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Constraints on rapidity-dependent initial conditions from charged-particle pseudorapidity densities and two-particle correlations

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Abstract

We study the initial three-dimensional spatial configuration of the quark-gluon plasma (QGP) produced in relativistic heavy-ion collisions using **centrality** and **pseudorapidity-dependent measurements** of the medium's **charged particle density and two-particle correlations**. A cumulant-generating function is first used to parametrize the rapidity dependence of local entropy deposition and **extend** arbitrary **boost-invariant initial conditions** to nonzero beam **rapidities**. The model is then *compared to p + Pb and Pb + Pb charged-particle pseudorapidity densities and two-particle pseudorapidity correlations* and systematically **optimized using Bayesian parameter estimation** to extract high-probability initial condition parameters. The *optimized initial conditions* are then *compared to a number of experimental observables including the pseudorapidity-dependent anisotropic flows, event-plane decorrelations, and flow correlations.* We find that the form of the initial local longitudinal entropy profile is well constrained by these experimental measurements.

This is continuation of previous work…
well-known paper:

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Applying Bayesian parameter estimation to relativistic heavy-ion collisions: Simultaneous characterization of the initial state and quark-gluon plasma medium

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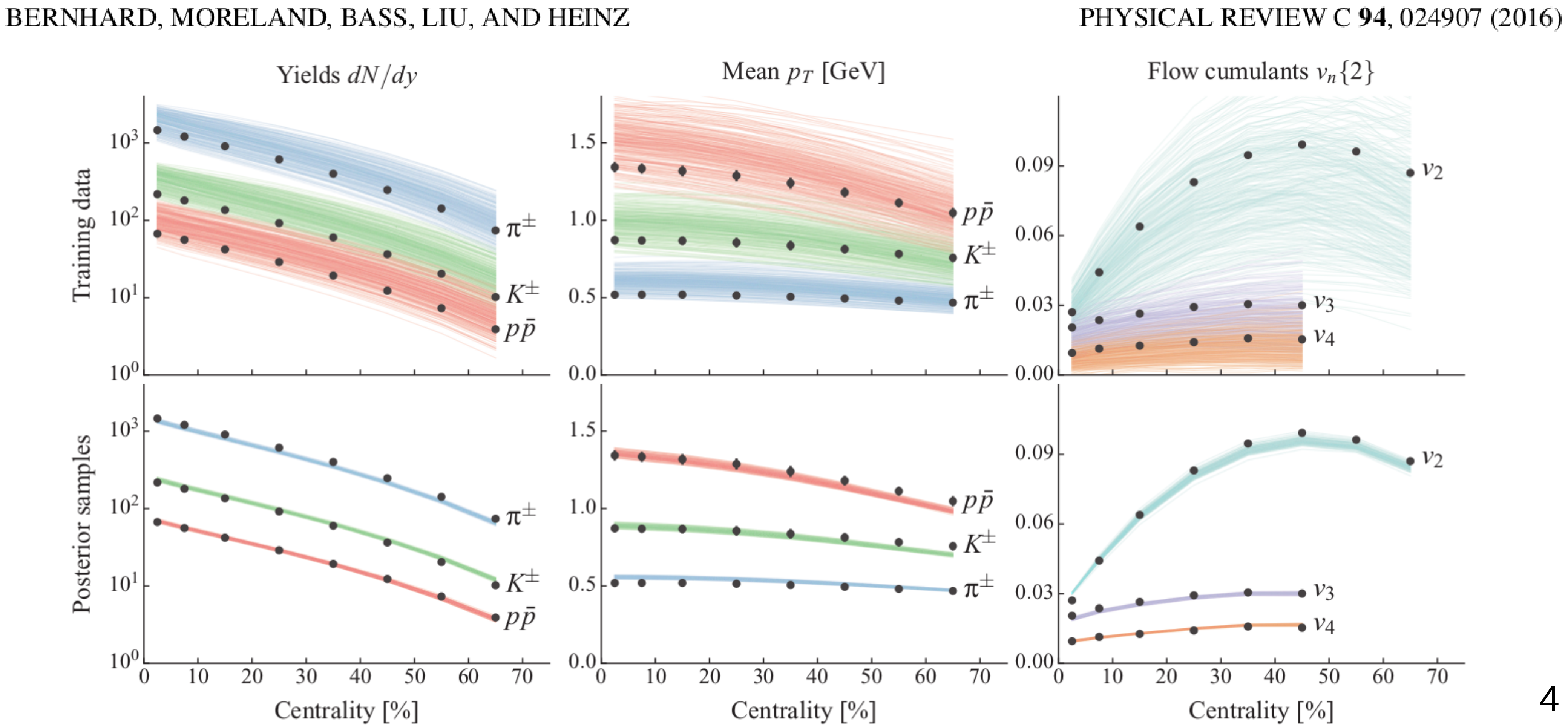
This is continuation of previous work…
well-known paper:



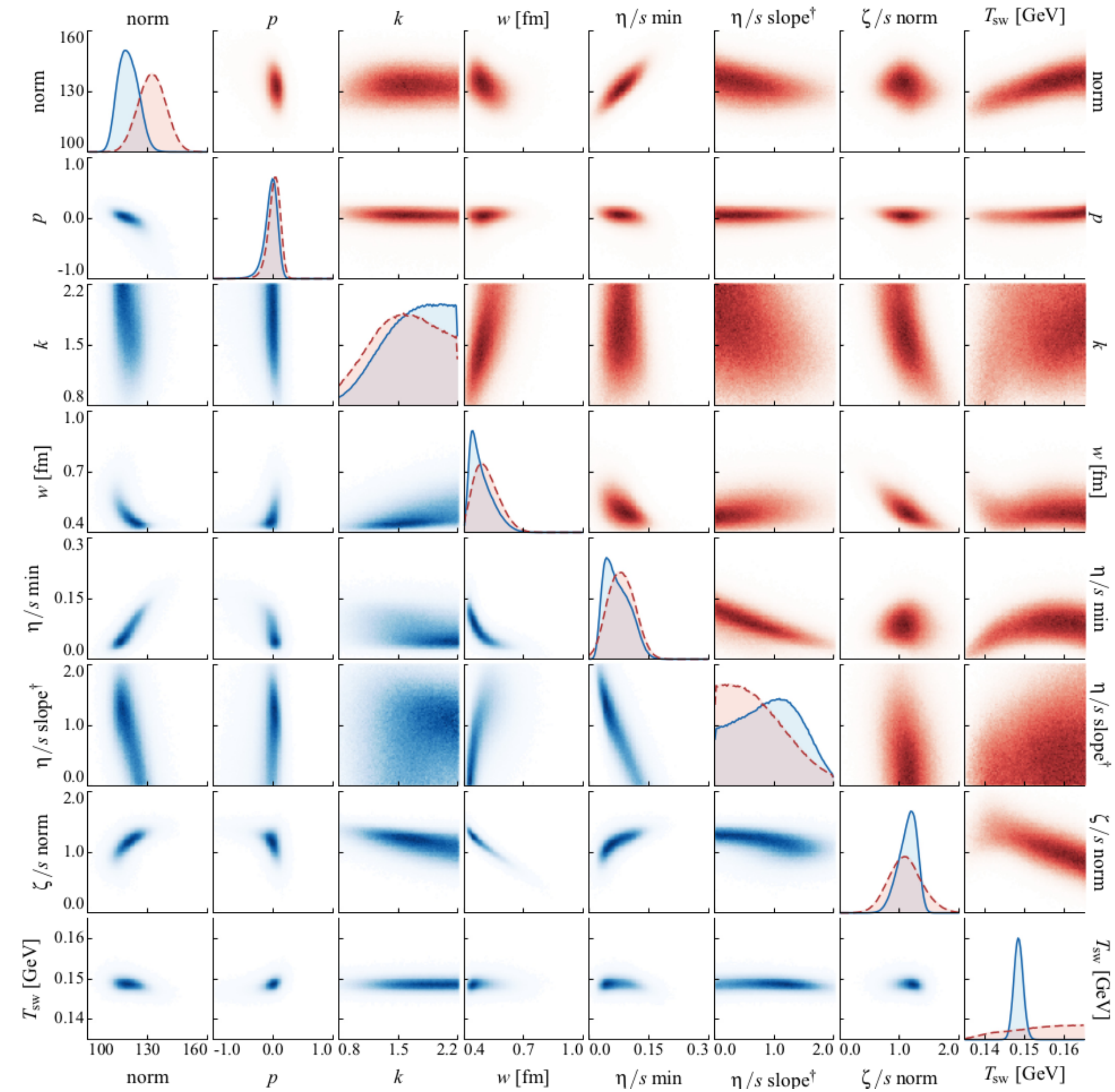


FIG. 7. Posterior distributions for the model parameters from calibrating to identified particles yields (blue, solid lines, lower triangle) and charged particles yields (red, dashed lines, upper triangle). The diagonal has marginal distributions for each parameter, while the off-diagonal contains joint distributions showing correlations among pairs of parameters. The units for η/s slope are [GeV $^{-1}$].

We choose a set of nine model parameters for estimation. Four control the parametric initial state:

(1) the overall normalization factor,
(2) entropy deposition parameter p from the generalized mean ansatz Eq. (14),
(3) γ shape parameter k, which sets nucleon multiplicity fluctuations in Eq. (12), and
(4) Gaussian nucleon width w from Eq. (11), which determines initial-state granularity.

The remaining five are related to the QGP medium:

(5−7) the three parameters (η/s hrg, min, and slope) in Eq. (4) that set the temperature dependence of the specific shear viscosity,
(8) normalization prefactor for the temperature dependence of bulk viscosity Eq. (5), and
(9) particlization temperature T_{switch}.

INITIAL CONDITION MODEL

$$s(\mathbf{x}, \eta_s)|_{\tau=\tau_0} \propto f(\mathbf{x}) g(\mathbf{x}, \eta_s),$$

Midrapidity calculation - parametric model TRENTo:
- Woods-Saxon density distributions
- random impact parameter
- Participant nucleons sampled according to the pairwise inelastic collision probability

$$\frac{d\sigma_{NN}^{\text{inel}}}{2\pi b\, db} = 1 - \exp[-\sigma_{gg} T_{pp}(b)],$$

$$T_{pp}(b) = \int d^2\mathbf{x}\, T_p(\mathbf{x})\, T_p(\mathbf{x} - \mathbf{b}). \quad T_p(\mathbf{x}) = \frac{1}{2\pi w^2} \exp\left(-\frac{\mathbf{x}^2}{2w^2}\right), \quad \tilde{T}_A(\mathbf{x}) = \sum_{i=1}^{N_{\text{part}}} w_i\, T_p(\mathbf{x} - \mathbf{x}_i).$$

w_i is a random weight sampled from a gamma distribution with unit mean and variance 1/k, where k is a tunable shape parameter

INITIAL CONDITION MODEL

$$s(\mathbf{x},\eta_s)|_{\tau=\tau_0} \propto f(\mathbf{x})g(\mathbf{x},\eta_s),$$

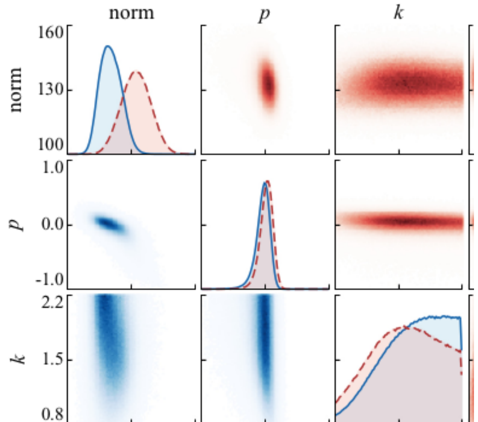
$$f(\mathbf{x}) \propto \left(\frac{\tilde{T}_A^p + \tilde{T}_B^p}{2}\right)^{1/p},$$

$$\propto \begin{cases} \max(\tilde{T}_A,\tilde{T}_B) & p \to +\infty, \\ (\tilde{T}_A + \tilde{T}_B)/2 & p = +1 \text{ (arithmetic)}, \\ \sqrt{\tilde{T}_A \tilde{T}_B} & p = 0 \text{ (geometric)}, \\ 2\,\tilde{T}_A\tilde{T}_B/(\tilde{T}_A + \tilde{T}_B) & p = -1 \text{ (harmonic)}, \\ \min(\tilde{T}_A,\tilde{T}_B) & p \to -\infty. \end{cases}$$

p = 1 is exactly a wounded nucleon model,

p = −0.67 original Kharzeev-Levin-Nardi (KLN) model

p= 0 both impact parameter dependent Glasma (IP-Glasma) model and event-by-event perturbative-QCD + saturation (EKRT) model [8]



INITIAL CONDITION MODEL

$$s(\mathbf{x},\eta_s)|_{\tau=\tau_0} \propto f(\mathbf{x})g(\mathbf{x},\eta_s),$$

$$g(\mathbf{x},\eta)\,d\eta = g(\mathbf{x},y)\,dy,$$

$$\frac{dy}{d\eta} = \frac{J\cosh\eta}{\sqrt{1+J^2\sinh^2\eta}}.$$

$$\eta_s = \frac{1}{2}\log\frac{t+z}{t-z} \sim \eta = \frac{1}{2}\log\frac{|\mathbf{p}|+p_z}{|\mathbf{p}|-p_z}$$

$$s(\mathbf{x},\eta_s)|_{\tau=\tau_0} \propto f(\mathbf{x})\,g(\mathbf{x},y)\,\frac{dy}{d\eta}.$$

g is defined by first three cumulants
inverse Fourier transform
of its cumulant-generating function

$$g(\mathbf{x},y) = \mathcal{F}^{-1}\{\tilde{g}(\mathbf{x},k)\},$$

$$\log\tilde{g} = i\mu k - \tfrac{1}{2}\sigma^2k^2 - \tfrac{1}{6}i\gamma\sigma^3k^3 + \cdots .$$

$$s(\mathbf{x},\eta_s) = s(\mathbf{x})[1+\eta_s\,\mathcal{A}(\mathbf{x})],$$

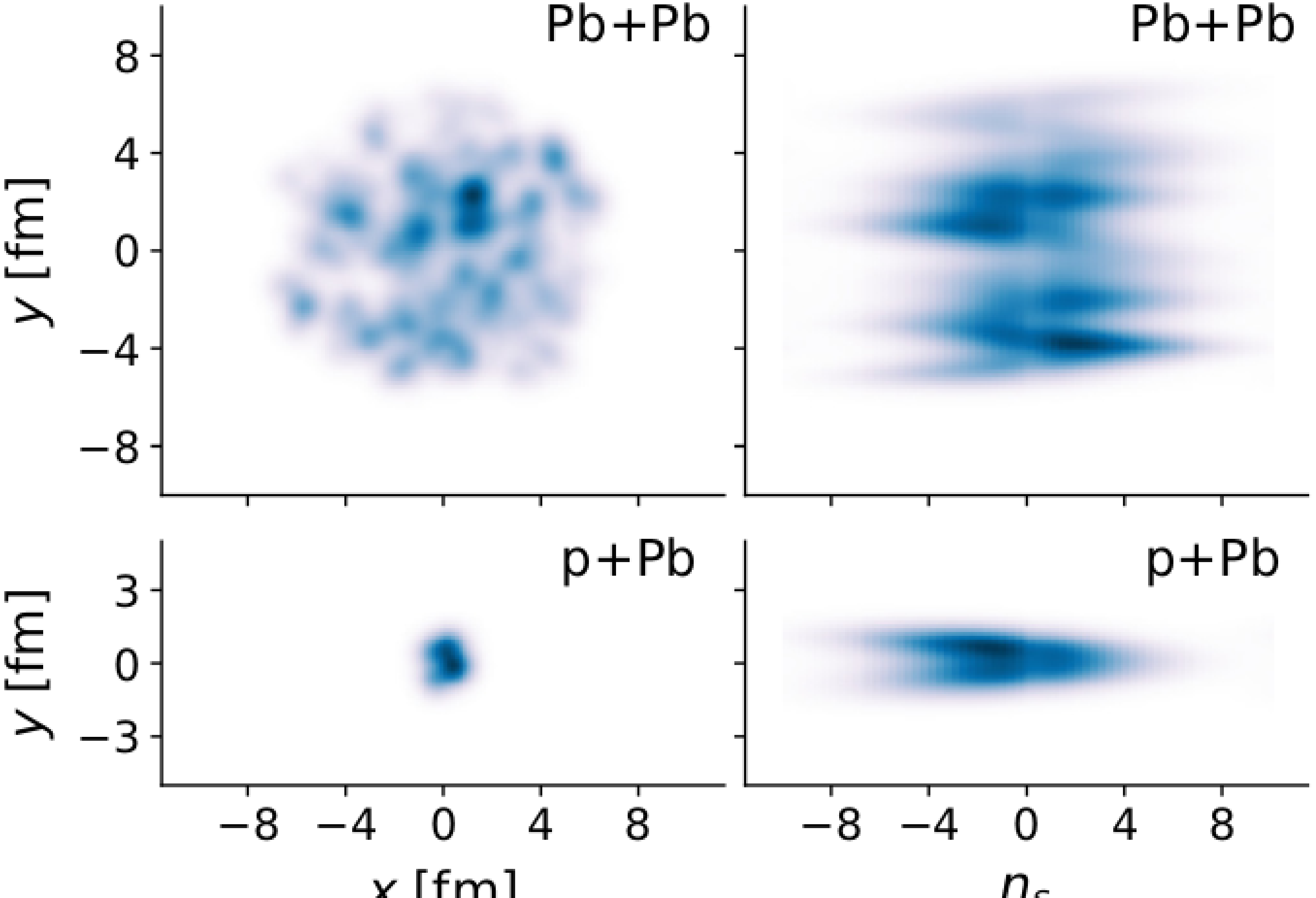
$$\mathcal{A}(T_A,T_B) = \gamma_0\frac{T_A-T_B}{T_A+T_B}$$

$$\mathcal{A}(T_A,T_B) = \gamma_0\frac{T_A-T_B}{T_0}$$

Model	Distribution cumulant		
	Mean μ	Std. σ	Skewness γ
Relative	$\frac{1}{2}\mu_0\ln\left(\frac{T_Ae^{y_b}+T_Be^{-y_b}}{T_Ae^{-y_b}+T_Be^{y_b}}\right)$	σ_0	$\gamma_0\frac{T_A-T_B}{T_A+T_B}$
Absolute	$\frac{1}{2}\mu_0\ln\left(\frac{T_Ae^{y_b}+T_Be^{-y_b}}{T_Ae^{-y_b}+T_Be^{y_b}}\right)$	σ_0	$\gamma_0(T_A-T_B)/T_0$

INITIAL CONDITION MODEL

Initial entropy density in sample Pb + Pb (upper) and p + Pb (lower) events



EXPERIMENTAL OBSERVABLES

1) centrality-dependent pseudorapidity densities $dN_{ch}/d\eta$ in Pb + Pb collisions with a wide pseudorapidity coverage $-3.5 < \eta < 5.0$ (by ALICE)

2) $dN_{ch}/d\eta$ in p + Pb collisions within $|\eta| < 2.7$ by ATLAS.

3) two-particle pseudorapidity correlation $C(\eta_1, \eta_2)$ by ATLAS

$$\frac{dN}{d\eta} = \left\langle \frac{dN}{d\eta} \right\rangle \left[1 + \sum_{n=0}^{\infty} a_n T_n\left(\frac{\eta}{Y}\right) \right],$$

$$T_n(x) = \sqrt{n + \frac{1}{2}}\, P_n(x).$$

$$C_N(\eta_1, \eta_2) = \frac{C(\eta_1, \eta_2)}{C_1(\eta_1) C_2(\eta_2)}, \qquad C_{1,2}(\eta_{1,2}) = \int_{-Y}^{Y} C(\eta_1, \eta_2) \frac{d\eta_{2,1}}{2Y}.$$

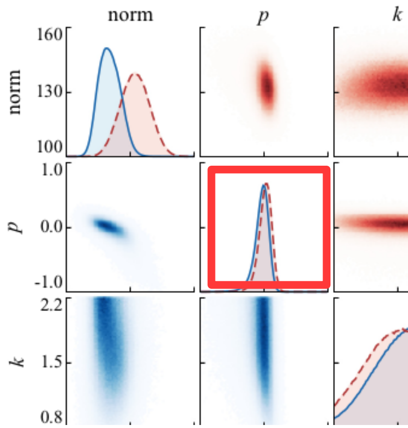
$$C_N(\eta_1, \eta_2) \sim 1 + \frac{3}{2} \langle a_1^2 \rangle \frac{\eta_1 \eta_2}{Y^2} + \cdots .$$

PARAMETERS

TABLE II. Input parameter ranges for the rapidity-dependent parametric initial condition model.

Parameter	Description	Range
$N_{p+\mathrm{Pb}}$	Overall $p+\mathrm{Pb}$ normalization	140.0–190.0
$N_{\mathrm{Pb+Pb}}$	Overall $\mathrm{Pb}+\mathrm{Pb}$ normalization	150.0–200.0
p[a]	Generalized mean parameter	−0.3–0.3
k	Multiplicity fluct. shape	1.0–5.0
w	Gaussian nucleon width	0.4–0.6
μ_0	Rapidity shift mean coeff.	0.0–1.0
σ_0	Rapidity width std. coeff.	2.0–4.0
γ_0	Rapidity skewness coeff.	0.0–10.0 (rel)
		0.0–3.6 (abs)
J	Pseudorapidity Jacobian param.	0.6–0.9

[a] *a priori* probability distributions fitted from Ref. [10] are applied on this parameter independently within the given ranges.



MODEL EMULATOR

$$\begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \ddots & \vdots \\ x_{m,1} & \cdots & x_{m,n} \end{pmatrix} \xrightarrow{\text{Model}} \begin{pmatrix} y_{1,1} & \cdots & y_{1,p} \\ \vdots & \ddots & \vdots \\ y_{m,1} & \cdots & y_{m,p} \end{pmatrix},$$

principal component analysis for observables

It is choosen to include q = 6, 6, and 4 principal components for dN_{PbPb} /dη, dN_{pPb} /dη, and rms a_1 which account for 99.5% of the observed variance.

emulator covariance function σ (x,x')

$$\sigma(\mathbf{x},\mathbf{x}') = \sigma_{\mathrm{GP}}^2 \exp\left[-\sum_{k=1}^{n} \frac{(x_k - x_k')^2}{2\ell_k^2}\right] + \sigma_n^2 \delta_{\mathbf{x}\mathbf{x}'},$$

maximize likelihood $$\log P = -\frac{1}{2}\mathbf{y}^\intercal \Sigma^{-1}\mathbf{y} - \frac{1}{2}\log|\Sigma| - \frac{m}{2}\log 2\pi$$

MODEL EMULATOR

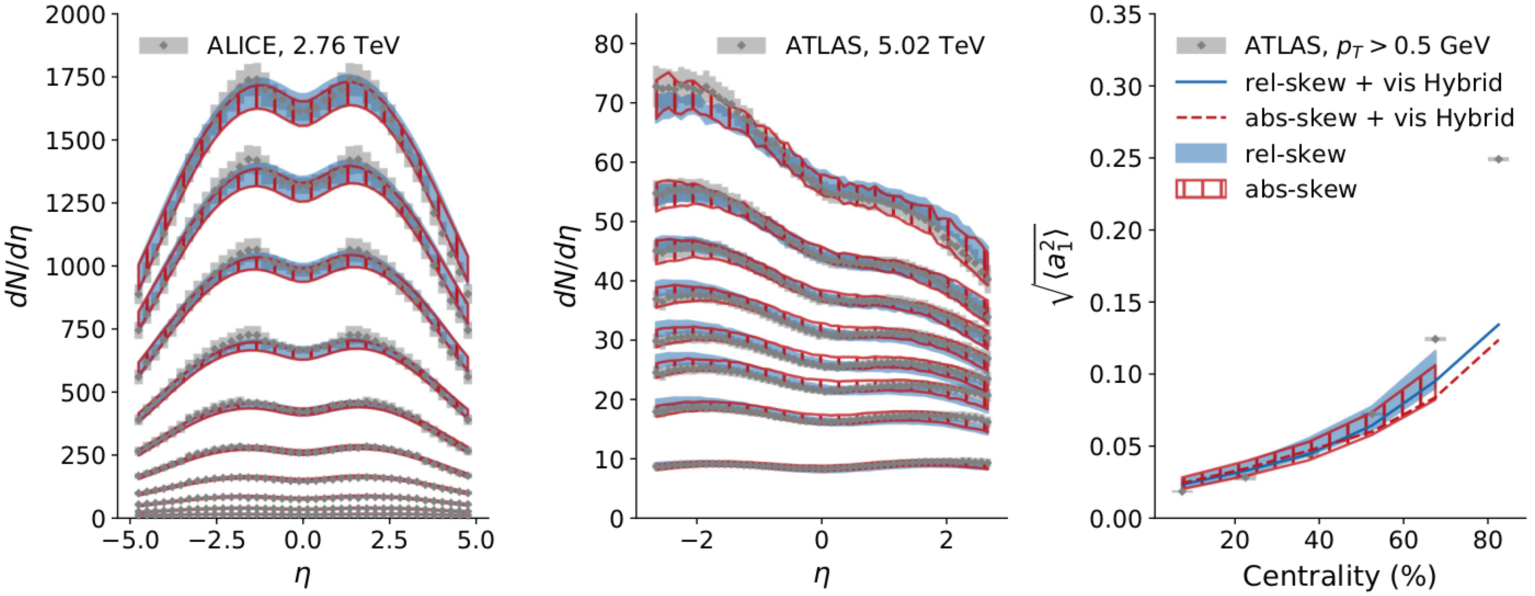
We assume a Gaussian form for the likelihood function,

$$\begin{aligned} P &= P(X, Y, \mathbf{y}_{\text{exp}} | \mathbf{x}_\star) \\ &= P(X, Z, \mathbf{z}_{\text{exp}} | \mathbf{x}_\star) \\ &\propto \exp\left\{ -\frac{1}{2}(\mathbf{z}_\star - \mathbf{z}_{\text{exp}})^\intercal \Sigma_z^{-1} (\mathbf{z}_\star - \mathbf{z}_{\text{exp}}) \right\}, \end{aligned}$$

$$\Sigma_z = \sigma I$$

$$\begin{aligned} \ln P &= \ln P_{\text{pPb}, dN/d\eta} + \ln P_{\text{PbPb}, dN/d\eta} \\ &\quad + \ln P_{\text{PbPb, rms } a_1} + \ln P_{\text{priori}}, \end{aligned}$$

RESULTS for observables



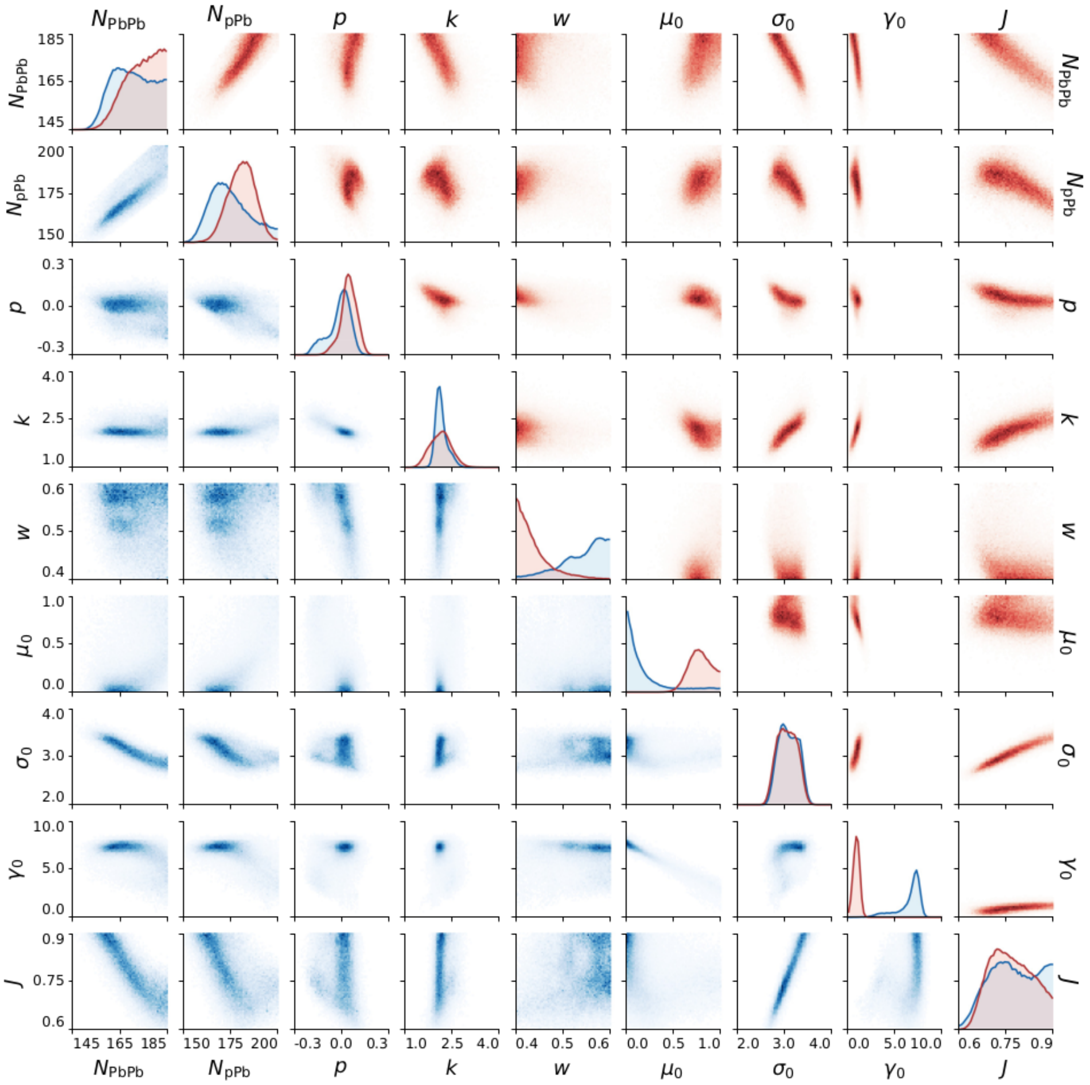
$$K = \frac{\int P(\text{Exp}|\text{Model I}, \vec{p}) d\vec{p}}{\int P(\text{Exp}|\text{Model II}, \vec{p}) d\vec{p}}.$$

$$K = \frac{\text{Relative-skewness model}}{\text{Absolute-skewness model}} = 2.5 \pm 0.2.$$

TABLE III. Interpretation of the scale of K.

K	Strength of evidence (supports model I)
$<10^{1/2}$	Negative (supports model II)
10^0 to $10^{1/2}$	Barely worth mentioning
$10^{1/2}$ to 10^1	Substantial
10^1 to $10^{3/2}$	Strong
$10^{3/2}$ to 10^2	Very strong
$>10^2$	Decisive

POSTERIOR DISTRIBUTION OF MODEL PARAMETERS



(1) Both models prefer the entropy deposition parameter p close to 0, consistent within the range of the prior distribution extracted from [10].

(2) The $p+p$ multiplicity fluctuation parameter is well constrained and distributed about $k = 2.0$ for both models. These k values are also consistent with the range of the previous estimates obtained from fits to $p+p$, $p+\mathrm{Pb}$, and $\mathrm{Pb}+\mathrm{Pb}$ multiplicity distributions at midrapidity [14].

(3) The relative-skewness model prefers a larger nucleon width than the absolute-skewness model. For future studies, one may also use more granular protons with subnucleonic structure instead of Gaussian protons.

(4) The calibrated relative-skewness model exhibits almost zero shift about the mean and large skewness, while the absolute-skewness model prefers a shift close to the center-of-mass rapidity and a moderate skewness.

relative-skewness (blue lower diagonal)
absolute-skewness (red upper diagonal)

POSTERIOR DISTRIBUTION OF MODEL PARAMETERS

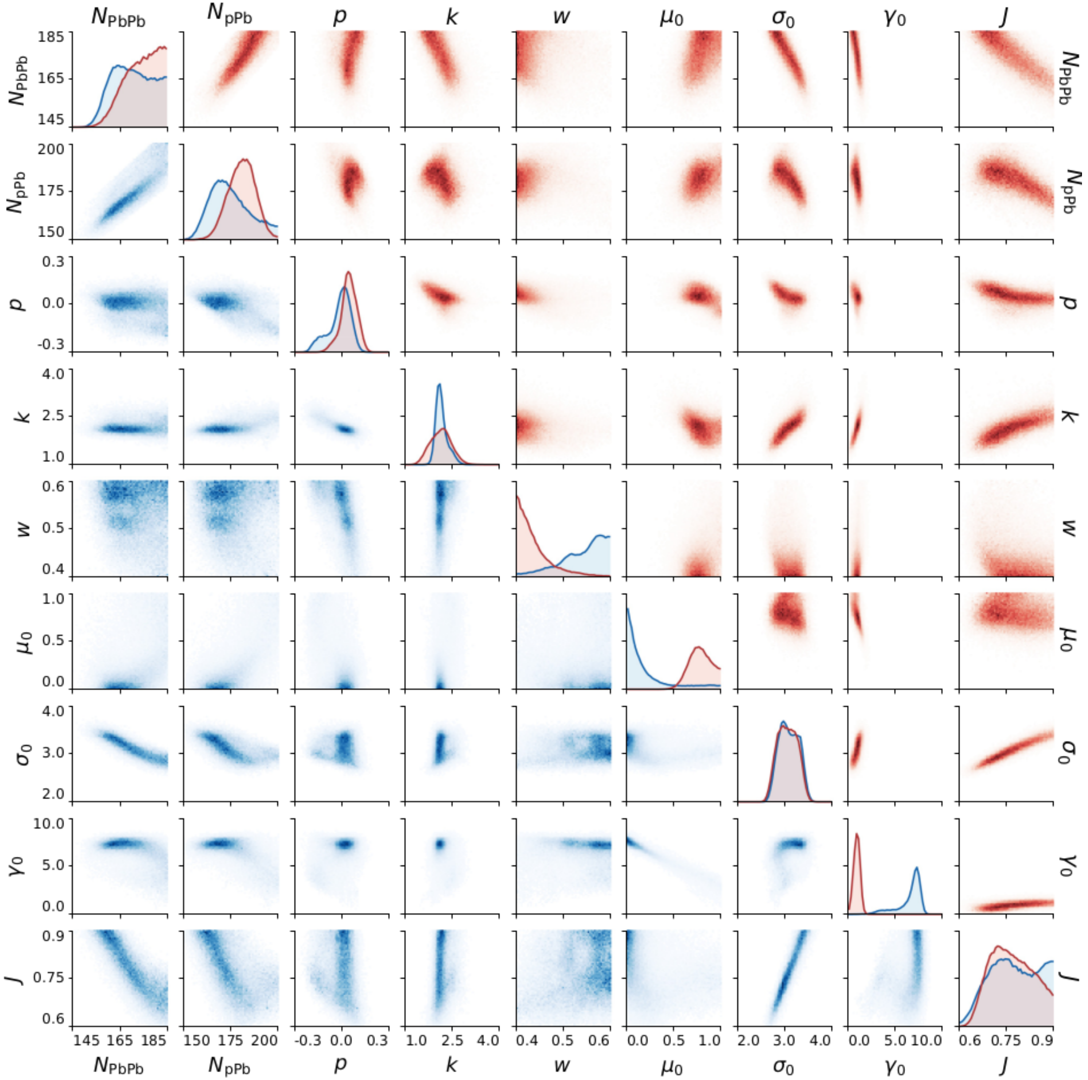


TABLE IV. Selected high-probability parameter sets.

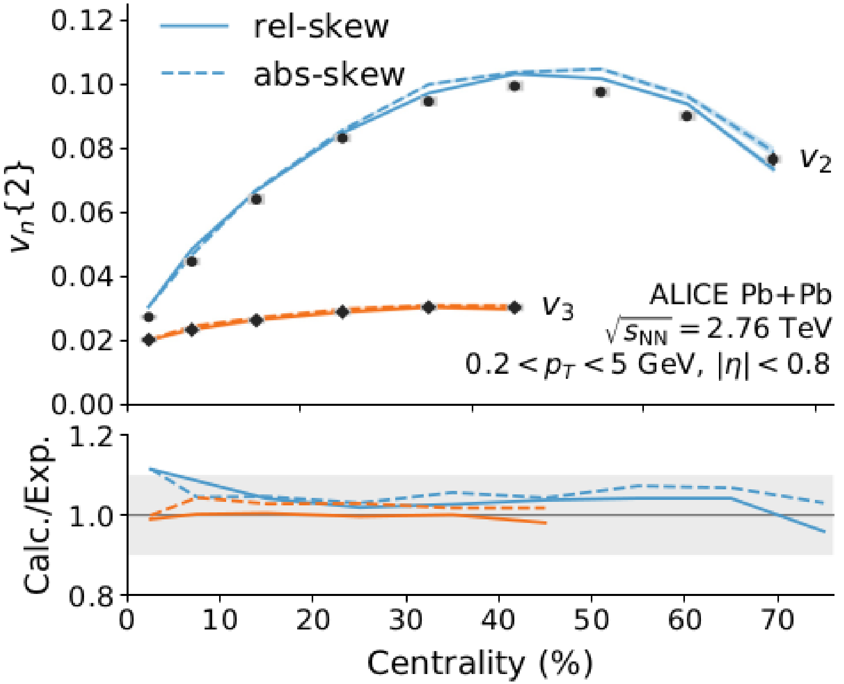
Parameter	rel-skew	abs-skew
N_{Pb+Pb} [a]	150.0	154.0
p	0.0	0.0
k	2.0	2.0
w	0.59	0.42
μ_0	0.0	0.75
σ_0	2.9	2.9
γ_0	7.3	1.0
J	0.75	0.75

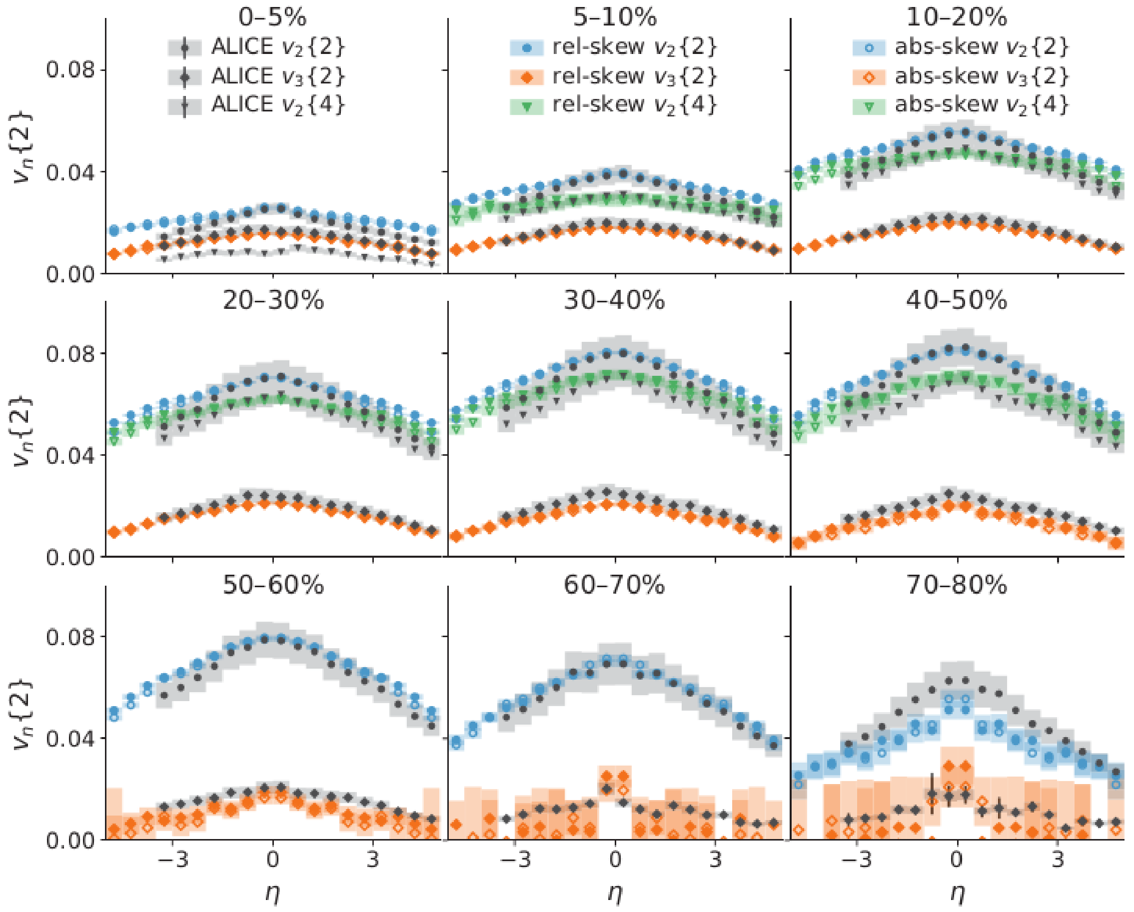
[a] Normalization tuned with ideal hydro is reduced when using viscous hydro.

relative-skewness (blue lower diagonal)
absolute-skewness (red upper diagonal)

PREDICTIONS FOR NOVEL OBSERVABLES

Anisotropic flows



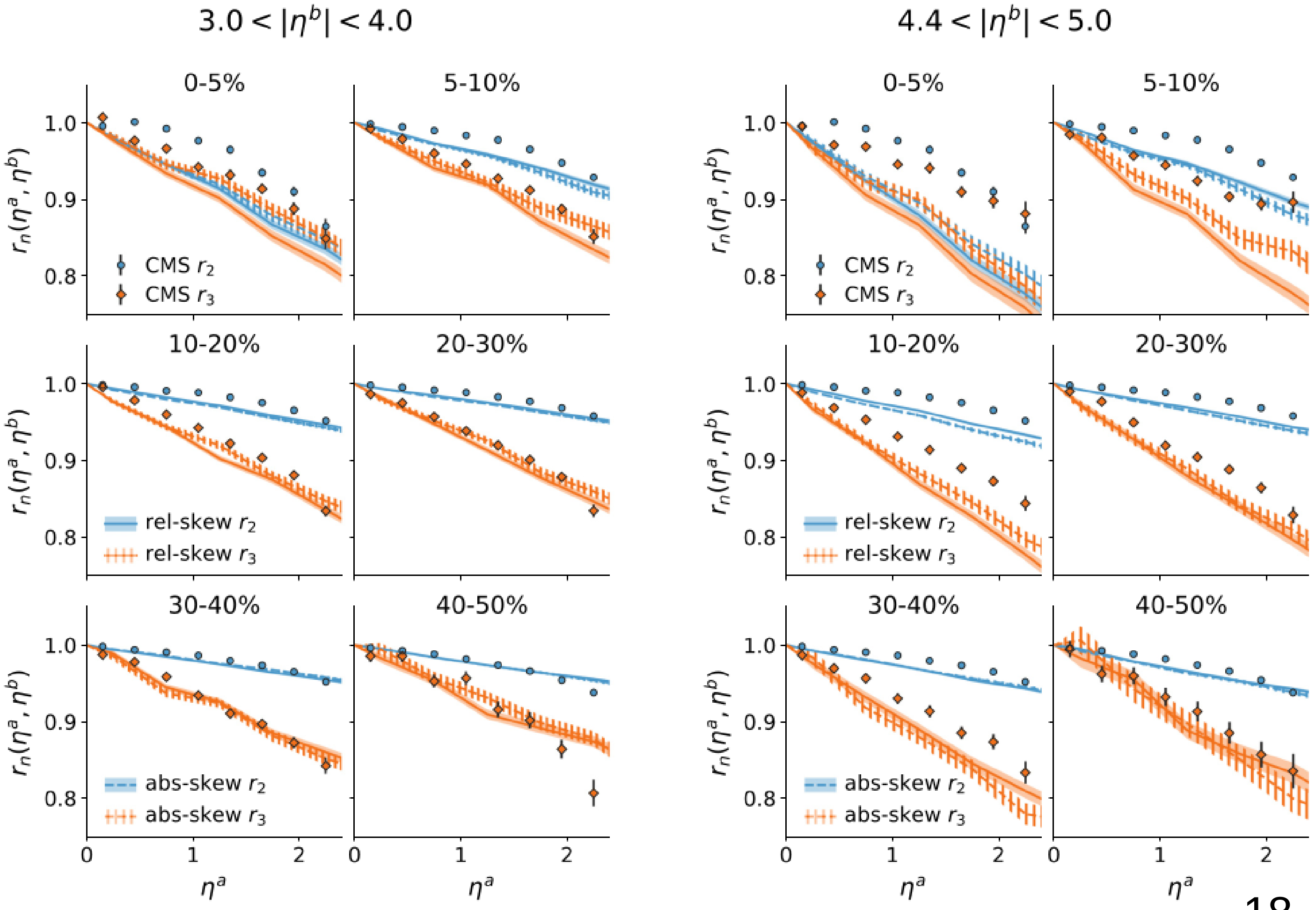


PREDICTIONS FOR NOVEL OBSERVABLES

Event-plane decorrelation

$$r_n(\eta^a, \eta^b) = \frac{V_{n\Delta}(-\eta^a, \eta^b)}{V_{n\Delta}(\eta^a, \eta^b)},$$

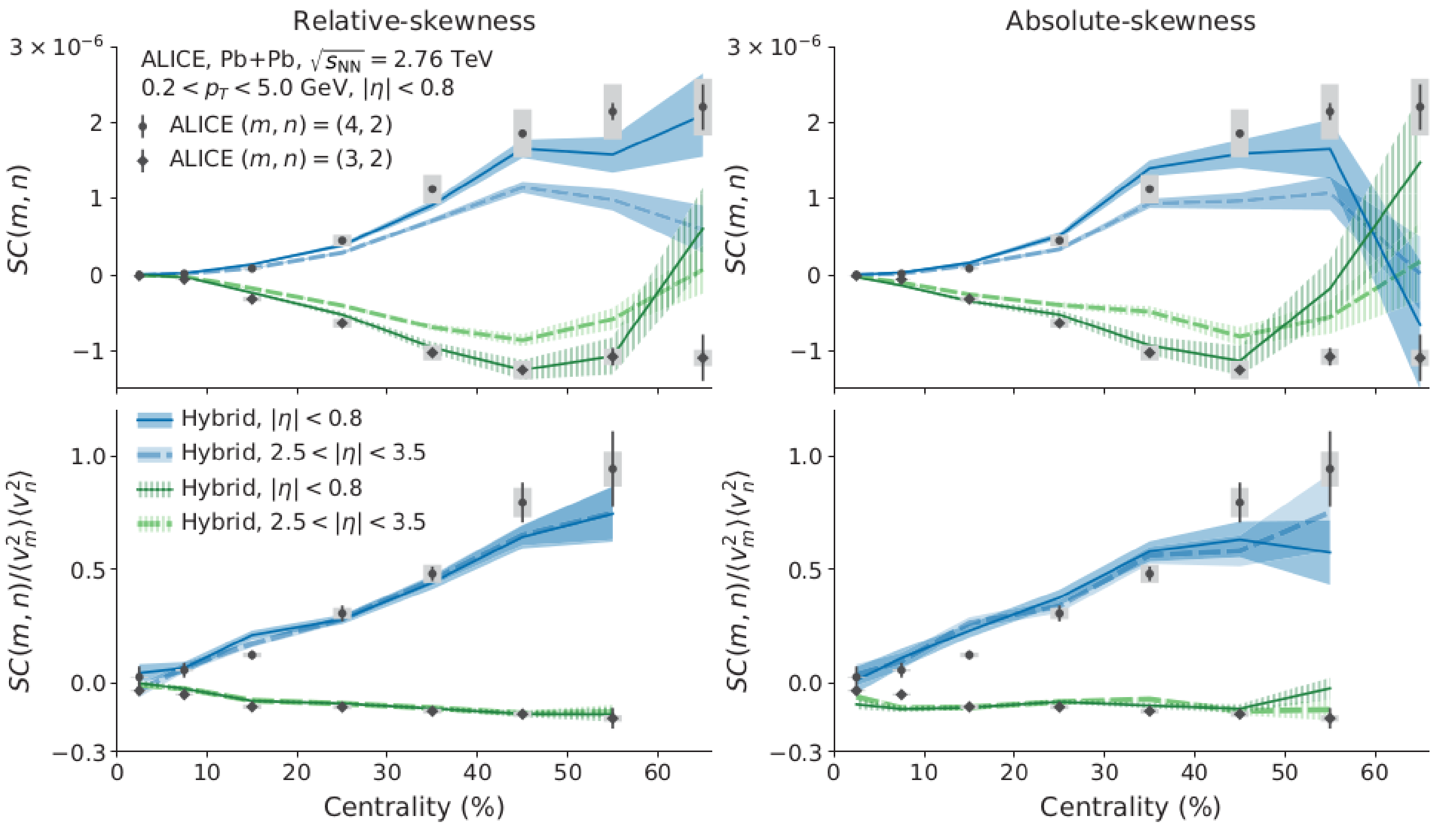
$$V_{n\Delta}(\eta^a, \eta^b) = \langle\langle \cos(n\Delta\phi) \rangle\rangle,$$



PREDICTIONS FOR NOVEL OBSERVABLES

Flow correlations
(symmetric cumulants)

$$\mathrm{SC}(m,n) = \langle\langle \cos(m\phi_1 + n\phi_2 - m\phi_3 - n\phi_4)\rangle\rangle - \langle\langle \cos[m(\phi_1-\phi_2)]\rangle\rangle\langle\langle \cos[n(\phi_1-\phi_2)]\rangle\rangle = \langle v_m^2 v_n^2\rangle - \langle v_m^2\rangle\langle v_n^2\rangle.$$



CONCLUSIONS

Authors proposed a new method to extend arbitrary initial condition models defined at midrapidity to forward and backward pseudorapidity.

initial entropy deposition is set as as a purely local function of nuclear participant densities, with the longitudinal profile reconstructed from generating-function cumulants.

Two models for the distribution's skewness are investigated:
- in one the skewness is proportional the relative nuclear thickness difference
- in other it is proportional to the absolute difference.

After the calibration, both models provide comparable descriptions of experimental $dN_{ch}/d\eta$ and rms a_1 (Legendre decomposition coefficient) data at LHC energy.

Then both models are used to study pseudorapidity-dependent anisotropic flows, event-plane decorrelations, and flow correlations in Pb + Pb collisions.

CONCLUSIONS

The model nicely describes integrated flows v_2 and v_3 at midrapidity as well as the pseudo-rapidity dependence of differential flow for different centrality classes.
The elliptic and triangular event-plane decorrelations with $3.0 < |\eta\ b| < 4.0$ are well explained except for the most central collisions, but both models overpredict the decorrelations with the reference particles $4.4 < |\eta_b| < 5.0$.

The general agreement of the present framework with pseudorapidity-dependent flows and event-plane decorrelations corroborates the use of relativistic viscous hydrodynamics in describing the QGP dynamics away from midrapidity region.

Future improvements:
- to add subnucleonic structure to the nuclear thickness functions
- in this work it was assumed that the multiplicity observables are insensitive to viscous effects and use ideal hydrodynamics in the model-to-data comparison process.
- for further calculations an oversimplified constant specific shear viscosity and zero bulk viscosity were used, although there have been many works suggesting preference for a temperature-dependent shear viscosity and finite bulk viscosity

SOME KITCHEN

APPENDIX: HYBRID MODEL SIMULATION

The (3+1)-D relativistic viscous hydrodynamics code VHLLE [46] is used for the QGP medium evolution. The equation-of-state (EoS) is obtained by interpolating a state-of-the-art lattice-QCD EoS [85] at high temperature with vanishing baryon density and a hadron resonance gas EoS at low temperature. We use a switching energy density $\varepsilon_s = 0.322\,\mathrm{GeV/fm^3}$ ($T_s \sim 0.154\,\mathrm{GeV}$) at which the hydrodynamic description is switched to the UrQMD transport description. The switching temperature T_s is the same as the EoS pseudocritical temperature $T_c = 0.154$ GeV. The hydrodynamic transport coefficients are given by

$$(\eta/s)(T > T_s) = 0.17\text{–}0.28, \tag{A1}$$

$$(\zeta/s)(T > T_s) = 0.0. \tag{A2}$$

For simplicity, there is no bulk viscosity and the shear viscosity to entropy ratio is a constant. Below T_s, the fluid is converted into hadrons and the time evolution is solved by UrQMD. No additional inputs for the transport coefficients are needed.

Bulk viscosity $\zeta/s = 0$ is set which precludes direct comparison with the
boost-invariant VISH2+1 hydrodynamics code

corresponding shear and bulk viscosities determined by the previous Bayesian analysis [10].
For the QGP specific shear viscosity, we choose constants QGP $\eta/s = 0.17$ and 0.19 for relative- and absolute-skewness models respectively,

which provide good descriptions of the data, although they are not a systematic best fit.

The resulting $v_2\{2\}$ and $v_3\{2\}$ agree with experimental data within 10% and verify that
the generating function rapidity extension recovers previous TRENTo initial condition results at midrapidity.

used $\eta/s = 0.25$–0.28 when comparing to ALICE measurements that are extrapolated to zero p_T.