Recoil scheme and logarithmic accuracy in agular-ordered parton showers

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IPPP Durham

Milan Christmas Meeting

19-20 December 2019

Based on Gavin Bewick, S.F.R., Peter Richardson and Mike Seymour [arxiv:1904.11866]

Introduction: Shower Monte Carlo generators

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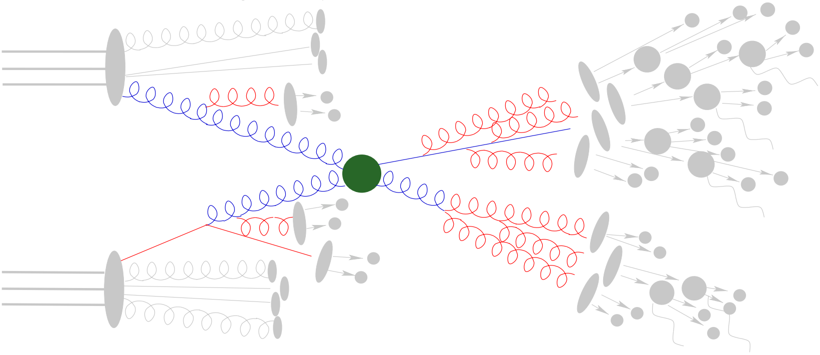
- The core of SMC is given by the Parton Shower.



Parton Showers in a nutshell (I)

- When a (quasi) collinear parton or a soft gluon is emitted, we have a **logarithmic** divergence:

$$\frac{d\sigma_{n+1}}{d\sigma_n} \propto \frac{d|\vec{k}|\ d\cos\theta_{pk}}{(p+k)^2 - m^2} = \frac{d|\vec{k}|}{|\vec{k}|} \frac{d\cos\theta_{pk}}{(\sqrt{|\vec{p}|^2 + m^2} - |\vec{p}|\cos\theta_{pk})}$$

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- in the (quasi-)**collinear** limit, the cross-section factorizes:

$$d\sigma_{n+1}(Q) = d\sigma_n(Q)\frac{\alpha_s}{2\pi}P_{\tilde{ij}\to i,j}(z)\frac{dt}{t}\frac{d\phi}{2\pi} \qquad \text{where } t = \{p_T^2, q_{\tilde{ij}}^2, E^2\theta^2, \dots\}$$

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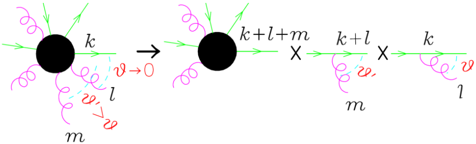
- Factorization can be applied recursively:



  - We define an ordering variable t: $Q > t_1 > t_2 > t_3 > \ldots > t_{\text{cutoff}}$
  - Emission probability $dP_{\tilde{ij}\to i,j}(t,z,\phi) = \frac{\alpha_s}{2\pi} P_{\tilde{ij}\to i,j}(z,t)\, dz \frac{dt}{t} \frac{d\phi}{2\pi}$
  - **Sudakov form factor**

$$\Delta(t_i, t_{i+1}) = \exp\left[-\int_{t_{i+1}}^{t_i} dt' \int_{z_{\min}(t')}^{z_{\max}(t')} dz \frac{\alpha_s}{2\pi} \frac{P_{\tilde{ij}\to i,j}(z,t')}{t'} \right]$$

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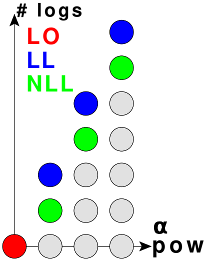

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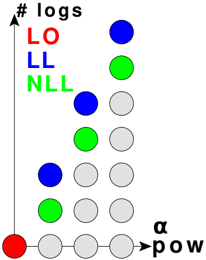

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- (quasi-)**collinear** splitting functions correctly describe all the **LL** (collinear AND soft) but only the collinear **NLL**.

$$\lim_{z\to 1} P_{i,g}(z,t) = \frac{2C_i}{1-z}\left[1 - \frac{(1-z)^2 m_i^2}{(1-z)^2 m_i^2 + |p_T^2|}\right]$$

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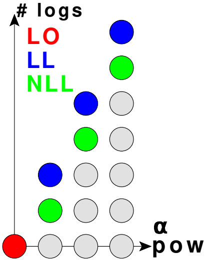

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- $\alpha_s = \alpha_s^{\mathrm{CMW}}(p_T) = \alpha_s^{\overline{\mathrm{MS}}}(p_T)\left[1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}(p_T)}{2\pi}\left(\left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{5}{9}n_f\right)\right]$

  allows to mimic all **LL** and **NLL**, except for those due to soft wide angle gluon emissions.

which is the logharitmic accuracy of parton showers?

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  From **Pythia** manual: *"While the final product is still not certified fully to comply with a NLO/NLL standard, it is well above the level of an unsophisticated LO/LL analytic calculation."*
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Logharitmic accuracy of parton showers: a fixed-order study, by Dasgupta, Dreyer, Hamilton, Monni and Salam, introduced approach for assessing the logarithmic accuracy of PS algorithms based on the ability to reproduce:

1. the singularity structure of multi-parton matrix elements
2. logarithmic resummation results

Logharitmic accuracy of parton showers: a fixed-order study (I)

- Case of study: double gluon emission, well separated in rapidity, in $e^+e^- \to q\bar{q}$ (all massless):

$$dP_2 = \frac{C_F^2}{2!}\prod_{i=1}^{2}\frac{2\alpha_s(p_{T,i})}{\pi}\frac{dp_{T,i}}{p_{T,i}}d\eta_i \qquad \text{where } \eta_i = -\log\left(\tan\left(\frac{\theta}{2}\right)\right)$$

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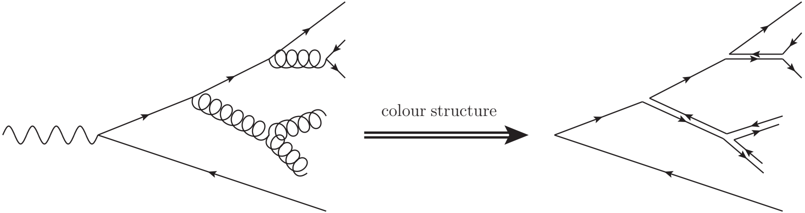
- **Dipole showers** implemented in the `Pythia8` and `Sherpa` generators were considered.



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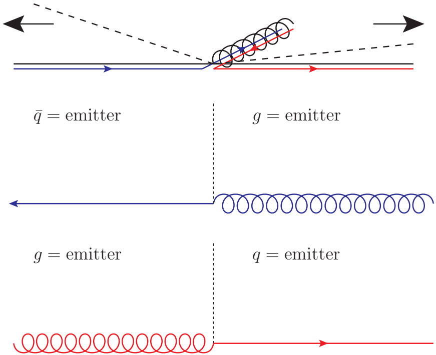
Issue with **dipole showers**



- Dipole frame: there are region where the second gluon looks closer to the first gluon
- When the gluon is identified as emitter:
  1. $\vec{p}_{T,1} \to \vec{p}_{T,1} - \vec{p}_{T,2}$
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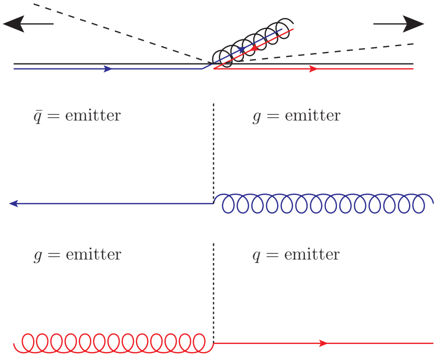

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What happens for the **angular-ordered** shower implemented in `Herwig7`??

Herwig7 angular-ordered parton shower

- The (anti-)quark is identified as **shower progenitor** and the anti-quark (quark) is its colour partner: each shower progenitor is showered independently in the frame where it is anti-collinear with the colour partner.

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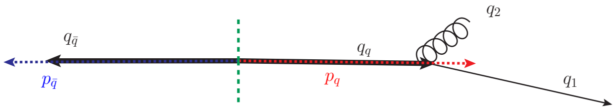

$$\begin{cases} q_1 &= z p_q + \beta_1 p_{\bar{q}} + p_T \\ q_2 &= (1-z) p_q + \beta_2 p_{\bar{q}} - p_T \\ q_q &= p_q + (\beta_1 + \beta_2)\, p_{\bar{q}} \end{cases}$$

$$\tilde{q}^2 = \frac{q_q^2}{z(1-z)} = \frac{2q_1 \cdot q_2}{z(1-z)} = \frac{p_T^2}{z^2(1-z)^2} \sim E_q^2 \theta^2$$

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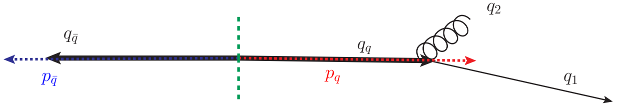

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- **Soft limit**: $1 - z \equiv \epsilon \to 0$, $\boxed{p_T \to \epsilon \tilde{q}, \quad \eta \to \log\left(\frac{Q}{\tilde{q}}\right)}$

$$dP^{\text{herwig}} \to 2C_F \frac{\alpha_s(\epsilon\tilde{q})}{\pi} \frac{d\tilde{q}}{\tilde{q}} \frac{d\epsilon}{\epsilon} = \boxed{2C_F \frac{\alpha_s(p_T)}{\pi} \frac{dp_T}{p_T} d\eta} \quad \checkmark$$

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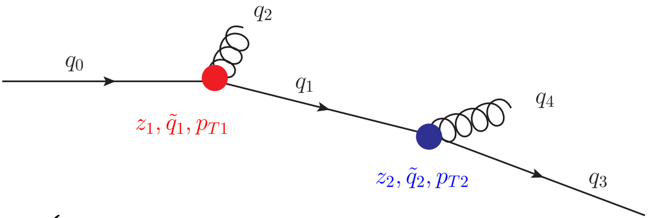
  2. $\boxed{\eta_2 > 0,\ |\eta_1 - \eta_2| \gg 1}$:

$|\eta_1 - \eta_2| \gg 1$: this suppress the gluon splitting;
angular ordering $\mathbf{z_1^2\tilde{q}_1^2} > \mathbf{\tilde{q}_2^2}$ imposes that the one with smallest rapidity comes first;

To do

We achieve the correct **colour factor**, we need to check the **recoil**

Herwig7 angular-ordered parton shower



$$
\begin{cases}
q_0 &= p_q + (\beta_2 + \beta_3 + \beta_4) p_{\bar{q}} \\
q_1 &= z_1 p_q + (\beta_3 + \beta_4) p_{\bar{q}} + p_{T1} \\
q_2 &= (1 - z_1) p_q + \beta_2 p_{\bar{q}} - p_{T1} \\
q_3 &= z_2 z_1 p_q + \beta_3 p_{\bar{q}} + z_2 p_{T1} + p_{T2} \\
q_4 &= (1 - z_2) z_1 p_q + \beta_4 p_{\bar{q}} + (1 - z_2) p_{T1} - p_{T2}
\end{cases}
$$

$$\boxed{\mathbf{q_0^2} = \frac{\mathbf{p_{T1}^2}}{z_1(1-z_1)} + \frac{q_1^2}{z_1}} \quad \Rightarrow \quad \text{Impossible to preserve simultaneously } q_q^2 \text{ and } p_{T1}^2$$

The choice of the preserved quantity determines the **recoil scheme**

p_T-preserving scheme

The original (and simplest) choice of hep-ph/0310083 (Gieseke, Stephens and Webber) is to preserve the **transverse momentum**:

$$\tilde{q}_i^2 = \frac{p_{Ti}^2}{z_i^2(1-z_i)^2}$$

- $\boxed{p_{Ti}^2 = z_i^2(1-z_i)^2\tilde{q}_i^2 \to \epsilon_i^2\tilde{q}_i^2}$ $\boxed{\eta_i \to \log\left(\frac{Q}{\tilde{q}_i}\right)}$

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q^2-preserving scheme

In Ref. 1708.01491 (Reichelt, Richardson and Siodmok) the **virtuality**-preserving scheme is introduced:

$$\tilde{q}_i^2 = \frac{q_i^2}{z_i(1-z_i)}$$

- The transverse momentum of the first emission is reduced

$$\boxed{p_{T1}^2} = \max\left[0, (1-z_1)\left[z_1^2(1-z_1)\tilde{q}_1^2 - z_2(1-z_2)\tilde{q}_2^2\right]\right]$$

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- Better description of the tail of the distributions and in general better agreement with data

Dot-product preserving scheme

In 1904.11866 we suggested something with intermediate properties

$$\tilde{q}^2 = \frac{2q_1 \cdot q_2}{z_i(1-z_i)}$$

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but $\tilde{q}_{i+1} < z_i \tilde{q}_i$ implied $p_{T1} > 0$ even for infinite emissions; the **double-soft** limit is correct

$$\boxed{p_{T1}^2 \to \epsilon_1^2 \left[\tilde{q}_1^2 - \epsilon_2 \tilde{q}_2^2\right] \to \epsilon_1^2 \tilde{q}_1^2}, \qquad \boxed{\eta_1 \to \log\left(\frac{Q}{\tilde{q}_1}\right)}$$

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- However, the virtuality still increases ...

$$\boxed{q_0^2 = z_1(1-z_1)\tilde{q}_1^2 + z_2(1-z_2)\tilde{q}_2^2}$$

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$$\frac{d\Phi_n(q,\bar{q},\ldots)}{d\Phi_2(q,\bar{q})} = \lambda\left(1, \frac{q_q^2}{s}, \frac{q_{\bar{q}}^2}{s}\right) \prod_{i=1}^{n} \frac{d\tilde{q}_i^2}{(4\pi)^2} z_i(1-z_i)dz_i$$

where $\lambda(1,a,b) = \sqrt{1-2(a+b)+(a^2-b^2)^2}$.

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- $\lambda \approx 1$ if the emissions are all soft or collinear and is far from 1 in the hard region of the spectrum.

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- In dipole-showers, the phase-space factorization is exact;
- In the `Herwig7` angular-ordered parton shower, the phase-space factorization is correct only in the soft or collinear limit. The exact formula for the case under analysis

$$\frac{d\Phi_n(q,\bar{q},\ldots)}{d\Phi_2(q,\bar{q})} = \lambda\left(1, \frac{q_q^2}{s}, \frac{q_{\bar{q}}^2}{s}\right) \prod_{i=1}^{n} \frac{d\tilde{q}_i^2}{(4\pi)^2} z_i(1-z_i)dz_i$$

  where $\lambda(1,a,b) = \sqrt{1-2(a+b)+(a^2-b^2)^2}$.
- $\lambda \approx 1$ if the emissions are all soft or collinear and is far from 1 in the hard region of the spectrum.
- We can accept the event with probability λ to improve the description of the tail of the distributions, (large virtualities) without spoiling the soft-collinear region (small virtualities).

Global recoil

- Prior the shower $p_i = \{\sqrt{m_i^2 + |\vec{q}_i|}, \vec{p}_i\}$ that satisfy

$$\sum_i \sqrt{m_i^2 + |\vec{p}_i|} = \sqrt{s}, \quad \sum_i \vec{p}_i = \vec{0}.$$

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- To achieve three-momentum conservation we can define for each particle a boost so that

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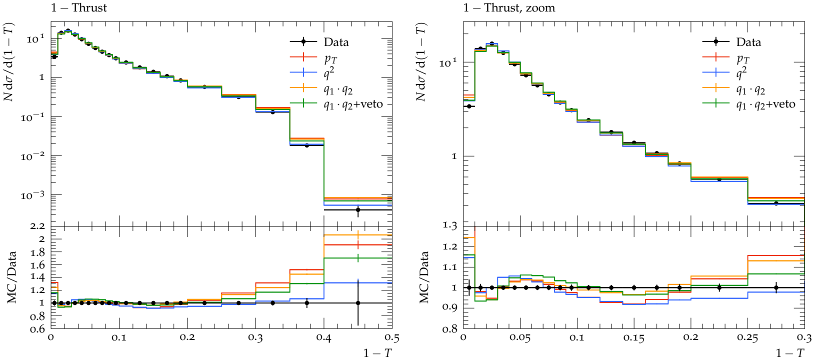
If only soft-emissions take place $q_i^2 = m_i^2 + \mathcal{O}(\epsilon)$, thus this boost gives subleading contributions

Selected LEP results

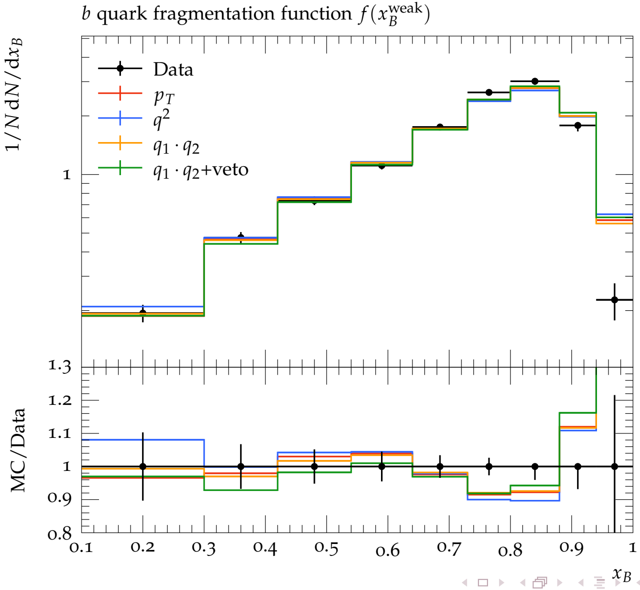
Thrust, DELPHI 1996



$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

Selected LEP results

Energy distribution of weakly-decaying b hadrons from DELPHI 2011



Last plot is one of the reasonw why we are currently studying processes involving **heavy quarks**:

1. $Q \to Qg$: radiation from hvq (important e.g. for b and t jet modelling)
2. $g \to Q\bar{Q}$: gluon splitting into hvq pair ($t\bar{t}g \to t\bar{t}b\bar{b}$ is an important background for $t\bar{t}H \to t\bar{t}b\bar{b}$)

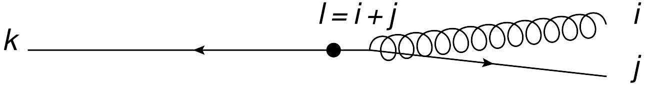
WWARN

What follows is still work in progress.

Suggestions are welcome!

Coherence: massless case

Coherence in the massless case



Definitions:

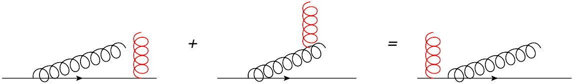
$$W_{ij} = -\frac{q_0^2}{2}\left(\frac{p_i}{p_i\cdot q} - \frac{p_i}{p_i\cdot q}\right)^2 = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_i)(1-\cos\theta_j)}$$

$W_{ij} = W_{ij}^{[i]} + W_{ij}^{[j]}$, where $\langle W_{ij}^{[i]}\rangle = \dfrac{\Theta(\theta_{ij}-\theta_i)}{1-\cos\theta_i}$

so that, after averaging over the azimuthal angle:

$$\begin{aligned} dP_{\text{soft}} \propto & -T_i\cdot T_j\,\langle W_{ij}\rangle - T_i\cdot T_k\,\langle W_{ik}\rangle - T_j\cdot T_k\langle W_{jk}\rangle \\ =& T_i^2\langle W_{ij}^{[i]}\rangle + T_j^2\langle W_{ij}^{[j]}\rangle + T_k^2\langle W_{lk}^{[k]}\rangle + T_l^2\langle W_{lk}^{[l]}\rangle \end{aligned}$$

that means



i.e. $\boxed{\theta_2 < \theta_1 \Leftrightarrow \tilde{q}_2 < z_1\tilde{q}_1}$

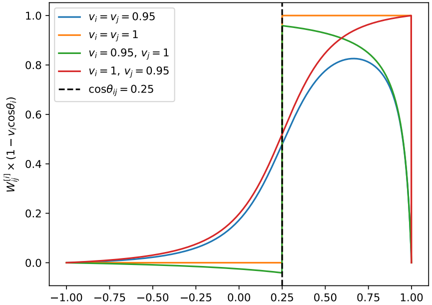
Coherence with massive quarks

- When the emitter or the recoiler is massive, the Θ-function gets smoothed out:

$$\langle W_{ij}^{[i]} \rangle = \frac{\Theta(\theta_{ij} - \theta_i)}{1 - \cos\theta_i} \to \frac{v_i}{2(1 - v_i)\cos\theta_i} \left[\frac{A_i}{v_i A_i + 1 - v_i^2} + \frac{B_i}{\sqrt{B_i^2 + \sin^2\theta_i (1 - v_j^2)}} \right]$$

with

$$A_i = v_i - \cos\theta_i, \qquad B_i = \cos\theta_i - v_j \cos\theta_{ij}$$



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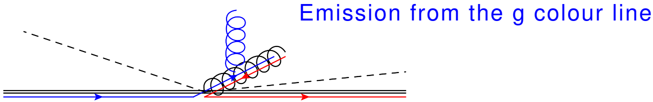
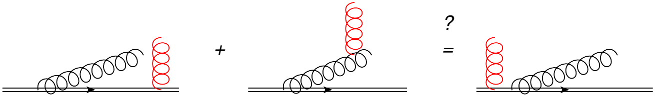
- In the dipole picture large angle radiation is emitted from the $\bar{q}g$ dipole: $v_i = 1$.



In the AO PS, the gluon is emitted from the quark: $v_i < 1$.



Gluon emissions from top

- Use always the massless splitting kernel and accept the last emission with probability:

$$p = \frac{P(z, \tilde{q}, m)}{P(z, \tilde{q}, m = 0)}$$

Gluon emissions from top

- Use always the massless splitting kernel and accept the last emission with probability:

$$p = \frac{P(z, \tilde{q}, m)}{P(z, \tilde{q}, m = 0)}$$

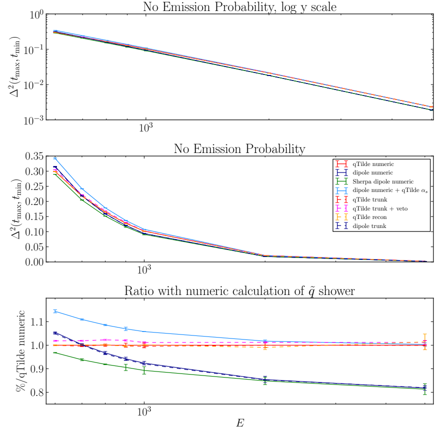
- When the last emission is discarded, try to generate a new one in the previous "forbidden" region

$$\tilde{q} > \tilde{q}' > z\tilde{q}$$

which was screened due to the angular-ordering condition

No emission probability in $e^+e^- \to t\bar{t}$ events.

recon = modified AO PS
trunk = `Hw7.2` default
• The **Sudakov** remains unchanged.
• α_s argument:
p_T^2 vs $p_T^2 + (1-z)^2 m^2$



This is check 0, much work is still needed!

Heavy quark pair production

Heavy quark pair production from gluon splitting:

- The choice of $\alpha_s(p_T)$ comes from the renormalization of the gluon field: so we must use $\alpha_s(p_T)$ every time we generate a new gluon line.
  When we have $g \to q\bar{q}$? The virtuality of the $q\bar{q}$-pair seems a more natural choice: $\boxed{\alpha_s(q^2)}$.

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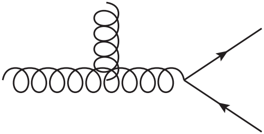
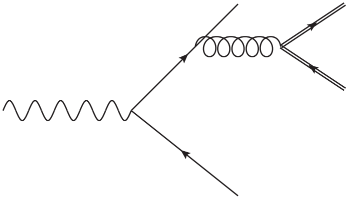
  When we have $g \to q\bar{q}$? The virtuality of the $q\bar{q}$-pair seems a more natural choice: $\boxed{\alpha_s(q^2)}$.
- Which ordering condition shall we apply here?

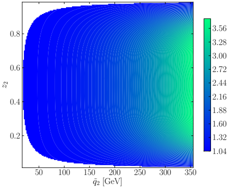


$$\theta_1 > \theta_2 \to \boxed{\tilde{q}_2^2 < z_1^2 \tilde{q}_1^2 + \frac{m^2}{z_2^2(1-z_2^2)}}$$

Heavy quark pair production

- `Herwig` overestimates $Q\bar{Q}$ production





$Q = 1$ TeV, $\tilde{q}_1 = 420$ GeV,
$m = 4.2$, $z_1 = 0.15$

$$\mathrm{d}\sigma_{\mathrm{hw}} = \sigma_0 \frac{\alpha_S}{2\pi} \frac{\mathrm{d}\tilde{q}_1^2}{\tilde{q}_1^2} \frac{\mathrm{d}\phi_1}{2\pi} \mathrm{d}z_1 P_{q\to qg}(z_1, q_1^2) \frac{\alpha_S}{2\pi} \frac{\mathrm{d}\tilde{q}_2^2}{\tilde{q}_2^2} \frac{\mathrm{d}\phi_2}{2\pi} \mathrm{d}z_2 P_{g\to b\bar{b}}(z_2, q_2^2)$$

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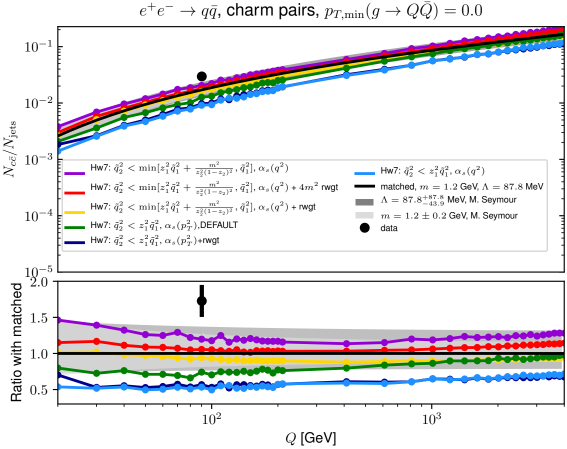
- The $g \to Q\bar{Q}$ splitting can lead to a substantial increment of the virtuality of the gluon's mother. Re-weighting factor that takes into account the variation of the virtuality of the first splitting?

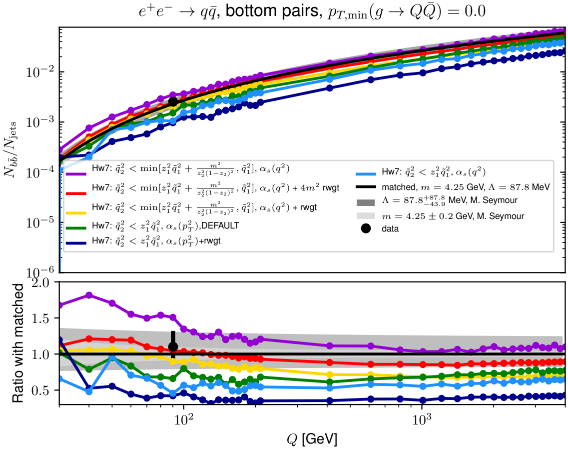
$$r = \frac{q_{1,\mathrm{orig}}^2 - m_1^2}{q_{1,\mathrm{orig}}^2 + q_2^2 - m_1^2} \leq \frac{q_{1,\mathrm{orig}}^2 - m_1^2}{q_{1,\mathrm{orig}}^2 + 4m_Q^2 - m_1^2}$$

- Analytic calculation (**NP B 436, 163 Seymour**) at the Z pole: $(17.13 \pm 6.7(\Lambda_5) \pm 4.6(m_c)) \times 10^{-3}$
- Final input parameters for the LEP/SLD heavy flavour analyses: $(29.6 \pm 3.8)^{10^-3}$



$b\bar{b}$ pairs at LEP

- Analytic calculation (**NP B 436, 163 Seymour**) at the Z pole: $(2.31 \pm 0.71(\Lambda_5) \pm 0.23(m_b)) \times 10^{-3}$
- Final input parameters for the LEP/SLD heavy flavour analyses: $(2.54 \pm 0.51) \times 10^{-3}$



Summary and Outlook

- We need a recoil scheme for final-state radiation able to describe multiple soft-collinear emissions that does not overpopulate the hard region of the spectrum;
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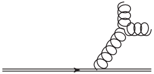
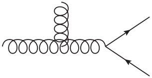
- The user needs to implement its own phase-space veto for more complicate processes (see e.g. `FullShowerVeto` documentation);
- Open issues:



  - ordering condition in case of massive recoilers: p_T can become negative also in the dot-scheme;
  - $g \to q\bar{q}$: argument of α_s and ordering condition for massive q;



  - b-quark fragmentation (does it depend on the PS or on the hadronization model?);

BACKUP

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5. Conclusions

Tuning

Each modification of the PS requires a new tuning of the hadronization parameters: interplay perturbative & non perturbative

Preserved	p_T	q^2	$q_i \cdot q_j$	$q_i \cdot q_j$+veto
Light-quark hadronization and shower parameters				
AlphaMZ ($\alpha_s^{\text{CMW}}(M_Z)$)	**0.1074**	**0.1244**	**0.1136**	**0.1186**
pTmin	0.900	1.136	0.924	0.958
ClMaxLight	4.204	3.141	3.653	3.649
ClPowLight	3.000	1.353	2.000	2.780
PSplitLight	0.914	0.831	0.935	0.899
PwtSquark	0.647	0.737	0.650	0.700
PwtDIquark	0.236	0.383	0.306	0.298
Bottom hadronization parameters				
ClMaxBottom	5.757	2.900	6.000	3.757
ClPowBottom	0.672	0.518	0.680	0.547
PSplitBottom	0.557	0.365	0.550	0.625
ClSmrBottom	0.117	0.070	0.105	0.078
SingleHadronLimitBottom	0.000	0.000	0.000	0.000
Charm hadronization parameters				
ClMaxCharm	4.204	3.564	3.796	3.950
ClPowCharm	3.000	2.089	2.235	2.559
PSplitCharm	1.060	0.928	0.990	0.994
ClSmrCharm	0.098	0.141	0.139	0.163
SingleHadronLimitCharm	0.000	0.011	0.000	0.000

Double soft-emission kinematics

- Phase-space veto and global recoil at the end give subleading corrections if only soft emissions take place.
- For two soft-collinear emissions we thus have the following Lund variables

Preserved quantity	p_T^2	q^2	$q_1 \cdot q_2$
p_{T1}^2	$\epsilon_1^2 \tilde{q}_1^2$	$\epsilon_1 \left[\epsilon_1 \tilde{q}_1^2 - \epsilon_2 \tilde{q}_2^2\right]$	$\epsilon_1^2 \tilde{q}_1^2$
η_1	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_1^2}\right)$	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_1^2 - \frac{\epsilon_2}{\epsilon_1}\tilde{q}_2^2}\right)$	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_1^2}\right)$
p_{T2}^2	$\epsilon_2^2 \tilde{q}_2^2$		
η_2	$\frac{1}{2}\log\left(\frac{Q^2}{\tilde{q}_2^2}\right)$		

- The kinematics of the second emission is always correct;
- The kinematic of the first emission is correct in the p_T and dot-product preserving schemes.

Double soft-emission kinematics

Emission of two-soft gluons with Lund variables k_{Ta}^2, η_a and k_{Tb}^2, η_b.

$$dP_2^{\text{exact}} = \frac{C_F^2}{2!}\frac{\alpha_s^2}{\pi^2}\left[\frac{dk_{Ta}^2}{k_{Ta}^2}d\eta_a\right]\left[\frac{dk_{Tb}^2}{k_{Tb}^2}d\eta_b\right]$$

$$\frac{dP_2^{\text{herwig}}}{dk_{Ta}^2\,d\eta_a\,dk_{Tb}^2\,d\eta_b} = \int \frac{C_F^2}{2!}\frac{\alpha_s^2}{\pi^2}\prod_{i=1}^{2}\left[\frac{d\tilde{q}_i^2}{\tilde{q}_i^2}\frac{d\epsilon_i}{\epsilon_i}\right]\Theta\big(\tilde{q}_1^2 - \tilde{q}_2^2\big)$$
$$\times\left[\delta(\eta_1-\eta_a)\delta(k_{T1}^2-k_{Ta}^2)\delta(\eta_2-\eta_b)\delta(k_{T2}^2-k_{Tb}^2) + a \leftrightarrow b\right]$$

- The p_T^2 and $q_1 \cdot q_2$ preserving schemes yield the correct double soft limit;
- For the q^2 scheme:

$$R = \frac{1}{1+\frac{k_{Tb}}{k_{Ta}}e^{\eta_a-\eta_b}} \times \Theta\left(\frac{k_{Tb}}{k_{Ta}} - 2\sinh(\eta_a-\eta_b)\right) + a \leftrightarrow b$$