Probing Colour Flow with Jet Pull

Chang Wu
Università di Genova & INFN Sezione di Genova

In collaboration with A. Larkoski, S. Marzani
Based on 1903.02275, 1911.05090, and ongoing work

Milan Christmas Meeting, December 20, 2019

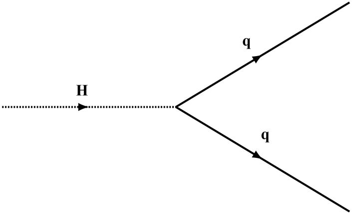
Motivation

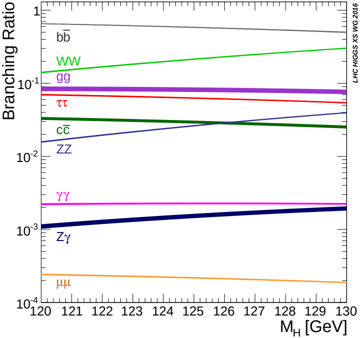
Processes with colour-singlet final states particle play a particularly important role in high-energy physics

- Better tools: Soft Drop[Larkoski, Marzani, Soyez and Thaler, 1402.2657]
- Better observables: Jet vetoes[ATLAS, 1203.5015], [Marzani, 1205.6808]

Understanding whether (new) particles carry colour charge:

- Largest BR for Higgs decay: $H \to b\bar{b}$





[LHC Higgs Cross XS WG, 1610.07922]

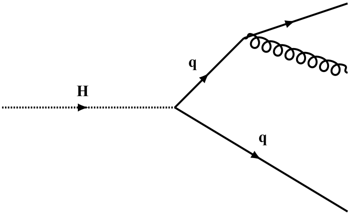
Motivation

Processes with colour-singlet final states particle play a particularly important role in high-energy physics

- Better tools: Soft Drop[Larkoski, Marzani, Soyez and Thaler, 1402.2657]
- Better observables: Jet vetoes[ATLAS, 1203.5015], [Marzani, 1205.6808]

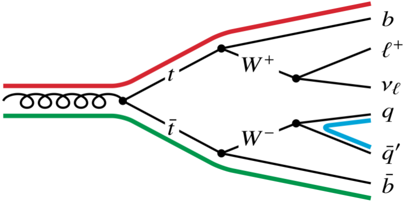
Understanding whether (new) particles carry colour charge:

- Largest BR for Higgs decay: $H \to b\bar{b}$



Non-trivial Leading Order

- Pull angle measurements: top quark decay



[ATLAS, 1805.02935]

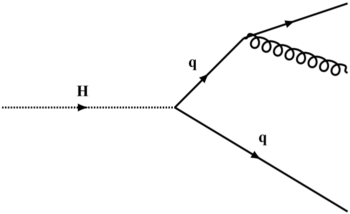
Motivation

Processes with colour-singlet final states particle play a particularly important role in high-energy physics

- Better tools: Soft Drop[Larkoski, Marzani, Soyez and Thaler, 1402.2657]
- Better observables: Jet vetoes[ATLAS, 1203.5015], [Marzani, 1205.6808]

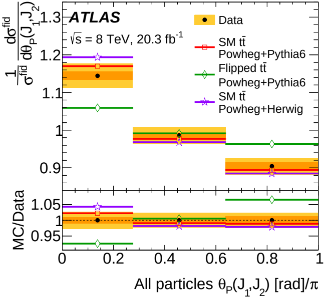
Understanding whether (new) particles carry colour charge:

- Largest BR for Higgs decay: $H \to b\bar{b}$



Non-trivial Leading Order

- Pull angle measurements: top quark decay



[ATLAS, 1506.05629]

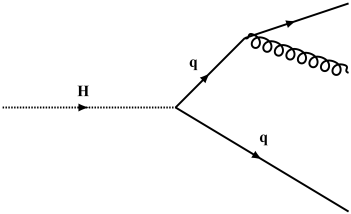
Motivation

Processes with colour-singlet final states particle play a particularly important role in high-energy physics

- Better tools: Soft Drop[Larkoski, Marzani, Soyez and Thaler, 1402.2657]
- Better observables: Jet vetoes[ATLAS, 1203.5015], [Marzani, 1205.6808]

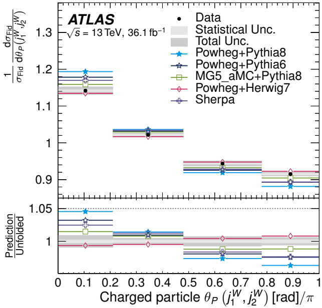
Understanding whether (new) particles carry colour charge:

- Largest BR for Higgs decay: $H \to b\bar{b}$



Non-trivial Leading Order

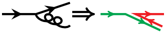
- Pull angle measurements: top quark decay



[ATLAS, 1805.02935]

Seeing in color with jet substructure

- Colour connection

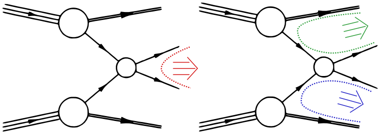


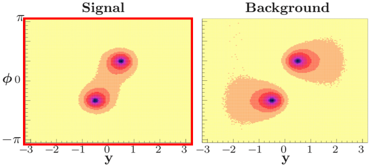
- Definition of pull vector:

$$\vec{t} = \sum_{i \in jet} \frac{p_T^i |r_i|}{p_T^{jet}} \vec{r_i}$$

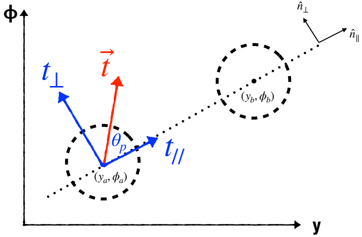
$$\vec{r_i} = (y_i - y_a, \phi_i - \phi_a)$$

- One dimensional projections
  - (t, θ_p): similar to q_T and ρ
  - $\left(|\vec{t} \cdot \hat{n}_{||}|, |\vec{t} \cdot \hat{n}_\perp|\right)$: similar to a_T and ϕ^*
  - $\left(\vec{t} \cdot \hat{n}_{||}, \vec{t} \cdot \hat{n}_\perp\right)$: asymmetry



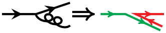


[Gallicchio, Schwartz, 1001.5027]



Seeing in color with jet substructure

- Colour connection

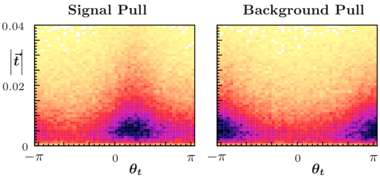


- Definition of pull vector:

$$\vec{t} = \sum_{i \in jet} \frac{p_T^i |r_i|}{p_T^{jet}} \vec{r}_i$$

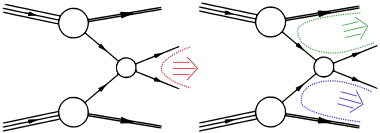
$$\vec{r}_i = (y_i - y_a, \phi_i - \phi_a)$$

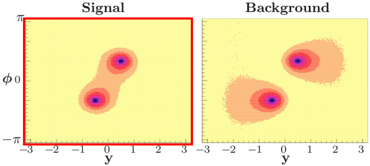
- One dimensional projections
  - (t, θ_p): similar to q_T and ρ



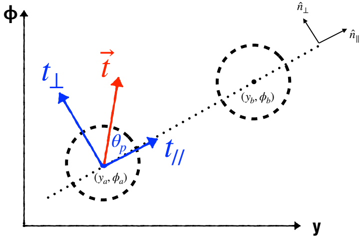
[Gallicchio, Schwartz, 1001.5027]

- **Theoretical challenge: IRC unsafe observable**





[Gallicchio, Schwartz, 1001.5027]

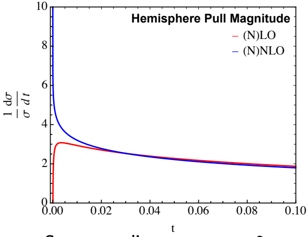


Outline

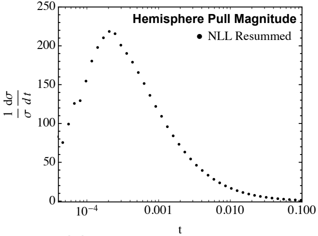
1. Basics of resummation
2. Challenges for precision QCD
   - Non-global logarithms
   - Sudakov safety
3. Probing colour flow with jet pull
   - Theory predictions for the pull angle
   - Safe projections of jet pull
4. Conclusions and outlook

Beyond fixed order perturbation theory

- In the region of soft-collinear, fixed order calculation spoiled by large logarithm enhancement $\alpha_s^m log^{2m-1} 1/t \sim 1$, need to be resummed at all-order



Spectrum diverges as $t \to 0$



Sudakov suppression as $t \to 0$

- From order-by-order to accuracy-by-accuracy

$$\frac{1}{\sigma}\frac{d\sigma^{fo}}{d\mathcal{O}} \propto \underbrace{O(1)}_{\text{LO}} + \underbrace{O(\alpha_S)}_{\text{NLO}} + \underbrace{O(\alpha_S^2)}_{\text{NNLO}} + \ldots$$

$$\propto exp(\underbrace{Lg_1(\alpha_S L)}_{\text{LL}} + \underbrace{g_2(\alpha_S L)}_{\text{NLL}} + \underbrace{\alpha_S g_3(\alpha_S L)}_{\text{NNLL}} + \ldots)$$

Principles of resummation

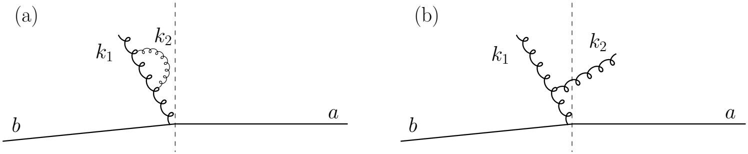
- Infrared and collinear (IRC) safety

IRC safety

The complete cancellation of infrared and collinear singularity require the IRC safe observable as

$$O_n(p_1 ... p_i ... p_n) \to O_{n-1}(p_1 ... p_{i-1}, p_{i+1} ... p_n), \ \ if\ p_i \to 0$$
$$O_n(p_1 ... p_i, p_j ... p_n) \to O_{n-1}(p_1 ... p_i + p_j ... p_n), \ \ if\ p_i \parallel p_j$$

- Globalness: an observable is defined to be global if it is sensitive to radiation anywhere in phase-space.



[Dasgupta and Salam, 0104277]

Transverse momentum resummation

Traditional approach:

- Simplify QCD amplitudes in soft/collinear limits
- Example: collinear section

$$\frac{d\sigma}{d\vec{q}_T} = \mathcal{V} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} d\left[k_i\right] \times P\left(z_i\right) \delta^{(2)} \left(\vec{q}_T - \sum_{n=0}^{\infty} \vec{q}_{Ti}\left(k_i\right) \right)$$

RGE approach:

- Factorization of the various scales involved

$$\frac{d\sigma}{d\vec{q}_T} = H\left(\mu, Q^2\right) \int d^2 k_c d^2 k_s \times J\left(\mu, \vec{k}_c\right) S\left(\mu, \vec{k}_s\right) \delta^{(2)} \left(\vec{q}_T - \vec{k}_c - \vec{k}_s\right)$$

- Solve the RG evolution between two scale:

$$\mu \frac{\partial F}{\partial \mu} = \gamma_F F$$

Resummation in Impact parameter space:

$$\frac{d\sigma}{d\vec{q}_T} = \frac{1}{2\pi} \int b_\perp db_\perp J_0\left(b_\perp q_T\right) e^{-R(b_\perp)}$$

Transverse momentum resummation

Traditional approach:

- Simplify QCD amplitudes in soft/collinear limits
- Example: collinear section

$$\frac{d\sigma}{d\vec{q}_T} = \mathcal{V}\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n} d\left[k_i\right] \times P\left(z_i\right)\delta^{(2)}\left(\vec{q}_T - \sum_{n=0}^{\infty}\vec{q}_{Ti}\left(k_i\right)\right)$$

RGE approach:

- Factorization of the various scales involved

$$\frac{d\sigma}{d\vec{q}_T} = H\left(\mu, Q^2\right)\int d^2k_c d^2k_s \times J\left(\mu, \vec{k}_c\right) S\left(\mu, \vec{k}_s\right)\delta^{(2)}\left(\vec{q}_T - \vec{k}_c - \vec{k}_s\right)$$

- Solve the RG evolution between two scale:

$$\mu\frac{\partial F}{\partial \mu} = \gamma_F F$$

Resummation in Impact parameter space:

$$\frac{d\sigma}{d\vec{t}} = \frac{1}{2\pi}\int bdbJ_0\left(bt\right)e^{-R(b)}$$

The NLL exponent is

$$-R_c\left(b\right) = Lf_1\left(\lambda\right) + f_2\left(\lambda\right)$$

$$f_1(\lambda) = -\frac{C_i}{2\pi\beta_0\lambda}\left[\left(1-2\lambda\right)\log\left(1-2\lambda\right) - 2\left(1-\lambda\right)\log\left(1-\lambda\right)\right]$$

$$f_{2c}(\lambda) = -\frac{C_iB_i}{\pi\beta_0}\log\left(1-\lambda\right) - \frac{C_iK}{4\pi^2\beta_0^2}\left[2\log\left(1-\lambda\right) - \log\left(1-2\lambda\right)\right] - \frac{C_i\beta_1}{2\pi\beta_0^3}\left[\log\left(1-2\lambda\right) - 2\log\left(1-\lambda\right) + \frac{1}{2}\log^2\left(1-2\lambda\right) - \log^2\left(1-\lambda\right)\right]$$

Outline

Non-global logarithms

- BMS equation: [Banfi, Marchesini and Smye, 0206076]

$$\partial g_{ab}(L) = \int_L \frac{d\Omega_j}{4\pi} W^j_{ab} \left[U_{abj}(L)\, g_{aj}(L)\, g_{jb}(L) - g_{ab}(L) \right]$$

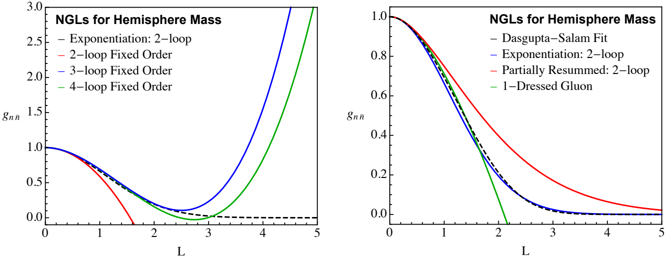
$$U_{abj}(L) = exp\left[L \int_R \frac{d\Omega_1}{4\pi} \left(W^1_{ab} - W^1_{aj} - W^1_{jb} \right) \right]$$

- Fixed-order result: [Schwartz and Zhu, 1403.4949]

$$g_{n\bar{n}}(L) = 1 - \frac{\pi^2}{24} L^2 + \frac{\zeta(3)}{12} L^3 + \frac{\pi^4}{34560} L^4 + \left(\frac{17\zeta(5)}{480} - \frac{\pi^3 \zeta(3)}{360} \right) L^5 + \cdots$$

$$L = \frac{\alpha_s}{\pi} N_c \log \frac{m_L}{m_R}$$

- Comparison between different approaches:



IRC safety vs Sudakov safety

- IRC unsafe observables:
  - Ratio Observable:[Larkoski and Thaler,1307.1699]

$$\frac{d\sigma}{dr} \equiv \int d\alpha d\beta \frac{d^2\sigma}{d\alpha d\beta} \delta\left(r - \frac{\alpha}{\beta}\right)$$

  - Pull angle:

$$\frac{d\sigma}{d\phi_p} = \int t dt d\phi_p \frac{d^2\sigma}{d\vec{t}} \delta\left(\phi_p - cos^{-1}\frac{t_x}{t}\right)$$

- Sudakov safety: with the help of all-order resummation, the Sudakov factor will act as the regulator for the double differential cross section
- Formalisms for the calculation:

$$\frac{1}{\sigma}\frac{d\sigma^{res}}{d\vec{t}} = \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{b}\cdot\vec{t}} e^{-R(b)}$$

IRC safety vs Sudakov safety

- IRC unsafe observables:
  - Ratio Observable:[Larkoski and Thaler,1307.1699]

$$\frac{d\sigma}{dr} \equiv \int d\alpha d\beta \frac{d^2\sigma}{d\alpha d\beta}\delta\left(r - \frac{\alpha}{\beta}\right)$$

  - Pull angle:

$$\frac{d\sigma}{d\phi_p} = \int tdtd\phi_p \frac{d^2\sigma}{d\vec{t}}\delta\left(\phi_p - cos^{-1}\frac{t_x}{t}\right)$$

- Sudakov safety: with the help of all-order resummation, the Sudakov factor will act as the regulator for the double differential cross section
- Joint probability approach:

$$p\left(\phi_p\right) \approx \int dt\, p_{res}\left(t\right) p_{fo}\left(\phi_p|t\right) = \int dt\, e^{-R(t)} \frac{d^2\sigma^{fo}}{dtd\phi_p}$$

We can avoid the puzzle by just resum the pull magnitude

Outline

1. Basics of resummation
2. Challenges for precision QCD
   - Non-global logarithms
   - Sudakov safety
3. Probing colour flow with jet pull
   - Theory predictions for the pull angle
   - Safe projections of jet pull
4. Conclusions and outlook

Resummation of the pull magnitude

- Consider the resummation of the magnitude of a vector

$$t = |\vec{t}| = |\sum_{i \in jet} \frac{p_T^i |r_i|}{p_T^{jet}} \vec{r}_i|$$

- For single gluon emission, scaling similar to the jet mass

$$t = |\vec{t}_1 + \vec{t}_2| = z\,(1-z)\,|1-2z|\theta^2$$

- NLL resummation:

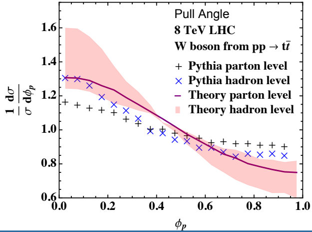
$$\frac{1}{\sigma}\frac{d\sigma}{dt} = t\int_0^\infty bdbJ_0\,(bt)\,e^{Lf_1(\lambda)+f_{2c}(\lambda)},\ \lambda = \alpha_s\beta_0 log\left(b\frac{e^{\gamma_E}}{2}\right)$$

  - situation very similar to well-known q_T resummation
  - the radiator f_1 and f_{2c} have the same functional form as jet mass

- Perturbative calculation

$$p_{pert}\,(t,\phi) = \int dcos\theta_{12} p_{res}\,(t)\,p_{fo}\,(t|\phi_p)\,p\,(cos\theta_{12})$$

$$p_{pert}\,(\phi) = \int dt p_{pert}\,(t,\phi)$$

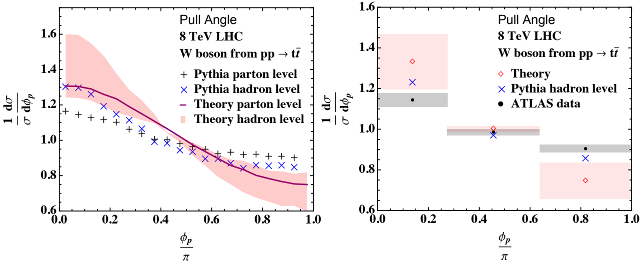


Comparison to the data

- Non-perturbative correction:

$$p_{np}\left(t,\phi_p\right) \sim tanh\left(\frac{1}{a\phi_p\left(2\pi-\phi_p\right)}\right)\delta\left(t-\frac{\Lambda_{QCD}}{E_J}\right),\ a\in(0,0.25)$$

- Our calculation is in fair agreement with the data, however it suffers from large theory uncertainties
  - perturbative uncertainties: Sudakov safe observables can't expand in terms of Feynman diagrams
  - non-pert. uncertainties: Lack of IRC safety prevents separation between perturbative and non-perturbative region



Safe uses of jet pull

- The new variable $t_{\|}$ and $t_{\perp}$ are IRC safe: analog from a_T distribution

  [Banfi, Dasgupta and Duran Delgado, 0909.5327]

$$a_T = |\sum_i k_{ti} sin\phi_i|, \; t_{\|} = |\sum_i t_i cos\phi_i|$$

- NLL resummation with NGLs

$$\frac{1}{\sigma}\frac{d\sigma}{dt_{\|}} = \frac{2}{\pi}\int_0^{\infty} db cos\left(bt_{\|}\right) e^{Lf_1(\lambda)+f_2(\lambda)} S_{ng}\left(\lambda\right)$$

- Measure the pull between the H subjets

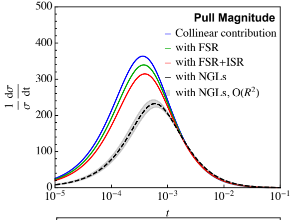
$$q\bar{q} \to H(\to b\bar{b})\, Z(\to l^+l^-).$$

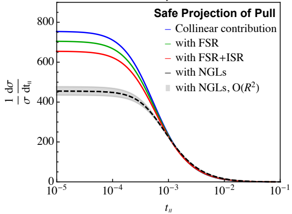
  - Impact of the different contributions
  - Non Sudakov suppression for $t_{\|}$

$$\frac{d\sigma}{dt_{\|}} = \frac{2}{\pi}\int_0^{\infty} db cos\left(bt_{\|}\right) e^{-\frac{\alpha_s C_F}{2\pi} log^2 b}$$

$$= \sqrt{\frac{2}{\alpha_s C_F}} e^{\frac{\pi}{2\alpha_s C_F}} + O\left(t_{\|}^2\right)$$

  - R^2 corrections to the NGLs



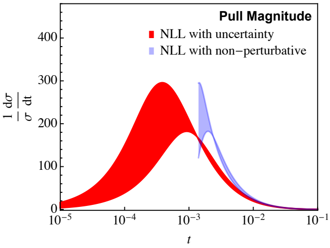


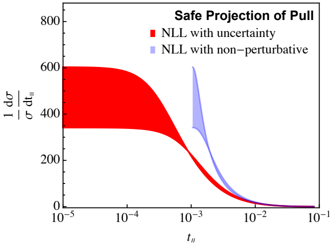
Towards phenomenology

- Non-perturbative effects: standard shape function

$$\frac{d^2\sigma^{\mathrm{np}}}{d\vec{t}^{\,2}} = \int_0^\infty dk_t \int_0^{2\pi} d\varphi\, F(k_t,\varphi)\, \frac{d^2\sigma^{\mathrm{pert}}}{d\vec{t}^{\,2}}\left(\vec{t} - \vec{t}_{\mathrm{np}}(k_t,\varphi)\right)$$
$$= \int_0^{2\pi} \frac{d\varphi}{2\pi}\, \frac{d^2\sigma^{\mathrm{pert}}}{d\vec{t}^{\,2}}\left(\vec{t} - \vec{t}_{\mathrm{np}}(k_t,\varphi)\right)$$

Theoretical predictions are affected by large uncertainties, the situation can be improved by matching with fixed-order result





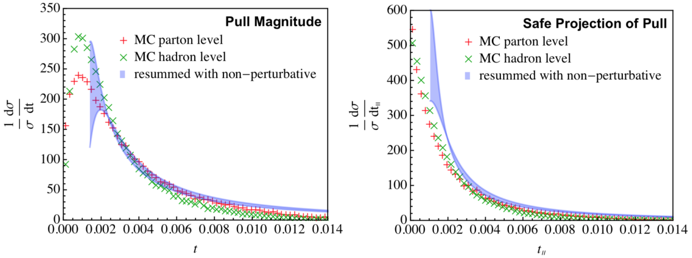
Towards phenomenology

- Non-perturbative effects: standard shape function

$$\frac{d^2\sigma^{\text{np}}}{d\vec{t}^{\,2}} = \int_0^\infty dk_t \int_0^{2\pi} d\varphi\, F(k_t,\varphi)\, \frac{d^2\sigma^{\text{pert}}}{d\vec{t}^{\,2}}\left(\vec{t} - \vec{t}_{\text{np}}(k_t,\varphi)\right)$$
$$= \int_0^{2\pi} \frac{d\varphi}{2\pi} \frac{d^2\sigma^{\text{pert}}}{d\vec{t}^{\,2}}\left(\vec{t} - \vec{t}_{\text{np}}(k_t,\varphi)\right)$$

Theoretical predictions are affected by large uncertainties, the situation can be improved by matching with fixed-order result

- Comparison to the Monte Carlo:



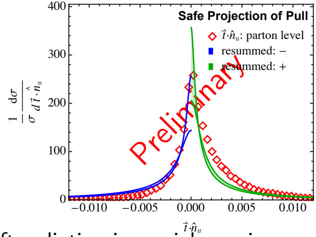
Asymmetries

- The magnitude of safe projection leads to a loss of information
- Fully exploit the radiation pattern: difference between towards and away from the other jet of interest

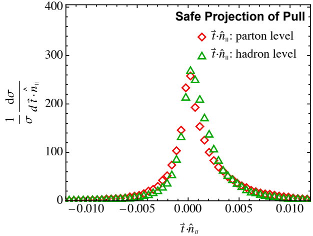
$$\mathcal{A}_{\parallel} = \frac{t_{\parallel}}{\sigma}\frac{d\sigma}{dt_{\parallel}}\Big|_{\vec{t}\cdot\hat{n}_{\parallel}>0} - \frac{t_{\parallel}}{\sigma}\frac{d\sigma}{dt_{\parallel}}\Big|_{\vec{t}\cdot\hat{n}_{\parallel}<0},$$

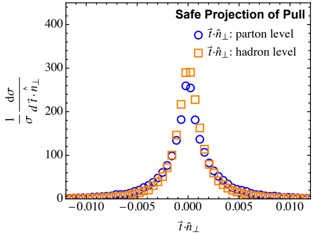
$$\mathcal{A}_{\perp} = \frac{t_{\perp}}{\sigma}\frac{d\sigma}{dt_{\perp}}\Big|_{\vec{t}\cdot\hat{n}_{\perp}>0} - \frac{t_{\perp}}{\sigma}\frac{d\sigma}{dt_{\perp}}\Big|_{\vec{t}\cdot\hat{n}_{\perp}<0}$$

- NLL Resummation



The soft radiation is crucial, precise prediction needs higher accuracy (N)NLO+NNLL





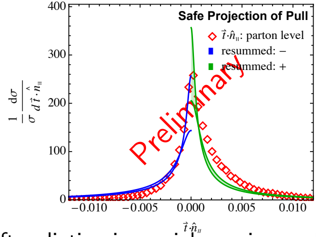
Asymmetries

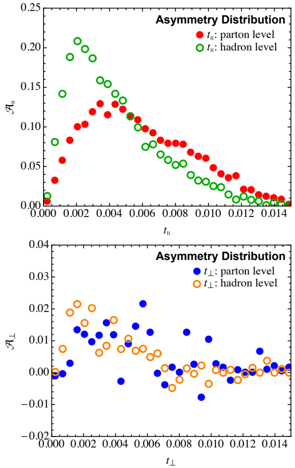
- The magnitude of safe projection leads to a loss of information
- Fully exploit the radiation pattern: difference between towards and away from the other jet of interest

$$\mathcal{A}_{\|} = \frac{t_{\|}}{\sigma}\frac{d\sigma}{dt_{\|}}\Big|_{\vec{t}\cdot\hat{n}_{\|}>0} - \frac{t_{\|}}{\sigma}\frac{d\sigma}{dt_{\|}}\Big|_{\vec{t}\cdot\hat{n}_{\|}<0},$$

$$\mathcal{A}_{\perp} = \frac{t_{\perp}}{\sigma}\frac{d\sigma}{dt_{\perp}}\Big|_{\vec{t}\cdot\hat{n}_{\perp}>0} - \frac{t_{\perp}}{\sigma}\frac{d\sigma}{dt_{\perp}}\Big|_{\vec{t}\cdot\hat{n}_{\perp}<0}$$

- NLL Resummation





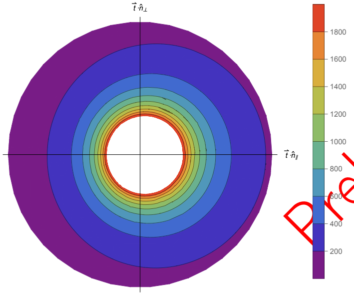
The soft radiation is crucial, precise prediction needs higher accuracy (N)NLO+NNLL

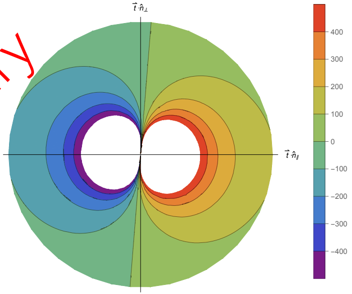
Asymmetry distributions:

$$\mathcal{A}_{\parallel} = \frac{\alpha_s C_F}{\pi}\left[\frac{4R}{\pi}\frac{\cos\beta\sinh\Delta y + \sin\beta\sin\Delta\phi}{\cos\Delta\phi - \cosh\Delta y} + O\left(R^3\right)\right] + \mathcal{O}\left(\alpha_s^2\right)$$

$$\mathcal{A}_{\perp} = \frac{\alpha_s C_F}{\pi}\left[\frac{4R}{\pi}\frac{\cos\beta\sin\Delta\phi - \sin\beta\sinh\Delta y}{\cos\Delta\phi - \cosh\Delta y} + O\left(R^3\right)\right] + \mathcal{O}\left(\alpha_s^2\right).$$

Based on the discussion above, we can also define the asymmetry cross section by subtracting the pull angle averaged result from the joint distribution.





Outline

Conclusions and Outlook

Summary:

- Jet pull is an interesting observable that can probe colour flow
- With the help of Sudakov safe techniques, we present the first theoretical prediction for the pull angle.
- Safe projections of jet pull: theoretical predictions are affected by large uncertainties, the situation can be improved by matching
- Fully exploit the radiation pattern can be achieved with the azimuthal asymmetry
- Asymmetries can play an important role in assessing subleading colour correlations

Work in Progress:

- The asymmetry introduces a new boundary in phase-space which renders the all-order structure of these observables richer
- Double differential resummation for the joint distribution
- Probing the full radiation pattern require higher order and accuracy: (N)NLO+NNLL
- Complete theory predictions: factorization with arbitrary color flow.

Thank you for your attention

Extra Slides

Perturbative expansion of the BMS equation

Leading NGLs arise from the strongly ordered gluon emissions

$$E_1 \gg E_2 \gg E_3 \gg \cdots \gg E_n$$

The multi-gluon emission amplitude is simplified, and at large N_c limit:

$$\begin{aligned} \left|\mathcal{M}_{ab}^{1\cdots m}\right|^2 &= \left|\langle p_1 \cdots p_m \middle| Y_a^\dagger Y_b \middle| 0\rangle\right|^2 \\ &= N_c^m g^{2m} \sum_{\text{perms of } 1\cdots m} \frac{(p_a \cdot p_b)}{(p_a \cdot p_1)(p_1 \cdot p_2)\cdots(p_m \cdot p_b)} \end{aligned}$$

And the expansion of BMS at 3-loop reads:

$$\begin{aligned} \partial g_{ab}^{(3)}(L) = \frac{L^2}{2} \int_\Omega 1_L 2_R 3_R W_{ab}^1 \left(W_{ab}^2 - W_{a1}^2 - W_{1b}^2\right)\left(W_{ab}^3 - W_{a1}^3 - W_{1b}^3\right) \\ + \int_\Omega 1_L W_{ab}^1 \left[g_{a1}^{(2)}(L) + g_{1b}^{(2)}(L) - g_{ab}^{(2)}(L)\right] \end{aligned}$$

Validation with EVENT2

- Double differential distribution:

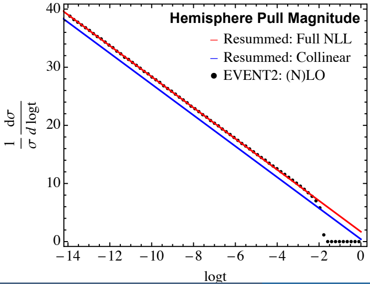
$$\frac{d^2\sigma}{dt\,d\phi_p} = \frac{\alpha_s}{\pi^2}\frac{C_F}{t}[\log\frac{4\tan^2\frac{R}{2}}{t} - \frac{3}{4}$$

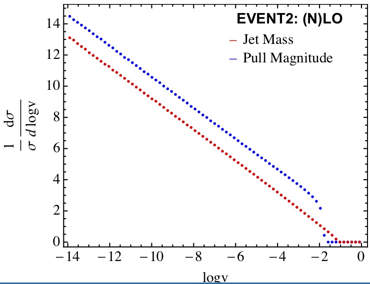
$$+ 2\cot\phi_p\ \tan^{-1}\frac{\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\sin\phi_p}{1-\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\phi_p} - \log(1+\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}} - 2\frac{\tan\frac{R}{2}}{\tan\frac{\theta_{12}}{2}}\cos\phi_p)]$$

- Structure of NLL' resummation:[M. Dasgupta G.P. Salam, hep-ph/0104277]

$$\Sigma(t) = (1+\alpha_s C_1^{(q)})S(\alpha_s L)e^{-R_q(\alpha_s C_F, L)} + \alpha_s C_1^{(g)} e^{-R_g(\alpha_s C_A, L)}$$

- Validation with EVENT2: (N)LO





Validation with EVENT2

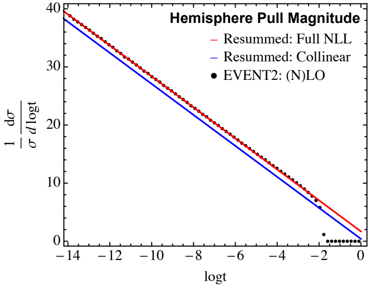
- Magnitude of pull:

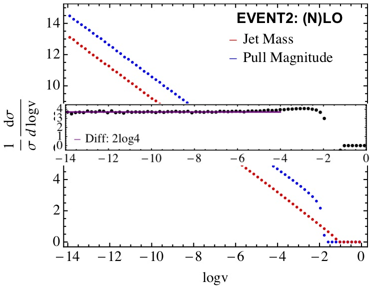
$$\frac{d\sigma}{dt} = \frac{\alpha_s}{\pi}\frac{C_F}{t}\left[\log\frac{1}{t} - \frac{3}{4} - \log\left(\frac{1-\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}}}{4\tan^2\frac{R}{2}}\right)\right].$$

- Structure of NLL' resummation:[M. Dasgupta G.P. Salam, hep-ph/0104277]

$$\Sigma(t) = (1 + \alpha_s C_1^{(q)})S(\alpha_s L)e^{-R_q(\alpha_s C_F, L)} + \alpha_s C_1^{(g)} e^{-R_g(\alpha_s C_A, L)}$$

- Validation with EVENT2: (N)LO





Validation with EVENT2

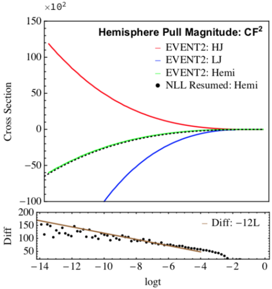
- Magnitude of pull:

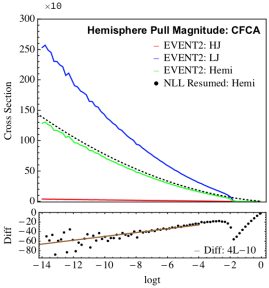
$$\frac{d\sigma}{dt} = \frac{\alpha_s}{\pi}\frac{C_F}{t}\left[\log\frac{1}{t} - \frac{3}{4} - \log\left(\frac{1-\frac{\tan^2\frac{R}{2}}{\tan^2\frac{\theta_{12}}{2}}}{4\tan^2\frac{R}{2}}\right)\right].$$

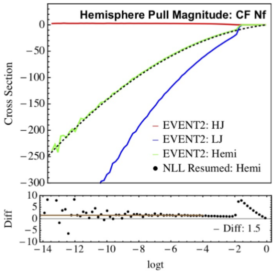
- Structure of NLL' resummation:[M. Dasgupta G.P. Salam, hep-ph/0104277]

$$\Sigma(t) = (1 + \alpha_s C_1^{(q)})S(\alpha_s L)e^{-R_q(\alpha_s C_F, L)} + \alpha_s C_1^{(g)} e^{-R_g(\alpha_s C_A, L)}$$

- Validation with EVENT2: (N)NLO







Double differential NLL resummation

- Missing NLL soft term for the joint distribution:

$$\frac{d\sigma^{NLO}}{tdtd\phi_p} = \frac{\alpha_s}{2\pi^2}\frac{C_F}{t^2}\left[log\frac{4tan^2\frac{R}{2}}{t} - \frac{3}{4} + f\left(\phi_p, R, \theta_{12}\right)\right]$$

$$\frac{d\sigma^{res\cdot exp}}{tdtd\phi_p} = \frac{\alpha_s}{2\pi^2}\frac{C_F}{t^2}\left[log\frac{4tan^2\frac{R}{2}}{t} - \frac{3}{4} - log\left(1 - a\left(R, \theta_{12}\right)\right)\right]$$

- Check pull magnitude

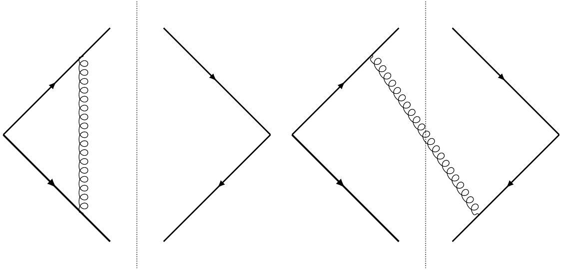
$$\int_0^{2\pi}\frac{d\phi_p}{2\pi}\frac{f\left(\phi_p\right) + log\left(1 - a^2\right)}{t^2} \equiv \int_0^{2\pi}\frac{d\phi_p}{2\pi}\frac{g\left(\alpha_s, \phi_p\right)}{t^2} = 0$$

- The missing term in conjugate space:

$$\int bdb\int\frac{d\phi_b}{\left(2\pi\right)^2}e^{-ibtcos\left(\phi_b-\phi\right)}M\left(\alpha_s, \phi_b\right) = \frac{\alpha_s C_F}{2\pi^2}\frac{g\left(\phi_p\right)}{t^2}$$

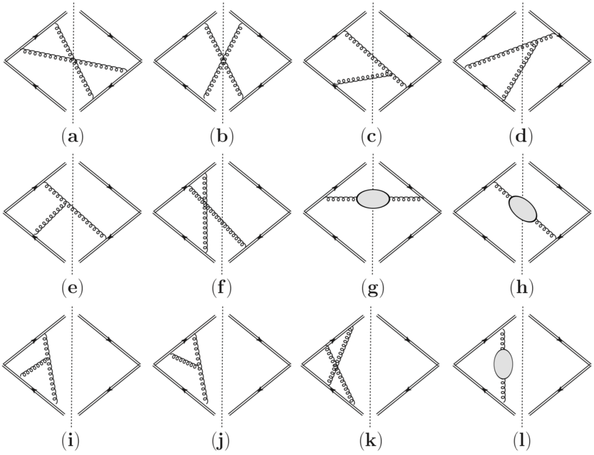
Soft gluon emission

- one-loop level:



Soft gluon emission

- two-loop level:



[Monni, Gehrmann and Luisoni, 1105.4560]