# Physics at LHC: SUperSYmmetry

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# Outline

- SUperSYmmetry: Brief introduction & Motivations
- Reminder of Standard Model (SM) Lagrangian
- SUSY phenomenology: Deeper look
  - "Constructing" the SUSY Lagrangian
  - > Different sectors of MSSM:
    - Squark & Slepton
    - > Chargino
    - Neutralino
    - Higgs

## Advised readings:

- "SUSY & Such" S. Dawson, arxiv:hep-ph/9612229v2
- "A supersymmetry primer" S. P. Martin, arxiv:hep-ph/9709356

# **Brief introduction & Motivations**



# **Supersymmetry:** Introduction words

#### "Generalize" the spin of known fields

#### **SUperSYmmetry :**

spin particle  $\frac{1}{2} \leftrightarrow$  spin partner 0 spin particle 1  $\leftrightarrow$  spin partner  $\frac{1}{2}$ 

Names		spin 0	spin $1/2$	Ν
squarks, quarks	Q	$(\widetilde{u}_L \ \widetilde{d}_L)$	$(u_L \ d_L)$	
$(\times 3 \text{ families})$	$\overline{u}$	$\widetilde{u}_R^*$	$u_R^\dagger$	gluir
	$\overline{d}$	$\widetilde{d}_R^*$	$d_R^\dagger$	winos.
sleptons, leptons	L	$(\widetilde{ u} \ \widetilde{e}_L)$	$( u \ e_L)$	
$(\times 3 \text{ families})$	$\overline{e}$	$\widetilde{e}_R^*$	$e_R^\dagger$	bino,
Higgs, higgsinos	$H_u$	$(H_u^+ \ H_u^0)$	$(\widetilde{H}^+_u \ \widetilde{H}^0_u)$	
	$H_d$	$(H^0_d \ H^d)$	$(\widetilde{H}^0_d \ \widetilde{H}^d)$	

Names	spin $1/2$	spin 1
gluino, gluon	$\widetilde{g}$	g
winos, W bosons	$\widetilde{W}^{\pm}$ $\widetilde{W}^{0}$	$W^{\pm} W^0$
bino, B boson	$\widetilde{B}^0$	$B^0$

Observed SUSY particles with same mass than Standard-Model partners ? No !

#### SUSY : A broken symmetry ! Physical sParticles: Mixture of super-partners

- Charginos ( $\chi^{\pm}$ ) / Neutralinos ( $\chi^{0}$ ) : Bino/Wino  $\leftrightarrow$  Higgs (charged/neutral)
- > Squarks, Sleptons : Mixture of  $f_L \leftrightarrow f_R$

**Supersymmetry**: The natural cure of Hierarchy problem

Discovery of a Higgs Boson:

 $m_{_{\rm H}} = 125 \text{ GeV/c}^2$ 

Consider Higgs mass correction from fermionic loop:

 $\begin{array}{l} \Lambda_{_{\rm UV}}: \mbox{ Energy-scale at which new physics alters the Standard-Model} \\ (momentum cut-off regulating the loop-integral) \\ \mbox{ If } \Lambda_{_{\rm UV}} \sim M_{_{\rm P}} \ \rightarrow \ \Delta m_{_{\rm H}}^2 \sim O(10^{30}) \mbox{ larger than } m_{_{\rm H}} \ !!! \\ \mbox{ And all Standard-Model masses indirectly sensitive to } \Lambda_{_{\rm UV}} \ !!! \end{array}$ 

$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2} \cdot \left[-2\Lambda_{UV}^2 + \ldots\right] \xrightarrow{\mathrm{H}} \left[-2\Lambda_{UV}^2 + \ldots\right]$$

 $\Delta m^2_{\ H}$  quadratic divergence cancelled : Hierarchy problem naturally solved !

# **Supersymmetry** & Coupling constants

In Gauge theories : Predict coupling constants at a scale Q once we measured them at another:

$$1/\alpha_{i}(Q) = 1/\alpha_{i}(M_{z}) + (b_{i}/2) \log[M_{z}/Q]$$

 $b_{i}$ : Function of  $N_{g}$ (=3) and  $N_{H}$ (Number of Higgs doublets)



In SUSY:  $N_{H}=2$  + New particles contributing to a different evolution of coupling constants ->  $b_{i}$ 's such that !

#### **SUSY can naturally be incorporated into Grand Unified Theories**



## Supersymmetry & Dark Matter

Most general SUSY lagrangian allows interactions leading to Baryon- & Lepton-number violation !

**Now if** sParticles were to exist at TeV scale:

Such interactions very seriously restricted by experimental observation !

In SUSY:  $N_{BL}$  conservation *can* be "protected" by new symmetry  $R_{p}$ :

- Eigenvalue: (-1)<sup>3(B-L)+s</sup>
  - +1 / -1 for SM / SUSY particles
- If R<sub>P</sub> conserved: Lightest Supersymmetric Particle (LSP) is stable In most SUSY scenarios, LSP is either:
  - > The lightest neutralino  $\chi^0$  (mixture of neutral Higgsinos / Bino / Wino)
  - Scalar neutrinos
- …In all cases a weakly interacting neutral particle

#### **SUSY** *can* have a natural candidate for the observed Cold Dark Matter : ~25% of mass of universe

# **Revisiting SM Lagrangian**



## **SM Lagrangian**

Let's put the QCD part aside & have a look at the EW part only

$$L_{EW} = L_{free+interaction} + L_{gauge} + L_{higgs} + L_{yukawa}$$

**SM Lagrangian:** Free & Interaction parts

$$\mathbf{L}_{\text{free+interaction}} = \boldsymbol{\Sigma}_{f} \mathbf{i} \left[ \boldsymbol{\psi}_{f}^{L} \boldsymbol{\gamma}^{\mu} \mathbf{D}_{\mu}^{L} \boldsymbol{\psi}_{f}^{L} + \boldsymbol{\psi}_{f}^{R} \boldsymbol{\gamma}^{\mu} \mathbf{D}_{\mu}^{R} \boldsymbol{\psi}_{f}^{R} \right]$$

→  $\psi_{f}^{L,R}$ : Left and Right fermion, CC, Dirac spinors

 $\label{eq:constraint} \begin{array}{l} \rightarrow \mbox{ Gauge-invariant derivatives:} \\ D^{\rm L}_{\ \mu} = \delta_{\mu} - \mbox{i } \mbox{g} \ (\mbox{$\tau_a/2$}) \ W^{\rm a}_{\ \mu} - \mbox{i } \mbox{g'} \ (\mbox{$Y_L/2$}) \ B_{\mu} \\ D^{\rm R}_{\ \mu} = \delta_{\mu} & - \mbox{i } \mbox{g'} \ (\mbox{$Y_L/2$}) \ B_{\mu} \end{array}$ 

→ g, g': Weak-isospin & -hypercharge couplings →  $W^{a}_{\mu}$ ,  $B_{\mu}$ : Weak-isospin & -hypercharge fields →  $\tau_{a}$ ,  $Y_{L,R}$ : Weak-isospin & -hypercharge quantum numbers, matrices **SM Lagrangian:** The gauge part

$$L_{gauge} = -(1/4) W^{a}_{\mu\nu} W^{a\mu\nu} - (1/4) B_{\mu\nu} B^{\mu\nu}$$

→ Gauge-invariant Weak-isospin & -hypercharge fields:

$$W^{a}_{\mu\nu} = \delta_{\mu}W^{a}_{\nu} - \delta_{\nu}W^{a}_{\nu} + \mathbf{g} \varepsilon_{abc} W^{b}_{\mu}W^{c}_{\nu}$$
$$B_{\mu\nu} = \delta_{\mu}B_{\nu} - \delta_{\nu}B_{\nu}$$

 $2^{nd}$  term of  $W^{a}_{\mu\nu}$ : Self-interacting character of Weakisospin interaction  $\rightarrow$  *This is the term allowing triboson couplings in SM* 

A similar term exists in QCD sector of SM: QCD is also non-abelian  $\rightarrow$  Allows self-coupling

**SM Lagrangian:** The Higgs part

$$\mathbf{L}_{\mathbf{Higgs}} = (\mathbf{D}_{\mu} \phi)^{+} (\mathbf{D}^{\mu} \phi) - \mathbf{V}(\phi)$$

# $\rightarrow 1^{st}$ term: Higgs $\leftrightarrow$ Boson interaction: Gives Boson masses Gives Higgs↔Boson couplings $\rightarrow$ V( $\phi$ ): Pure Higgs interaction: Mass: $m_{_{\rm H}} = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$ Coupling: Calculate :-D The lagrangian has to be SU(2)xU(1) invariant $\rightarrow$ 4 scalar <u>real</u> fields: $\phi = (\phi^+, \phi^0)$ $\phi^+ = (1/\sqrt{2})(\phi_1 + i\phi_2)$ $\phi^0 = (1/\sqrt{2})(\phi_3 + i\phi_4)$

SM Lagrangian: Yukawa

$$L_{yukawa} = -G_{d}(\bar{u},\bar{d})_{L}(\phi^{+},\phi^{0}) d_{R} - G_{u}(\bar{u},\bar{d})_{L}(-\bar{\phi}^{0},\phi^{-}) u_{R}$$
  
+ hermitian-conjugate

(u,d): Up & Down doublets of <u>quarks / leptons</u>

Once Higgs sector is EW-broken:  $\phi = (1/\sqrt{2})(0,v+H) \rightarrow \text{``Confers''} \text{ mass to fermions:}$   $L_{yukawa} = -m_d \overline{d}_L d_R (1+H/v) - m_u \overline{u}_L u_R (1+H/v)$ because:  $m_f = G_f v/\sqrt{2}$ 

For neutrinos:  $m = G_v v / \sqrt{2} \sim 0$ 

# "Constructing" the SUSY Lagrangian



# **MSSM:** Writing the Lagrangian

#### **Recipe to build the particle content and Lagrangian:**

- Each SM fermion f has 2 chiral superpartners:  $f_{L} \& f_{R}$
- SM fermions and SUSY sfermions are regrouped in superfields



- Gauge superfields: "Simply" containing the SM gauge fields and their SUSY partners
- Gauge superfields: Respecting the SU(3) x SU<sub>L</sub>(2) x U(1)

Superfields of Gauge & Matter, by definition, respect the gauge symmetries extended from the SM

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{Q}$	3	2	$\frac{1}{6}$	$(u_L, d_L),  (\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	3	1	$-\frac{2}{3}$	$\overline{u}_R,~ ilde{u}_R^*$
$\hat{D}^c$	3	1	$\frac{1}{3}$	$\overline{d}_R,  ilde{d}_R^*$
$\hat{L}$	1	2	$-\frac{1}{2}$	$(\nu_L, e_L), (\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^{c}$	1	1	1	$\overline{e}_R, \ \widetilde{e}_R^*$
$\hat{H}_1$	1	2	$-\frac{1}{2}$	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	$\frac{1}{2}$	$(H_2,  ilde{h}_2)$

Superfield	SU(3)	$SU(2)_L$	$U(1)_Y$	Particle Content
$\hat{G}^a$	8	1	0	$g,~ ilde{g}$
$\hat{W}^i$	1	3	0	$W_i, \ \tilde{\omega}_i$
$\hat{B}$	1	1	0	$B,  ilde{b}$

#### The interaction part:

$$\mathcal{L}_{int} = -\sqrt{2} \sum_{i,A} g_A \left[ S_i^* T^A \overline{\psi}_{iI} \lambda_A + \text{h.c.} \right] - \frac{1}{2} \sum_A \left( \sum_i g_A S_i^* T^A S_i \right)^2$$

- Interaction-specific quantum number
- S<sub>i</sub>: Scalar fields: Squarks & Sleptons
- $\psi_i$ : Higgsinos
- >  $\lambda_{A}$ : Gauge <u>fermions</u>

The gauge invariant derivative part: Same as introduced in SM, but generalized to superfields

The kinetic part:

$$\mathcal{L}_{KE} = \sum_{i} \left\{ (D_{\mu} S_{i}^{*}) (D^{\mu} S_{i}) + i \overline{\psi}_{i} D \psi_{i} \right\} + \sum_{A} \left\{ -\frac{1}{4} F_{\mu\nu}^{A} F^{\mu\nu A} + \frac{i}{2} \overline{\lambda}_{A} D \lambda_{A} \right\}$$

# **MSSM:** SM ↔ MSSM correspondance

Fermion	Scalar	Gauge field
$\frac{\mathbf{SM}}{\mathrm{i}\overline{f}}\gamma^{\mu}D_{\mu}f+$	$(D_{\mu} \phi)^{+}(D^{\mu} \phi)$ SM: Higgs	- (1/4) F <sup>μν</sup>
<u>MSSM</u> (include	es what is above)	
$i \psi \gamma^{\mu} D_{\mu} \psi +$ MSSM: Higgsinos	$(D_{\mu} S_{i})^{+}(D^{\mu} S_{i})$ Squarks & Sleptons	- (1/4) $F_{\mu\nu}F^{\mu\nu}$ Same as above
+(i/2) $\lambda_{A}^{-} \gamma^{\mu} D_{\mu} \lambda$ Gauge fermions	Ă	

# SUSY: Let's minimally break it: Broken & effective MSSM



## SUSY breaking

#### How is it broken ? We don't know... did not discover it (yet)...

How we *think* it's broken: Models/Implications by/for the

theorists/experimentalists

MSSM

**mSUGRA** Spontaneous Super-Gravity breaking: More constrained  $\rightarrow$  5 parameters @ breaking scale -> RGEs  $\rightarrow$  Our mass spectrum

- m<sub>0</sub>: Scalar mass
- m<sub>1/2</sub>: Fermion mass
- >  $\mu$ : Higgs parameter ( $\mu H_1 H_2$ )
- A: Tri-linear squark/slepton mixing term

$$tan\beta =  /$$

Parametrizing our ignorance of SUSY breaking, i.e. no hypothesis: Un-constrained  $\rightarrow$  124 parameters

- >  $\tan\beta / \mu / M_A$  (pseudoscalar Higgs boson mass)
- ▶ M<sub>L1,2,3</sub>: Controls slepton masses
- M<sub>Q1,2,3</sub>: Controls squark masses
- M<sub>1.2</sub>: Controls neutralino/chargino sectors

This is the most general Lagrangian we can write, hence the large number of unknowns: Only the spin hypothesis has been made Pedrame Bargassa – LIP Lisbon

## **MSSM:** Effective Lagrangian

- We don't know <u>how</u> SUSY is broken, but can write the most general broken effective Lagrangian
- Soft: The breaking of the symmetry is taken care of by introducing "soft" mass terms for scalars & gauginos: Soft because no reintroduction of quadratic divergence
- Maximal dimension of soft operators:  $\leq 3 \rightarrow$  Mass terms, Bilinear & Trilinear terms

$$\begin{split} -\mathcal{L}_{soft} &= \boxed{m_{1}^{2} \mid H_{1} \mid^{2} + m_{2}^{2} \mid H_{2} \mid^{2}}_{=} - \boxed{B\mu\epsilon_{ij}(H_{1}^{i}H_{2}^{j} + \text{h.c.})} + \boxed{\tilde{M}_{Q}^{2}(\tilde{u}_{L}^{*}\tilde{u}_{L} + \tilde{d}_{L}^{*}\tilde{d}_{L})} \\ &+ \widetilde{M}_{u}^{2}\tilde{u}_{R}^{*}\tilde{u}_{R} + \widetilde{M}_{d}^{2}\tilde{d}_{R}^{*}\tilde{d}_{R} + \widetilde{M}_{L}^{2}(\tilde{e}_{L}^{*}\tilde{e}_{L} + \tilde{\nu}_{L}^{*}\tilde{\nu}_{L}) + \widetilde{M}_{e}^{2}\tilde{e}_{R}^{*}\tilde{e}_{R}} \\ &+ \frac{1}{2} \left[ \boxed{M_{3}\overline{\tilde{g}}\tilde{g}} + M_{2}\overline{\tilde{\omega}_{i}}\tilde{\omega}_{i} + M_{1}\overline{\tilde{b}}\tilde{b} \right] + \frac{g}{\sqrt{2}M_{W}}\epsilon_{ij} \left[ \frac{M_{d}}{\cos\beta}A_{d}H_{1}^{i}\tilde{Q}^{j}\tilde{d}_{R}^{*} \right] \\ &+ \frac{M_{u}}{\sin\beta}A_{u}H_{2}^{j}\tilde{Q}^{i}\tilde{u}_{R}^{*} + \frac{M_{e}}{\cos\beta}A_{e}H_{1}^{i}\tilde{L}^{j}\tilde{e}_{R}^{*} + \text{h.c.} \right] \quad . \end{split}$$

Specificity of SUSY: Writing the most general Lagrangian, generalizing the spins of fields, SUCH that quadratic divergences are always shut down

# Squark & Slepton sector



## **MSSM:** Squark & Slepton sector

# Physical states are 2 scalar mass-eigenstates: Mixtures of left- & -right chiral superpartners (scalars) of SM quark and leptons

Let's pick-up example of the top sector: If  $[f_L - f_R]$  chiral basis:

$$M_{\tilde{t}}^{2} = \begin{pmatrix} \tilde{M}_{Q}^{2} + M_{T}^{2} + M_{Z}^{2}(\frac{1}{2} - \frac{2}{3}\sin^{2}\theta_{W})\cos 2\beta & M_{T}(A_{T} + \mu\cot\beta) \\ M_{T}(A_{T} + \mu\cot\beta) & \tilde{M}_{U}^{2} + M_{T}^{2} + \frac{2}{3}M_{Z}^{2}\sin^{2}\theta_{W}\cos 2\beta \end{pmatrix}$$

- $\sim \widetilde{M}_{Q}$ : Left squark mass
- $\sim \widetilde{\mathrm{M}}_{_{\mathrm{U}}}$ : Right squark mass
- A<sub>T</sub>: Trilinear coupling specific to the top sector
- $M_Q = M_T$ : Mass of the SM particle
- µ: Higgs (bilinear) mixing parameter
- β: Higgs vev-specific parameter (see in a couple of slides): Plays a role in the mixing



# Chargino sector



#### **MSSM:** Chargino sector

#### Physical states are 2 fermionic mass-eigenstates: Mixtures of charged winos and charged higgsinos, which are SUSY eigenstates

In the charged [wino – higgsino] basis:

$$M_{\tilde{\chi}^{\pm}} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & -\mu \end{pmatrix}$$

- M<sub>2</sub>: Mass of the wino
- μ: Higgs (bilinear) mixing parameter

# Neutralino sector



#### **MSSM:** Neutralino sector

Physical states are 4 fermionic mass-eigenstates: Mixtures of neutral winos  $w^0$ , bino b, and 2 neutral higgsinos, which are SUSY eigenstates

In the neutral  $[b - w^0 - h^0_1 - h^0_2]$  basis:

$$M_{\tilde{\chi}_{i}^{0}} = \begin{pmatrix} M_{1} & 0 & -M_{Z}\cos\beta\sin\theta_{W} & M_{Z}\sin\beta\sin\theta_{W} \\ 0 & M_{2} & M_{Z}\cos\beta\cos\theta_{W} & -M_{Z}\sin\beta\cos\theta_{W} \\ -M_{Z}\cos\beta\sin\theta_{W} & M_{Z}\cos\beta\sin\theta_{W} & 0 & \mu \\ M_{Z}\sin\beta\sin\theta_{W} & -M_{Z}\sin\beta\cos\theta_{W} & \mu & 0 \end{pmatrix}$$

- M<sub>1</sub>: Mass of the bino
- M<sub>2</sub>: Mass of the wino
- μ: Higgs (bilinear) mixing parameter

<u>Exercise</u>: Qualitatively gauge the influence of each parameters in the mass-matrix above on the "type" of neutralinos

#### **EXERCISES**

1/ Install the SuSpect software on your computer: This one of the only SUSY spectrum calculators with parametrized MSSM (pMSSM) parameters as input: You don't have 124, but 27 parameters to play with ;-)

2/ Just play with different parameters and follow evolution of the generated masses

2i) What are the most sensitive parameters for different types of particles ?

2ii) Once you get an idea for 2i): For a set of frozen parameters, produce plots showing evolution of the physical masses, say , as function of pMSSM parameters

- For 2i) & 2ii), let's pick-up:
  - $\rightarrow$  The lightest neutralino
  - $\rightarrow$  The chargino
  - $\rightarrow$  The lightest stop and stau
  - $\rightarrow$  The lighest Higgs

3/ Once your fingers are well warmed-up with pMSSM, produce the points on the following page :-D

# **Stop decays:** Different diagrams for different domains

$$\begin{split} \widetilde{\mathbf{t}}_{1} \rightarrow \mathbf{b} \ \mathbf{W}^{+} \ \widetilde{\boldsymbol{\chi}}_{1}^{\ 0} & \widetilde{\mathbf{t}}_{1} \rightarrow \mathbf{b} \ \widetilde{\boldsymbol{\chi}}_{1}^{\ +} & \widetilde{\mathbf{t}}_{1} \rightarrow \mathbf{t} \ \widetilde{\boldsymbol{\chi}}_{1}^{\ 0} \\ \\ \underbrace{\mathbf{W}_{2}^{200}}_{\text{setar top quark mass [GeV/c]}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} & \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} & \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}^{\text{Scalar top quark mass [GeV/c]}} \\ \mathbf{Conditions:} \\ \underbrace{\mathbf{W}_{1} < \mathbf{X}_{1}^{\ 0} < \widetilde{\mathbf{X}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 0} < \widetilde{\mathbf{X}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 0} < \widetilde{\mathbf{X}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 0} \\ \underbrace{\mathbf{W}_{140}}_{1}^{\ 0} < \widetilde{\mathbf{X}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 0} < \widetilde{\mathbf{W}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 1} \\ \underbrace{\mathbf{W}_{100}}_{\text{scalar top quark mass [GeV/c]}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{100}}_{\text{scalar top quark mass [GeV/c]}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{\text{top 200 300 400 500 600 700 800}}_{\text{scalar top quark mass [GeV/c]}}^{\text{MSSM grid points}} \\ \underbrace{\mathbf{W}_{140}}_{1} < \underbrace{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}}_{1} < \mathbf{W}_{1}^{\ 1} \\ \underbrace{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{X}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 1} \\ \underbrace{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 1} \\ \underbrace{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 1} \\ \underbrace{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} < \mathbf{W}_{1}^{\ 1} \\ \underbrace{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} < \widetilde{\mathbf{W}_{1}^{\ 1} <$$

Make  $\widetilde{\chi}_{1}^{+}$  virtual