# M-Theory as a Dynamical System

Georgios Linardopoulos

Institute of Nuclear & Particle Physics
National Center for Scientific Research "Demokritos"

INPP Annual Meeting
NCSR, "Demokritos", November 15th 2019

# Research Themes, References & Collaborators

# AdS\CFT Correspondence & Integrability

- Work on Holographic Defects in collaboration with M. de Leeuw and C. Kristjansen (my INPP talk last month):

[1]. M. de Leeuw, C. Kristjansen, and G. Linardopoulos. "Scalar One-point Functions and Matrix Product States of AdS/dCFT". In: *Phys.Lett.* **B781** (2018), p. 238. arXiv: 1802.01598 [hep-th].

[2]. M. de Leeuw, C. Kristjansen, and G. Linardopoulos. "One-point Functions of Non-protected Operators in the $SO(5)$ Symmetric D3-D7 dCFT". In: *J. Phys.* **A50** (2017), p. 254001. arXiv: 1612.06236 [hep-th].

- Work on the Spectral Problem of AdS/CFT with M. Axenides and E. Floratos:

[3]. M. Axenides, E. Floratos, and G. Linardopoulos. "The Omega-Infinity Limit of Single Spikes". In: *Nucl. Phys.* **B907** (2016), p. 323. arXiv: 1511.03587 [hep-th],

in collaboration with E. Floratos:

[4]. E. Floratos and G. Linardopoulos. "Large-Spin and Large-Winding Expansions of Giant Magnons and Single Spikes". In: *Nucl.Phys.* **B897** (2015), p. 229. arXiv: 1406.0796 [hep-th],

in collaboration with E. Floratos and G. Georgiou:

[5]. E. Floratos, G. Georgiou, and G. Linardopoulos. "Large-Spin Expansions of GKP Strings". In: *JHEP* **03** (2014), p. 018. arXiv: 1311.5800 [hep-th],

and single-authored work:

[6]. G. Linardopoulos. "Classical Strings and Membranes in the AdS/CFT Correspondence". http://www.didaktorika.gr/eadd/handle/10442/35838?locale=en. PhD thesis. National and Kapodistrian University of Athens, 2015.

[7]. G. Linardopoulos. "Large-Spin Expansions of Giant Magnons". In: *PoS* (CORFU2014), p. 154. arXiv: 1502.01630 [hep-th].

# M-Theory and Relativistic Membranes

- Work on the BMN Matrix Model in collaboration with M. Axenides and E. Floratos (the present talk):

  [8]. M. Axenides, E. Floratos, and G. Linardopoulos. "Multipole stability of spinning M2 branes in the classical limit of the BMN matrix model". In: *Phys. Rev.* **D97** (2018), p. 126019. arXiv: 1712.06544 [hep-th].

  [9]. M. Axenides, E. Floratos, and G. Linardopoulos. "M2-brane Dynamics in the Classical Limit of the BMN Matrix Model". In: *Phys. Lett.* **B773** (2017), p. 265. arXiv: 1707.02878 [hep-th].

- Work on Relativistic Membranes with M. Axenides and E. Floratos (see talks on 2012 & 2013 annual meetings of INPP):

  [10]. M. Axenides, E. Floratos, and G. Linardopoulos. "Stringy Membranes in AdS/CFT". In: *JHEP* **08** (2013), p. 089. arXiv: 1306.0220 [hep-th].

- Work on Minimal Hypersurfaces with J. Hoppe and T. Turgut:

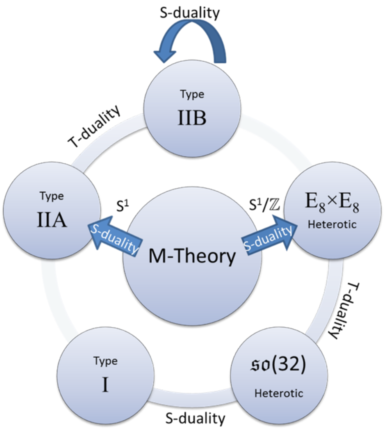
  [11]. J. Hoppe, G. Linardopoulos, and T.O. Turgut. "New Minimal Hypersurfaces in $\mathbb{R}^{(k+1)(2k+1)}$ and $S^{2k^2+3k}$". In: *Mathematische Nachrichten* (2016). arXiv: 1602.09101 [math.DG].

# What is M-theory?

# Why do we need M-theory?

- By the end of the first superstring revolution (1984-1994), five different 10-dimensional superstring theories had emerged:

  Types I, II (IIA, IIB), Heterotic ($\mathfrak{so}(32)$, $E_8 \times E_8$).

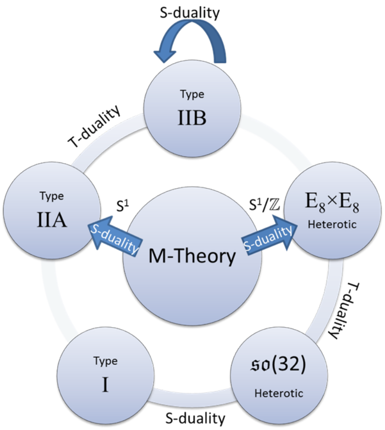


# Why do we need M-theory?

- By the end of the first superstring revolution (1984-1994), five different 10-dimensional superstring theories had emerged:

  Types I, II (IIA, IIB), Heterotic ($\mathfrak{so}(32)$, $E_8 \times E_8$).

- During the second superstring revolution (1994-2003), it was found that the 5 superstring theories are linked with dualities (T-duality, S-duality, U-duality, mirror symmetry)...

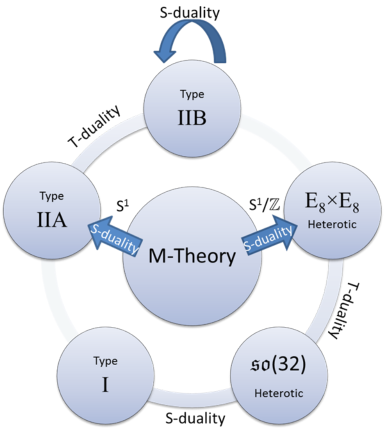


# Why do we need M-theory?

- By the end of the first superstring revolution (1984-1994), five different 10-dimensional superstring theories had emerged:

  Types I, II (IIA, IIB), Heterotic ($\mathfrak{so}(32)$, $E_8 \times E_8$).

- During the second superstring revolution (1994-2003), it was found that the 5 superstring theories are linked with dualities (T-duality, S-duality, U-duality, mirror symmetry)...
- What is more, it was realized that all five 10-dimensional superstring theories were just limiting cases of an 11-dimensional theory...

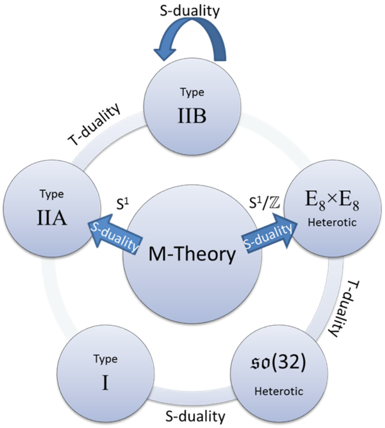


# Why do we need M-theory?

- By the end of the first superstring revolution (1984-1994), five different 10-dimensional superstring theories had emerged:

  Types I, II (IIA, IIB), Heterotic ($\mathfrak{so}(32)$, $E_8 \times E_8$).

- During the second superstring revolution (1994-2003), it was found that the 5 superstring theories are linked with dualities (T-duality, S-duality, U-duality, mirror symmetry)...
- What is more, it was realized that all five 10-dimensional superstring theories were just limiting cases of an 11-dimensional theory...
- This theory has been called "M-theory" and it is obtained in the strong-coupling limit ($g_s \to \infty$) of (IIA) superstring theory...

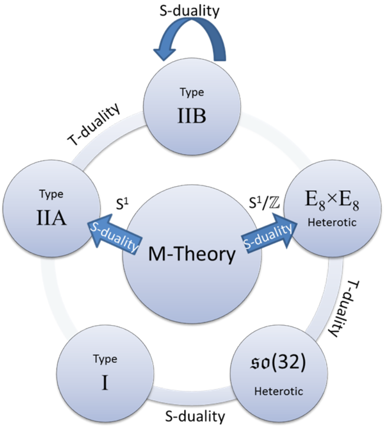


# Why do we need M-theory?

- By the end of the first superstring revolution (1984-1994), five different 10-dimensional superstring theories had emerged:

  Types I, II (IIA, IIB), Heterotic ($\mathfrak{so}(32)$, $E_8 \times E_8$).

- During the second superstring revolution (1994-2003), it was found that the 5 superstring theories are linked with dualities (T-duality, S-duality, U-duality, mirror symmetry)...
- What is more, it was realized that all five 10-dimensional superstring theories were just limiting cases of an 11-dimensional theory...
- This theory has been called "M-theory" and it is obtained in the strong-coupling limit ($g_s \to \infty$) of (IIA) superstring theory...
- The letter "M" stands for "magic, mystery and matrix" according to one of its founders, Edward Witten...

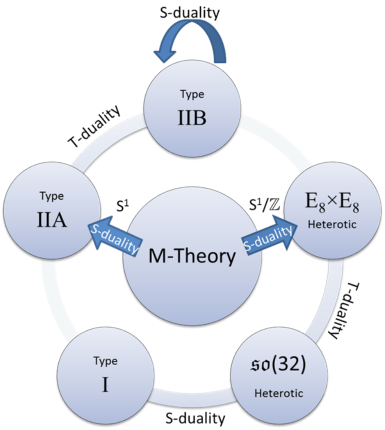


# Why do we need M-theory?

- By the end of the first superstring revolution (1984-1994), five different 10-dimensional superstring theories had emerged:

  Types I, II (IIA, IIB), Heterotic ($\mathfrak{so}(32)$, $E_8 \times E_8$).

- During the second superstring revolution (1994-2003), it was found that the 5 superstring theories are linked with dualities (T-duality, S-duality, U-duality, mirror symmetry)...
- What is more, it was realized that all five 10-dimensional superstring theories were just limiting cases of an 11-dimensional theory...
- This theory has been called "M-theory" and it is obtained in the strong-coupling limit ($g_s \to \infty$) of (IIA) superstring theory...
- The letter "M" stands for "magic, mystery and matrix" according to one of its founders, Edward Witten...
- Others have associated "M" with "membranes"...

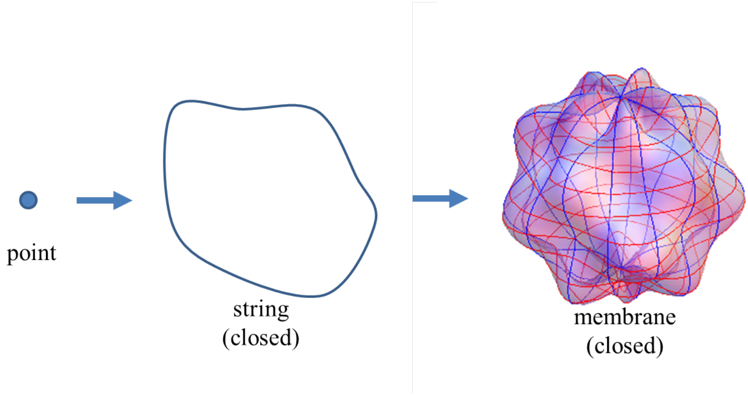


# Membranes

- The idea behind membrane theory is simple: replace 1-dimensional strings with 2-dimensional membranes...

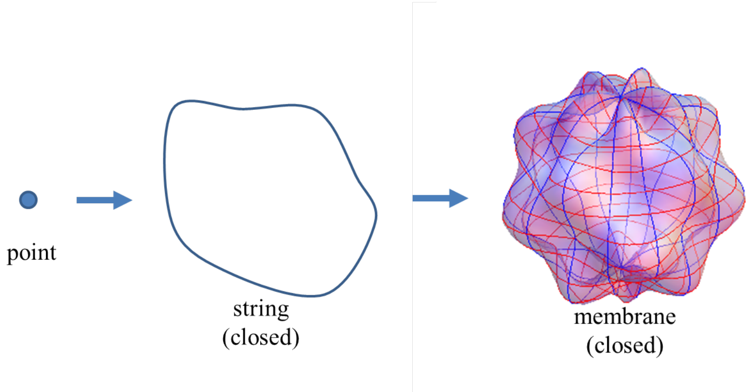
# Membranes

- The idea behind membrane theory is simple: replace 1-dimensional strings with 2-dimensional membranes...



# Membranes

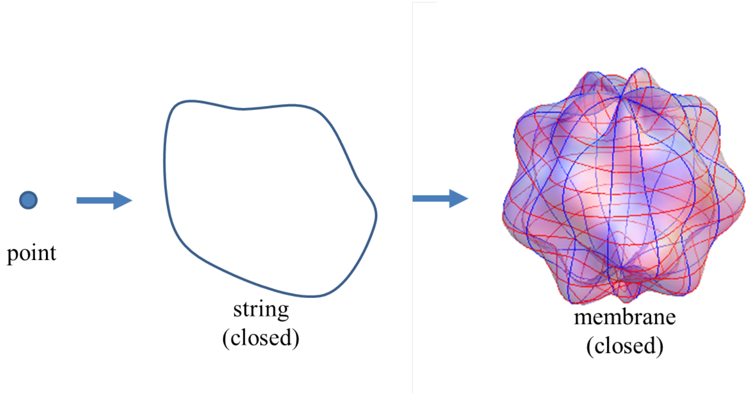
- The idea behind membrane theory is simple: replace 1-dimensional strings with 2-dimensional membranes...




- Like strings, membranes can be supersymmetrized... we thus obtain supermembranes...

# Membranes

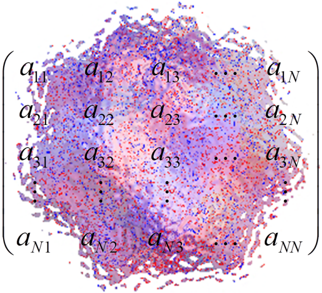
- The idea behind membrane theory is simple: replace 1-dimensional strings with 2-dimensional membranes...




- Like strings, membranes can be supersymmetrized... we thus obtain supermembranes...
- There are reasons to believe that membranes (or "M2-branes") are the fundamental objects of 11-dimensional M-theory, just like strings are the fundamental objects of 10-dimensional string theory...

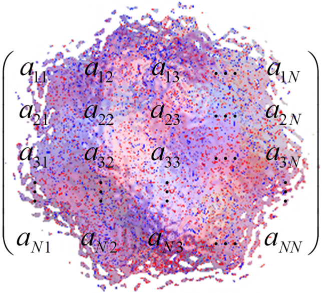
# M(atrix) theory

- According to the matrix theory conjecture of Banks, Fischler, Shenker and Susskind (BFSS, 1996), a theory of matrix-quantized supermembranes provides a realization of M-theory in flat space-time...



# M(atrix) theory

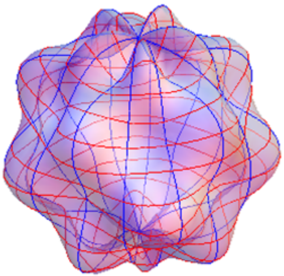
- According to the matrix theory conjecture of Banks, Fischler, Shenker and Susskind (BFSS, 1996), a theory of matrix-quantized supermembranes provides a realization of M-theory in flat space-time...
- In the matrix-model language, membranes are fuzzy objects represented by $N \times N$ matrices...



$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2N} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3N} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{N1} & a_{N2} & a_{N3} & \cdots & a_{NN} \end{pmatrix}$$

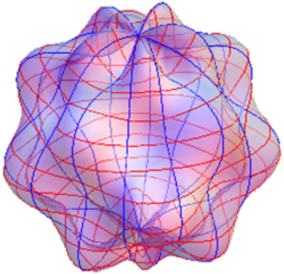
# M(atrix) theory

- According to the matrix theory conjecture of Banks, Fischler, Shenker and Susskind (BFSS, 1996), a theory of matrix-quantized supermembranes provides a realization of M-theory in flat space-time...
- In the matrix-model language, membranes are fuzzy objects represented by $N \times N$ matrices...
- In the limit of large matrix dimensions ($N \to \infty$), matrix models are known to reduce to membrane theories...



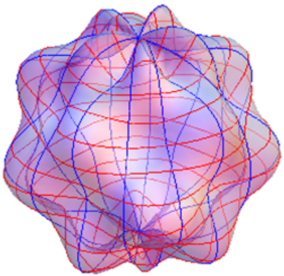
# M(atrix) theory

- According to the matrix theory conjecture of Banks, Fischler, Shenker and Susskind (BFSS, 1996), a theory of matrix-quantized supermembranes provides a realization of M-theory in flat space-time...
- In the matrix-model language, membranes are fuzzy objects represented by $N \times N$ matrices...
- In the limit of large matrix dimensions ($N \to \infty$), matrix models are known to reduce to membrane theories...
- In 2002, Berenstein, Maldacena and Nastase (BMN) proposed a reformulation of the BFSS matrix model on a particular type of weakly curved space-times that are known as plane-waves...



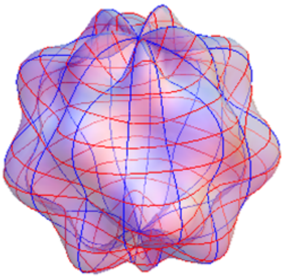
# M(atrix) theory

- According to the matrix theory conjecture of Banks, Fischler, Shenker and Susskind (BFSS, 1996), a theory of matrix-quantized supermembranes provides a realization of M-theory in flat space-time...
- In the matrix-model language, membranes are fuzzy objects represented by $N \times N$ matrices...
- In the limit of large matrix dimensions ($N \to \infty$), matrix models are known to reduce to membrane theories...
- In 2002, Berenstein, Maldacena and Nastase (BMN) proposed a reformulation of the BFSS matrix model on a particular type of weakly curved space-times that are known as plane-waves...
- The BMN matrix model is a deformation of the BFSS matrix model by mass terms and a flux (Myers) term...



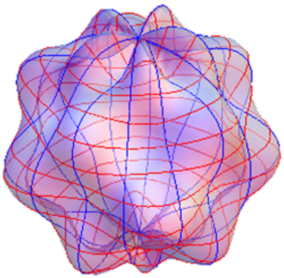
# M(atrix) theory

- According to the matrix theory conjecture of Banks, Fischler, Shenker and Susskind (BFSS, 1996), a theory of matrix-quantized supermembranes provides a realization of M-theory in flat space-time...
- In the matrix-model language, membranes are fuzzy objects represented by $N \times N$ matrices...
- In the limit of large matrix dimensions ($N \to \infty$), matrix models are known to reduce to membrane theories...
- In 2002, Berenstein, Maldacena and Nastase (BMN) proposed a reformulation of the BFSS matrix model on a particular type of weakly curved space-times that are known as plane-waves...
- The BMN matrix model is a deformation of the BFSS matrix model by mass terms and a flux (Myers) term...
- In the large-$N$ limit, the BMN matrix model is again known to reduce to a theory of membranes inside an 11-dimensional plane-wave background...



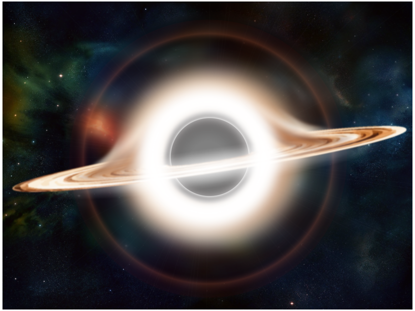
# M(atrix) theory

- According to the matrix theory conjecture of Banks, Fischler, Shenker and Susskind (BFSS, 1996), a theory of matrix-quantized supermembranes provides a realization of M-theory in flat space-time...
- In the matrix-model language, membranes are fuzzy objects represented by $N \times N$ matrices...
- In the limit of large matrix dimensions ($N \to \infty$), matrix models are known to reduce to membrane theories...
- In 2002, Berenstein, Maldacena and Nastase (BMN) proposed a reformulation of the BFSS matrix model on a particular type of weakly curved space-times that are known as plane-waves...
- The BMN matrix model is a deformation of the BFSS matrix model by mass terms and a flux (Myers) term...
- In the large-$N$ limit, the BMN matrix model is again known to reduce to a theory of membranes inside an 11-dimensional plane-wave background...
- Very recently, M(atrix) theory has been applied to the study of chaotic phenomena that take place on the horizons of black holes...



# The black hole membrane paradigm

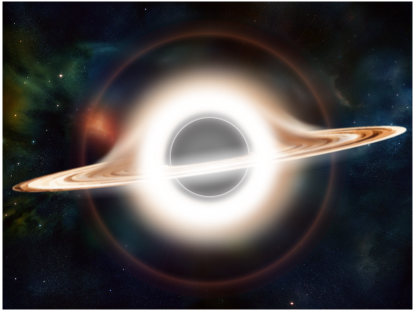
- Black holes (BHs) are regions of spacetime where the force of gravity is so strong that nothing (not even light) can escape...



Gargantua black hole - *Interstellar* movie.

# The black hole membrane paradigm

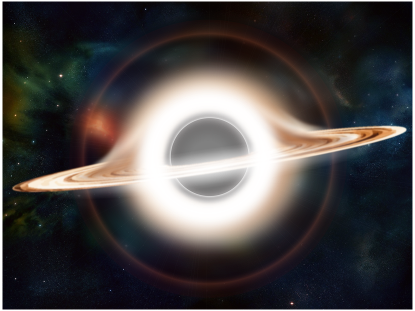
- Black holes (BHs) are regions of spacetime where the force of gravity is so strong that nothing (not even light) can escape...
- The 2-dimensional surface beyond which it is (classically) impossible for matter or information to escape the gravitational pull of a BH is known as the event horizon...



Gargantua black hole - *Interstellar* movie.

# The black hole membrane paradigm

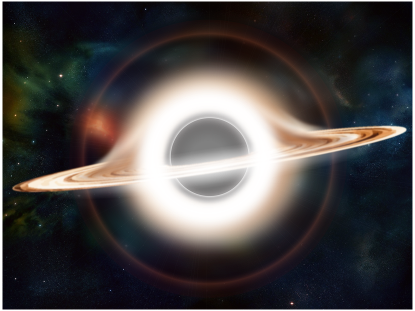
- Black holes (BHs) are regions of spacetime where the force of gravity is so strong that nothing (not even light) can escape...
- The 2-dimensional surface beyond which it is (classically) impossible for matter or information to escape the gravitational pull of a BH is known as the event horizon...
- In 1974, Stephen Hawking predicted that it is (quantum-mechanically) possible for BHs to emit thermal radiation...



Gargantua black hole - *Interstellar* movie.

# The black hole membrane paradigm

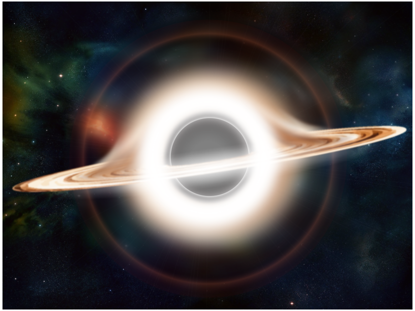
- Black holes (BHs) are regions of spacetime where the force of gravity is so strong that nothing (not even light) can escape...
- The 2-dimensional surface beyond which it is (classically) impossible for matter or information to escape the gravitational pull of a BH is known as the event horizon...
- In 1974, Stephen Hawking predicted that it is (quantum-mechanically) possible for BHs to emit thermal radiation...
- The BH slowly evaporates... Because Hawking radiation is purely thermal, all the information that is stored in the BH seems to get lost...



Gargantua black hole - *Interstellar* movie.

# The black hole membrane paradigm

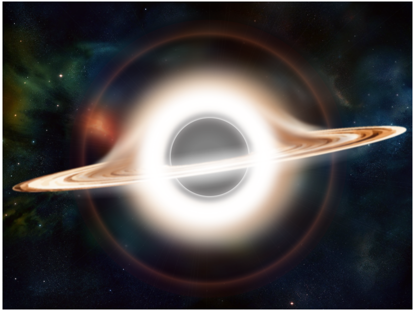
- Black holes (BHs) are regions of spacetime where the force of gravity is so strong that nothing (not even light) can escape...
- The 2-dimensional surface beyond which it is (classically) impossible for matter or information to escape the gravitational pull of a BH is known as the event horizon...
- In 1974, Stephen Hawking predicted that it is (quantum-mechanically) possible for BHs to emit thermal radiation...
- The BH slowly evaporates... Because Hawking radiation is purely thermal, all the information that is stored in the BH seems to get lost...
- To resolve the BH information paradox we ultimately need to understand the mechanisms of information processing in BHs...



Gargantua black hole - *Interstellar* movie.

# The black hole membrane paradigm

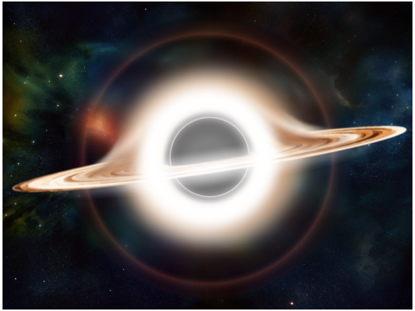
- Black holes (BHs) are regions of spacetime where the force of gravity is so strong that nothing (not even light) can escape...
- The 2-dimensional surface beyond which it is (classically) impossible for matter or information to escape the gravitational pull of a BH is known as the event horizon...
- In 1974, Stephen Hawking predicted that it is (quantum-mechanically) possible for BHs to emit thermal radiation...
- The BH slowly evaporates... Because Hawking radiation is purely thermal, all the information that is stored in the BH seems to get lost...
- To resolve the BH information paradox we ultimately need to understand the mechanisms of information processing in BHs...
- One such mechanism is *fast scrambling* or ultra-fast thermalization... In general it is believed that chaotic phenomena are a dominant feature of BH horizons...



Gargantua black hole - *Interstellar* movie.

# The black hole membrane paradigm

- Black holes (BHs) are regions of spacetime where the force of gravity is so strong that nothing (not even light) can escape...
- The 2-dimensional surface beyond which it is (classically) impossible for matter or information to escape the gravitational pull of a BH is known as the event horizon...
- In 1974, Stephen Hawking predicted that it is (quantum-mechanically) possible for BHs to emit thermal radiation...
- The BH slowly evaporates... Because Hawking radiation is purely thermal, all the information that is stored in the BH seems to get lost...
- To resolve the BH information paradox we ultimately need to understand the mechanisms of information processing in BHs...
- One such mechanism is *fast scrambling* or ultra-fast thermalization... In general it is believed that chaotic phenomena are a dominant feature of BH horizons...
- Because it is inherently nonlocal, M(atrix) theory turns out to be a valuable tool in the study of BH information processing...



Gargantua black hole - *Interstellar* movie.

# Membranes as dynamical systems

# Membranes in plane-wave backgrounds

# General setup

- Consider the 11-dimensional maximally supersymmetric plane-wave background:

$$ds^2 = -2dx^+ dx^- - \left[\frac{\mu^2}{9}\sum_{i=1}^{3} x^i x^i + \frac{\mu^2}{36}\sum_{j=1}^{6} y^j y^j\right] dx^+ dx^+ + dx^i dx^i + dy^j dy^j, \qquad F_{123+} = \mu.$$

# General setup

- Consider the 11-dimensional maximally supersymmetric plane-wave background:

$$ds^2 = -2dx^+dx^- - \left[\frac{\mu^2}{9}\sum_{i=1}^{3} x^i x^i + \frac{\mu^2}{36}\sum_{j=1}^{6} y^j y^j\right] dx^+dx^+ + dx^i dx^i + dy^j dy^j, \qquad F_{123+} = \mu.$$

- The Hamiltonian of a bosonic relativistic membrane in the above background reads

$$\boxed{H = \frac{T}{2}\int d^2\sigma \left[p_x^2 + p_y^2 + \frac{1}{2}\left\{x^i, x^j\right\}^2 + \frac{1}{2}\left\{y^i, y^j\right\}^2 + \left\{x^i, y^j\right\}^2 + \frac{\mu^2 x^2}{9} + \frac{\mu^2 y^2}{36} - \frac{\mu}{3}\epsilon_{ijk}\left\{x^i, x^j\right\}x^k\right]},$$

in the so-called light-cone gauge ($x_+ = \tau$).

# General setup

- Consider the 11-dimensional maximally supersymmetric plane-wave background:

$$ds^2 = -2dx^+dx^- - \left[\frac{\mu^2}{9}\sum_{i=1}^{3} x^i x^i + \frac{\mu^2}{36}\sum_{j=1}^{6} y^j y^j\right] dx^+ dx^+ + dx^i dx^i + dy^j dy^j, \qquad F_{123+} = \mu.$$

- The Hamiltonian of a bosonic relativistic membrane in the above background reads

$$\boxed{H = \frac{T}{2}\int d^2\sigma \left[p_x^2 + p_y^2 + \frac{1}{2}\left\{x^i, x^j\right\}^2 + \frac{1}{2}\left\{y^i, y^j\right\}^2 + \left\{x^i, y^j\right\}^2 + \frac{\mu^2 x^2}{9} + \frac{\mu^2 y^2}{36} - \frac{\mu}{3}\epsilon_{ijk}\left\{x^i, x^j\right\} x^k\right]},$$

in the so-called light-cone gauge ($x_+ = \tau$).

- The corresponding equations of motion and the Gauß-law constraint are given by:

$$\ddot{x}_i = \left\{\left\{x_i, x_j\right\}, x_j\right\} + \left\{\left\{x_i, y_j\right\}, y_j\right\} - \frac{\mu^2}{9}x_i + \frac{\mu}{2}\epsilon_{ijk}\left\{x_j, x_k\right\}, \qquad \sum_{i=1}^{3}\left\{\dot{x}^i, x^i\right\} + \sum_{j=1}^{6}\left\{\dot{y}^j, y^j\right\} = 0$$

$$\ddot{y}_i = \left\{\left\{y_i, y_j\right\}, y_j\right\} + \left\{\left\{y_i, x_j\right\}, x_j\right\} - \frac{\mu^2}{36}y_i.$$

# The ansatz

- The following $\mathfrak{so}(3)$-invariant ansatz automatically satisfies the Gauß-law constraint:

$$x_i = \tilde{u}_i(\tau)\, e_i, \qquad y_j = \tilde{v}_j(\tau)\, e_j, \qquad y_{j+3} = \tilde{w}_j(\tau)\, e_j, \qquad i, j = 1, 2, 3.$$

Collins-Tucker (1976), Harmark-Savvidy (2000)

# The ansatz

- The following $\mathfrak{so}(3)$-invariant ansatz automatically satisfies the Gauß-law constraint:

$$x_i = \tilde{u}_i(\tau)\, e_i, \qquad y_j = \tilde{v}_j(\tau)\, e_j, \qquad y_{j+3} = \tilde{w}_j(\tau)\, e_j, \qquad i, j = 1, 2, 3.$$

Collins-Tucker (1976), Harmark-Savvidy (2000)

The coordinates $e_i$ are given by

$$(e_1, e_2, e_3) \equiv (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta), \qquad \phi \in [0, 2\pi), \quad \theta \in [0, \pi],$$

satisfy the $\mathfrak{so}(3)$ Poisson algebra and are orthonormal:

$$\{e_i, e_j\} = \epsilon_{ijk}\, e_k, \qquad \int e_i\, e_j\, d^2\sigma = \frac{4\pi}{3}\, \delta_{ij}.$$

# The ansatz

- The following $\mathfrak{so}(3)$-invariant ansatz automatically satisfies the Gauß-law constraint:

$$x_i = \tilde{u}_i(\tau)\, e_i, \qquad y_j = \tilde{v}_j(\tau)\, e_j, \qquad y_{j+3} = \tilde{w}_j(\tau)\, e_j, \qquad i, j = 1, 2, 3.$$

Collins-Tucker (1976), Harmark-Savvidy (2000)

The coordinates $e_i$ are given by

$$(e_1, e_2, e_3) \equiv (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta), \qquad \phi \in [0, 2\pi), \quad \theta \in [0, \pi],$$

satisfy the $\mathfrak{so}(3)$ Poisson algebra and are orthonormal:

$$\{e_i, e_j\} = \epsilon_{ijk}\, e_k, \qquad \int e_i\, e_j\, d^2\sigma = \frac{4\pi}{3}\, \delta_{ij}.$$

- As we will see, the above ansatz leads to an interesting dynamical system with stable and unstable solutions that describe rotating and pulsating membranes of spherical topology.

# The ansatz

- The following $\mathfrak{so}(3)$-invariant ansatz automatically satisfies the Gauß-law constraint:

$$x_i = \tilde{u}_i(\tau)\, e_i, \qquad y_j = \tilde{v}_j(\tau)\, e_j, \qquad y_{j+3} = \tilde{w}_j(\tau)\, e_j, \qquad i,j = 1,2,3.$$

Collins-Tucker (1976), Harmark-Savvidy (2000)

The coordinates $e_i$ are given by

$$(e_1, e_2, e_3) \equiv (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta), \qquad \phi \in [0, 2\pi), \quad \theta \in [0, \pi],$$

satisfy the $\mathfrak{so}(3)$ Poisson algebra and are orthonormal:

$$\{e_i, e_j\} = \epsilon_{ijk}\, e_k, \qquad \int e_i\, e_j\, d^2\sigma = \frac{4\pi}{3}\, \delta_{ij}.$$

- As we will see, the above ansatz leads to an interesting dynamical system with stable and unstable solutions that describe rotating and pulsating membranes of spherical topology.
- Similar work in flat space has previously been carried out by (Axenides-Floratos, 2007).

# The Hamiltonian system

- Here's the Hamiltonian of the membrane:

$$H = \frac{2\pi T}{3}\Big[\tilde{p}_u^2 + \tilde{p}_v^2 + \tilde{p}_w^2 + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) +$$
$$+\tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big], \quad \tilde{r}_j^2 \equiv \tilde{v}_j^2 + \tilde{w}_j^2, \; j = 1, 2, 3.$$

# The Hamiltonian system

- Here's the Hamiltonian of the membrane:

$$H = \frac{2\pi T}{3}\Big[\tilde{p}_u^2 + \tilde{p}_v^2 + \tilde{p}_w^2 + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big], \quad \tilde{r}_j^2 \equiv \tilde{v}_j^2 + \tilde{w}_j^2, \; j = 1, 2, 3.$$

- The Hamiltonian has an obvious $SO(2) \times SO(2) \times SO(2)$ symmetry in the coordinates $\tilde{v}_i$ and $\tilde{w}_i$ so that any solution will preserve three $SO(2)$ angular momenta $\ell_i$ $(i = 1, 2, 3)$. The kinetic terms are written as:

$$\tilde{p}_v^2 + \tilde{p}_w^2 = \sum_{i=1}^{3}\left(\dot{\tilde{r}}_i^2 + \frac{\ell_i^2}{\tilde{r}_i^2}\right).$$

# The Hamiltonian system

- Here's the Hamiltonian of the membrane:

$$H = \frac{2\pi T}{3}\Big[\tilde{p}_u^2 + \tilde{p}_v^2 + \tilde{p}_w^2 + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + $$
$$+ \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big], \quad \tilde{r}_j^2 \equiv \tilde{v}_j^2 + \tilde{w}_j^2,\ j = 1, 2, 3.$$

- The Hamiltonian has an obvious $SO(2) \times SO(2) \times SO(2)$ symmetry in the coordinates $\tilde{v}_i$ and $\tilde{w}_i$ so that any solution will preserve three $SO(2)$ angular momenta $\ell_i$ $(i = 1, 2, 3)$. The kinetic terms are written as:

$$\tilde{p}_v^2 + \tilde{p}_w^2 = \sum_{i=1}^{3}\left(\dot{\tilde{r}}_i^2 + \frac{\ell_i^2}{\tilde{r}_i^2}\right).$$

- The effective potential of the membrane becomes:

$$V_{\text{eff}} = \frac{2\pi T}{3}\Big[\frac{\ell_1^2}{\tilde{r}_1^2} + \frac{\ell_2^2}{\tilde{r}_2^2} + \frac{\ell_3^2}{\tilde{r}_3^2} + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + $$
$$+ \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big].$$

# The Hamiltonian system

- Here's the Hamiltonian of the membrane:

$$H = \frac{2\pi T}{3}\Big[\tilde{p}_u^2 + \tilde{p}_v^2 + \tilde{p}_w^2 + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big], \quad \tilde{r}_j^2 \equiv \tilde{v}_j^2 + \tilde{w}_j^2, \; j = 1, 2, 3.$$

- The Hamiltonian has an obvious $SO(2) \times SO(2) \times SO(2)$ symmetry in the coordinates $\tilde{v}_i$ and $\tilde{w}_i$ so that any solution will preserve three $SO(2)$ angular momenta $\ell_i$ $(i = 1, 2, 3)$. The kinetic terms are written as:

$$\tilde{p}_v^2 + \tilde{p}_w^2 = \sum_{i=1}^{3}\left(\dot{\tilde{r}}_i^2 + \frac{\ell_i^2}{\tilde{r}_i^2}\right).$$

- The effective potential of the membrane becomes:

$$V_{\text{eff}} = \frac{2\pi T}{3}\Big[\frac{\ell_1^2}{\tilde{r}_1^2} + \frac{\ell_2^2}{\tilde{r}_2^2} + \frac{\ell_3^2}{\tilde{r}_3^2} + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big].$$

- There are four distinct types of terms: • (1) $\ell$-terms (repulsive)

# The Hamiltonian system

- Here's the Hamiltonian of the membrane:

$$H = \frac{2\pi T}{3}\Big[\tilde{p}_u^2 + \tilde{p}_v^2 + \tilde{p}_w^2 + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big], \quad \tilde{r}_j^2 \equiv \tilde{v}_j^2 + \tilde{w}_j^2,\ j = 1, 2, 3.$$

- The Hamiltonian has an obvious $SO(2) \times SO(2) \times SO(2)$ symmetry in the coordinates $\tilde{v}_i$ and $\tilde{w}_i$ so that any solution will preserve three $SO(2)$ angular momenta $\ell_i$ $(i = 1, 2, 3)$. The kinetic terms are written as:

$$\tilde{p}_v^2 + \tilde{p}_w^2 = \sum_{i=1}^{3}\left(\dot{\tilde{r}}_i^2 + \frac{\ell_i^2}{\tilde{r}_i^2}\right).$$

- The effective potential of the membrane becomes:

$$V_{\text{eff}} = \frac{2\pi T}{3}\left[\frac{\ell_1^2}{\tilde{r}_1^2} + \frac{\ell_2^2}{\tilde{r}_2^2} + \frac{\ell_3^2}{\tilde{r}_3^2} + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\right].$$

- There are four distinct types of terms: • (1) $\ell$-terms (repulsive) • (2) quartic interaction terms (attractive)

# The Hamiltonian system

- Here's the Hamiltonian of the membrane:

$$H = \frac{2\pi T}{3}\Big[\tilde{p}_u^2 + \tilde{p}_v^2 + \tilde{p}_w^2 + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big], \quad \tilde{r}_j^2 \equiv \tilde{v}_j^2 + \tilde{w}_j^2, \ j = 1, 2, 3.$$

- The Hamiltonian has an obvious $SO(2) \times SO(2) \times SO(2)$ symmetry in the coordinates $\tilde{v}_i$ and $\tilde{w}_i$ so that any solution will preserve three $SO(2)$ angular momenta $\ell_i$ $(i = 1, 2, 3)$. The kinetic terms are written as:

$$\tilde{p}_v^2 + \tilde{p}_w^2 = \sum_{i=1}^{3}\left(\dot{\tilde{r}}_i^2 + \frac{\ell_i^2}{\tilde{r}_i^2}\right).$$

- The effective potential of the membrane becomes:

$$V_{\text{eff}} = \frac{2\pi T}{3}\Big[\frac{\ell_1^2}{\tilde{r}_1^2} + \frac{\ell_2^2}{\tilde{r}_2^2} + \frac{\ell_3^2}{\tilde{r}_3^2} + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big].$$

- There are four distinct types of terms: • (1) $\ell$-terms (repulsive) • (2) quartic interaction terms (attractive) • (3) $\mu$-terms (attractive)

# The Hamiltonian system

- Here's the Hamiltonian of the membrane:

$$H = \frac{2\pi T}{3}\Big[\tilde{p}_u^2 + \tilde{p}_v^2 + \tilde{p}_w^2 + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big], \quad \tilde{r}_j^2 \equiv \tilde{v}_j^2 + \tilde{w}_j^2, \; j = 1, 2, 3.$$

- The Hamiltonian has an obvious $SO(2) \times SO(2) \times SO(2)$ symmetry in the coordinates $\tilde{v}_i$ and $\tilde{w}_i$ so that any solution will preserve three $SO(2)$ angular momenta $\ell_i$ $(i = 1, 2, 3)$. The kinetic terms are written as:

$$\tilde{p}_v^2 + \tilde{p}_w^2 = \sum_{i=1}^{3}\left(\dot{\tilde{r}}_i^2 + \frac{\ell_i^2}{\tilde{r}_i^2}\right).$$

- The effective potential of the membrane becomes:

$$V_{\text{eff}} = \frac{2\pi T}{3}\Big[\frac{\ell_1^2}{\tilde{r}_1^2} + \frac{\ell_2^2}{\tilde{r}_2^2} + \frac{\ell_3^2}{\tilde{r}_3^2} + \tilde{u}_1^2\tilde{u}_2^2 + \tilde{u}_2^2\tilde{u}_3^2 + \tilde{u}_3^2\tilde{u}_1^2 + \tilde{r}_1^2\tilde{r}_2^2 + \tilde{r}_2^2\tilde{r}_3^2 + \tilde{r}_3^2\tilde{r}_1^2 + \tilde{u}_1^2\left(\tilde{r}_2^2 + \tilde{r}_3^2\right) + \tilde{u}_2^2\left(\tilde{r}_3^2 + \tilde{r}_1^2\right) + \\ + \tilde{u}_3^2\left(\tilde{r}_1^2 + \tilde{r}_2^2\right) + \frac{\mu^2}{9}\left(\tilde{u}_1^2 + \tilde{u}_2^2 + \tilde{u}_3^2\right) + \frac{\mu^2}{36}\left(\tilde{r}_1^2 + \tilde{r}_2^2 + \tilde{r}_3^2\right) - 2\mu\tilde{u}_1\tilde{u}_2\tilde{u}_3\Big].$$

- There are four distinct types of terms: • (1) $\ell$-terms (repulsive) • (2) quartic interaction terms (attractive) • (3) $\mu$-terms (attractive) • (4) cubic Myers/flux terms (repulsive).

# Membranes as dynamical systems

# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system...

# Membranes as dynamical systems

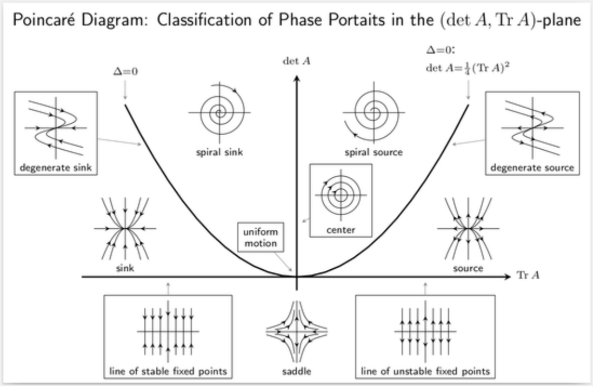
- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...

# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...

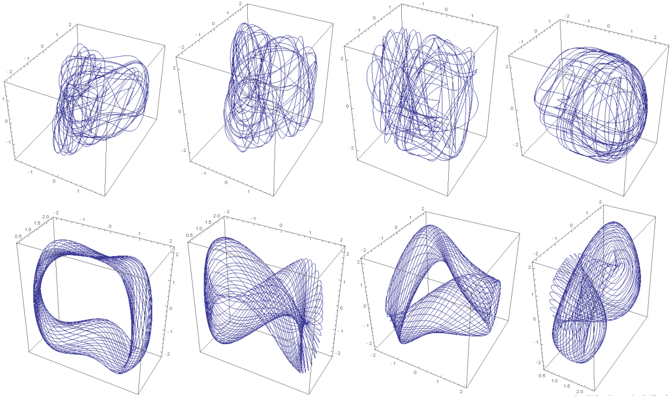
# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points...



# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points... We are also interested in charting periodic orbits...



# Membranes as dynamical systems

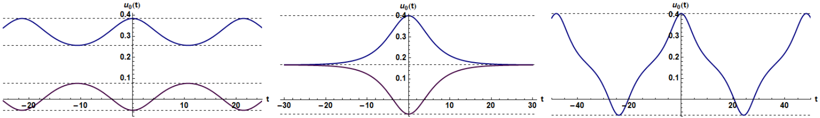
- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points... We are also interested in charting periodic orbits...
- Our system is obviously chaotic (i.e. sensitively dependent on ICs)...

# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points... We are also interested in charting periodic orbits...
- Our system is obviously chaotic (i.e. sensitively dependent on ICs)... But does it have integrable sectors?

# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points... We are also interested in charting periodic orbits...
- Our system is obviously chaotic (i.e. sensitively dependent on ICs)... But does it have integrable sectors? If yes, can we find analytic solutions?

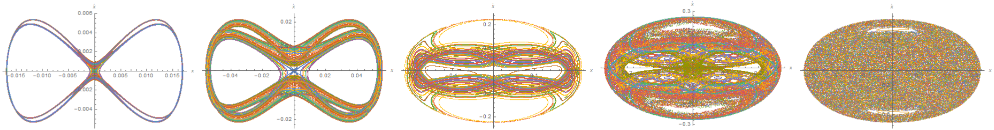


# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points... We are also interested in charting periodic orbits...
- Our system is obviously chaotic (i.e. sensitively dependent on ICs)... But does it have integrable sectors? If yes, can we find analytic solutions?
- We also want to understand the chaotic regions...

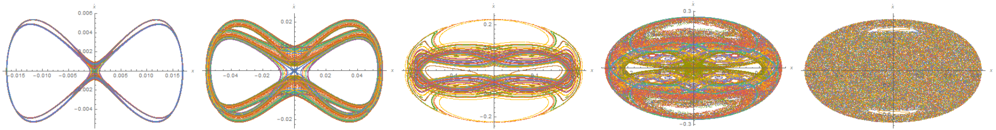
# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points... We are also interested in charting periodic orbits...
- Our system is obviously chaotic (i.e. sensitively dependent on ICs)... But does it have integrable sectors? If yes, can we find analytic solutions?
- We also want to understand the chaotic regions... Is the system weakly chaotic or strongly chaotic?

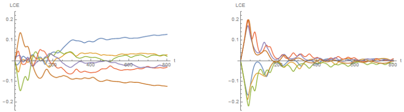


# Membranes as dynamical systems

- We have thus managed to reduce the dynamics of the M2-brane to a 9d Hamiltonian dynamical system... Due to its high degree of complexity, it is obviously hopeless to try to solve it exactly...
- Dynamical systems theory focuses instead on the long-term qualitative behavior of dynamical systems...
- The first goal is to find and classify the fixed points... We are also interested in charting periodic orbits...
- Our system is obviously chaotic (i.e. sensitively dependent on ICs)... But does it have integrable sectors? If yes, can we find analytic solutions?
- We also want to understand the chaotic regions... Is the system weakly chaotic or strongly chaotic?



What is its spectrum of Lyapunov characteristic exponents?



# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...
- Uplift to the matrix model...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...
- Uplift to the matrix model...
- Black holes & the information paradox...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...
- Uplift to the matrix model...
- Black holes & the information paradox... fast scrambling...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...
- Uplift to the matrix model...
- Black holes & the information paradox... fast scrambling... eigenstate thermalization hypothesis (ETH)...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...
- Uplift to the matrix model...
- Black holes & the information paradox... fast scrambling... eigenstate thermalization hypothesis (ETH)... Maldacena-Shenker-Stanford (MSS) bound...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...
- Uplift to the matrix model...
- Black holes & the information paradox... fast scrambling... eigenstate thermalization hypothesis (ETH)... Maldacena-Shenker-Stanford (MSS) bound... BH entropy...

# Outlook

- Radial & angular perturbation analysis in the $SO(3) \times SO(6)$ case in (Axenides-Floratos-G.L., 2017, 2018).
- Analysis of higher orders in perturbation theory... instability cascade... (Axenides-Floratos-G.L., in progress).
- Analysis of chaotic behavior near the separatrix... (Axenides-Floratos-G.L., in progress).
- Study of chaotic dynamics... KAM theory... Entanglement entropy...
- Full classification of all the spherical tops based on their solvability... (Axenides-Floratos-G.L., in progress).
- Quantum chaos... out-of-time-ordered correlators (OTOCs)...
- Generalization to M5-branes... Search for toroidal solutions... Study the topology change phenomenon...
- Uplift to the matrix model...
- Black holes & the information paradox... fast scrambling... eigenstate thermalization hypothesis (ETH)... Maldacena-Shenker-Stanford (MSS) bound... BH entropy...

Ευχαριστώ!