Power corrections for slicing methods in QCD

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based on :: arXiv:1906.09044, with L. Cieri and C. Oleari

Outline

Precision physics @ LHC

▷▷ LHC is entering a high-precision phase

It is mandatory a deep understanding of the underlying theory, i.e. to improve the prediction accuracy by at least an order of magnitude:

$$\text{[QCD] NLO} \rightarrow \text{NNLO (or even N}^3\text{LO)}$$

- ▷ evaluation of multi-loop amplitudes, with complexity growing with the n. of scales involved → progresses in massive and many-legs processes
- ▷ automatization of infrared singularities cancellation: real and virtual amplitudes combine for IR-safe observables to give a finite result (KLN)
  - local subtraction → more complex but exact (*ex.* antennae, stripper, nested s-c, colorful, P2B, Torino, ...)
  - slicing methods → simpler but approximate, need to check cutoff independence

Slicing methods

Slicing methods introduce a resolution parameter, $\lambda_{\rm cut}$, separating the integration into two regions:

- *below* $\lambda_{\rm cut}$: obtained as an expansion of a resummed formula
- *above* $\lambda_{\rm cut}$: obtained by a MC integration (no singularities above the cut)

$$\sigma^{\rm (N)NLO} = \int_0^{\lambda_{\rm cut}} {\rm d}\lambda \, \frac{{\rm d}\sigma^{\rm (N)NLO}}{{\rm d}\lambda} + \int_{\lambda_{\rm cut}}^{\lambda_{\rm max}} {\rm d}\lambda \, \frac{{\rm d}\sigma^{\rm (N)NLO}}{{\rm d}\lambda}$$

Ex.

- **$\mathbf{q_{\rm T}}$-subtraction** :: $\lambda = q_{\rm T}$, the transverse momentum of a particle set
  [Catani, Grazzini; Bozzi et al.]
- **N-jettiness** :: $\lambda = \mathcal{T}_N$, an event-shape variable describing final-state jets
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Power corrections (PCs)

$$\sigma^{\rm (N)NLO} = \int_0^{\lambda_{\rm cut}} {\rm d}\lambda \, \frac{{\rm d}\sigma^{\rm (N)NLO}}{{\rm d}\lambda} + \int_{\lambda_{\rm cut}}^{\lambda_{\rm max}} {\rm d}\lambda \, \frac{{\rm d}\sigma^{\rm (N)NLO}}{{\rm d}\lambda}$$

- **theory side** :: PCs, i.e. new non-trivial terms, increase the understanding of the perturbative behaviour of QCD cross sections
- **practical side** :: PCs make the numerical implementation of the subtraction more robust, weakening the dependence on the cutoff
  - $\lambda_{\rm cut}$ too small $\to$ integration difficulties above the cutoff; larger $\lambda_{\rm cut}$ $\to$ preferable, if we control PCs in $\lambda_{\rm cut}$
  - PCs are more relevant as the perturbative order increases $\to$ however, we begin with the NLO XS

Some references for Next-to-Leading PCs

- Tackmann et al., Boughezal, Isgrò, Petriello within SCET (2017-) $\to$ Andrea's talk
- Laenen et al., within treshold resummation (2015-)

Framework

Color-singlet (F) production @ NLO in α_{S}

$$\mathrm{Born} :: \quad p + p \to F\left(Q^2\right) + X$$

- $p + p \to Z + j$
  - $q\,(\bar{q}) + g \to Z + q\,(\bar{q})$
  - $q + \bar{q} \to Z + g$ "diagonal channel"
- $p + p \to H + j \quad (m_{\mathrm{top}} \to \infty)$
  - $q\,(\bar{q}) + g \to H + q\,(\bar{q})$
  - $g + g \to H + g$ "diagonal channel"
  - $q + \bar{q} \to H + g$

Method

In order to extract the PCs, we study the behaviour of the real contribution at small $\lambda \equiv q_{\rm T}$ of F, first at parton level: [$a, b \equiv$ initial-state partons]

$$\hat{\sigma}^{<}_{ab}(z) \equiv \int_0^{\left(q_{\rm T}^{\rm cut}\right)^2} dq_{\rm T}^2 \, \frac{d\hat{\sigma}_{ab}(q_{\rm T}, z)}{dq_{\rm T}^2}\,, \qquad z \equiv \frac{Q^2}{\hat{s}}$$

Since we know the total cross section, we may refer to the above-$q_{\rm T}^{\rm cut}$ region

$$\hat{\sigma}^{>}_{ab}(z) = \int_{\left(q_{\rm T}^{\rm cut}\right)^2}^{\left(q_{\rm T}^{\rm max}\right)^2} dq_{\rm T}^2 \, \frac{d\hat{\sigma}_{ab}(q_{\rm T}, z)}{dq_{\rm T}^2}$$

$\rightarrow$ two kinds of terms:

- singular (logarithmically-enhanced) terms, known for a while
- vanishing, i.e. the **power corrections**, whose general structure is unknown and which we have analytically computed as a series in $a \equiv \frac{\left(q_{\rm T}^{\rm cut}\right)^2}{Q^2}$

Method

At hadron level, the reality of the parton level XS's restricts the z-integration

$$\begin{aligned}\sigma_{ab}^{<} &= \tau\int_{\tau}^{1-f(a)}\frac{dz}{z}\,\mathcal{L}_{ab}\Big(\frac{\tau}{z}\Big)\,\frac{1}{z}\,\hat{\sigma}_{ab}^{<}(z)\\ &\equiv \tau\int_{\tau}^{1}\frac{dz}{z}\,\mathcal{L}_{ab}\Big(\frac{\tau}{z}\Big)\,\hat{\sigma}^{(0)}\hat{R}_{ab}(z)\,, \qquad \tau\equiv\frac{Q^2}{S}\end{aligned}$$

Here we are interested in

$$\hat{R}_{ab}(z) = \delta_{\rm B}\,\delta(1-z) + \sum_{n=1}^{\infty}\Big(\frac{\alpha_{\rm S}}{2\pi}\Big)^{n}\,\hat{R}_{ab}^{(n)}(z)$$

whose known structure at first order in $\alpha_{\rm S}$ is

$$\hat{R}_{ab}^{(1)}(z) = \log^2(a)\,\hat{R}_{ab}^{(1,2,0)}(z) + \log(a)\,\hat{R}_{ab}^{(1,1,0)}(z) + \hat{R}_{ab}^{(1,0,0)}(z) + \mathcal{O}\Big(a^{\frac{1}{2}}\log a\Big)$$

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** the same method was used by Catani et al., at leading power in a, to extract
→ soft constant of the $q_{\rm T}$-subtraction hard function
→ second-order collinear coefficient functions for $q_{\rm T}$-resummation

[see 1106.4652, 1209.0158]

Method & Results

At hadron level, the reality of the parton level XS's restricts the z-integration

$$\sigma_{ab}^{>(1)} = \tau \int_{\tau}^{1-f(a)} \frac{dz}{z}\, \mathcal{L}_{ab}\left(\frac{\tau}{z}\right) \frac{1}{z}\, \hat{\sigma}_{ab}^{>(1)}(z)$$

$$\equiv \tau \int_{\tau}^{1} \frac{dz}{z}\, \mathcal{L}_{ab}\left(\frac{\tau}{z}\right) \hat{\sigma}^{(0)}\, \hat{G}_{ab}^{(1)}(z)$$

which gives

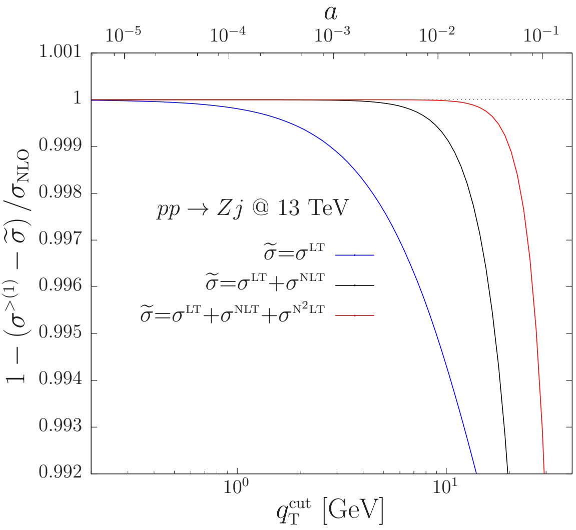
$$\hat{G}_{ab}^{(1)}(z) = \log^2(a)\, \hat{G}_{ab}^{(1,2,0)}(z) + \log(a)\, \hat{G}_{ab}^{(1,1,0)}(z) + \hat{G}_{ab}^{(1,0,0)}(z)$$
$$+ a\log(a)\, \hat{G}_{ab}^{(1,1,2)}(z) + a\, \hat{G}_{ab}^{(1,0,2)}(z)$$
$$+ a^2 \log(a)\, \hat{G}_{ab}^{(1,1,4)}(z) + a^2\, \hat{G}_{ab}^{(1,0,4)}(z) + \mathcal{O}\left(a^{\frac{5}{2}} \log(a)\right)$$

- ▷ no odd-power corrections of $\sqrt{a}$
- ▷ NLT and N^2LT terms are at most linearly dependent on $\log(a)$

Comments

- Our integration method can be extended to any order of PCs in a
  - higher-order plus distributions
  - at numerical level Chebyshev polynomials for calculating $\frac{\partial \mathcal{L}_{ab}}{\partial z}$
- Application to the q_{T}-subtraction method
  - the method suffers from a residual $q_{\mathrm{T}}^{\mathrm{cut}}$-dependence
  - our PCs can be directly used for the counter-term
  - the method uses $\hat{R}^{(1)}_{ab}(z) = -\hat{G}^{(1)}_{ab}(z)$, plus Born-like terms
- Contribution from the soft expansion
  - the (universal) leading soft term leads to known leading logs
  - a claim about the sub-leading soft terms and the sub-leading logarithmic PCs does **not** hold order-by-order in the final-state parton energy

Numerical results :: $pp \to Z + j$ @ 13 TeV



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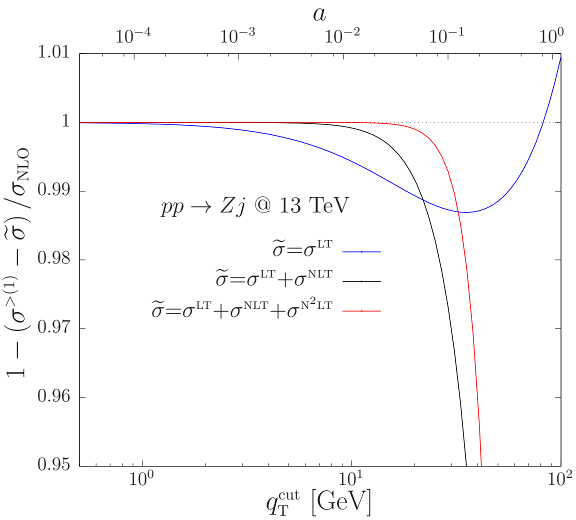

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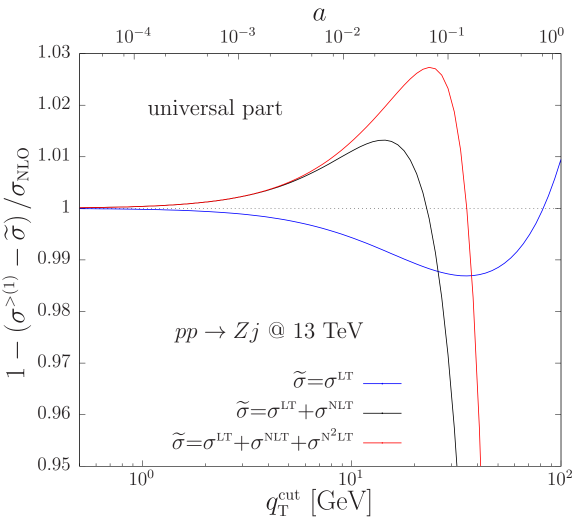
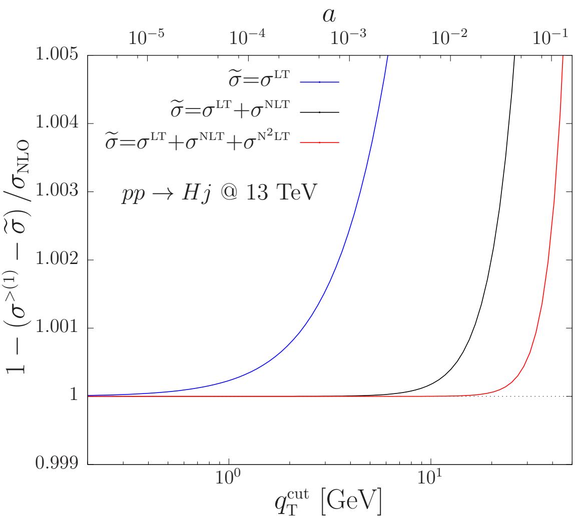


Numerical results :: $pp \to H + j$ @ 13 TeV



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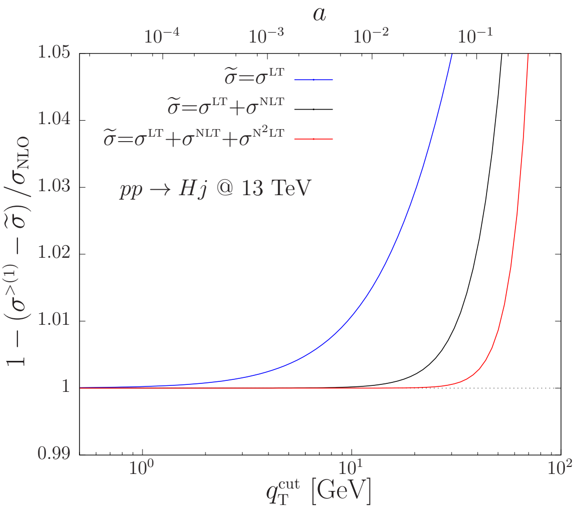

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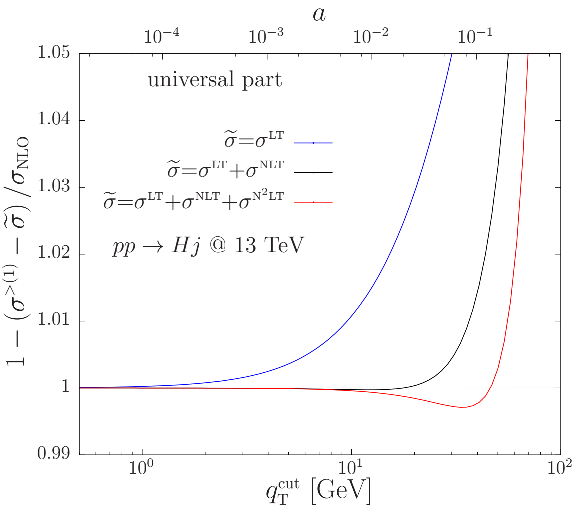

Conclusions

- we reproduced the known logarithms from collinear and soft regions of the PS, along with vanishing contributions, i.e. the new PCs in $q_{\mathrm{T}}^{\mathrm{cut}}$
- we noticed the **absence of odd-powers** in $q_{\mathrm{T}}^{\mathrm{cut}}$, claiming that this is likely to be true at all orders in $q_{\mathrm{T}}^{\mathrm{cut}}$, while false for more exclusive quantities
- we kept track of the universal part of the result, connected with the IR singular behaviour, also studying the higher-order soft expansion which does **not** yield a universal interpretation
- numerically speaking, H production shows a **larger sensitivity** on the cutoff w.r.t. Z production, although for the most part of universal origin
- our result may be crucial in the understanding of the q_{T}-resummation structure at subleading orders

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Some ongoing and future work

- NLO PCs for $pp \to (H \to 2\gamma) + j$, with cuts on $p_{\mathrm{T}}(\gamma)$
- NNLO inclusive PCs for color-singlet production
  - ☺ useful for a would-be local version of the q_{T}-subtraction method
  - ☹ much more involved than second-order collinear functions

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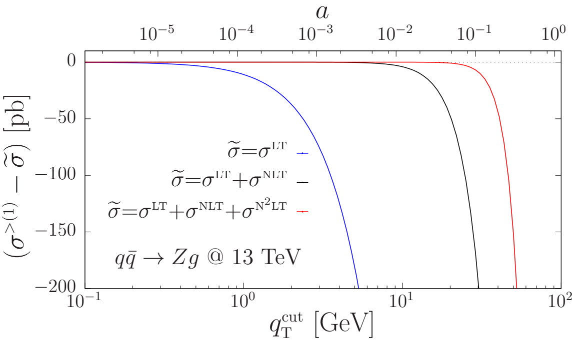

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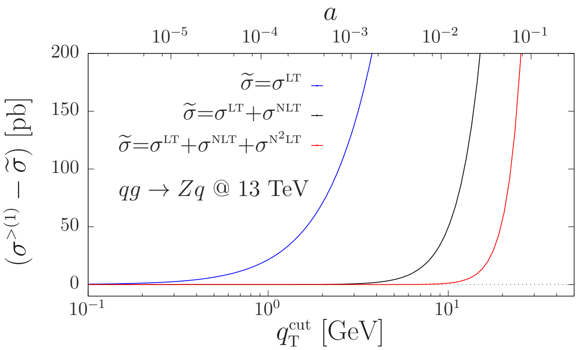

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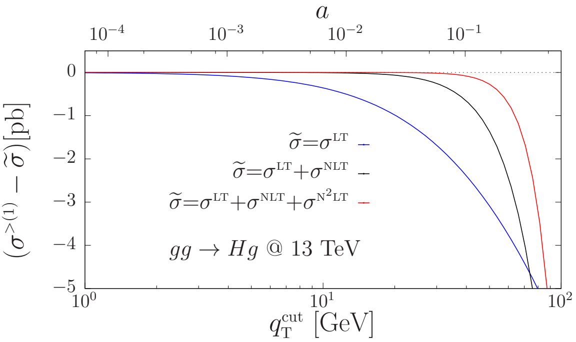

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