Higher-order predictions for large-x splitting functions and coefficient functions[1]

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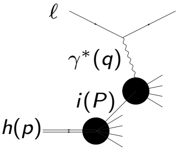
[1] arXiv:0912.0369 NP B 832 (2010) 152-227 (G.S, Moch, Vermaseren, Vogt) and to appear (G.S, Vogt)

Outline

- Introduction: Deep Inelastic Scattering in perturbative QCD
- Large-x double-log behaviour of splitting and coefficient functions
- Singlet physical evolution kernels: single-logarithmic enhancement
  - (F_2, F_ϕ): $c^{(3)}_{2,i}, c^{(3)}_{\phi,i} \longrightarrow$ prediction of highest 3 logarithms of $P^{(3)}_{qg,gq}$
  - (F_2, F_L): $P^{(3)}_{qg,gq} \longrightarrow$prediction of highest 3 logs of $c^{(3)}_{L,i}$
- Derivation of the leading double-logarithmic contributions to 'off-diagonal' splitting and coefficient functions to all orders
- Summary & Outlook

Deep Inelastic Scattering

- Inclusive lepton-hadron DIS $\longrightarrow$ partonic content of hadrons.



Bjorken variable: $x = Q^2/(2p \cdot q)$

Parton momentum: $P = \xi p$

Resolution: $Q^2 = -q^2$

- $$F_a(\xi, Q^2) = C_{a,i}\left(\alpha_s(\mu^2, \mu^2/Q^2)\right) \otimes f_i(\xi, \mu^2)$$

  μ: renormalization/mass-fact. scale. $\otimes$: Mellin convolution
  $f_i(\eta, Q^2)$: momentum distribution functions for (anti-)quarks and gluons
  $\longrightarrow$ Generally not defined in terms of physical process

- PDFs at different scales related by *evolution equations*

$$\frac{d}{d \ln \mu^2} f_i(\xi, \mu^2) = P_{ij}(\xi, \mu^2) \otimes f_j(\xi, \mu^2)$$

- P_{ij}: splitting function: perturbation series in the couplings

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \alpha_s^4 P^{(3)} + \dots$$

- Coefficient functions ($n_a = 0, 1$ for $F_{2/\phi}$ and F_L respectively)

$$C_a = \alpha_s^{n_a}\left[c_a^{(0)} + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \alpha_s^3 c_a^{(3)} + \dots\right]$$

Large-x Logarithms in Splitting and Coefficient Functions

- In the modified minimal subtraction ($\overline{\text{MS}}$) scheme:

  $P^{(\ell)}_{qq,gg}$: terms up to $(1-x)^{-1}$

  $P^{(\ell)}_{qg,gq}$: terms up to $\ln^{2\ell}(1-x)$

  $c^{(\ell)}_{2,q/\phi,g}$: terms up to $(1-x)^{-1}\ln^{2\ell-1}(1-x)$

  $c^{(\ell)}_{2,g/\phi,q}$: terms up to $\ln^{2\ell-1}(1-x)$

  $c^{(\ell)}_{L,q}$: terms up to $\ln^{2\ell}(1-x)$

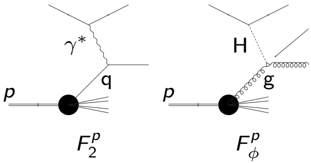
  $c^{(\ell)}_{L,g}$: terms up to $(1-x)\ln^{2\ell}(1-x)$

  $\longrightarrow$ stable diagonal splitting functions at large-x
- Everything else: double logarithmic enhancement at large-x

- Alternative approach:
  - Eliminate partons from description of structure function
    $\longrightarrow$ dependence on Q^2 then given in terms of Physical Evolution Kernels
  - Preview: Kernels can be shown to only contain single logs
- Advantage: Single-log behaviour of physical kernels $\longrightarrow$ prediction of double-log terms in higher order splitting and coefficient functions!

Singlet Physical Evolution Kernel for (F_2, F_ϕ)

[Furmanski, Petronzio (82)]

- 

effective Higgs-coupling $\sim G^{\mu\nu} G_{\mu\nu}$ in heavy-top limit

$$\mathbf{F} = \begin{pmatrix} F_2 \\ F_\phi \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} P_{\mathrm{qq}} & P_{\mathrm{qg}} \\ P_{\mathrm{gq}} & P_{\mathrm{gg}} \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} C_{2,\mathrm{q}} & C_{2,\mathrm{g}} \\ C_{\phi,\mathrm{q}} & C_{\phi,\mathrm{g}} \end{pmatrix}$$

- Perform scheme transformation through transformation matrix Z with

$$Z_{ik} = \delta_{ik} + \sum_{\ell=1}^{\infty} a_s^\ell Z_{ik}^{(\ell)}, \quad a_s \equiv \frac{\alpha_s}{4\pi}$$

- Results in transformed coefficient functions $\widetilde{C}$ and parton distributions $\widetilde{q}$

$$F = C \cdot q = CZ^{-1} \cdot Zq = \widetilde{C} \cdot \widetilde{q}$$

$\longrightarrow$ transformed splitting functions

$$\mathbf{P}' = \mathbf{P} + \left(\beta \frac{d\mathbf{Z}}{da_s} + [\mathbf{Z}, \mathbf{P}]\right) \mathbf{Z}^{-1}$$

- Choose $Z = C$: P'_{ij} are Physical Evolution Kernels, K_{ij}

Singlet Physical Evolution Kernel for (F_2, F_ϕ)

- Matrix evolution equation for singlet structure function

$$\frac{d}{d\ln Q^2} F = \mathbf{K}\, F \equiv \sum_{\ell=0}^{\infty} a_s^{\ell+1} \begin{pmatrix} K_{22}^{(\ell)} & K_{2\phi}^{(\ell)} \\ K_{\phi 2}^{(\ell)} & K_{\phi\phi}^{(\ell)} \end{pmatrix} \cdot \begin{pmatrix} F_2 \\ F_\phi \end{pmatrix}$$

- Large-x behaviour at NLO, NNLO $\rightarrow$ single logarithmic enhancement:
  $K_{22,\phi\phi}^{(\ell)} \sim (1-x)^{-1} \ln^{(\ell)}(1-x)$, $K_{2\phi,\phi 2}^{(\ell)} \sim (1-x)^{0} \ln^{(\ell)}(1-x)$
  Conjecture: persists to N^3LO at least (c.f. non-singlet (NS) case)
- $\rightarrow$ Prediction of $\ln^{6,5,4}(1-x)$ of $P_{qg,gq}^{(3)}$ (and $\ln^{5,4,3}(1-x)$ of $P^{(3)}_{ps,gg|_{C_F}}$)
- Example ($(1-x)^0$ part (for brevity), $C_{AF} = C_A - C_F$):

$$\begin{aligned} P_{\rm qg}^{(3)}(x) = {} & \ln^6(1-x) \cdot 0 \\ & + \ln^5(1-x)\Big[\frac{22}{27} C_{AF}^3 n_f - \frac{14}{27} C_{AF}^2 C_F n_f - \frac{4}{27} C_{AF}^2 n_f^2 \Big] \\ & + \ln^4(1-x)\Big[\Big(\frac{293}{27} - \frac{80}{9}\zeta_2\Big) C_{AF}^3 n_f + \Big(\frac{4477}{162} - 8\zeta_2\Big) C_{AF}^2 C_F n_f \\ & - \frac{13}{81} C_{AF} C_F^2 n_f - \frac{116}{81} C_{AF}^2 n_f^2 + \frac{17}{81} C_{AF} C_F n_f^2 - \frac{4}{81} C_{AF} n_f^3 \Big] \\ & + \mathcal{O}\left(\ln^3(1-x)\right) \end{aligned}$$

$\rightarrow$ Extension to all orders in $(1-x)$: see arXiv:0912.0369

(F_2, F_L) Physical Kernels

[Catani(97), Blümlein, Ravindran and van Neerven(00)]

- Recall: For (F_2, F_ϕ) system, transformation matrix

$$\mathbf{Z}^{(\ell)}(\mathbf{N}) = \begin{pmatrix} C_{2,q} & C_{2,g} \\ C_{\phi,q} & C_{\phi,g} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \sum_{\ell=1} \mathbf{a}_s^\ell \begin{pmatrix} c_{2,q}^{(\ell)} & c_{2,g}^{(\ell)} \\ c_{\phi,q}^{(\ell)} & c_{\phi,g}^{(\ell)} \end{pmatrix}$$

- For (F_2, F_L) system, transformation matrix

$$\mathbf{Z}^{(\ell)}(\mathbf{N}) = \begin{pmatrix} C_{2,q} & C_{2,g} \\ \widehat{C}_{L,q} & \widehat{C}_{L,g} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & \widehat{c}_{L,g}^{(0)} \end{pmatrix} + \sum_{\ell=1} \mathbf{a}_s^\ell \begin{pmatrix} c_{2,q}^{(\ell)} & c_{2,g}^{(\ell)} \\ \widehat{c}_{L,q}^{(\ell)} & \widehat{c}_{L,g}^{(\ell)} \end{pmatrix}$$

  where $\widehat{C}_{L,i} = C_{L,i}/a_s c_{L,q}^{(0)}$, hence $\widehat{C}_{L,q}^{(k)} \sim \ln^k N$, $\widehat{C}_{L,g}^{(k)} \sim N^{-1} \ln^k N$. (Normalization differs from [BRvN(00)])

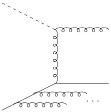
- Physical evolution kernels $\to$ single logarithmic enhancement to NNLO also!
- $P_{qg}^{(3)}$ predictions from (F_2, F_ϕ) + physical kernel single-log enhancement of $(F_2, F_L) \to \ln^{6,5,4}(1-x)$ predictions of 4-loop coefficient function $c_{L,i}^{(3)}$.
- Agrees with/extends NS treatment of [Moch, Vogt (09)] ($C_F = 0$ part only for $C_{L,g}$)

Leading Double Logarithms To All Orders

- Single-log enhancement from cancellation of double-log quantities of $\overline{\text{MS}}$ splitting and coefficient functions $\longrightarrow$ all-order relation for $\{P^{(n)}, c^{(n)}\}$
- Quantities extracted order by order in α_s from ϵ-expansion of unfactorized photon/Higgs-parton forward scattering amplitudes.
- ϵ^{ℓ}, $\ell < -1$: lower order terms. ϵ^{-1}: contains new split. function.
  ϵ^{0}: contains new coeff. function. ϵ^{ℓ}, $\ell > 0$: required for higher orders.
- LL terms in $T^{(n)}_{\phi,q}$ and $T^{(n)}_{2,g}$ at any power of ϵ:

  $T^{(n)}_{\phi,q}$: $L_{n,\ell} C_F (C_F^{n-1} + C_F^{n-2} C_A + \ldots C_A^{n-1}) \epsilon^{-n+\ell} \frac{1}{N} \ln^{n+\ell+1} N$

  $T^{(n)}_{2,g}$: $L_{n,\ell} n_F (C_F^{n-1} + C_F^{n-2} C_A + \ldots C_A^{n-1}) \epsilon^{-n+\ell} \frac{1}{N} \ln^{n+\ell+1} N$
- Same $L_{n,\ell}$ for both cases $\longrightarrow$ can infer LL terms of $P^{(n)}_{qg}$, $P^{(n)}_{gq}$ and $C_{2,g}, C_{\phi,q}$ from just one of the colour factors!
- Consider $\alpha_s^n C_F^n$ (i.e. pure-C_F) contributions to $T_{\phi,q}$ in Mellin N-space $\longrightarrow$ used for extraction of P_{gq} and $C_{\phi,q}$:



$$T^{(n)}_{\phi,q} \overset{LL}{=} T^{(1)}_{\phi,q} T^{(n-1)}_{2,q} \frac{1}{n}$$

$$\overset{LL}{=} T^{(1)}_{\phi,q} (T^{(1)}_{2,q})^{n-1} \frac{1}{(n-1)!} \frac{1}{n}$$

$$= T^{(1)}_{\phi,q} (T^{(1)}_{2,q})^{n-1} \frac{1}{n!}$$

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$$\longrightarrow \quad T_{\phi,q} \overset{LL}{=} \frac{T^{(1)}_{\phi,q}}{T^{(1)}_{2,q}} [e^{\alpha_s T^{(1)}_{2,q}} - 1] \quad (*)$$

Leading Double Logarithms To All Orders

- $T^{(1)}_{\phi,q}$ and $T^{(1)}_{2,q}$ leading-log contributions known to ALL powers in ϵ.

$$T^{(1)}_{\phi,q}(x) \stackrel{LL}{=} -2C_F \frac{1}{\epsilon}(1-x)^{-\epsilon} \stackrel{M-trf.}{\longrightarrow} -\frac{2}{N} C_F \frac{1}{\epsilon} e^{\epsilon \ln N}$$

$$T^{(1)}_{2,q}(x) \stackrel{LL}{=} -4C_F \frac{1}{\epsilon}(1-x)^{-1-\epsilon} + virt.corr. \stackrel{M-trf.}{\longrightarrow} 4C_F \frac{1}{\epsilon^2}(e^{\epsilon \ln N} - 1)$$

- Through mass factorization, $T = CZ$, can calculate first 4 powers in ϵ of $T_{\phi,q}$ at all-orders in α_s $\rightarrow$ assume $(*)$ holds to all powers in ϵ.
- Mass Factorization $T_{\phi,q} = C_{\phi,q} Z_{qq} + C_{\phi,g} Z_{gq}$
- Elimination of all non-contributing terms to the LL simplifies problem:
  - $P^{(n)}_{qq,gg} \sim \ln N + \ldots \rightarrow$ drop $P^{(n>0)}_{qq,gg}$
  - $P^{(n)}_{qg,gq} \sim 1/N \rightarrow$ only one off-diagonal splitting function per term
  - $\rightarrow$ all-order expression for double-log Z_n with $P = \left(\frac{d}{d \ln \mu^2} Z\right) Z^{-1}$
- $\rightarrow$ Extract $P^{(n)}_{gq}$, $c^{(n)}_{\phi,q}$ and $\epsilon^{\ell>0}$ terms order by order:
  $P^{(2n+1)}_{gq} = 0$ for $n \geq 1$, $P^{(2n)}_{gq} = \tilde{L}_n C_F (C_A - C_F)^{2n}$, $C_{\phi,q} \neq 0 \; \forall n$
- Results explain 'mysterious' accidental disappearance of $\ln^6(1-x)$ in $P^{(3)}_{qg}$ $\rightarrow$ occurs at every odd power. $\tilde{L}_4 = -4/135$, $\tilde{L}_6 = 16/42525$, ...

The (F_2, F_ϕ) Physical Kernel Revisited

- Physical kernel given by $\mathbf{K} = \mathbf{ZPZ}^{-1}$ where

$$\mathbf{Z_n} = \begin{pmatrix} C_{2,q}^{(n)} & C_{2,g}^{(n)} \\ C_{\phi,q}^{(n)} & C_{\phi,g}^{(n)} \end{pmatrix}, \quad \mathbf{Z}^{-1} = \frac{\mathbf{1}}{\mathbf{Z_{qq}Z_{gg}}} \begin{pmatrix} Z_{gg} & -Z_{qg} \\ -Z_{gq} & Z_{qq} \end{pmatrix},$$

$$\mathbf{P^{(0)}} = \begin{pmatrix} P_{qq}^{(0)} & P_{qg}^{(0)} \\ P_{gq}^{(0)} & P_{gg}^{(0)} \end{pmatrix}, \quad \mathbf{P^{(n)}} = \begin{pmatrix} 0 & P_{qg}^{(n)} \\ P_{gq}^{(n)} & 0 \end{pmatrix}$$

- Leading double logs of off-diagonal physical kernels vanish for

$$\frac{1}{C_{\phi,g}}\left[C_{2,q}P_{qg} + C_{2,g}(P_{gg}^{(0)} - P_{qq}^{(0)})\right] = 0$$
$$\frac{1}{C_{2,q}}\left[C_{\phi,g}P_{gq} + C_{\phi,q}(P_{qq}^{(0)} - P_{gg}^{(0)})\right] = 0 \quad (*)$$

- Splitting and coefficient functions extracted from $T_{\phi,q}$, $T_{2,g}$(unrelated to physical kernel): $(*)$ holds to all orders in α_s
$\rightarrow$ first step to proof of physical kernel single-log enhancement that was conjectured.

Summary & Outlook

- $\overline{MS}$ splitting and coefficient functions: double-log enhanced (except $P_{qq,gg}$)
- Singlet physical evolution kernels for (F_2, F_ϕ) and (F_2, F_L): single-log behaviour (proven to NNLO and conjectured to all orders in α_s)
- Prediction for $\ln^{6,5,4}(1-x)$ in N^3LO splitting functions $P^{(3)}_{qg,gq}$ (from (F_2, F_ϕ)) and coefficient functions $c^{(3)}_{L,i}$ (from (F_2, F_L))
- $\rightarrow$ Alone not sufficient for phenomenology, but important for future approximations based on fixed-N moments (c.f. van Neerven, Vogt (00), first N^3LO moment calculation: Baikov, Chetyrkin (06))
- Iterative structure of leading double-log contributions uncovered for $T_{\phi,q}$ and $T_{2,g}$ $\rightarrow$ all-order coefficients of $P_{qg,gq}$ and $C_{2,g}$, $C_{\phi,q}$
  $\rightarrow$ Confirms physical kernel conjecture
- At least partial extension to next one (or two?) logarithms should be possible $\rightarrow$ work in progress.