Semi-inclusive DIS at small x : TMD parton distributions and saturation

Cyrille Marquet

Institut de Physique Théorique

CEA/Saclay

based on:
C.M., B.-W. Xiao and F. Yuan, *Phys. Lett.* B**682** (2009) 207, arXiv:0906.1454
and work in progress

Motivations

- cross sections in the Bjorken limit of QCD

$$s \to \infty \;,\quad Q^2 \to \infty$$

$$Q^2/s = x \;\; \text{fixed}$$

are expressed as a $1/Q^2$ "twist" expansion

$$d\sigma = \sum_p f_p \otimes d\hat{\sigma} + O(1/Q^2)$$

collinear factorization: parton content of proton described by k_T-integrated distributions

sufficient approximation for most high-p_T processes

TMD factorization: involves transverse-momentum-dependent (TMD) distributions

needed in particular cases, TMD-pdfs are process dependent

- cross sections in the Regge limit of QCD

$$s \to \infty \;,\quad x \to 0$$

$$xs = Q^2 \;\; \text{fixed}$$

are expressed as a $1/s$ "eikonal" expansion

$$d\sigma = \sum_p f_p \otimes d\hat{\sigma} + O(1/s)$$

k_T factorization: parton content described by unintegrated parton distributions (u-pdfs)

we would like to understand: - the connection between TMD & k_T factorizations
- how TMD-pdfs and u-pdfs are related

Outline

- SIDIS in the small-x limit
  semi-inclusive DIS (SIDIS) in the dipole picture
  k_T factorization in momentum representation
  the large-Q^2 limit of the small-x result

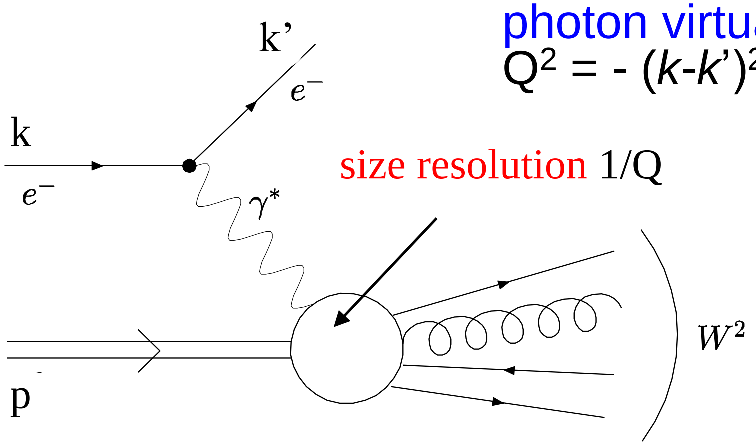
- SIDIS in the large-Q^2 limit
  TMD factorization for SIDIS
  the small-x limit of the large-Q^2 result

- Equivalence of TMD & k_T factorizations in SIDIS
  in the overlaping domain of validity
  the TMD quark distribution in terms of the unintegrated gluon distribution

- Breaking of TMD & k_T factorizations in di-jet production
  are they related ?
  at small x we understand very well why k_T factorization breaks down
  can this help us understand the TMD factorization breaking?

SIDIS in the small-x limit

The dipole factorization in DIS



size resolution 1/Q

photon virtuality
$Q^2 = -(k-k')^2 > 0$

ep center-of-mass energy
$S = (k+P)^2$
$\gamma^* p$ center-of-mass energy
$W^2 = (k-k'+P)^2$

$$x_B = \frac{Q^2}{2P.(k-k')} = \frac{Q^2}{W^2 - M_h^2 + Q^2}$$

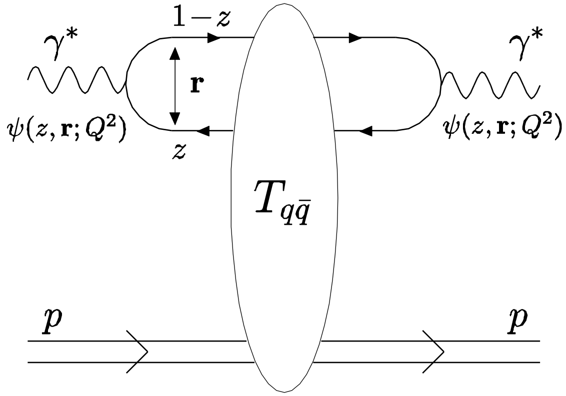
$$y = \frac{P.(k-k')}{P.k} = \frac{Q^2/x_B}{S - M_h^2}$$

- the cross section at small x

Mueller (1990), Nikolaev and Zakharov (1991)

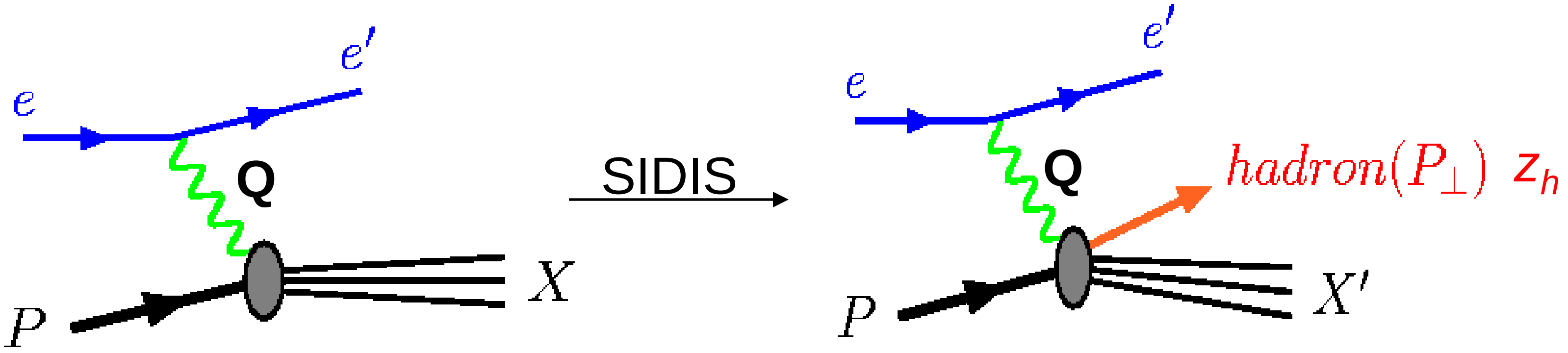
$$\sigma_{T,L}^{\gamma^* p \to X} = 2\int d^2r\, dz\, |\psi_{T,L}(z,\mathbf{r};Q^2)|^2 \underbrace{\int d^2b\, T_{q\bar{q}}(\mathbf{r},\mathbf{b},x_B)}_{\text{dipole-hadron cross-section}}$$

overlap of $\gamma^* \to q\bar{q}$
splitting functions



at small *x*, the dipole cross section is comparable to that of a pion, even though $r \sim 1/Q \ll 1/\Lambda_{QCD}$

The dipole factorization in SIDIS

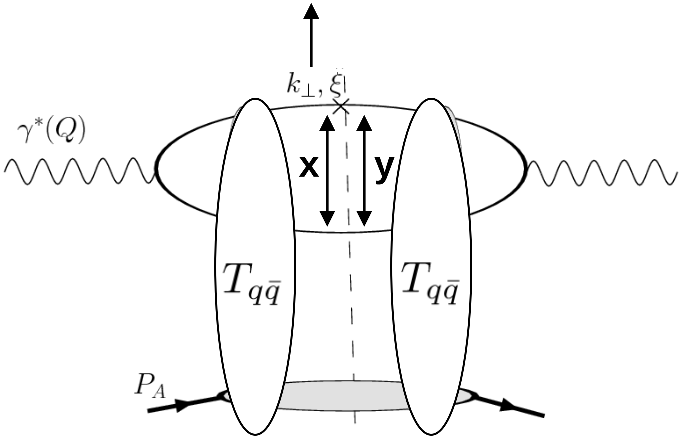


- the cross section at small x

$$\Phi(\xi,\mathbf{x},\mathbf{y};Q^2)=\psi(\xi,\underset{\uparrow}{\mathbf{x}};Q^2)\psi^*(\xi,\underset{\uparrow}{\mathbf{y}};Q^2)$$

dipoles in amplitude / conj. amplitude

fragmentation into hadron



$$\frac{d\sigma^{\gamma^* p\to hX}}{dz_h d^2P_\perp}=\frac{d\sigma_{T,L}^{\gamma^* p\to qX}}{d\xi d^2k_\perp}\left(k_\perp=\frac{\xi}{z_h}P_\perp\right)\otimes D_{h/q}(z_h/\xi)$$

$$\frac{d\sigma_{T,L}^{\gamma^* p\to qX}}{d\xi d^2k_\perp}=\int\frac{d^2x}{2\pi}\frac{d^2y}{2\pi}\,e^{-ik_\perp\cdot(\mathbf{x}-\mathbf{y})}\Phi_{T,L}(\xi,\mathbf{x},\mathbf{y};Q^2)\int d^2b\,[T_{q\bar{q}}(\mathbf{x},x_B)+T_{q\bar{q}}(\mathbf{y},x_B)-T_{q\bar{q}}(\mathbf{x}-\mathbf{y},x_B)]$$

McLerran and Venugopalan, Mueller, Kovchegov and McLerran (1999)

Cross section in momentum space

- the lepto-production cross section

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}}=\frac{\alpha_{em}^2N_c}{2\pi^3x_BQ^2}\sum_f e_f^2\int_{z_h}\frac{dz}{z}\frac{D(z)}{z^2}\overbrace{\int d^2bd^2q_\perp F(q_\perp,x_B)\,\mathcal{H}\left(\xi=\frac{z_h}{z},k_\perp=\frac{P_\perp}{z}\right)}^{k_T\text{ factorization}}$$

phase space $d\mathcal{P}=dx_BdQ^2dz_hdP_\perp^2$

the unintegrated gluon distribution

$$F(q_\perp,x_B)=\int\frac{d^2r}{(2\pi)^2}\,e^{-iq_\perp\cdot\mathbf{r}}[1-T_{q\bar{q}}(\mathbf{r},x_B)]$$

F.T. of photon wave function

$\epsilon_f^2=\xi(1-\xi)Q^2$

massless quarks

$$\mathcal{H}(\xi,k_\perp)\;=\;\left(1-y+\frac{y^2}{2}\right)(\xi^2+(1-\xi)^2)\left|\frac{k_\perp}{k_\perp^2+\epsilon_f^2}-\frac{k_\perp-q_\perp}{(k_\perp-q_\perp)^2+\epsilon_f^2}\right|^2$$

photon T

$$+(1-y)4\xi^2(1-\xi)^2Q^2\left(\frac{1}{k_\perp^2+\epsilon_f^2}-\frac{1}{(k_\perp-q_\perp)^2+\epsilon_f^2}\right)^2$$

photon L

The x evolution of the u-pdf

- the Balitsky-Kovchegov (BK) evolution

Balitsky (1996), Kovchegov (1998)

$$\frac{d}{dY} f_Y(k) = \bar{\alpha} \underbrace{\int \frac{dk'^2}{k'^2}\left[\frac{k'^2 f_Y(k') - k^2 f_Y(k)}{|k^2 - k'^2|} + \frac{k^2 f_Y(k)}{\sqrt{4k'^4 + k^4}}\right]}_{\text{BFKL}} - \bar{\alpha} f_Y^2(k)$$

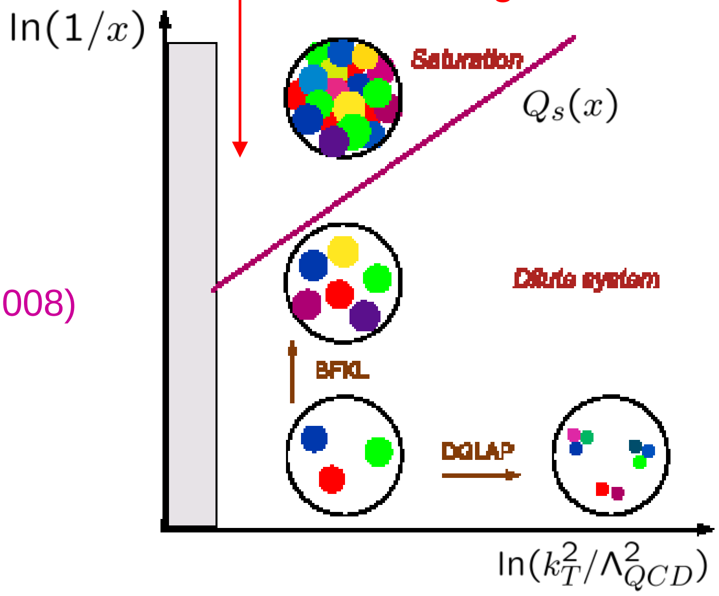
$$Y = \ln\left(\frac{1}{x}\right)$$

$$\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

here $f_Y(k)$ is not exactly the u-pdf but a slightly modified F.T. of $T_{q\bar{q}}$

$$f_Y(k) = \int \frac{d^2r}{2\pi r^2}\, e^{ik.\mathbf{r}}\, T_{q\bar{q}}(\mathbf{r}, Y)$$

non-linearity important when the gluon density becomes large



BK evolution at NLO has been recently calculated

Balitsky-Chirilli (2008)

- in the saturation regime

the evolution of the u-pdf becomes non-linear

in general cross sections become non-linear functions of the gluon distribution

however, SIDIS is a special case in which the kT-factorization formula written previously still holds

the distribution of partons as a function of *x* and k_T

Large-Q^2 limit of small-x result

- keeping the leading $1/Q^2$ term:

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}}\Big|_{P_\perp^2\ll Q^2}=\frac{\alpha_{em}^2N_c}{2\pi^3Q^4x_B}\sum_f e_f^2\left(1-y+\frac{y^2}{2}\right)\frac{D(z_h)}{z_h^2}\int d^2bd^2q_\perp F(q_\perp,x_B)A(q_\perp,k_\perp=P_\perp/z_h)$$

simple function

only transverse photons

$$A(q_\perp,k_\perp)=\int d\xi\left|\frac{k_\perp|k_\perp-q_\perp|}{(1-\xi)k_\perp^2+\xi(k_\perp-q_\perp)^2}-\frac{k_\perp-q_\perp}{|k_\perp-q_\perp|}\right|^2$$

- the saturation regime can still be probed

the cross section above has contributions to all orders in $Q_s^2/P_\perp^2$

even if Q^2 is much bigger than Q_s^2, the saturation regime will be important when $P_\perp^2\sim Q_s^2$

in fact, thanks to the existence of Q_s, the limit $|P_\perp|\to 0$ is finite, and computable with weak-coupling techniques ($Q_s\gg\Lambda_{QCD}$)

eventually true at small x

SIDIS in the large-Q^2 limit

TMD factorization

- the cross section can be factorized in 4 pieces

Collins and Soper (1981), Collins, Soper and Sterman (1985), Ji, Ma and Yuan (2005)

$$\frac{d\sigma(ep \to e'hX)}{d\mathcal{P}} = \frac{4\pi\alpha_{em}^2}{Q^2}\left(1-y+\frac{y^2}{2}\right)\int d^2k_\perp d^2p_{1\perp} d^2\lambda_\perp$$

$$q(x_B, k_\perp; x_B\zeta) D(z_h, p_{1\perp}; \hat{\zeta}/z_h) \longrightarrow$$

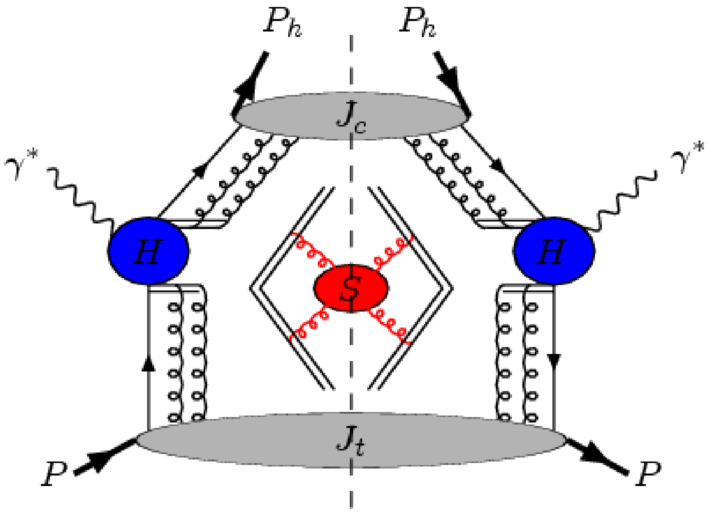
TMD ff

TMD quark distribution

$$S(\lambda_\perp;\rho) H(Q^2, x_B, z_h; \rho)\delta^{(2)}(z_h k_\perp + p_{1\perp} + \lambda_\perp - p_\perp)$$

soft factor

hard part



valid to leading power in $1/Q^2$ and to all orders in α_s

(the gluon TMD piece is power-suppressed)

however we shall only discuss the leading α_s order

The TMD quark distribution

- operator definition

$$q(x,k_\perp)=\frac{1}{2}\int\frac{d^2\xi_\perp d\xi^-}{(2\pi)^2}e^{-ixP^+\xi^- -ik_\perp\cdot\xi_\perp}\langle P|\bar{\Psi}(\xi)\mathcal{L}_\xi\gamma^+\mathcal{L}_0\Psi(0)|P\rangle$$

quark fields also have transverse separation

Wilson lines needed for gauge invariance

- how factorization works

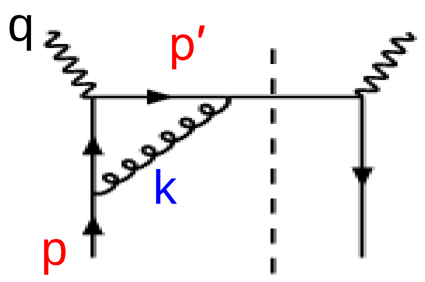
possible regions for the gluon momentum

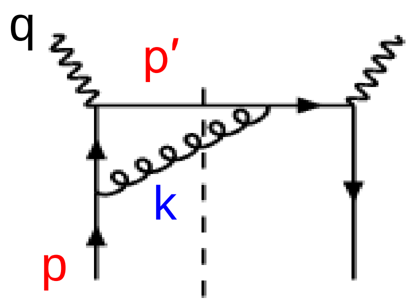
k collinear to p (parton distribution)

k collinear to p’ (parton fragmentation)

k soft (soft factor)

k hard (α_s correction)



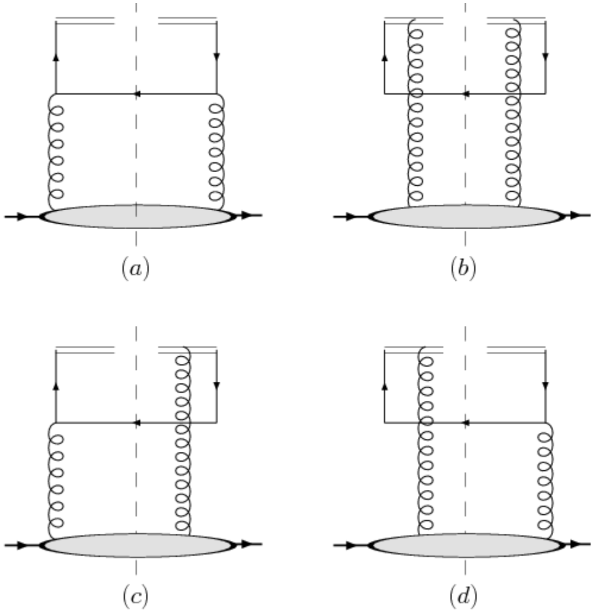


Small-x limit of large-Q^2 result

- at small-x, the leading contribution reads:

$$\frac{d\sigma(ep\to e'hX)}{d\mathcal{P}}\Big|_{x_B\ll 1}=\frac{4\pi\alpha_{em}^2}{Q^4}\sum_f e_f^2\left(1-y+\frac{y^2}{2}\right)\frac{D(z_h)}{z_h^2}\;q(x_B,P_\perp/z_h)$$

- and the TMD quark distribution comes from gluon splitting



$$xq(x,k_\perp)=\frac{N_c}{8\pi^4}\int d^2b\,d^2q_\perp F(q_\perp,x)A(q_\perp,k_\perp)$$

gluon distribution (a priori two-gluon exchange)

gluon to quark splitting

however, comparison with the small-x calculation shows that saturation/multiple scatterings can be included in this TMD formula, simply by calculating $F(q_\perp,x)$ to all orders in $Q_s^2/P_\perp^2$

The Q^2 evolution of the TMD-pdf

- the Collins-Soper-Sterman (CSS) evolution

Collins, Soper and Sterman (1985)

or how the TMD-pdf changes with the increase of the factorization scale $x_B\varsigma$, which in practice is chosen to be Q

- in the small-x limit

Idilbi, Ji, Ma and Yuan (2004)

the evolution simplifies (double leading logarithmic approximation)

$$q(x,k_\perp;Q^2)=\int\frac{d^2r}{(2\pi)^2}e^{ik_\perp\cdot r}e^{-S(Q^2,Q_0^2,r)}\int d^2k'_\perp e^{-ik'_\perp\cdot r}q(x,k'_\perp;Q_0^2)$$

$$S(Q^2,Q_0^2,r)=\ln\frac{Q^2}{Q_0^2}\left[\frac{\alpha_sC_F}{4\pi}\ln(Q^2Q_0^2r^4)+c_0r^2\right]$$

DLLA

non-perturbative contribution

Korchemsky and Sterman (1995)

Equivalence between TMD and k_T factorizations in SIDIS

TMD-pdf / u-pdf relation

- at small *x* and large Q^2

the two results for the SIDIS cross section are identical, with

$$xq(x,k_\perp) = \frac{N_c}{4\pi^4}\int d^2b d^2q_\perp F(q_\perp,x)\left[1-\frac{k_\perp\cdot(k_\perp-q_\perp)}{k_\perp^2-(k_\perp-q_\perp)^2}\ln\left(\frac{k_\perp^2}{(k_\perp-q_\perp)^2}\right)\right]$$

TMD-pdf

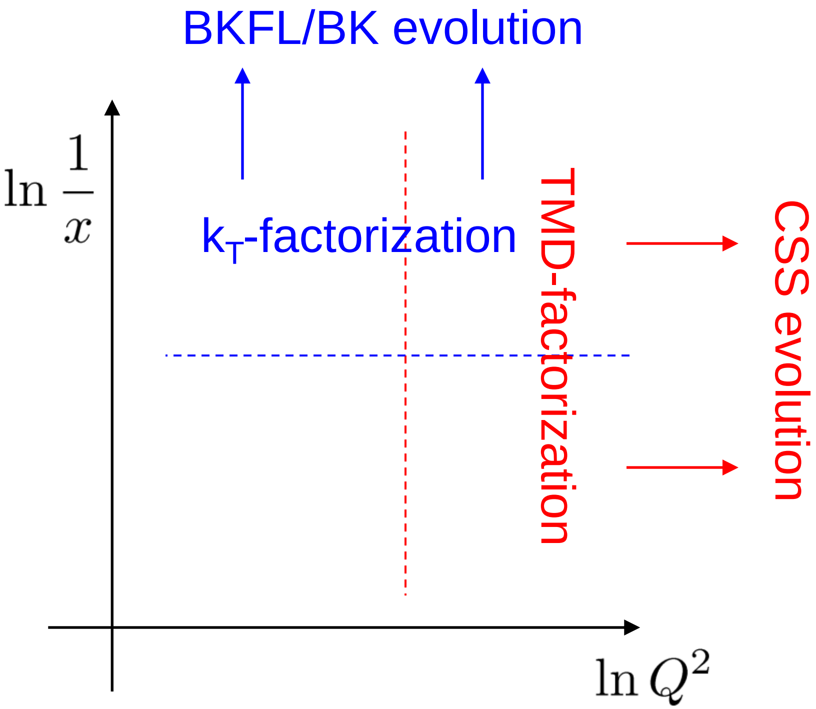
u-pdf

in the overlaping domain of validity,
TMD & kT factorization are consistent

- the saturation regime

the TMD factorization can be used in the saturation regime, when $k_\perp^2 \sim Q_s^2$

there $xq(x,k_\perp) \rightarrow \text{const.}$



x evolution of the TMD-pdf

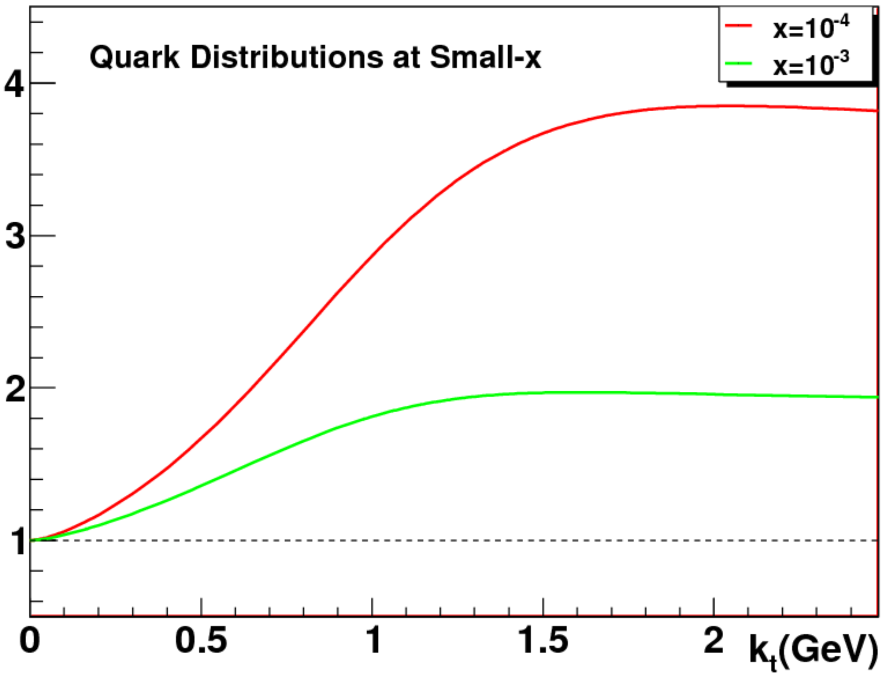
- from small *x* to smaller *x*

$$\frac{xq(x,k_\perp)}{x_0 q(x_0,k_\perp)}$$

$$x_0 = 10^{-2}$$

at small k_t

$$xq(x,0) = c$$



at large k_t

$$\frac{Q_s^2(x)}{Q_s^2(x_0)}$$

not full BK evolution here, but GBW parametrization

$$F(q_\perp, x) = e^{-q_\perp^2/Q_s^2(x)}/Q_s^2(x) \qquad Q_s^2(x) = (3.10^{-4}/x)^{0.28}\ \mathrm{GeV}^2$$

Golec-Biernat and Wusthoff (1998)

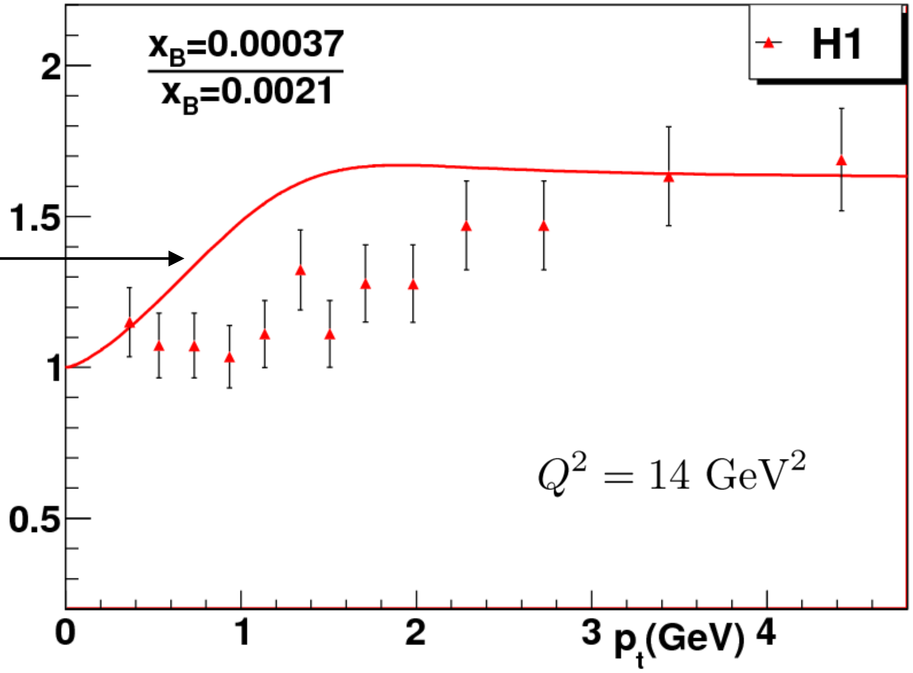
HERA data probe saturation

- ratio of SIDIS cross sections at two different values of *x*

H1 collaboration (1997)

$$\left.\frac{d\sigma(ep\rightarrow e'hX)}{d\mathcal{P}}\right|_{\substack{x_B\ll 1\\ Q^2\gg P_\perp^2}} \simeq q(x_B,P_\perp)$$

our (crude) calculation



one can do much better with actual BK evolution and quark fragmentation

the data show the expected trend

- at future EIC's

the SIDIS measurement provides direct access to the transverse momentum distribution of partons in the proton/nucleus, and the saturation regime can be easily investigated

Q^2 evolution of the TMD-pdf

- the GBW parametrization at 10 GeV^2 evolved to larger Q^2

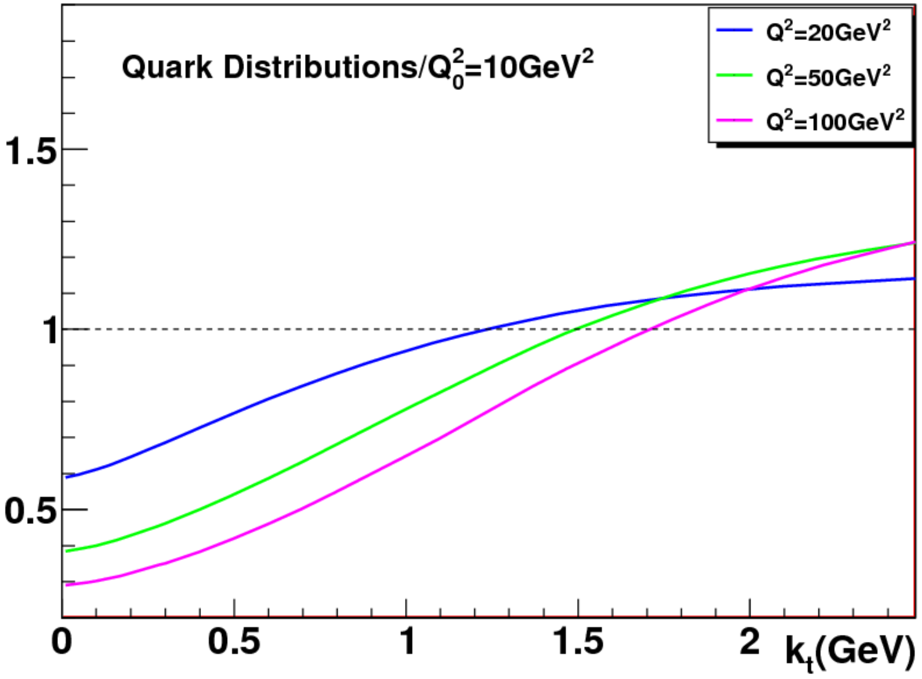
$$xq(x,k_\perp,Q_0^2) = \frac{N_c}{4\pi^4}\int d^2b\,d^2q_\perp F(q_\perp,x)\left[1-\frac{k_\perp\cdot(k_\perp-q_\perp)}{k_\perp^2-(k_\perp-q_\perp)^2}\ln\left(\frac{k_\perp^2}{(k_\perp-q_\perp)^2}\right)\right]$$

$$e^{-q_\perp^2/Q_s^2(x)}/Q_s^2(x)$$

$$x = 3.10^{-4}$$

not full CSS evolution but DLLA

the transverse momentum distribution becomes harder when Q^2 increases



Breaking of TMD and k_T factorizations in di-jet production

C.M., Venugopalan, Xiao and Yuan, work in progress

TMD factorization at large Q^2 ?

- non-universality of the TMD-pdf

the TMD distributions involved in di-jet production and SIDIS are different

Bacchetta, Bomhof, Mulders and Pijlman (2005)
Collins and Qiu, Vogelsang and Yuan (2007)
Rogers and Mulders, Xiao and Yuan (2010)

breaking of TMD factorization:

one cannot use information extracted

from one process to predict the other

in this approach the breaking of TMD factorization is a problem

- is there a better approach ? at small-x, maybe yes

in the Color Glass Condensate (CGC)/dipole picture, we also notice that k_T factorization is broken, but this is not an obstacle

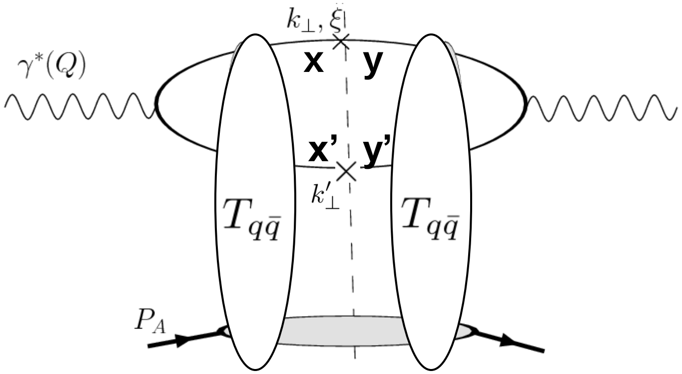
we can consistently bypass the problem,
and define improved pdfs to recover universality

k_T factorization at small-x ?

- the di-jet cross section in the dipole picture

$$\frac{d\sigma_{T,L}^{\gamma^* p\to q\bar q X}}{d^2k_\perp d^2k'_\perp} = \int \frac{d^2x}{2\pi}\frac{d^2y}{2\pi}\frac{d^2x'}{2\pi}\frac{d^2y'}{2\pi}\, e^{-ik_\perp\cdot(\mathbf{x}-\mathbf{y})} e^{-ik'_\perp\cdot(\mathbf{x}'-\mathbf{y}')}\int d\xi\;\Phi_{T,L}(\xi,\mathbf{x}-\mathbf{x}',\mathbf{y}-\mathbf{y}';Q^2)$$
$$\times\left[T_{q\bar q}(\mathbf{x}-\mathbf{x}',x_B)+T_{q\bar q}(\mathbf{y}-\mathbf{y}',x_B)-T_{q\bar q\bar q q}(\mathbf{x},\mathbf{x}',\mathbf{y}',\mathbf{y},x_B)\right]$$

because of the 4-point function $T_{q\bar q\bar q q}$, there is no k_T factorization (unless saturation and multiple scatterings can be safely neglected)



- SIDIS was a special case

in SIDIS, the $k'_\perp$ integration sets **x'=y'**,
and then $T_{q\bar q\bar q q}(\mathbf{x},\mathbf{x}',\mathbf{x}',\mathbf{y},x_B)=T_{q\bar q}(\mathbf{x}-\mathbf{y},x_B)$

this cancellation of the interactions involving the spectator antiquark in SIDIS is what led to k_T factorization

with dijets, this does not happen, and as expected,
the cross section is a non-linear function of the u-pdf

Can the CGC rescue the OPE ?

- on the breaking of k_T factorization at small-x

this breaking of k_T factorization is expected, understood, and can be bypassed

a more involved factorization should be used, with more a appropriate description of the parton content of the proton (in terms of classical fields)

one can still use information extracted
from one process to predict the other

- can this understanding help us with the TMD-pdf problem ?

expanding the small-x di-jet cross section at large Q^2,
one should be able to identify a TMD quark distribution

we expect that this TMD-pdf will be different from the one obtained in SIDIS
(we should recover the non universality)

however the calculation will show us how to compute one from the other,
and therefore show us how to work around the TMD-factorization breaking

Conclusions

- considering the SIDIS process, we have shown that

TMD factorization (valid at large Q^2)
and k_T factorization (valid at small x)

are consistent with each other in the overlaping domain of validity

- the SIDIS measurement provides direct access to the transverse momentum distribution of partons

the saturation regime, characterized by $Q_s^2 \simeq \Lambda_{QCD}^2 (A/x)^{1/3}$,
can be easily investigated

even if Q^2 is much bigger than Q_s^2,
the saturation regime will be important when $P_\perp^2 \sim Q_s^2$

- this is an encouraging start, but now we would like to understand the relations between TMD and k_T factorization breaking

k_T factorization breaking at small x is no obstacle, so perhaps we can learn from the CGC how to work around the TMD factorization breaking