# Intra-bunch motion for TMCI: a simple theoretical approach

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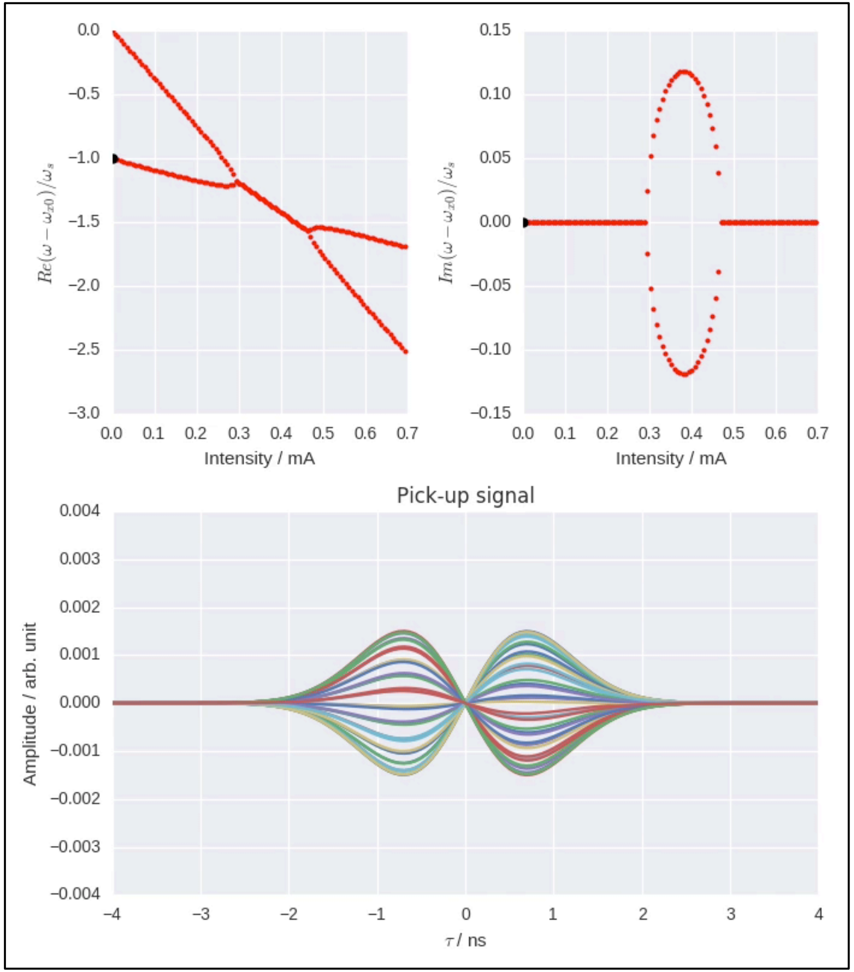
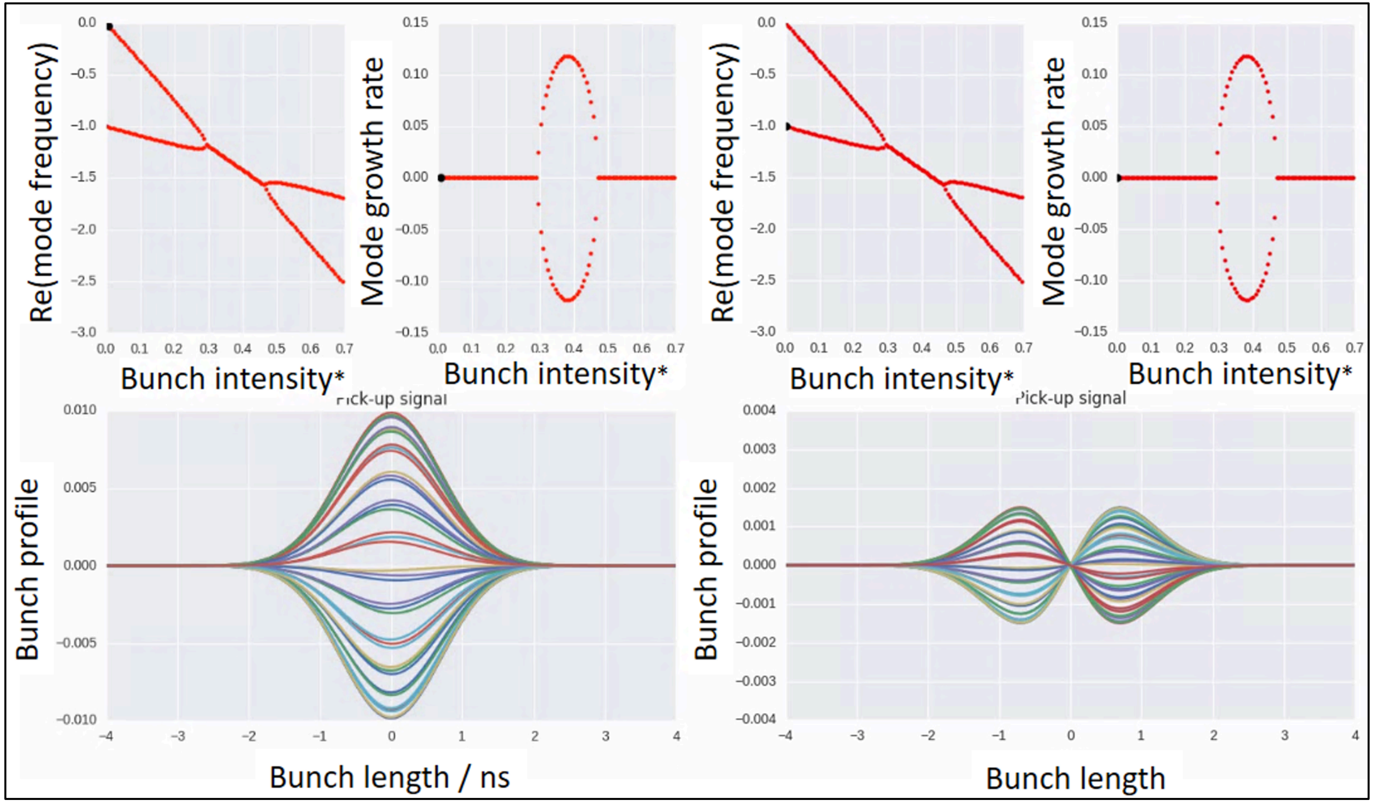
- Reminder from DavidA who showed the result from DELPHI Vlasov solver => See e.g. his PHD defence (https://indico.cern.ch/event/830231/attachments/1921272/3178320/2019-09-05_PhD_defense_presentation_v7.pptx)

- Goal here => Try and explain it with a simple analytical model
  - Reminder from general approach with GALACTIC Vlasov solver
  - Simple analytical model

# Reminder from DavidA's result with DELPHI
*(movie)*



# Reminder from DavidA's result with DELPHI



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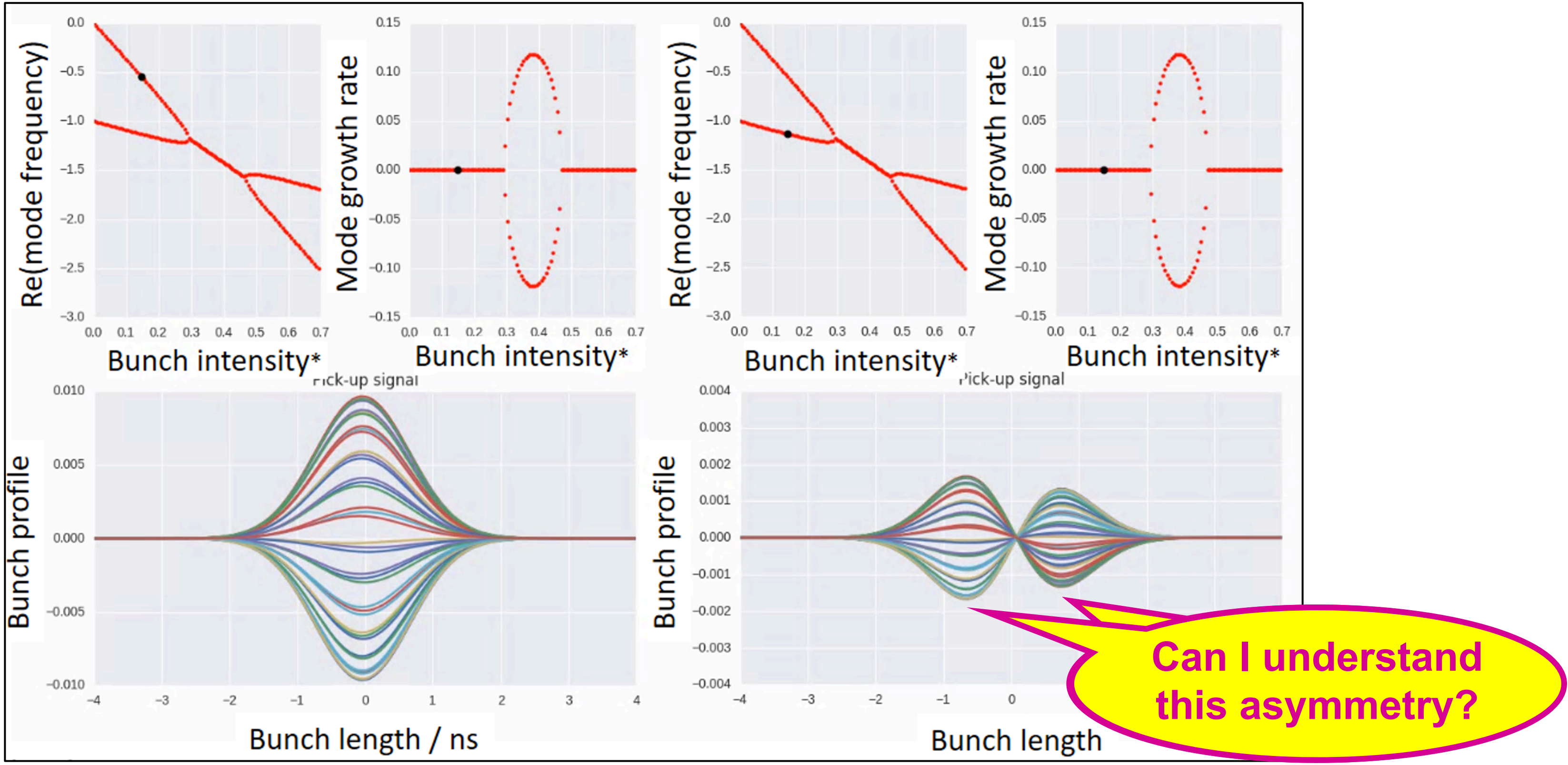

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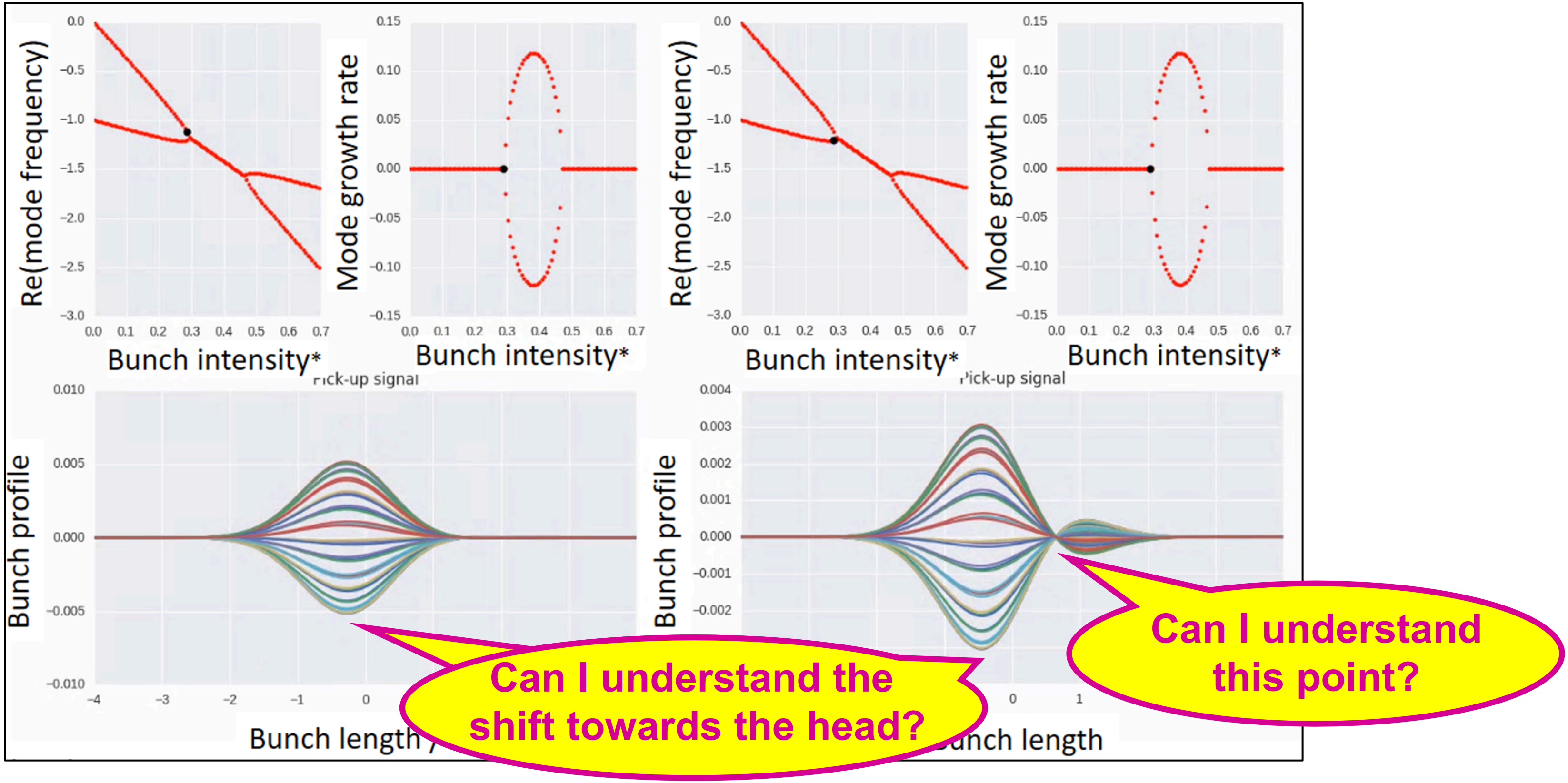
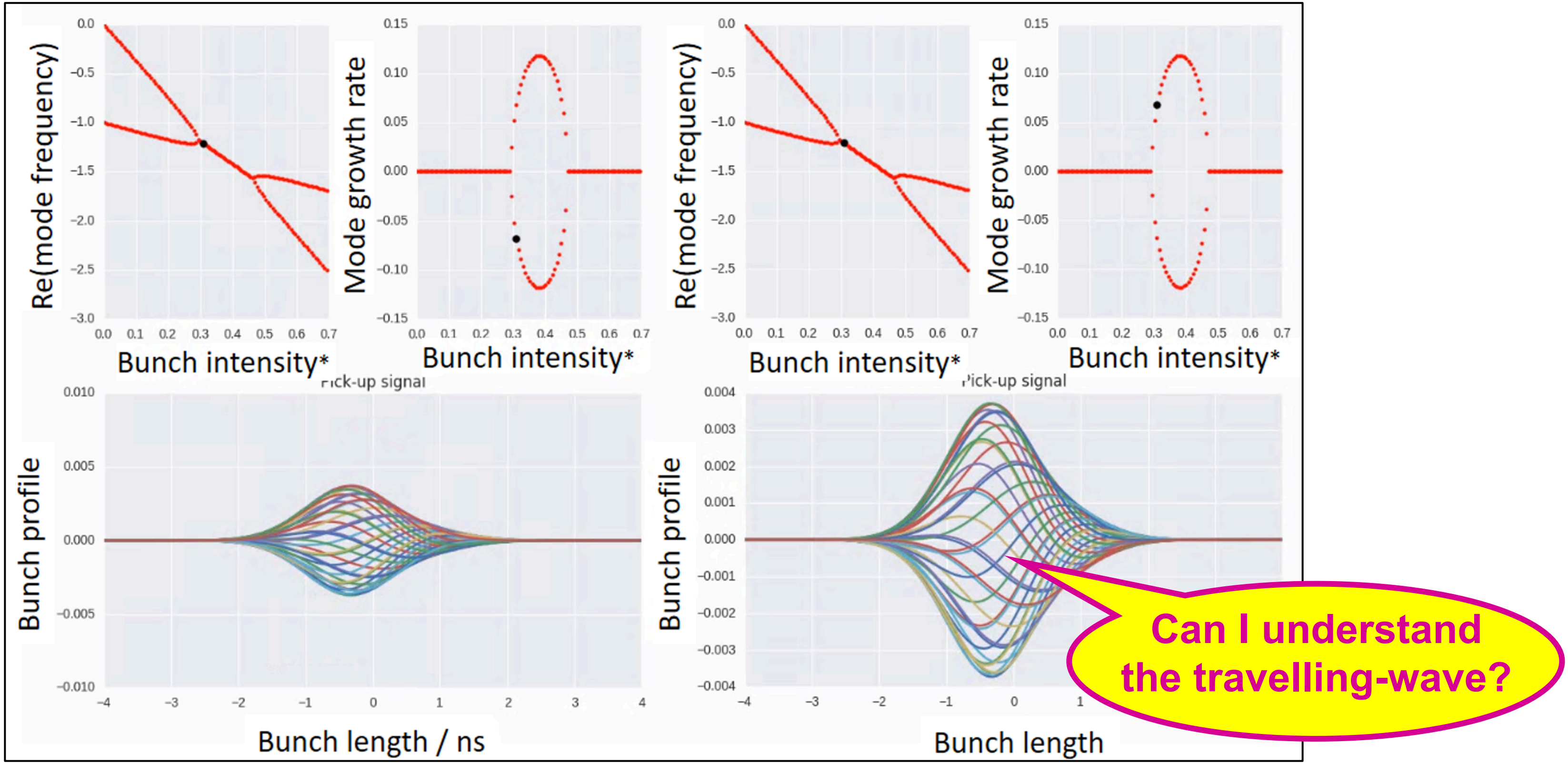


# Reminder from DavidA's result with DELPHI



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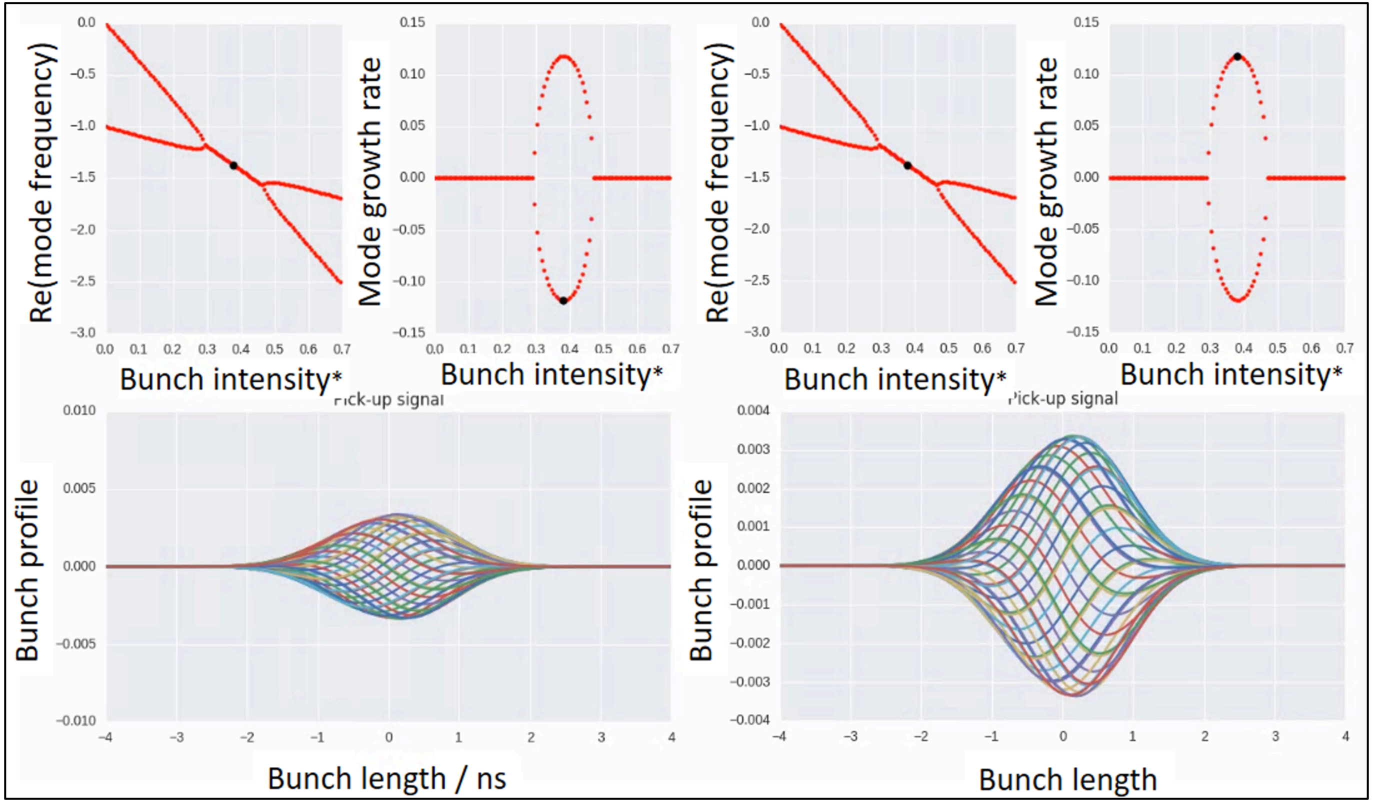

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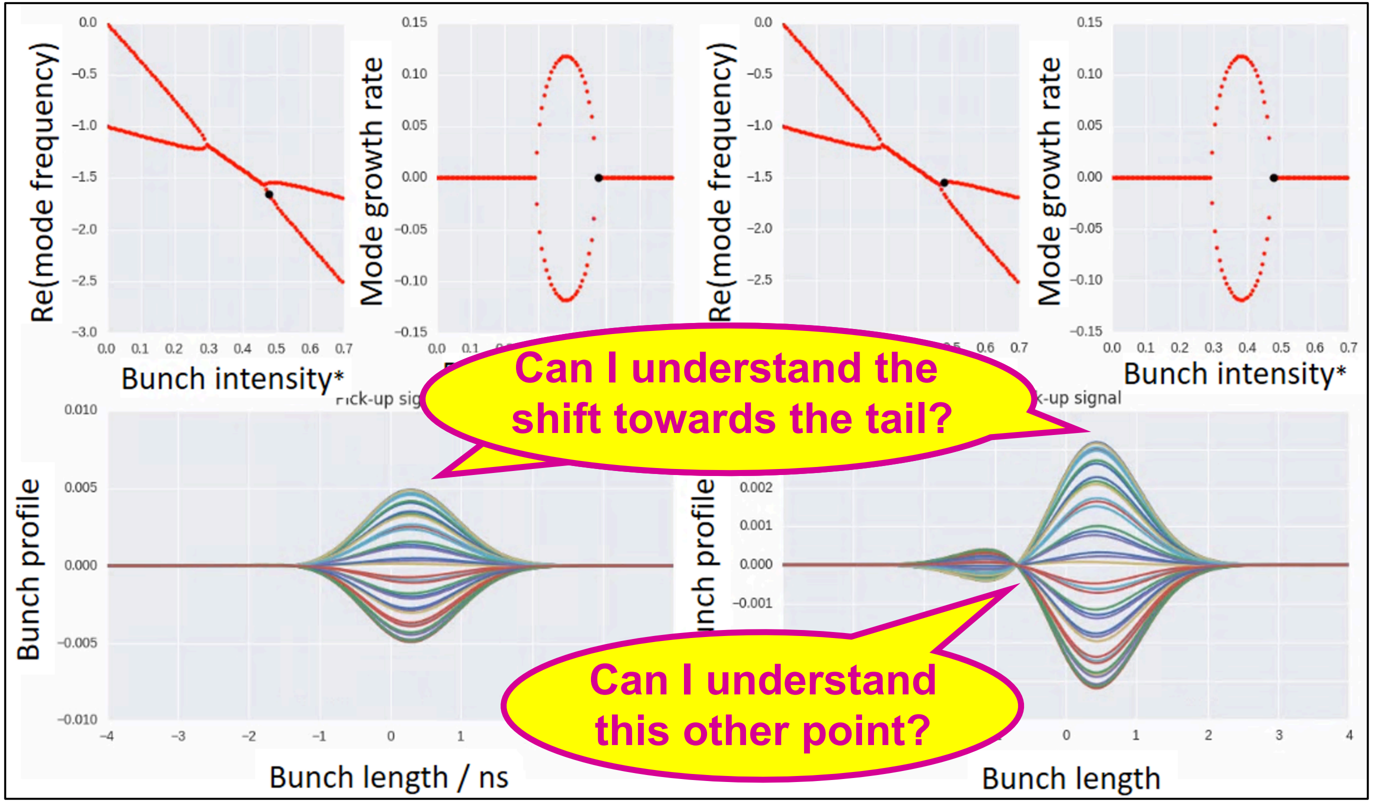
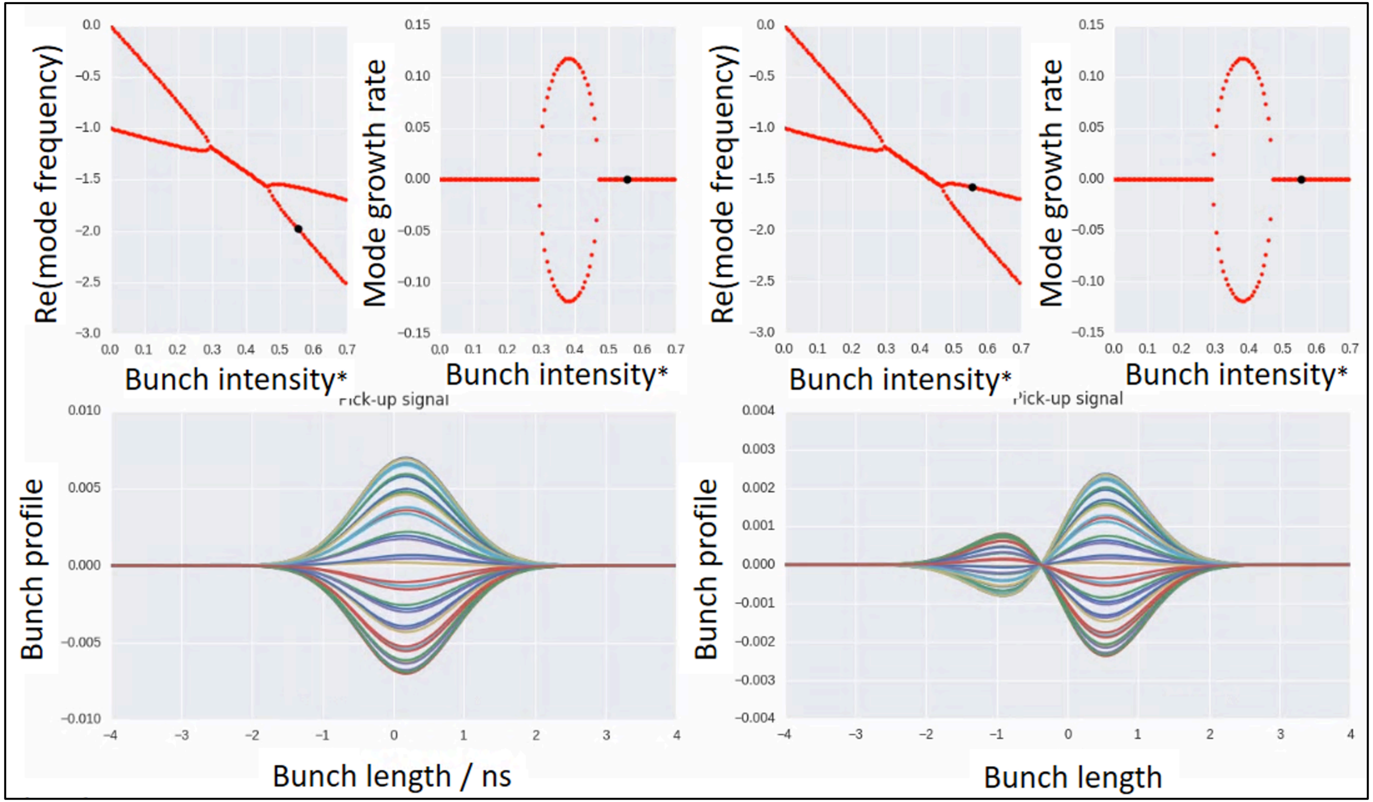


# Reminder from DavidA's result with DELPHI



# Reminder from general approach with GALACTIC Vlasov solver

$$\sigma_x(l) = \sum_{i,j} a_{ij}\, \sigma_{x,ij}(l)$$

Low-intensity Eigenvectors

$$\frac{\omega_c}{\omega_s} a_{kl} = H^x a_{ij}$$

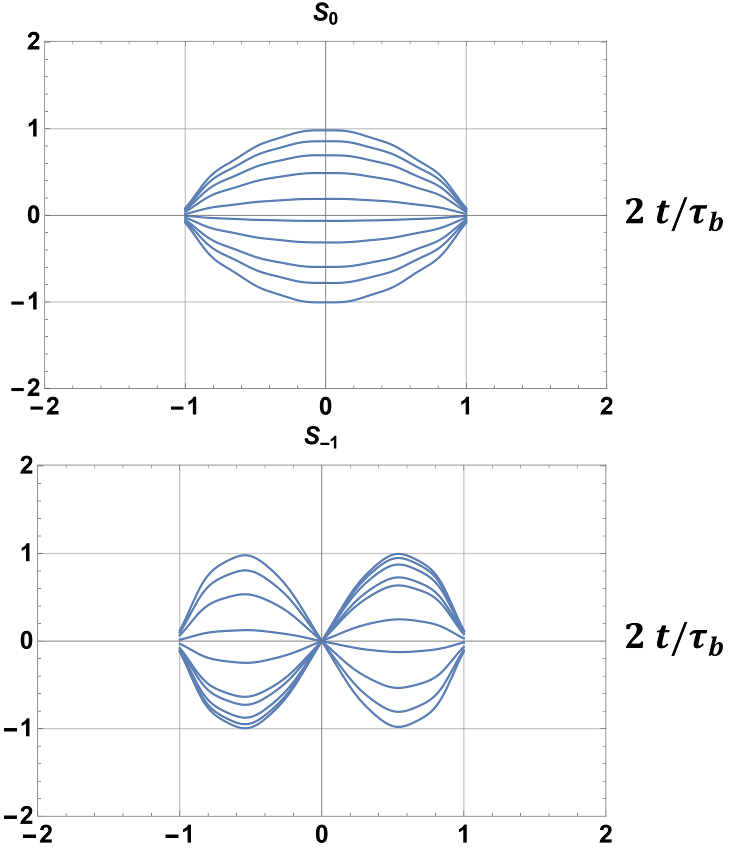
**Matrix to be diagonalised:**

**1) Eigenvalues give the mode frequency shifts (Re and Im)**

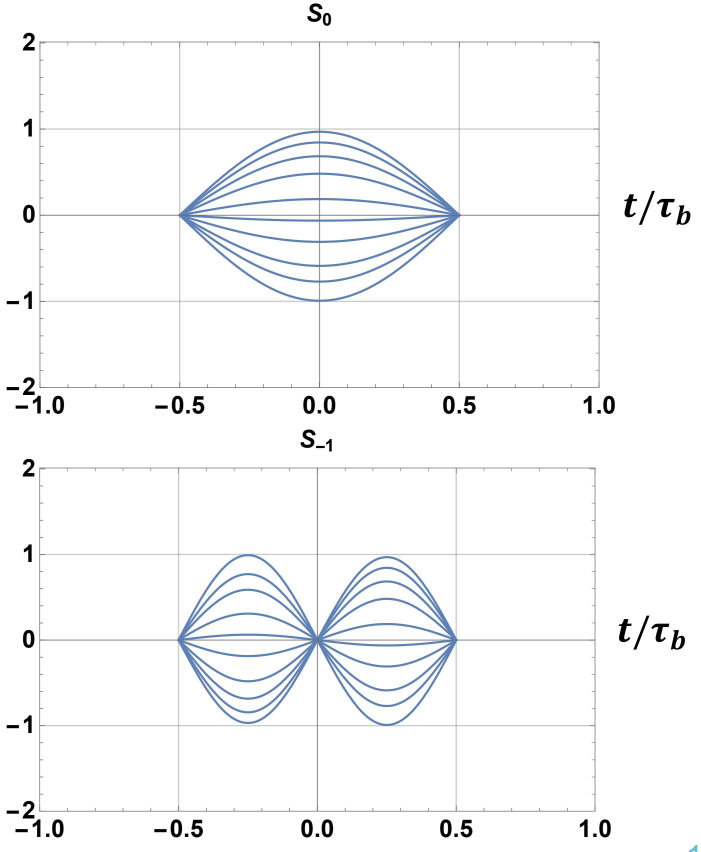
**2) Eigenvectors give the coeffcients $a_{ij}$ to be used in the equation on the left to be able to plot the intra-bunch signal**

# Reminder from general approach with GALACTIC Vlasov solver

Solutions of the Eigenvalue problem at low intensity



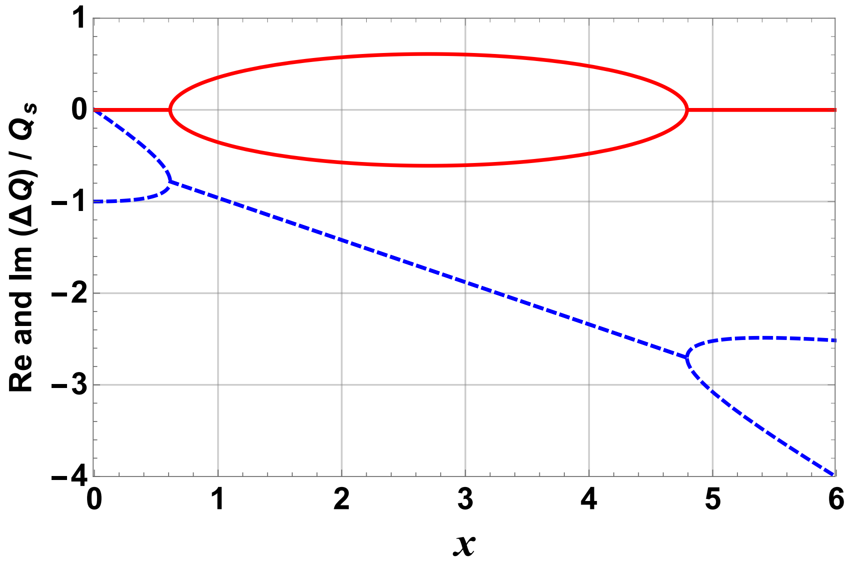

Approximation by sinusoidal modes



# Simple analytical model

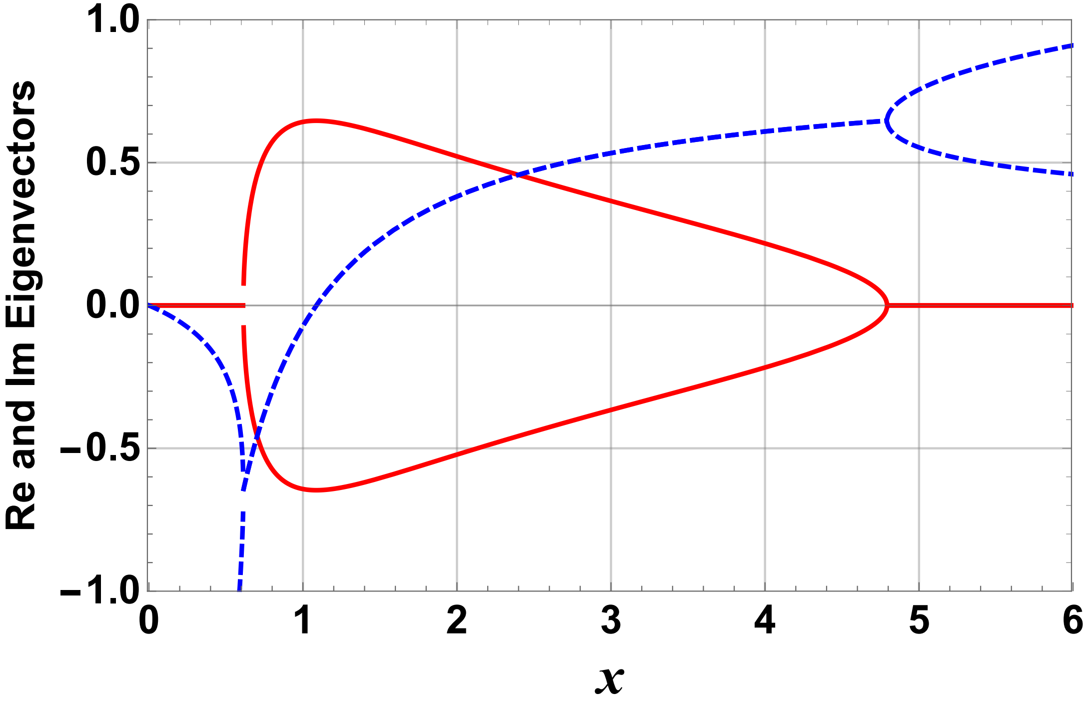
- If I use the simple model I used in the past to study the destabilising effect of the LHC transverse damper for $Q' = 0$ (deduced from studies with the GALACTIC Vlasov solver, using a water-bag bunch) but without damper
  - Eigenvalues of the $2 \times 2$ matrix given below

$$\begin{pmatrix} -1 & -0.23\, j\, x \\ -0.55\, j\, x & -0.92\, x \end{pmatrix}$$



# Simple analytical model

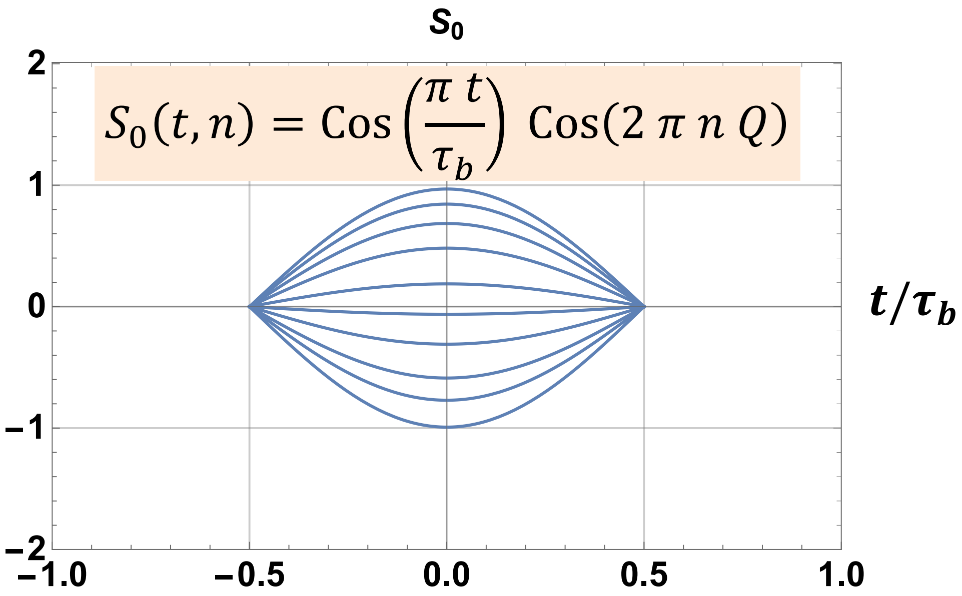
- Eigenvectors (of the previous 2×2 matrix)

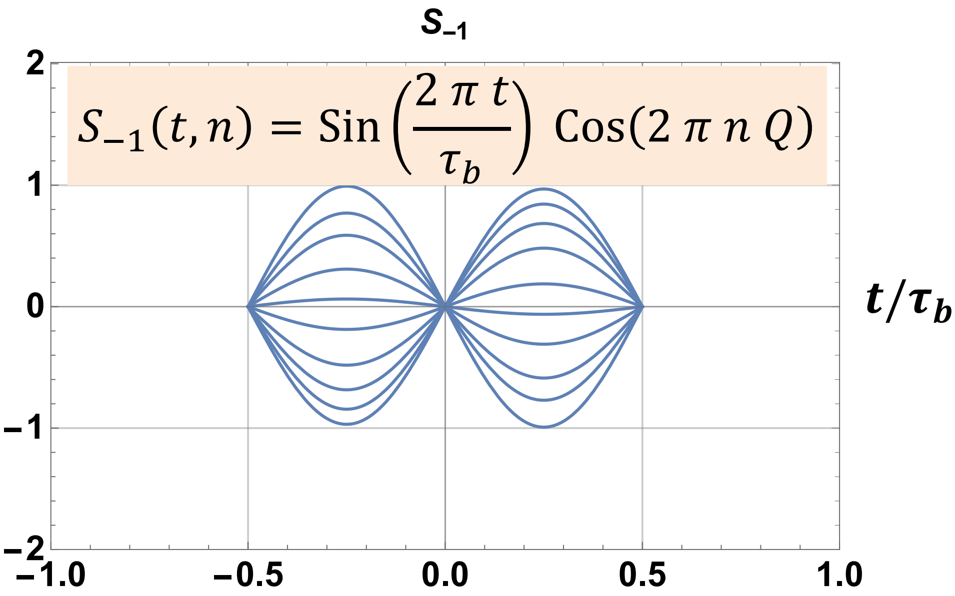


# Simple analytical model

- Considering the 2 modes independently (i.e. low-intensity case) and approximating them by sinusoidal modes

**Mode 0 alone** — **Mode -1 alone**



# Simple analytical model

=> Considering the 2 modes together

# Simple analytical model

- Below TMCI => $\text{Signal} \propto a_0 S_0 - a_{-1} S_{-1}$ ($a_0$ and $a_{-1}$ reals)

- At TMCI threshold => $\text{Signal} \propto a\,(S_0 - S_{-1})$ as $a_0 = a_{-1} = a$ (real)

  - Signal is 0 at both bunch extremities

  - Signal is also 0 when $\text{Cos}\left(\frac{\pi\, t}{\tau_b}\right) - \text{Sin}\left(\frac{2\,\pi\, t}{\tau_b}\right) = 0 \Rightarrow t = \frac{\tau_b}{6}$

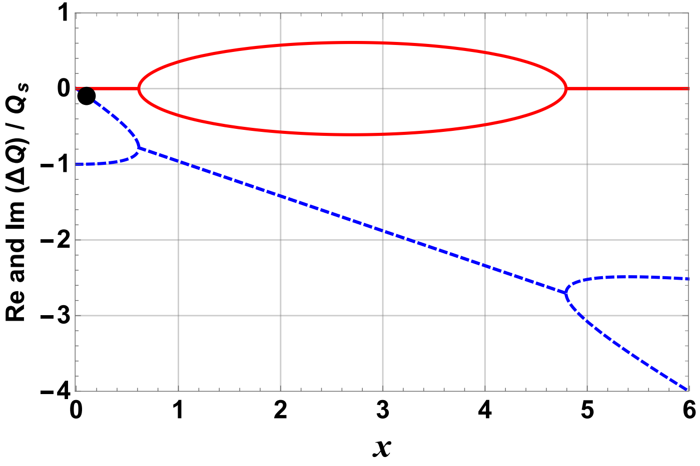
# Simple analytical model

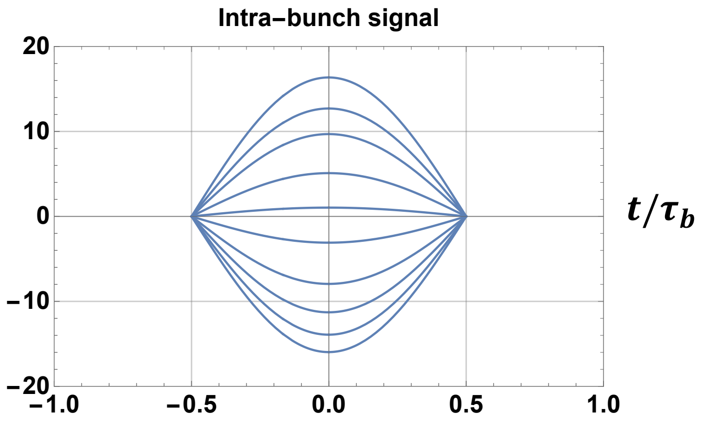
- Above TMCI => $\text{Signal} \propto (a + jb)\, S_0 - (a - jb)\, S_{-1}$ ($a$ and $b$ reals)

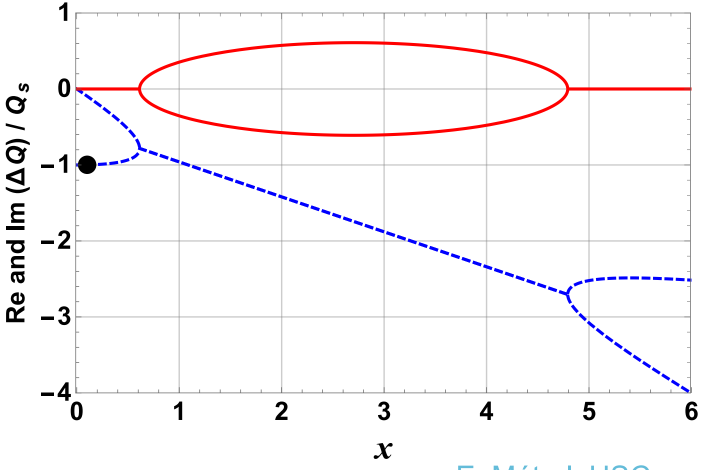
$$\propto \sqrt{\left\{a\left[\mathrm{Cos}\left(\frac{\pi\, t}{\tau_b}\right) - \mathrm{Sin}\left(\frac{2\,\pi\, t}{\tau_b}\right)\right]\right\}^2 + \left\{b\left[\mathrm{Cos}\left(\frac{\pi\, t}{\tau_b}\right) + \mathrm{Sin}\left(\frac{2\,\pi\, t}{\tau_b}\right)\right]\right\}^2}\ \mathrm{Cos}[2\,\pi\, n\, Q + \varphi(t)]$$

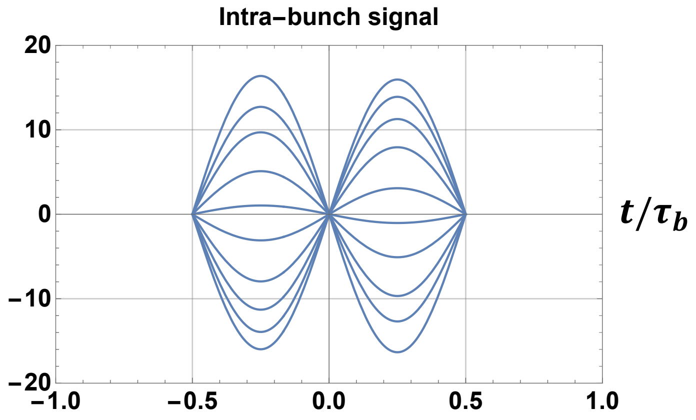
with $\varphi(t) = \mathrm{ArcTan}\left\{\frac{b\left[\mathrm{Cos}\left(\frac{\pi\, t}{\tau_b}\right) + \mathrm{Sin}\left(\frac{2\,\pi\, t}{\tau_b}\right)\right]}{a\left[\mathrm{Cos}\left(\frac{\pi\, t}{\tau_b}\right) - \mathrm{Sin}\left(\frac{2\,\pi\, t}{\tau_b}\right)\right]}\right\}$

# Simple analytical model ($x = 0.1$)

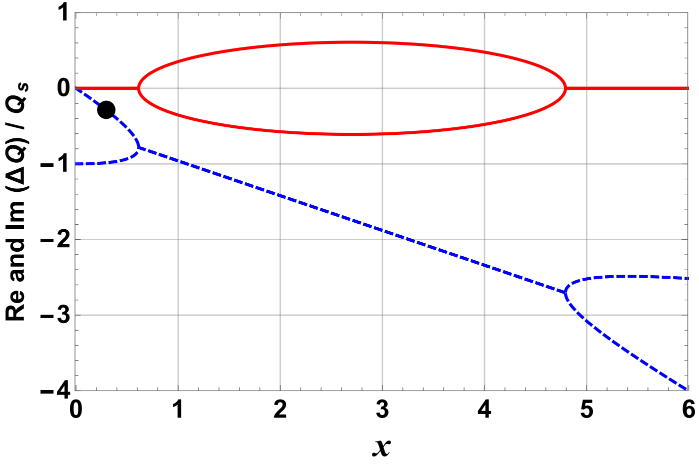


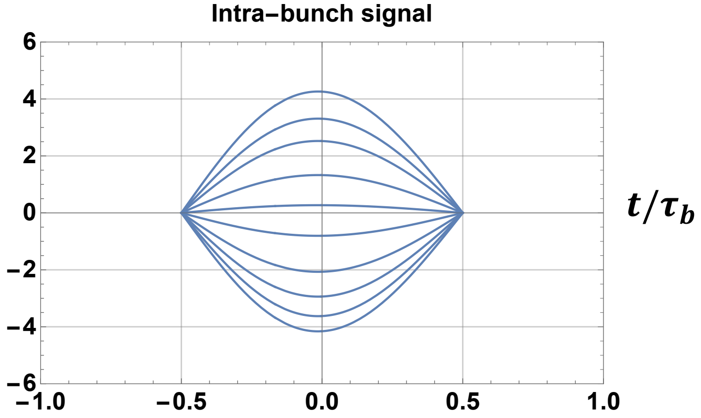



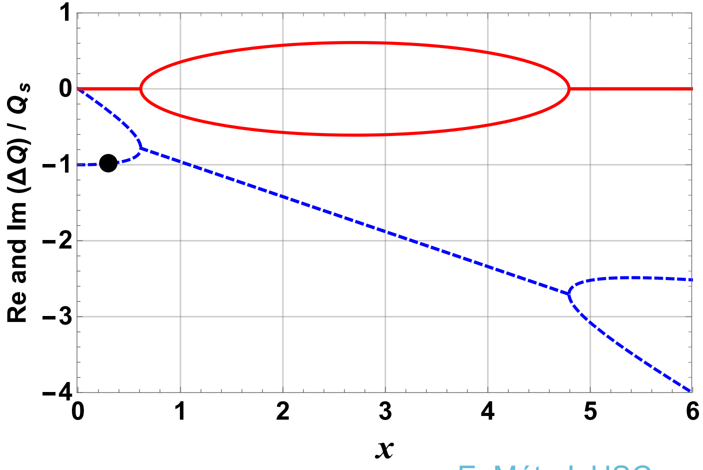



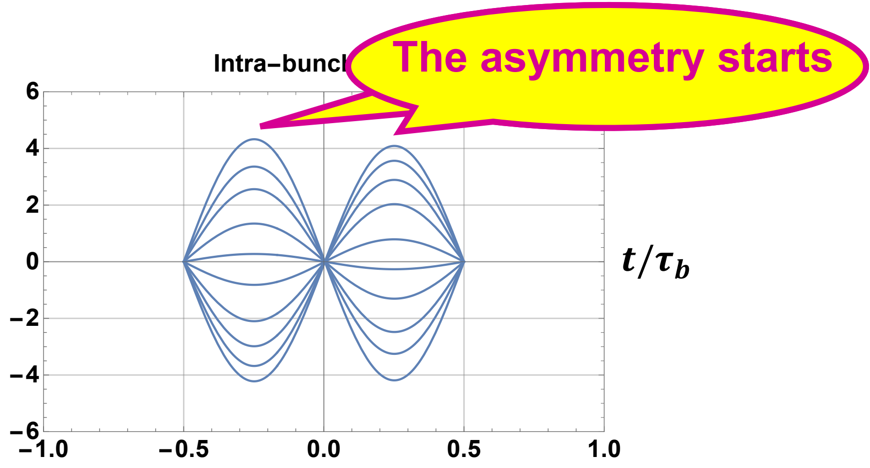



# Simple analytical model ($x = 0.3$)

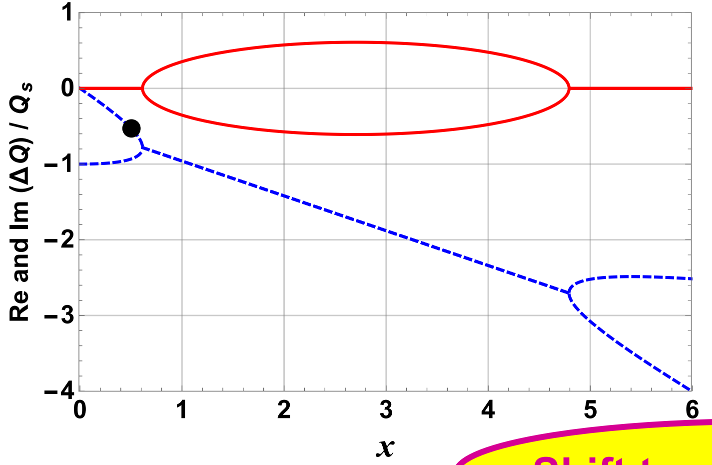


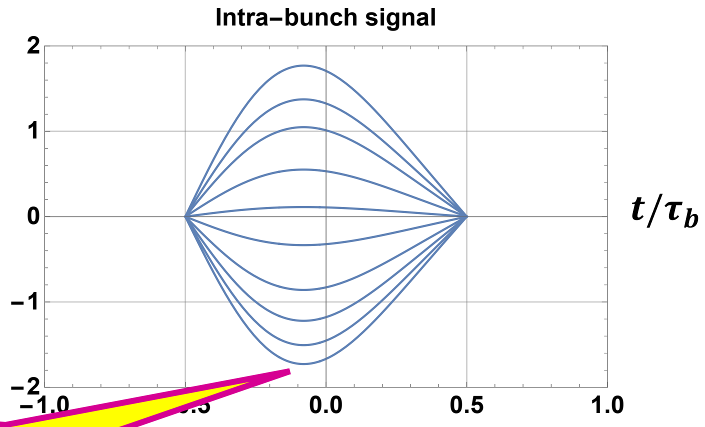



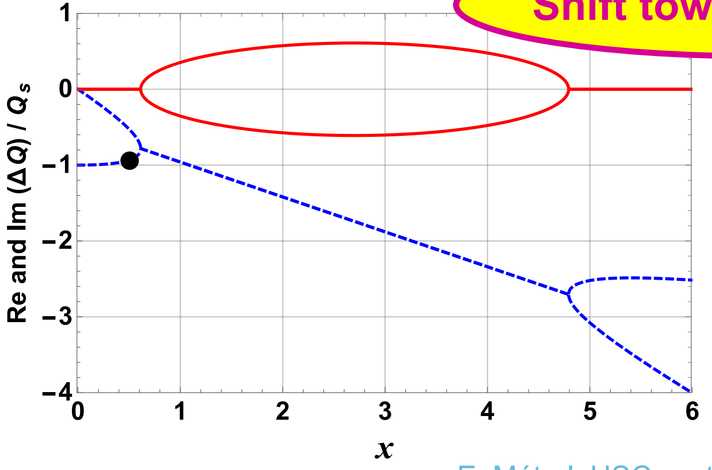



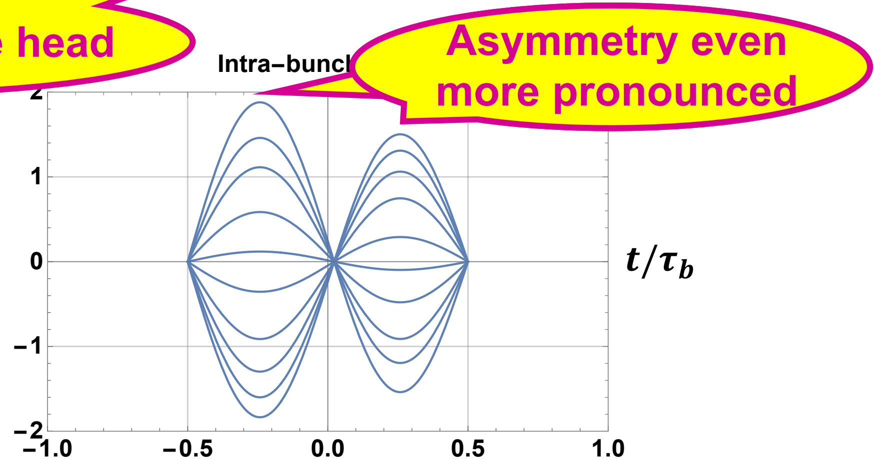



# Simple analytical model ($x = 0.5$)



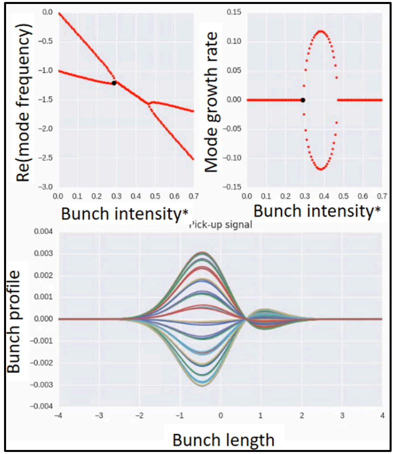



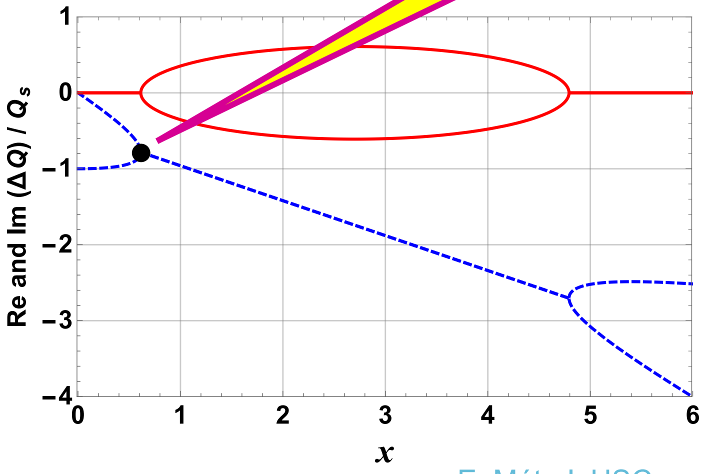



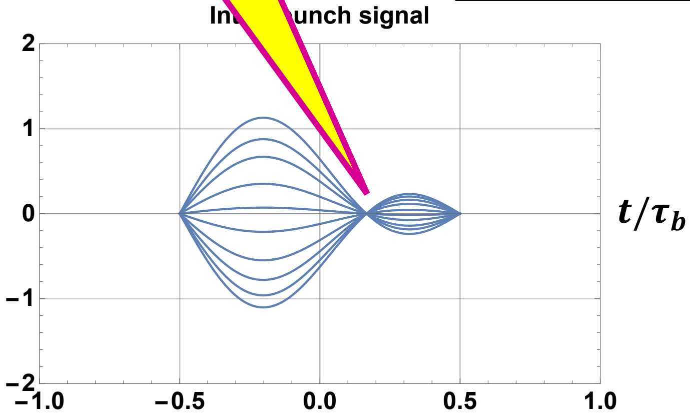



# Simple analytical model ($x = 0.613$)

This point is given by $\frac{\tau_b}{6}$

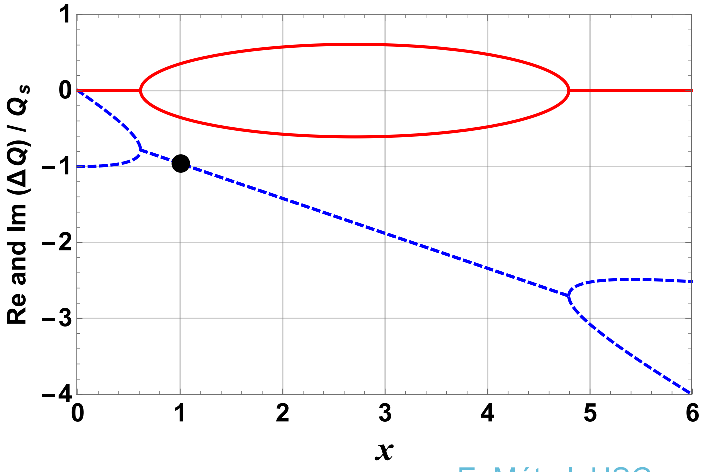


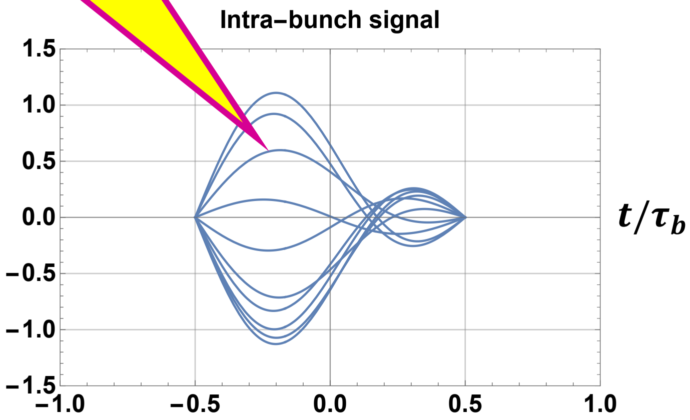



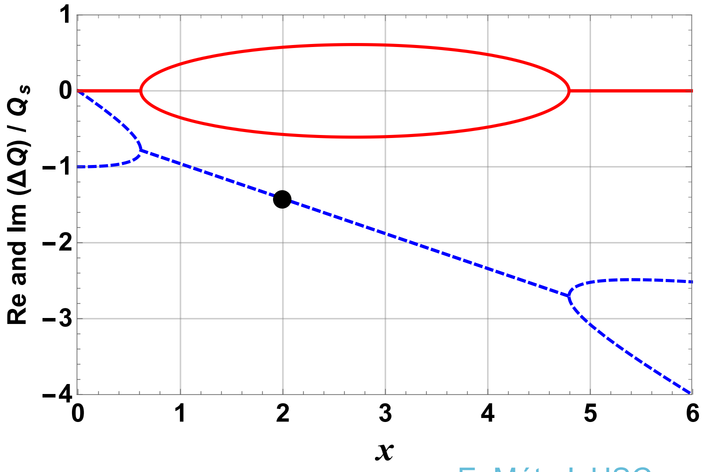
# Simple analytical model ($x = 1.0$)

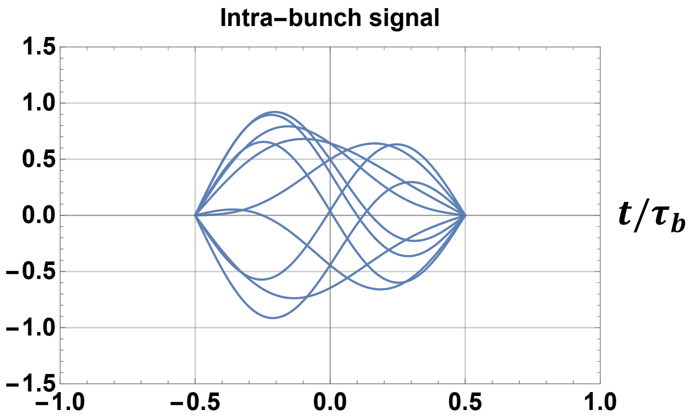
**Travelling-wave comes from the fact that the eigenvectors have now both a Re and Im part => The phase $\varphi(t)$ appears**



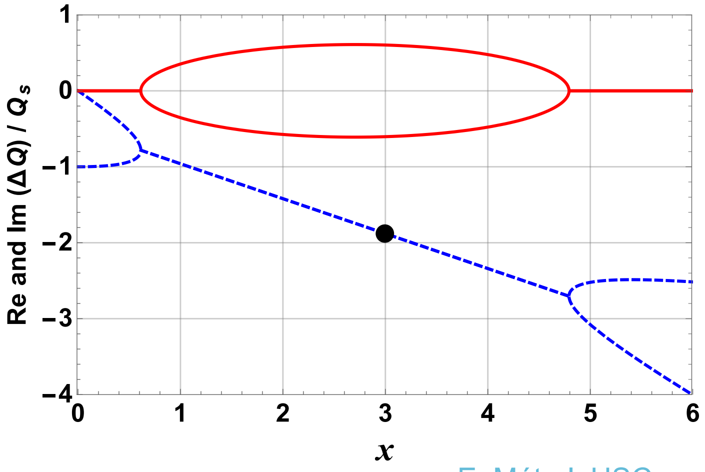



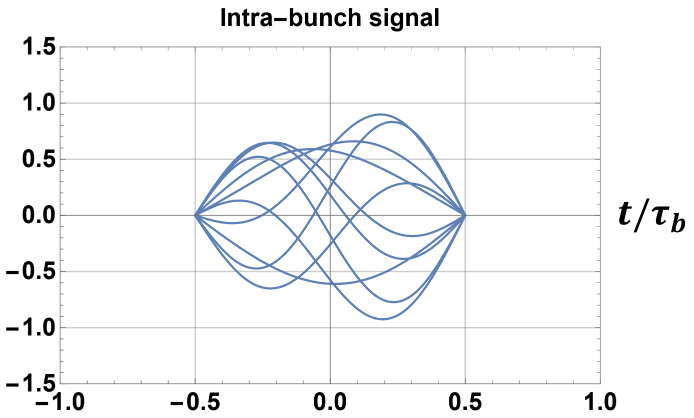
# Simple analytical model ($x = 2.0$)



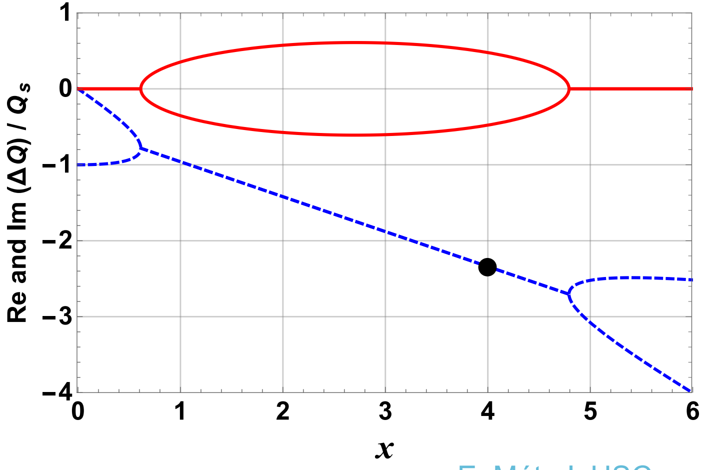



# Simple analytical model ($x = 3.0$)

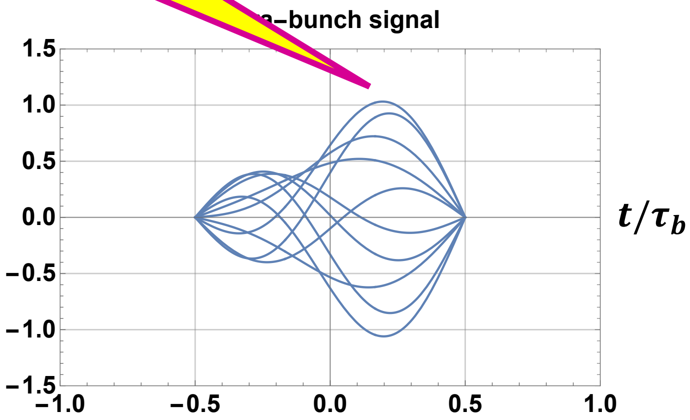


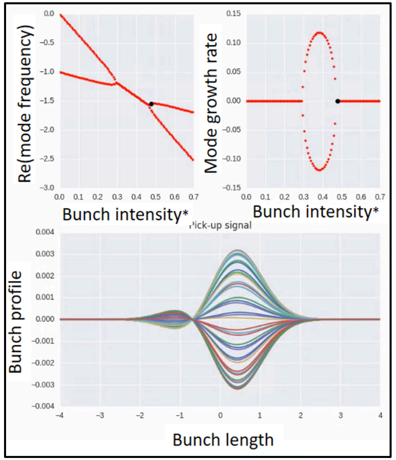
# Simple analytical model ($x = 4.0$)



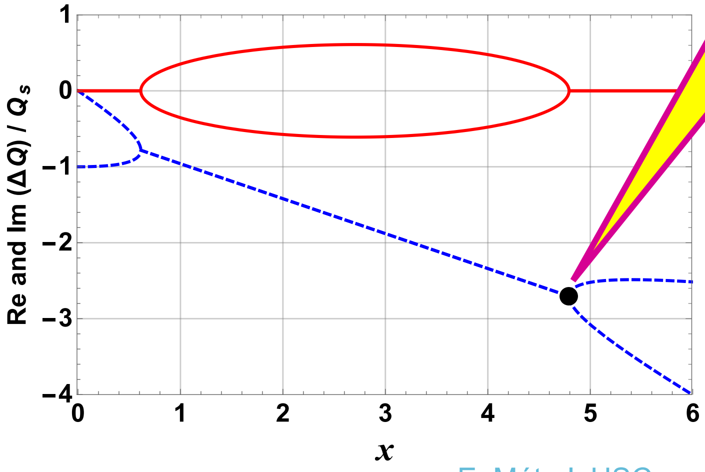

Shift towards the tail

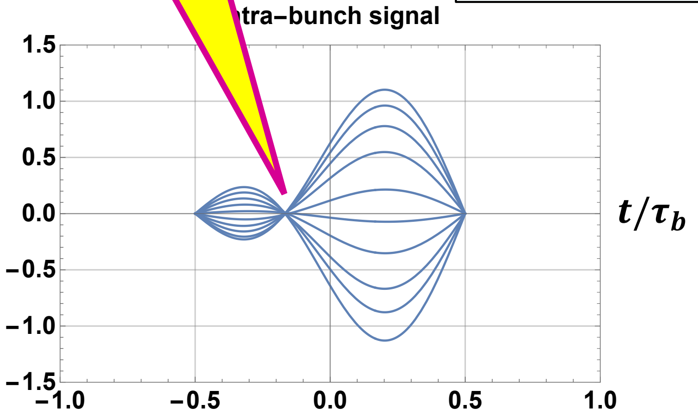


# Simple analytical model ($x = 4.8$)

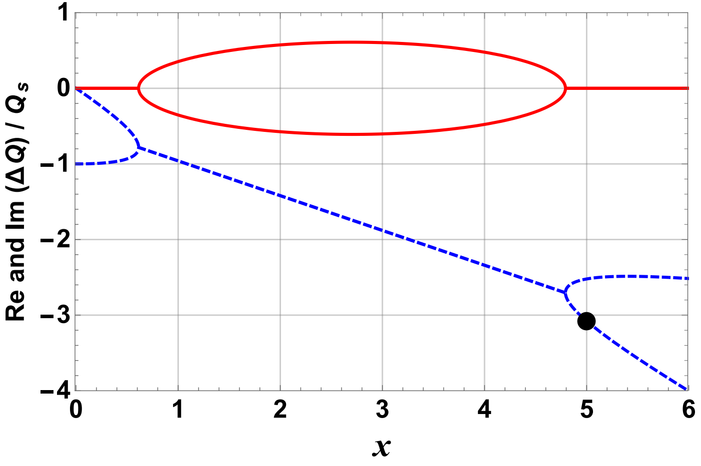


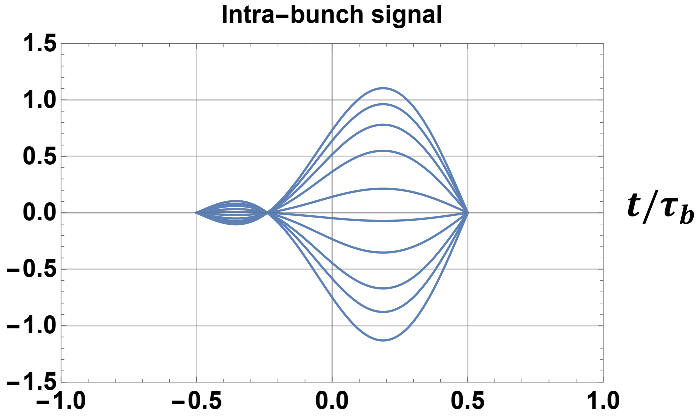

**This point is given by $-\frac{\tau_b}{6}$**

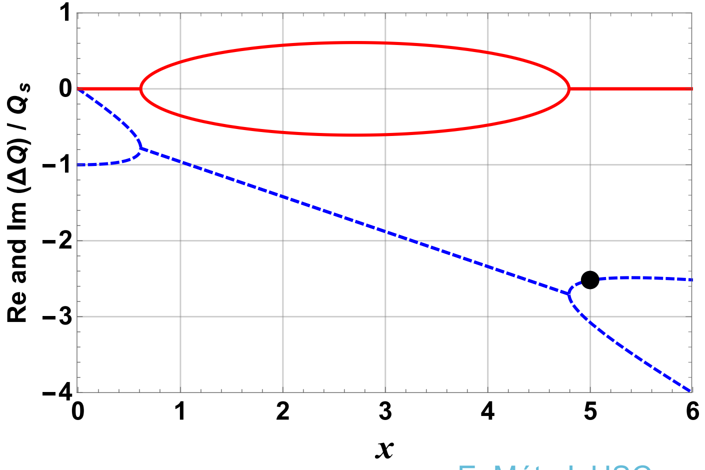


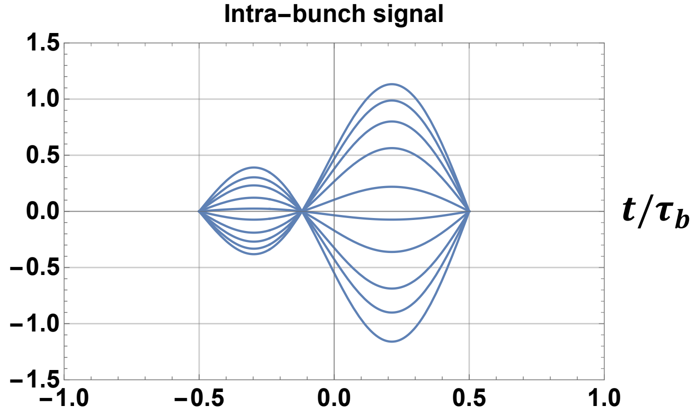



# Simple analytical model ($x = 5.0$)

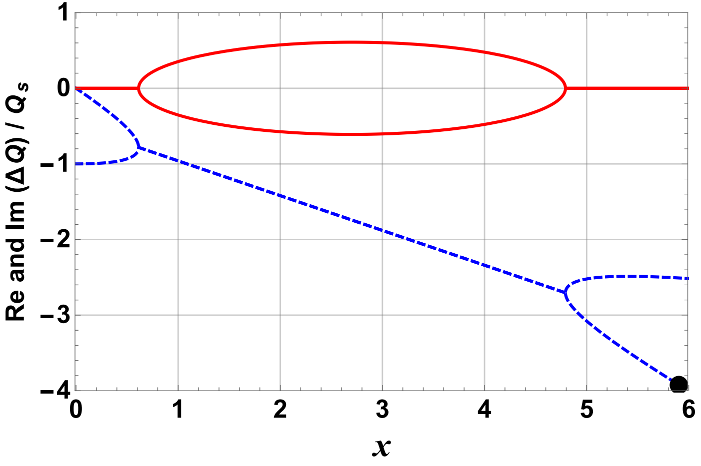


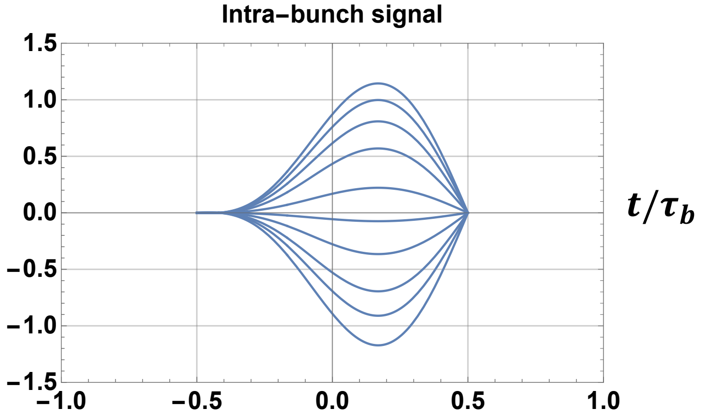



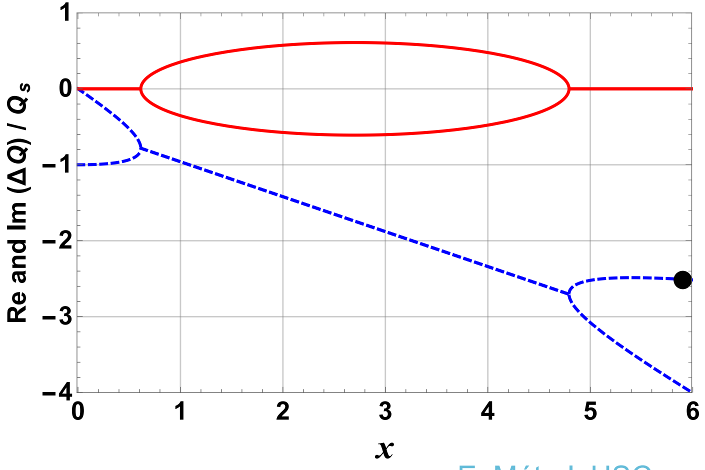



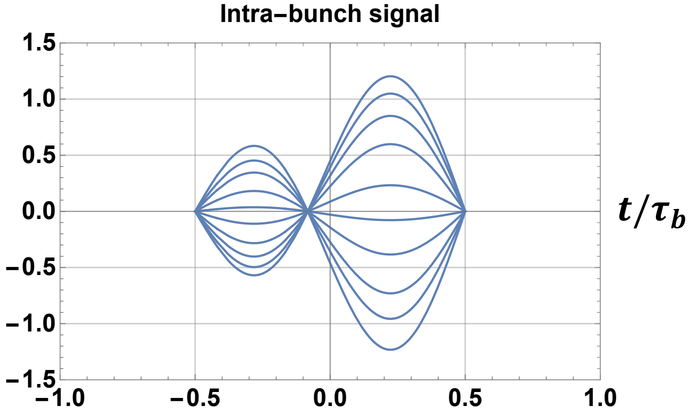



# Simple analytical model ($x = 5.9$)



# Conclusion and next step

- The intra-bunch motion for TMCI (and its main features below-at-above TMCI threshold) can be explained with a simple analytical model

- Next => Finalise and present the fully self-consistent result from GALACTIC Vlasov solver