# Direct determination of $\sin \theta_{\text {eff }}^{\text {lept }}$ at hadron colliders 

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## Introduction

$\theta_{W}$ rules the mixing of $B$ and $W$ fields
determination of $\sin \theta_{W}$
■ indirect: $\sin \theta$ can be computed from $\alpha, G_{\mu}, M_{Z}, m_{H}, m_{t}$

- direct: 4-fermion processes at the $Z$ resonance


$$
g_{L}=\frac{I_{3}-s_{W}^{2} Q}{s_{W} c_{W}}, \quad g_{R}=-\frac{s_{W}}{c_{W}} Q
$$

measurable from forward/backward asymmetries

## $\sin \theta_{\text {eff }}^{\text {lept }}$ at LEP (1)



- cross sections and distributions parametrized in terms of pseudo-observables
- pseudo-observables fitted from data
$■ \sin \theta_{\text {eff }}^{\text {lept }}$ derived from tree-level like relations between pseudo-observables

The LEP pseudo-observables approach assumes factorization of on-shell production and on-shell decay for the $Z$

Some of the constraints come from $e^{+} e^{-} \rightarrow$ hadrons with separation of hadron flavours

## $\sin \theta_{\text {eff }}^{\text {lept }}$ at LEP (2)



## $\sin \theta_{\text {eff }}^{\text {lept }}$ at the LHC (1)



Same process as at LEP, with swapped IS and FS
Pseudo-observables approach is NOT used at the LHC:

- $M_{l l}$ window $[50,120] \mathrm{GeV}$ (factorized approach?)
- quark flavour not under control

■ additional uncertainties from PDFs

At the LHC $\sin \theta_{\text {eff }}^{\text {lept }}$ is measured using template fits

## $\sin \theta_{\text {eff }}^{\text {lept }}$ at the LHC (2)

measured from invariant-mass forward-backward asymmetry

$$
\begin{gathered}
A_{F B}\left(M_{l l}\right)=\frac{F\left(M_{l l}\right)-B\left(M_{l l}\right)}{F\left(M_{l l}\right)+B\left(M_{l l}\right)} \\
F=\int_{0}^{1} \mathrm{~d} \cos \theta^{*} \frac{\mathrm{~d} \sigma}{\mathrm{~d} \cos \theta^{*}}, \quad B=\int_{-1}^{0} \mathrm{~d} \cos \theta^{*} \frac{\mathrm{~d} \sigma}{\mathrm{~d} \cos \theta^{*}}
\end{gathered}
$$

$\theta^{*}$ measured in the Collins-Soper frame
using template fits

- measure $A_{F B}\left(M_{l l}\right)$
- generate Monte Carlo samples with different values of $\sin \theta_{W}$
- fit the template to the data
measured $\sin \theta_{W}$ is the one of the sample that describes best the data


## $\sin \theta_{\text {eff }}^{\text {lept }}$ and EW corrections

- calculations are usually done in the on-shell scheme
- in the $O S$ scheme $\sin \theta$ is constant at all orders

$$
\sin \theta_{O S}^{2}=1-\frac{M_{W}^{2}}{M_{Z}^{2}}
$$

- in the direct determination of $\sin \theta$ we want to extract $\sin \theta$ from the strength of the Zff coupling that is NOT constant at H.O.

$$
\sin \theta_{\mathrm{eff}}^{2}=\frac{1}{4}\left(1-\operatorname{Re} \frac{g_{V}}{g_{A}}\right)
$$

$■ \sin \theta_{\text {eff }}^{2}=\kappa_{l} \sin \theta_{O S},\left(\kappa_{l}=1\right.$ at LO $)$

## Template fits for $\sin \theta_{\mathrm{eff}}^{\text {lept }}$ and EW corrections

- Accuracy goal on $\sin ^{2} \theta_{W}$ is $10^{-4}$ : EW corrections mandatory
- $\sin \theta_{W}$ can always be used as input parameter for fits at LO
- The typical input schemes used at the LHC are $\left(\alpha / G_{\mu}, M_{W}, M_{Z}\right)$ : $\sin \theta_{W}$ is a derived quantity

In order to perform a fit at NLO EW and have a clean way to estimate the EW uncertainties, a new input parameter scheme should be used with $\sin \theta_{W}$ as free parameter

$$
\left(\alpha / G_{\mu}, \sin \theta, M_{Z}\right)
$$

## NC DY in the $\left(\alpha / G_{\mu}, \sin \theta, M_{Z}\right)$ scheme


(a)
(b)
(c)
bare diagrams don't change

(d)


CT diagrams don't change
w.r.t. the on-shell scheme, different expression for the countertem functions $\frac{\delta s_{W}^{2}}{s_{W}^{2}}$ and $\Delta r$

## Renormalization conditions

$$
\begin{aligned}
& \frac{i e}{2 s_{W} c_{W}} \gamma^{\mu}\left[g_{V}^{l}-g_{A}^{l} \gamma_{5}\right], \\
& g_{V}=\frac{g_{L}+g_{R}}{2}, \quad g_{A}=\frac{g_{L}-g_{R}}{2} \\
& \text { at } \mathrm{LO} \sin \theta_{\mathrm{eff}}^{2}=\frac{1}{4}\left(1-\operatorname{Re} \frac{g_{V}}{g_{A}}\right)
\end{aligned}
$$

the renormalization condition is

$$
\left.\sin \theta_{\mathrm{eff}}^{2}\right|_{N L O}=\left.\sin \theta_{\mathrm{eff}}^{2}\right|_{L O} \quad \Rightarrow \quad \frac{g_{V}+\delta g_{V}}{g_{A}+\delta g_{A}}=\frac{g_{V}}{g_{A}}
$$

## $\delta \sin \theta_{\mathrm{eff}}$

$$
\frac{\delta \sin \theta_{\text {eff }}^{2}}{\sin \theta_{\text {eff }}^{2}}=\operatorname{Re}\left\{\frac{\cos \theta_{\text {eff }}}{\sin \theta_{\text {eff }}} \frac{\Sigma_{T}^{A Z}\left(M_{Z}^{2}\right)}{M_{Z}^{2}}+\left(1-\frac{Q_{l}}{I_{3}^{l}} \sin \theta_{\text {eff }}^{2}\right)\left[\delta V^{L}-\delta V^{R}\right]\right\}
$$

$\delta V^{L / R}=$ bare vertex diagrams+fermion w.f. renorm.

- $\delta^{Q E D} g_{L}=\delta^{Q E D} g_{R}$ : affected only by weak corrections

■ no enhancement from logs of fermion masses

- no dependence on $\Delta \rho$ (no $m_{t}^{2}$ enhancement)


## $\Delta r, \Delta \tilde{r}$


computed from the NLO EW corrections to $\mu$-decay after subtracting 1-loop QED corrections in the Fermi model

$$
\begin{aligned}
& \Delta r=\Delta r\left(\alpha, M_{W}, M_{Z}\right) \\
& \Delta \tilde{r}=\Delta r\left(\alpha, \sin \theta, M_{Z}\right)
\end{aligned}
$$

$\Delta \tilde{r}$
$-\mathrm{LO} \simeq \frac{-1}{c_{W}^{2} M_{Z}^{2}}$

- $\mathrm{CT} \simeq \frac{\delta M_{Z}^{2}}{c_{W}^{2} M_{Z}^{4}}-\frac{2}{M_{Z}^{2}} \frac{s_{W}^{2}}{c_{W}^{2}} \frac{\delta \tilde{s}_{W}}{\tilde{s}_{W}}$
- $\Delta \tilde{r}=\Delta \alpha(s)-\Delta \rho+\Delta \tilde{r}_{\text {remn }}$
- $\mathrm{LO} \simeq \frac{-1}{M_{W}^{2}}$
- $\mathrm{CT} \simeq \frac{\delta M_{W}^{2}}{M_{W}^{4}}$
- $\Delta r=\Delta \alpha(s)-\frac{c_{W}^{2}}{s_{W}^{2}} \Delta \rho+\Delta r_{\text {remn }}$


## The $\left(G_{\mu}, \sin \theta_{\text {eff }}, M_{Z}\right)$ scheme: numerical results (1)




■ sensitivity dominated by LO behaviour

■ NLO EW corrections are smaller in the $\left(G_{\mu}, \sin \theta_{\text {eff }}, M_{Z}\right)$ scheme
■ H.O. effects smaller in the $\left(G_{\mu}, \sin \theta_{\text {eff }}, M_{Z}\right)$ scheme

## Universal fermionic corrections (H.O.) (1)

■ Leading fermionic corrections to DY come from $\Delta \alpha$ and $\Delta \rho$

- They can be included at 2-loop rescaling the relevant parameters in the LO amplitudes (subtracting the terms $\mathcal{O}(\alpha)$ already present at NLO)

■ In the OS scheme:

$$
\alpha_{0} \rightarrow \frac{\alpha_{0}}{1-\Delta \alpha\left(M_{Z}^{2}\right)}, s_{W}^{2} \rightarrow s_{W}^{2}\left(1+\frac{\delta s_{W}^{2}}{s_{W}^{2}}\right)=s_{W}^{2}+\Delta \rho c_{W}^{2}
$$

$g_{L}$ and $g_{R}$ diagrams receive different corrections

- In the $\sin \theta$ scheme:

$$
\alpha_{0} \rightarrow \frac{\alpha_{0}}{1-\Delta \alpha\left(M_{Z}^{2}\right)}, G_{\mu} \rightarrow G_{\mu}(1+\Delta \rho)^{2}
$$

## Universal fermionic corrections (H.O.) (2)

$$
\Delta \rho=\frac{\Sigma_{T}^{Z Z}(0)}{M_{Z}^{2}}-\frac{\Sigma_{T}^{W W}(0)}{M_{W}^{2}}
$$

$\Delta \rho$ in H.O. calculation:

$$
\begin{aligned}
& \Delta \rho=3 x_{t}\left[1+\rho^{(2)} x_{t}\right]\left[1-\frac{2 \alpha_{S}}{9 \pi}\left(\pi^{2}+3\right)\right] \\
& 3 x_{t}=\frac{3 \sqrt{2} G_{\mu} m_{t}^{2}}{16 \pi^{2}}=\Delta \rho^{(1)} \\
& \text { including 2-loop EW and QCD effects }
\end{aligned}
$$

## The $\left(G_{\mu}, \sin \theta_{\text {eff }}, M_{Z}\right)$ scheme: numerical results (2)


smaller parametric uncertainties from $m_{t}$ dependence compared to the OS scheme
$m_{t}^{2}$ dependence from $\Delta \rho$

- OS scheme: $\Delta \rho$ enters $\Delta r$ and $\delta s_{W}$. EW corrections affect $\gamma$ and $Z$ diagrams in a different way.
$■ \sin \theta$ scheme: $\Delta \rho$ enters only $\Delta r$. Overall effect, cancels in $A_{F B}$.


## Future steps

■ systematic evaluation of the impact of several classes of available radiative corrections

- estimate of the impact of missing higher orders on the observables, and translation into a $\sin \theta_{\text {eff }}$ shift

It is crucial to understand which corrections/ approximations/ contributions must be under control with an accuracy target of $10^{-4}$ on $\sin ^{2} \theta_{\text {eff }}$, e.g.

■ photon induced processes?
■ treatment of resonances and decay widths?

## Conclusions

■ We developed the $\left(G_{\mu}, \sin \theta_{\text {eff }}, M_{Z}\right)$ renormalization scheme, suitable for the extraction of $\sin \theta_{\text {eff }}$ at NLO EW

■ The NLO EW (and H.O.) corrections in the $\left(G_{\mu}, \sin \theta_{\text {eff }}, M_{Z}\right)$ are smaller than in the OS schemes

■ Smaller parametric dependence on $m_{t}$ in the $\left(G_{\mu}, \sin \theta_{\text {eff }}, M_{Z}\right)$ scheme

- Future development:
- assessment of the uncertainties form PS, matching and mixed QCD-EW effects
- systematic comparison against OS-schemes/other codes (distribution level)
- study of potential sources of uncertainties $\geq 10^{-4}$ on $\sin ^{2} \theta_{\text {eff }}$


## Backup Slides

## Collins-Soper frame

$\theta^{*}$ measured in the Collins-Soper frame

$$
\cos \theta^{*}=\frac{2 p_{z}^{l l}}{\left|p_{z}^{l l}\right|} \frac{P_{l^{-}}^{+} P_{l^{+}}^{-}-P_{l^{-}}^{-} P_{l^{+}}^{+}}{m_{l l} \sqrt{m_{l l}^{2}+p_{T l l}^{2}}}, \quad P^{ \pm}=\frac{1}{\sqrt{2}}\left(E \pm p_{z}\right)
$$

