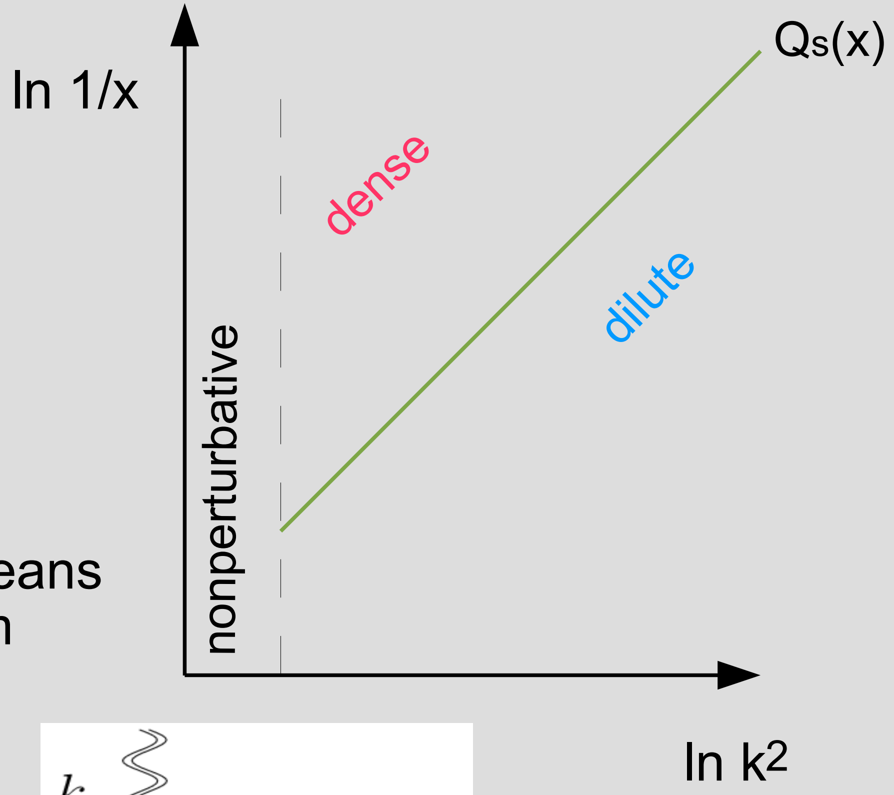
This has to obey unitarity bound- should not grow faster than power of energy

Saturation

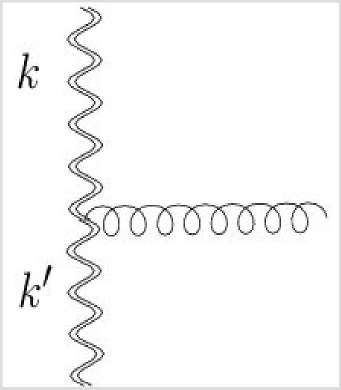
Saturation – way to fulfill unitarity requirements in high energy limit of QCD

Should change the power like growth of Hadronic cross section at high energies from power like to logarithmic

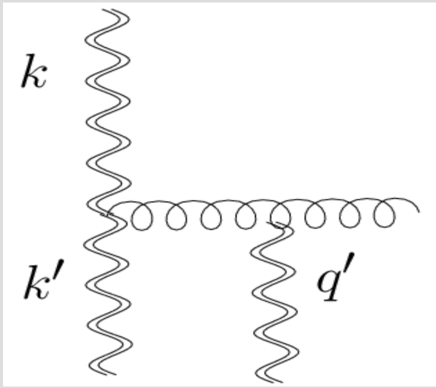
On microscopic – gluon – level it means that gluons at high energies apart from



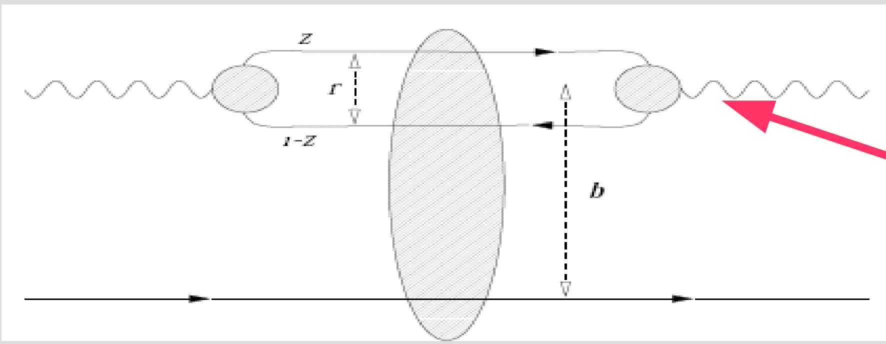
splitting



recombine



GBW dipole model



wave function (coordinate space)

$$F_2(x,Q^2) \sim \psi \otimes N \otimes \psi$$

$$N(x,r,b) = \theta(b_0 - b)\left[1 - \exp\left(-\frac{r^2}{4R_0^2}\right)\right]$$

$$R_0(x) = \frac{1}{Q_0}\left(\frac{x}{x_0}\right)^{\lambda/2} \qquad Q_s(x) = 1/R_0(x)$$

Golec-Biernat, Wusthoff '99

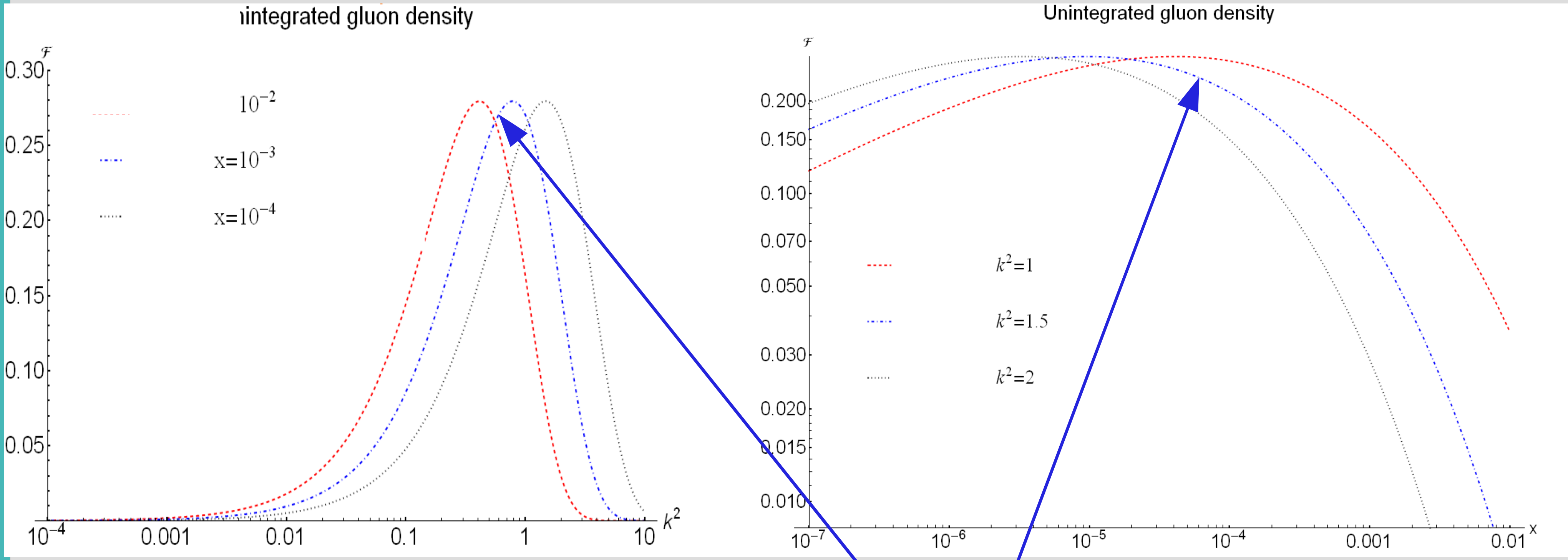
- Successful phenomenological applications — Golec-Biernat, Wusthoff '99,....
- Extensions to include impact parameter dependence — Kowalski, Tenney '03,....
- Extensions to include QCD evolution — Bartels, Golec-Biernat, Kowalski '02

Unintegrated gluon density

One can calculate gluon density using:

$$f(x,k^2,b) = \frac{N_c}{4\alpha_s\pi^2}k^4\nabla_k^2\int\frac{d^2\mathbf{r}}{2\pi}\exp(-i\mathbf{k}\cdot\mathbf{r})\frac{N(x,r,b)}{r^2}$$

Braun'98, Kwiecinski ,KK'01



- Both distributions have maximum
- One can define saturation as a scale for which gluon density has a maximum

Transport equation

$$\partial_x f(x,k^2,b) = \frac{\lambda f(x,k^2,b)(1-R_0^2(x)k^2)}{xQ_0^2}$$

$$\partial_{k^2} \frac{f(x,k^2,b)}{k^2} = \frac{f(x,k^2,b)(1-R_0^2(x)k^2)}{k^4 Q_0^2}$$

$$\mathcal{F}(x,k^2,b) = f(x,k^2,b)/k^2$$

$$Y = \ln x_0/x \quad L = \ln k^2/Q_0^2$$

Equation: $$\partial_Y \mathcal{F}(Y,L,b) + \lambda \partial_L \mathcal{F}(Y,L,b) = 0$$

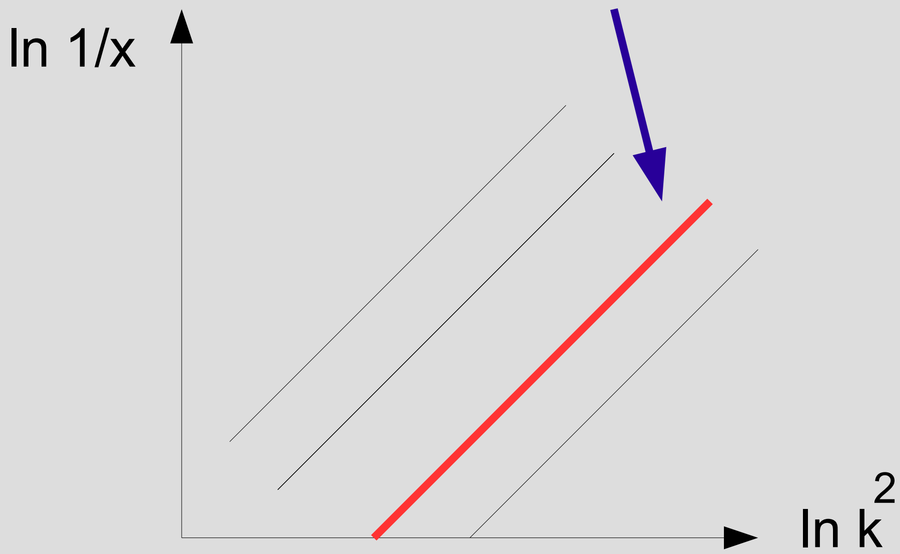
Solution: $$\mathcal{F}(Y,L,b) = \mathcal{F}_0(L-\lambda Y,b)$$

Transport equation for gluon density

With initial condition:

$$\mathcal{F}(x=x_0, k^2, b) = \frac{N_c}{2\pi^2\alpha_s}\theta(b_0 - b)k^2 \exp(-k^2)$$

We get the GBW gluon density
Critical line of GBW - characteristics of the transport equation
GBW fit chooses particular one line from family of characteristics



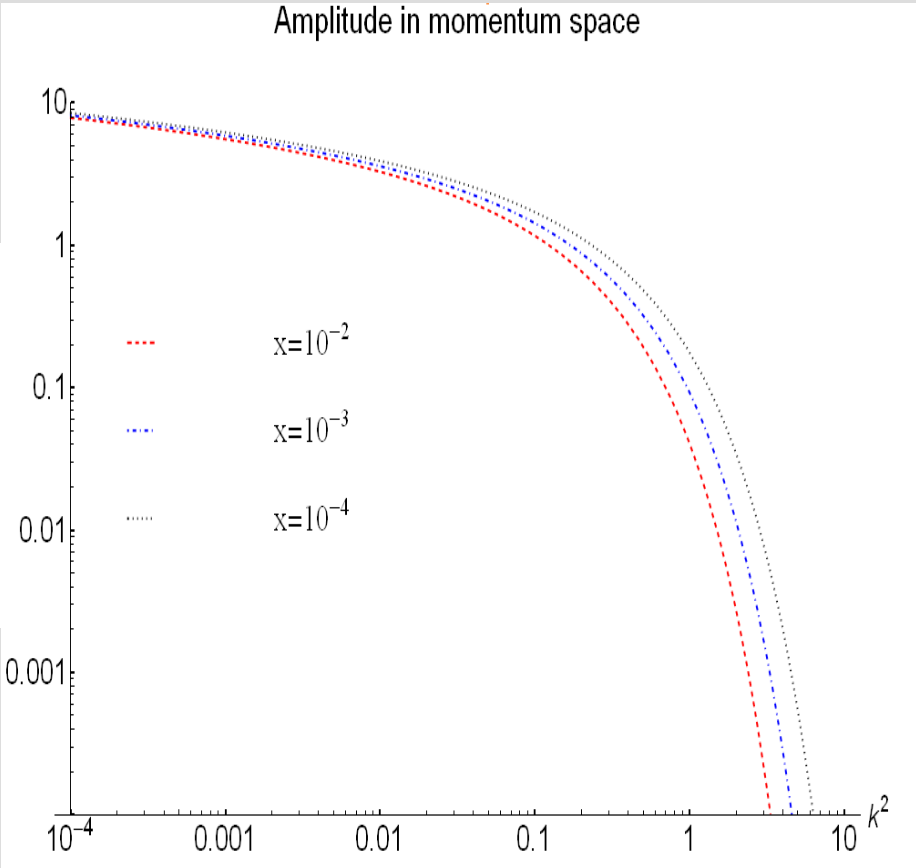
Transport equation for dipole amplitude in momentum space

The same equation for momentum space representation of dipole amplitude

$$\phi(x,k^2,b) = \int \frac{d^2\mathbf{r}}{2\pi} \exp(-i\mathbf{k}\cdot\mathbf{r}) \frac{N(x,r,b)}{r^2}$$

$$\phi(x,k^2,b) = \frac{1}{2}\theta(b_0-b)\,\Gamma\left[0, \frac{k^2}{Q_0^2}\left(\frac{x_0}{x}\right)^\lambda\right]$$

$$\partial_Y \phi(Y,L,b) + \lambda \partial_L \phi(Y,L,b) = 0$$



Link this to nonlinear QCD evolution equation....

BK in diffusion approximation

$$\partial_Y \phi(Y,k^2,b) = \overline{\alpha}\chi\left(-\frac{\partial}{\partial_{\log k^2}}\right)\phi(Y,k^2,b) - \overline{\alpha}\phi^2(Y,k^2,b)$$

Charcteristic function (integral kernel) of BFKL

Expand kernel up to second order

$$\chi(-\partial_L)\phi(Y,L,b) = \left[\chi(\gamma_c) + (-\partial_L - \gamma_c)\chi'(\gamma_c) + \frac{1}{2!}(-\partial_L - \gamma_c)^2\chi''(\gamma_c) + \ldots\right]\phi(Y,L,b)$$

Solve equation

Munier, Peschanski '03

$$\phi(Y,k^2,b) = \theta(b_0 - b)\sqrt{\frac{2}{\overline{\alpha}\chi''(\gamma_c)}}\ln\left(\frac{k^2}{Q_s^2(Y)}\right)\left(\frac{k^2}{Q_s^2(Y)}\right)^{\gamma_c - 1}\exp\left[-\frac{1}{2\overline{\alpha}\chi''(\gamma_c)Y}\ln^2\left(\frac{k^2}{Q_s^2(Y)}\right)\right]$$

Front interior: $k \sim Q_s$, Y large

$$\phi(Y,k^2,b) = \theta(b_0 - b)\sqrt{\frac{2}{\overline{\alpha}\chi''(\gamma_c)}}\left(\frac{k^2}{Q_s^2(Y)}\right)^{\gamma_c - 1}\log[k^2/Q_s^2(Y)]$$

BK as transport equation

$$\phi(Y,k^2,b)=\theta(b_0-b)\sqrt{\frac{2}{\overline{\alpha}\chi''(\gamma_c)}}\left(\frac{k^2}{Q_s^2(Y)}\right)^{\gamma_c-1}\log[k^2/Q_s^2(Y)]$$

Munier, Peschanski '03

Differentiate w.r.t. ln1/x and lnk/k0 to obtain:

$$\partial_Y\phi(Y,L,b)+\lambda_{BK}\partial_L\phi(Y,L,b)=0$$

where:

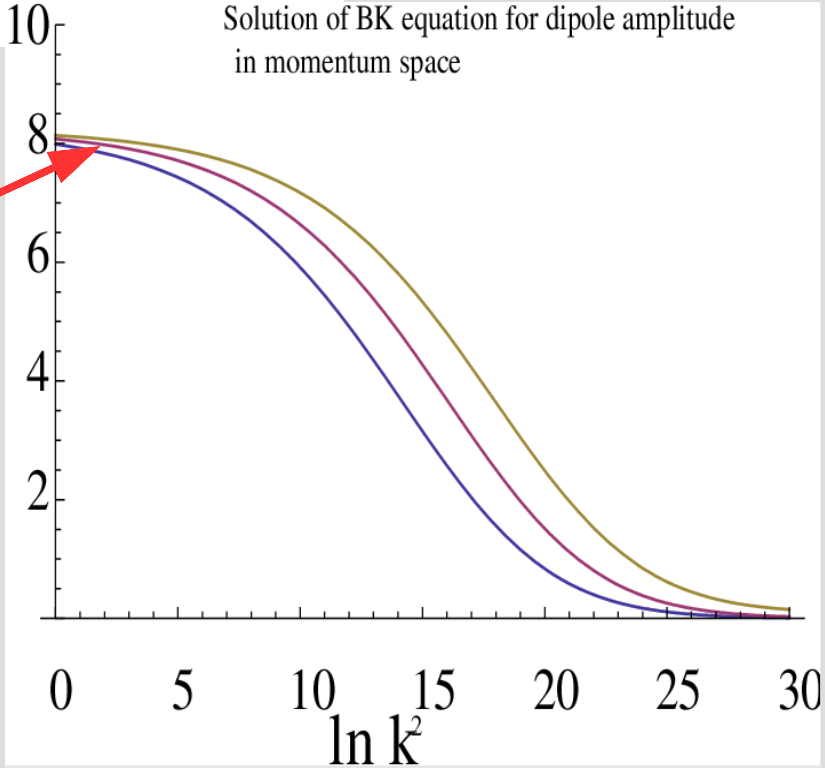
$$\lambda_{BK}=-\overline{\alpha}\chi'(\gamma_c)$$

BK in diffusion approximation

$$\partial_Y \phi(Y,k^2,b) = \overline{\alpha}\chi\left(-\frac{\partial}{\partial_{\log k^2}}\right)\phi(Y,k^2,b) - \overline{\alpha}\phi^2(Y,k^2,b)$$

$$\chi(-\partial_L)\phi(Y,L,b) = \left[\chi(\gamma_c) + (-\partial_L - \gamma_c)\chi'(\gamma_c) + \frac{1}{2!}(-\partial_L - \gamma_c)^2\chi''(\gamma_c) + \ldots\right]\phi(Y,L,b)$$

Saturation for small kt



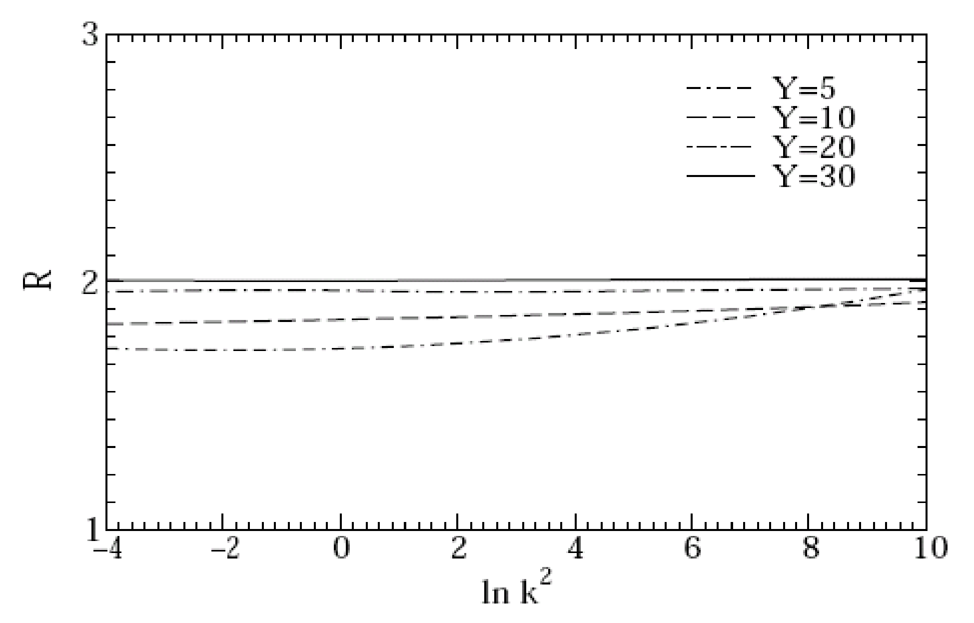
BK in diffusion approximation

$$\frac{\partial_Y \phi(Y,L,b)}{\partial_L \phi(Y,L,b)} = \bar{\alpha}\chi_0'(\gamma_c) - \bar{\alpha}\frac{D\phi(Y,L,b) - \phi^2(Y,L,b)}{\partial_L \phi(Y,L,b)}$$

Don't redefine variables to remove first derivative. It has physical meaning

D $\equiv$ diffusion + splitting + energy conservation

$$R \equiv \frac{D\phi(Y,L,b) - \phi^2(Y,L,b)}{\chi_0'(\gamma_c)\partial_L \phi(Y,L,b)}$$



The nonlinearity causes the second term to be:

$$2\bar{\alpha}\chi_0'(\gamma_c)$$

So we again get as in GBW case :

$$\partial_Y \phi(Y,L,b) + \lambda_{BK}\partial_L \phi(Y,L,b) = 0$$

Conclusions

Eqution to which GBW model is a solution has been found

Link to BK has been established

GBW is motivated by pQCD

Success of GBW – initial condition with saturation + wave equation which does preserve it