Multiple interactions and generalized parton distributions

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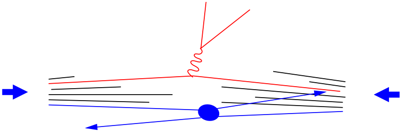
Multi-parton interactions

- generically take place in hadron-hadron collisions
- effects average out in sufficiently inclusive quantities
  but do affect final state properties

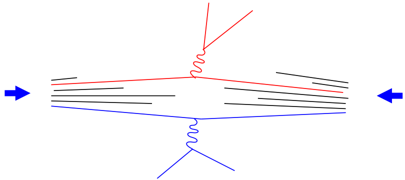


- estimated to be important for many LHC processes
  see e.g. Procs. of the Workshop on HERA and the LHC, 2005 and 2008
- many studies in the literature (theory + experiment)
  but so far no systematic derivation in QCD
- this talk: discuss some steps in this direction

work in progress with A. Schäfer

Theoretical framework

- consider gauge boson pair production (pairs of γ^*, W, Z)



- keep transverse gauge boson momentum differential
  - since are interested in final-state details
  - need k_T dependent parton distributions

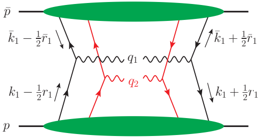
talk in final-state session, Tue 11:15:

power counting, perturbative dynamics, evolution, Sudakov effects

this talk:

spin aspects, relation with generalized parton distributions

Basic structure: cross section



- cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \Bigg[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i \bar{x}_i s\big) \Bigg]$$

$$\times \Bigg[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i) \Bigg] \int d^2\boldsymbol{y}_1\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_i)\, \bar{F}(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y}_i)$$

$\hat{\sigma}_i =$ parton-level cross section

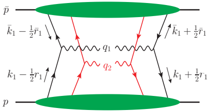
$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) =$ k_T dependent two-parton distribution

$\boldsymbol{y}_1 =$ Fourier conjugate to momentum differences $\boldsymbol{r}_1$ and $\bar{\boldsymbol{r}}_1$

- result follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation involved
- $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2$ in cross sect. $\rightarrow$ collinear distributions

$$F(x_i, \boldsymbol{y}_1) = \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)$$

Operator definitions



- k_T dependent distribution

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$
$$\times\, 2p^+ \int dy_1^- \, \langle p| \bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2 \, q\big(\tfrac{1}{2}z_2\big) \, \bar{q}\big(y_1 - \tfrac{1}{2}z_1\big)\Gamma_1 \, q\big(y_1 + \tfrac{1}{2}z_1\big)|p\rangle \Big|_{z_i^+ = y_1^+ = 0}$$

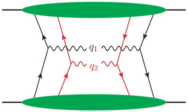
Wilson lines to be inserted as for k_T dependent single-parton distr's

- collinear distributions

$$F(x_i, \boldsymbol{y}_1) = \int \frac{dz_2^-}{2\pi} \, e^{ix_2 z_2^- p^+} \int \frac{dz_1^-}{2\pi} \, e^{ix_1 z_1^- p^+}$$
$$\times\, 2p^+ \int dy_1^- \, \langle p| \bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2 \, q\big(\tfrac{1}{2}z_2\big) \, \bar{q}\big(y_1 - \tfrac{1}{2}z_1\big)\Gamma_1 \, q\big(y_1 + \tfrac{1}{2}z_1\big)|p\rangle \Big|_{\substack{z_i^+ = y_1^+ = 0 \\ \boldsymbol{z}_i = \boldsymbol{0}}}$$

still $\boldsymbol{y}_1 \neq \boldsymbol{0}$ $\Rightarrow$ finite transverse distance between two partons
$\Rightarrow$ not a twist-four operator
but product of two twist-two operators

Spin structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2 \boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1 \boldsymbol{k}_1}$$

$$\times \, 2p^+ \int dy_1^- \, \langle p| \bar{q}(-\tfrac{1}{2} z_2) \Gamma_2 \, q(\tfrac{1}{2} z_2) \, \bar{q}(y_1 - \tfrac{1}{2} z_1) \Gamma_1 \, q(y_1 + \tfrac{1}{2} z_1) |p\rangle$$

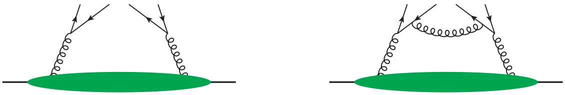
- at leading twist for quarks: $\Gamma_i = \frac{1}{2}\gamma^+, \; \frac{1}{2}\gamma^+\gamma_5, \; \frac{1}{2}\sigma^{+\alpha}$
- spin correlations even in unpolarized target, e.g.

  $$\Gamma_1 = \Gamma_2 = \tfrac{1}{2}\gamma^+\gamma_5 \quad \Leftrightarrow \quad q_1^{\uparrow} q_2^{\uparrow} + q_1^{\downarrow} q_2^{\downarrow} - q_1^{\uparrow} q_2^{\downarrow} - q_1^{\downarrow} q_2^{\uparrow}$$

  note: not suppressed by hard scattering in double Drell-Yan
- transverse spin correlations from $\Gamma_1 = \Gamma_2 = \frac{1}{2}\sigma^{+\alpha}$
  $\rightsquigarrow$ correlated decay planes of the two bosons
- to parameterize $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)$ for unpol. target: 30 scalar functions
  still 8 functions to parameterize $F(x_i, \boldsymbol{y}_1)$
- expect spin effects to decrease for small x_1, x_2
  but there is a counter-example:

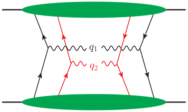
Short-distance dynamics

- consider region $\Lambda \ll q_T \ll Q$, where $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent distr'n = hard scattering $\otimes$ collinear distr'n
  hard scattering closely related to DGLAP splitting functions
- for $|\boldsymbol{r}_1| \sim |\boldsymbol{q}_i|$, i.e. $|\boldsymbol{y}| \ll 1/\Lambda$ small
  (is not the generic case, but part of cross section)



equal contributions to $\Gamma_1 = \Gamma_2 = \frac{1}{2}\gamma^+$ and $\Gamma_1 = \Gamma_2 = \frac{1}{2}\gamma^+\gamma_5$
$\rightsquigarrow$ q and $\bar{q}$ polarizations 100% correlated

Color structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$
$$\times\, 2p^+ \int dy_1^- \, \langle p | \bar{q}(-\tfrac{1}{2}z_2)\Gamma_2\, q(\tfrac{1}{2}z_2)\, \bar{q}(y_1 - \tfrac{1}{2}z_1)\Gamma_1\, q(y_1 + \tfrac{1}{2}z_1)|p\rangle$$

- operators $\bar{q}_2\, q_2$ and $\bar{q}_1\, q_1$ can couple to color singlet or octet:

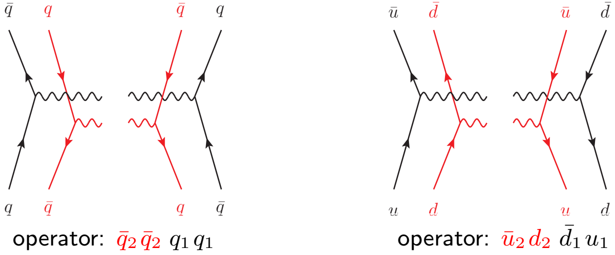
$$F_1 \longrightarrow (\bar{q}_2 \mathbb{1} q_2)\,(\bar{q}_1 \mathbb{1} q_1)$$
$$F_8 \longrightarrow (\bar{q}_2 t^a q_2)\,(\bar{q}_1 t^a q_1) = \tfrac{1}{2}(\bar{q}_2 \mathbb{1} q_1)\,(\bar{q}_1 \mathbb{1} q_2) - \tfrac{1}{2N_c}\,(\bar{q}_2 \mathbb{1} q_2)\,(\bar{q}_1 \mathbb{1} q_1)$$

- octet not suppressed in gauge boson pair production
- gauge invariance $\rightsquigarrow$ Wilson lines between field pairs in color singlet after $\int d^2\boldsymbol{k}_i$ still complicated Wilson line structure for $(\bar{q}_2 \mathbb{1} q_1)\,(\bar{q}_1 \mathbb{1} q_2)$

spin and color correlations discussed by M. Mekhfi, PRD32 (1985)
but apparently not followed up in literature

Interference effects

- so far: distributions with operators $\bar{q}_2 q_2 \; \bar{q}_1 q_1$
  $\rightsquigarrow$ double parton densities if coupled to color singlet
- but also have interference contributions (no probability interpretation)



- must be included in cross section formula
  but is discarded in existing estimates

Approximation by single-parton distributions

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$

$$\times \, 2p^+ \int dy_1^- \; \langle p| \bar{q}(-\tfrac{1}{2}z_2) \Gamma_2 \, q(\tfrac{1}{2}z_2) \; \bar{q}(y_1 - \tfrac{1}{2}z_1) \Gamma_1 \, q(y_1 + \tfrac{1}{2}z_1) |p\rangle$$

- between $\bar{q}_2 \, q_2$ and $\bar{q}_1 \, q_1$ insert complete set $\sum_X |X\rangle\langle X|$ of states
- if assume that single-proton states $|p\rangle\langle p|$ dominate in $\sum |X\rangle\langle X|$ then $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \approx$ product of single-quark distributions

$$\langle p| \bar{q}_2 \, q_2 \; \bar{q}_1 \, q_1 |p\rangle \approx \sum_{p'} \langle p| \bar{q}_2 \, q_2 |p'\rangle \, \langle p'| \bar{q}_1 \, q_1 |p\rangle$$

- transverse momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ differ
  $\rightsquigarrow$ generalized parton distributions (GPDs) at zero skewness
- in physical terms: neglect correlations between parton 1 and 2

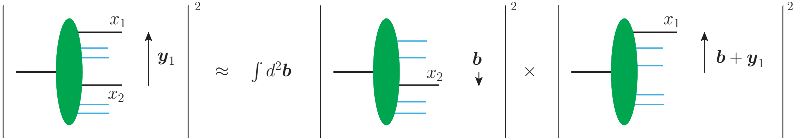
Approximation by single-parton distributions

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) = \int \frac{dz_2^- \, d^2\boldsymbol{z}_2}{(2\pi)^3} \, e^{ix_2 z_2^- p^+ - i\boldsymbol{z}_2\boldsymbol{k}_2} \int \frac{dz_1^- \, d^2\boldsymbol{z}_1}{(2\pi)^3} \, e^{ix_1 z_1^- p^+ - i\boldsymbol{z}_1\boldsymbol{k}_1}$$
$$\times \, 2p^+ \int dy_1^- \, \langle p| \bar{q}(-\tfrac{1}{2}z_2)\Gamma_2 \, q(\tfrac{1}{2}z_2) \, \bar{q}(y_1 - \tfrac{1}{2}z_1)\Gamma_1 \, q(y_1 + \tfrac{1}{2}z_1)|p\rangle$$

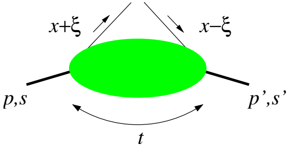
- between $\bar{q}_2 \, q_2$ and $\bar{q}_1 \, q_1$ insert complete set $\sum_X |X\rangle\langle X|$ of states
- if assume that single-proton states $|p\rangle\langle p|$ dominate in $\sum |X\rangle\langle X|$ then $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \approx$ product of single-quark distributions
- especially simple for collinear distributions:

$$F(x_i, \boldsymbol{y}_1) \;\approx\; \int d^2\boldsymbol{b} \; f(x_2, \boldsymbol{b}) \, f(x_1, \boldsymbol{b} + \boldsymbol{y}_1)$$

with $f(x, \boldsymbol{b}) =$ impact-parameter dependent distribution (M. Burkardt '02)



Reminder: GPDs and impact parameter



- at given skewness ξ can trade Mandelstam t for transv. mom. transfer $\boldsymbol{\Delta}$

$$H(x,\xi,\boldsymbol{\Delta}) \xrightarrow{\text{Fourier trf.}} H(x,\xi,\boldsymbol{b})$$

$\boldsymbol{b}$ = impact parameter = distance of struck parton from proton center

- for $\xi = 0$ have density interpretation: $H(x,0,\boldsymbol{b}) = f(x,\boldsymbol{b})$
- important difference:
  - distance $\boldsymbol{y}_1$ in multi-parton distr's appears under integral
    $d\sigma = \ldots \int d^2\boldsymbol{y}_1\, F(x_i,\boldsymbol{k}_i,\boldsymbol{y}_i)\, \bar{F}(\bar{x}_i,\bar{\boldsymbol{k}}_i,\boldsymbol{y}_i)$
  - $\boldsymbol{\Delta}$ in GPDs experimentally observable

Multi-parton distributions and non-forward matrix elements

- slide 10 was oversimplified
  when insert proton states $|p\rangle\langle p|$ have two spin combinations:

$$\langle p_\uparrow|\bar{q}_2\, q_2\ \bar{q}_1\, q_1|p_\uparrow\rangle$$
$$\approx \sum_{p'} \Big[\langle p_\uparrow|\bar{q}_2\, q_2|p'_\uparrow\rangle\, \langle p'_\uparrow|\bar{q}_1\, q_1|p_\uparrow\rangle + \langle p_\uparrow|\bar{q}_2\, q_2|p'_\downarrow\rangle\, \langle p'_\downarrow|\bar{q}_1\, q_1|p_\uparrow\rangle \Big]$$

  includes proton helicity flip: $H(x_1,\ldots)H(x_2,\ldots) + E(x_1,\ldots)E(x_2,\ldots)$

- can also insert $\sum_X |X\rangle\langle X|$ in distributions describing interference, e.g.
  - fermion number: $\bar{q}_2\,\bar{q}_2\ q_1\, q_1 = \bar{q}_2\, q_1\ \bar{q}_2\, q_1$
  - color: $(\bar{q}_2 t^a q_2)\ (\bar{q}_1 t^a q_1) = \frac{1}{2}(\bar{q}_2 \mathbb{1} q_1)\ (\bar{q}_1 \mathbb{1} q_2) - \frac{1}{6}\,(\bar{q}_2 \mathbb{1} q_2)\ (\bar{q}_1 \mathbb{1} q_1)$

  get single-quark matrix elements of form $\langle p'|\,\bar{q}_2\, q_1|p\rangle$
  $\rightsquigarrow$ different longitudinal momenta of partons: nonzero skewness
  $\rightsquigarrow$ k_T dependent GPDs even for $\int d^2\boldsymbol{k}_1\ \int d^2\boldsymbol{k}_2\ F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1)$

Summary

- multi-parton distributions have nontrivial aspects:
  - spin and color structure
  - interference in fermion number and quark flavor

  size of these effects presently unknown
- distrib's depend on transverse distance between partons
- under assumptions can relate to GPDs, which
  - contain experimentally accessible information about transverse distribution of partons and its correlation with long. momenta
  - offer a way to estimate size of interference-type multi-parton distr's
  - allow study of general patterns such as correlations between x and b

$\rightsquigarrow$ may provide one piece of input for describing mult. interactions