GEANT4
A SIMULATION TOOLKIT

Version 10.5

Magnetic Fields

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Geant4 Advanced Course at CERN

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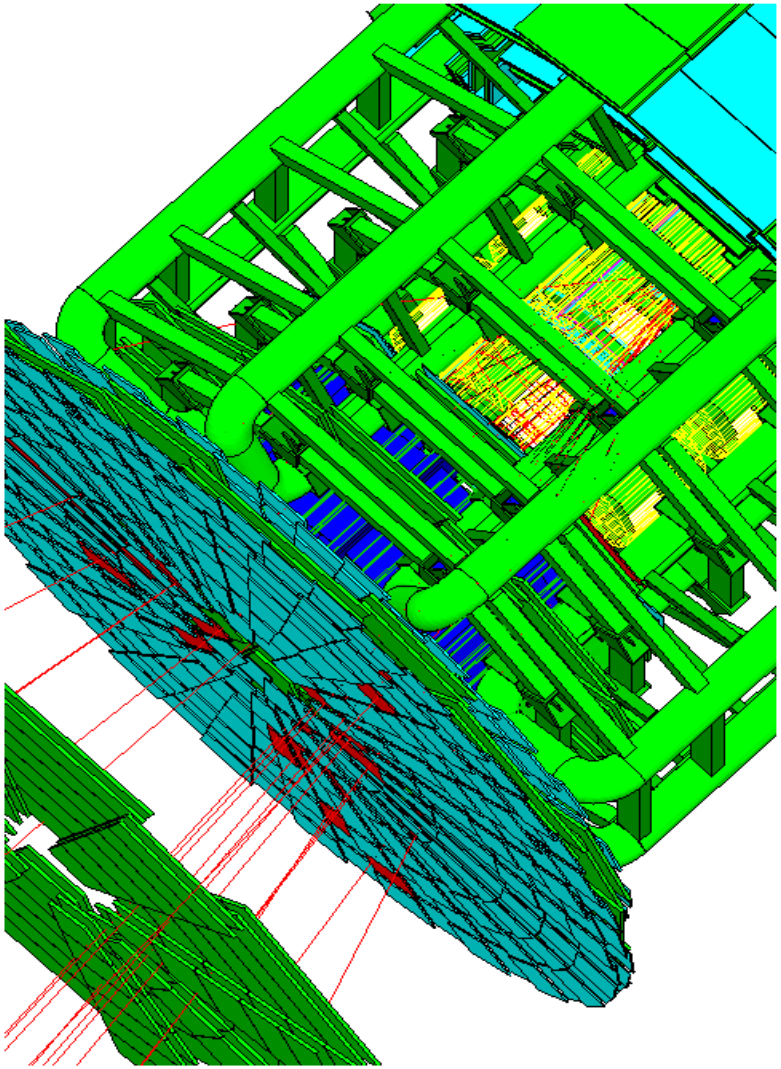
- Magnetic field
- Integration of trajectories in field
- Other types of field

Many slides adapted from M. Asai (SLAC)



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Defining a magnetic field

Basics 1: Defining a Magnetic field

- How to create a (magnetic) field ? Instantiate it in the *ConstructSDandField*() method of your *DetectorConstruction*
  - Uniform field :
    - Use an object of the G4UniformMagField class

```
G4MagneticField* magField =
      new G4UniformMagField(G4ThreeVector(1.*tesla,0.,0.);
```

  - Non-uniform field :
    - Create your own concrete class derived from G4MagneticField and implement **`GetFieldValue`** method.

```
void MyField::GetFieldValue(
        const double Point[4], double *field) const
```

    - Point[0..2] are x,y,z position in global coordinates, Point[3] is time
    - field[0..2] are output x,y,z components of magnetic field (in G4 units)

Basics 2: How to assign a field to the whole detector

- The *global field manager* is the one associated with the 'world' volume
  - it already exists, before G4VUserDetectorConstruction is called,
  - it is created / set in G4TransportationManager.
- To associate your field with the world, you must obtain that global field manager:

```
auto tm = G4TransportationManager::GetTransportationManager();

G4Fieldmanager* globalpFieldManager = tm->GetFieldManager();
```

- Then you assign the field to it

```
G4Field* field= new G4UniformField(…); // B/E or other field
globalpFieldManager->SetDetectorField(field);
```

- Alternative: get the field manager from the world volume you created – if you kept a pointer to it as a data member in your G4VUserDetectorConstuction:

```
G4VPhysicalVolume* fMyWorld; // In class declaration, e.g. MyDetectorConstruction.hh)
fMyWorld = new … ; // In the Construct() method
G4Fieldmanager* globalpFieldManager = fMyWorld->GetFieldManager();
```

Note: In an (advanced) use case with parallel worlds the *global field manager* is associated with the primary geometry (the 'mass' geometry in which nearly all materials are assigned.)

Refinement: Region and local fields

- Other volumes can override a global field
  - An alternative field manager can be associated with any logical volume – or a 'region'
    - The field must accept position in global coordinates and return the value of the field in global coordinates

```
auto localpFieldManager = new G4FieldManager(myEMfield);
Region->setFieldManager(localpFieldManager);
```

The region Field Manager overrides the global one (if any.)

```
logVolume->setFieldManager(localpFieldManager, true);
```

The logical Volume's Field Manager overrides the region and the global (if any.)

Note that the assignment affects also sub-volumes contained in **`logVolume`**:

- By default only sub-volumes that do not yet have a field manager.
- Using '**`true`**' for the second argument asks it to push the field to all the sub-volumes, even if a daughter volume has its own field manager.

CUSTOMIZING FIELD PROPAGATION

Choosing an appropriate integration method for your field

Setting precision parameters

Magnetic field (2)

In the DetectorConstruction's *ConstructSDandField*() method, after creating a field

```
G4MagneticField* fMyField = new MyMagneticField();
 G4Fieldmanager* pFieldManager = new G4FieldManager();
 pFieldManager->SetDetectorField(fMyField);
```

a user must create an integration method. There is a very easy way:

```
// Use the default method - only for magnetic fields
pFieldManager->CreateChordFinder(fMyField);

G4bool forceToContained = true; //overwrites any existing field managers
fMagLogicalVol->SetFieldManager(pFieldManager, forceToContained); // in
 daughters
// Register the field and its manager for deleting at the end
G4AutoDelete::Register(fMagneticField);
G4AutoDelete::Register(pFieldManager);
```

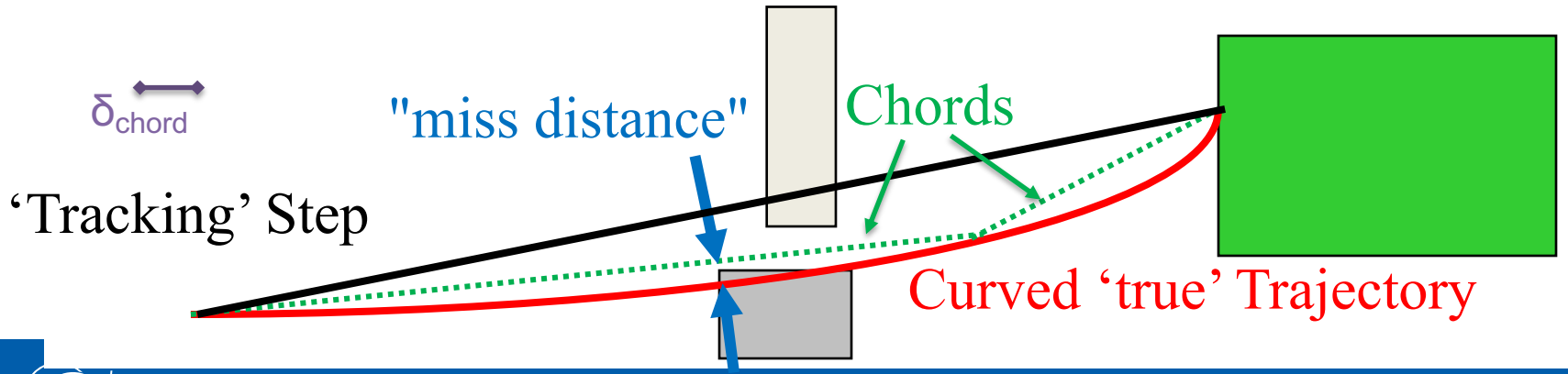
- /example/basic/B5 is a good starting point

Field integration: overview of Geant4 approach

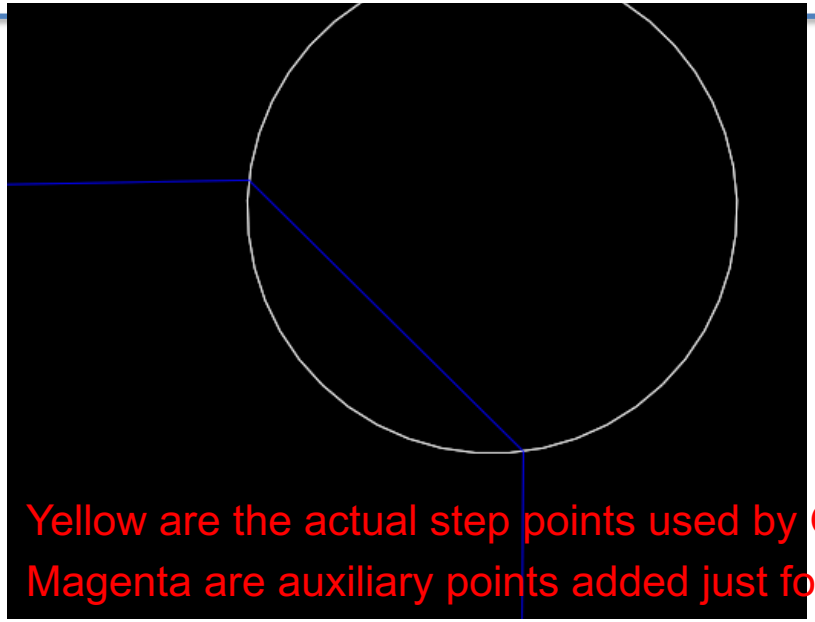
- To propagate a particle in an external field (magnetic, electric, both or other), we integrate numerically its equation of motion.
- We use Runge-Kutta integration method for the ordinary differential equations
  - Several Runge-Kutta 'steppers' are available.
- In specific cases other integration methods can also be used:
  - In a uniform field, using the analytical solution – a helix (*G4ExactHelix*).
  - In a smooth but varying field, with RK+helix.
  - An alternative multi-step integration method, Bulirsch-Stoer.
- As it calculates the track's motion in a field, Geant4 breaks up its curved path into linear chord segments.
  - Choosing chord segments so that they approximate the curved path within a given tolerance.

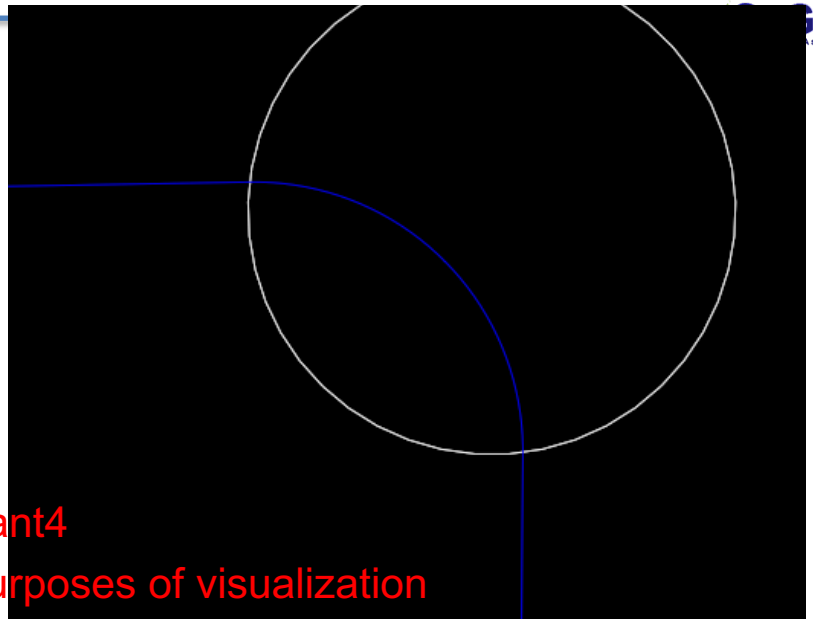
Tracking in field

- The chords are used to interrogate the G4Navigator, to check whether/where a track has crossed a volume boundary.
- One physics/tracking step can create several chords.
  - In gases or vacuum, a ‘physics’ step can span several helix turns.
- The user controls the accuracy of the volume intersection,
  - By setting a parameter δ_{chord} to limit the “miss distance” It is the
    - maximum acceptable error in approximating the curved track by chords,
    - maximum depth inside a volume that a curved track could enter and yet the volume is still missed (not crossed by the series of chords.)
  - It is quite expensive in CPU performance to set too small a “miss distance”.



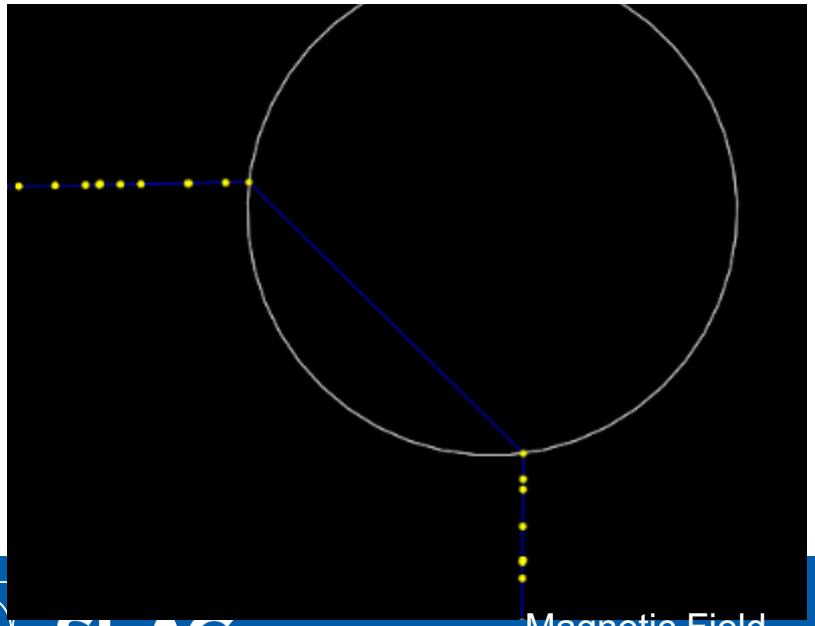
Regular versus Smooth Trajectory

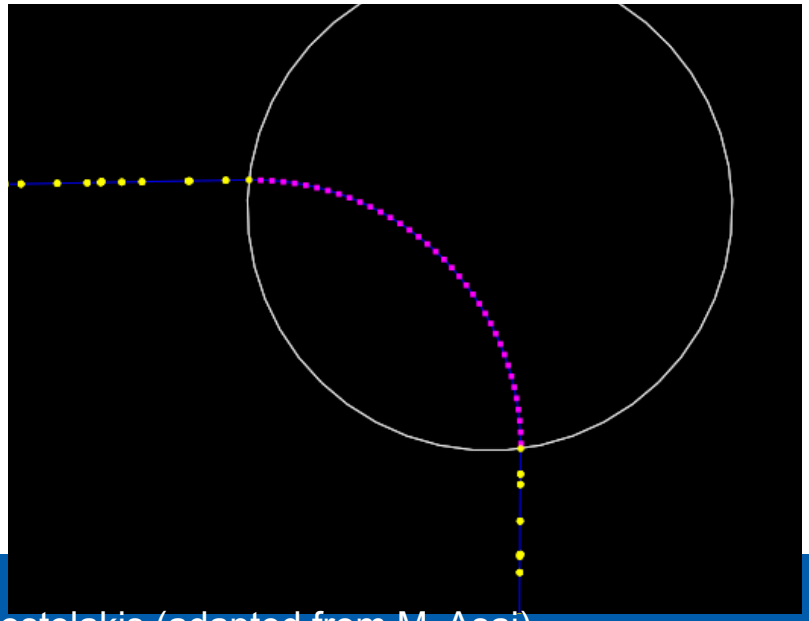




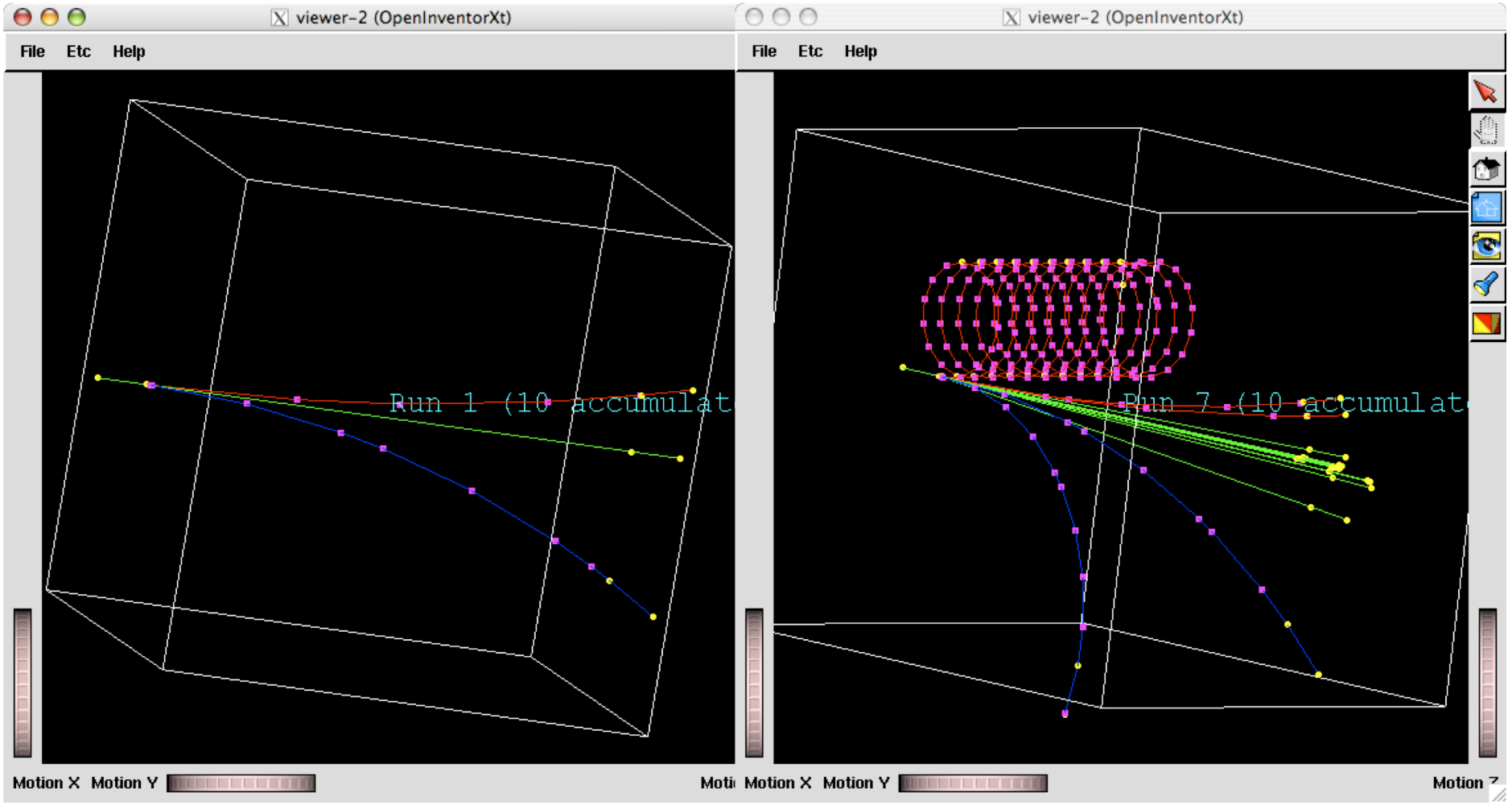
Yellow are the actual step points used by Geant4

Magenta are auxiliary points added just for purposes of visualization





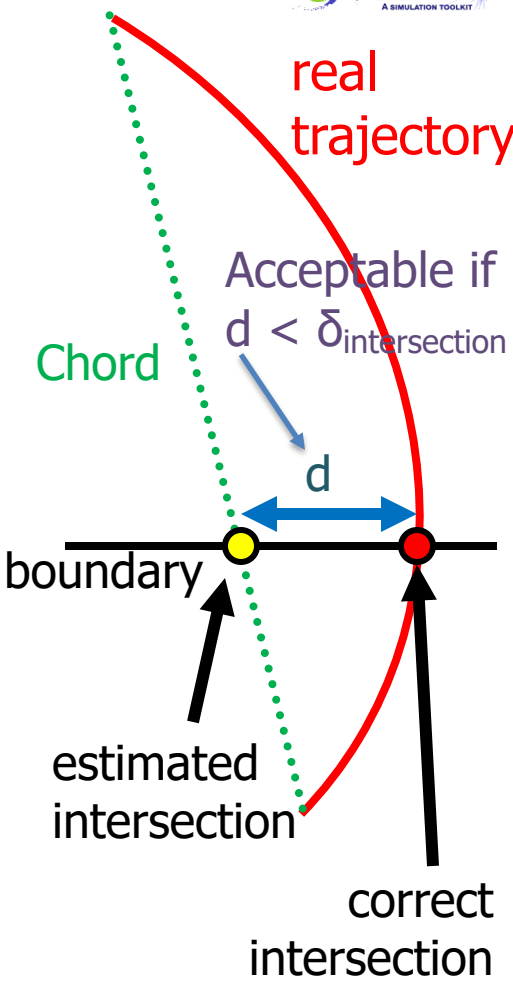
Smooth Trajectory Makes Big Difference for Trajectories that Loop in a Magnetic Field



- Yellow dots are the actual step points used by Geant4
- Magenta dots are auxiliary points added just for the purpose of visualization

Tunable parameters

- In addition to the "miss distance" there are two more parameters which the user can set in order to adjust the accuracy (and performance) of tracking in a field.
  - These parameters govern the accuracy of the intersection with a volume boundary and the accuracy of the integration of other steps.
- The "delta intersection" parameter is the accuracy to which an intersection with a volume boundary is calculated.
  - **Important**: it is used to **limit the bias** that our algorithm (for boundary crossing in a field) exhibits: the intersection point is always on the 'inside' of the curve.
  - Set its **value** much **smaller** than your acceptable error, to limit the cumulative effect of this bias (after the total number of volume crossings in the track's path.)



Tunable parameters

- The most important accuracy parameter is the maximum relative tolerance ε_{max} for the integration error
  - ε_{max} limits the estimated error for large steps: $|\Delta x| < \varepsilon_{max}\, s$ and $|\Delta p| < \varepsilon_{max}\, |p|$
- The "delta one step" parameter is accuracy for endpoint of integration steps that do **not intersect** a volume boundary.
  - It also limits on the estimated error of the endpoint of each physics step (essentially it is < 1000 $\delta_{1\ step}$.)
  - Values of $\delta_{intersection}$ and $\delta_{1\ step}$ should be within one order of magnitude.
- These tunable parameters can be set by

```
ptrChordFinder->SetDeltaChord( missDistance );
ptrFieldManager->SetDeltaIntersection( deltaIntersection );
ptrFieldManager->SetDeltaOneStep( deltaOneStep );
ptrFieldManager->SetEpsilonMax( epsilonMax );
ptrFieldManager->SetEpsilonMin( 0.1 * epsilonMax );
```

- Further details are described in Section 4.3 (Electromagnetic Field) of the Geant4 Application Developers Guide.

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Additional Fields & Integration Methods

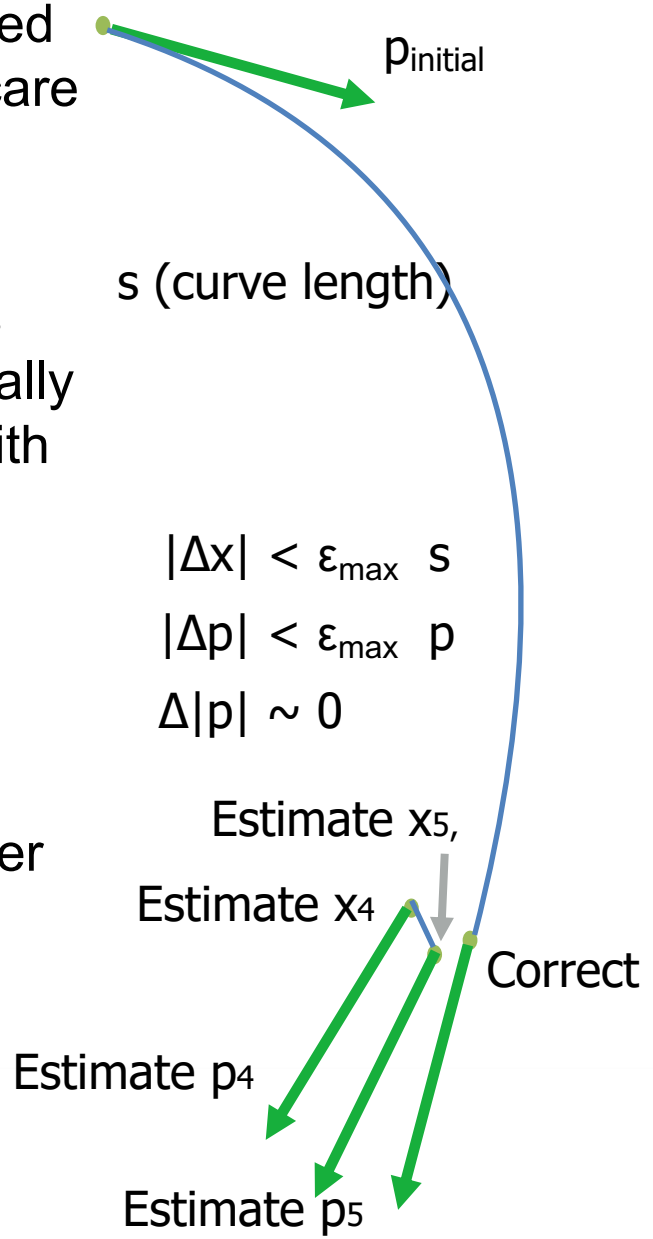
Field integration – usual and custom methods
Alternative types of field

Customizing field integration

- Runge-Kutta (RK) integration is used to compute the motion of a charged track in a any type of field: magnetic, electric, combined EM, gravitational or a mix.
- There are two kinds of RK steppers are available in Geant4:
  - General-purpose steppers, applicable to any field type, usable for any ODE.
  - Specialized steppers, applicable only to charged particle motion in **pure magnetic** fields.
- RK steppers related to the order of the Taylor expansion from which they derive:
  - low 1st, 2nd or 3rd order are considered 'low-order' steppers – with one field evaluation per stage (i.e. 1 initial + 2 other evaluations for 3rd order method).
  - 4th order (4 evals/step) & 5th order (6-7 evals/step) are the typical 'sweet' spot
  - higher order require many more stages (evaluations) per step – their increased accuracy is relevant for very specialized applications.
- An integration method must estimate both an end state (position, momentum, maybe polarisation) and an integration error for each state variable
  - Old (pre 1967) 'simple' methods estimated error by having the step

Integrating efficiently

- Given a detector's field B(x,y,z) [or B+E] we need to integrate the trajectory of each track, taking care
  - to stay within the relative accuracy ε_{max}
  - to be fast – in particular using as few calls as possible to the field evaluation method (typically expensive in arithmetic operations / time – with an interpolation or function evaluation)
- Typically choose Runge-Kutta methods
  - No memory / history needed – it "self starts"
  - Adjusts easily to change(s) of momentum after (frequent) collisions / interactions
  - Ability to adjust step size



Customizing field integration

- Runge-Kutta (RK) integration is used to compute the motion of a charged track in a any type of field: magnetic, electric, combined EM, gravitational or a mix.
- Many general steppers are available applicable to any equation / field. Some steppers are specialized, applicable only to pure magnetic fields.
- of low and high order, and specialized steppers for pure magnetic fields.
- Default in G4 is the general purpose G4DormandPrince745 an embedded 4^{th}-5^{th} order RK stepper. (Embedded = compares 4^{th} & 5^{th} order to estimate error.)
  - It typically uses 6 field evaluates per field step, and provides the derivative .
  - Earlier Geant4 versions (<10.4) had G4ClassicalRK4 as default – this is just as general and very robust, but requires 11 field evaluations per step.
- If the field has very rough or smooth, lower or higher order steppers may obtain the results of same quality using fewer computing cycles.

Explicit Runge-Kutta methods

- "Integrate" $dy/dx = F(x, y)$ from x_0 to x_0+h
- Uses evaluations of $F(x, y)$
  - $f_i = F(x_0 + a_i h , y_0 + h \Sigma_{j<i} b_{ij} f_j)$
  - $y_{estim}(x_0 + h) = \Sigma_i c_i f_i$
- Each method has a 'tableau' made up by coefficients a_i, b_{ij}, c_i, c'_i
- Key Parameters of an RK method:
  - Number of 'stages' = number of evaluations of the derivative F()
  - 'Order' N: the expected scaling of the errors ~ h^N
  - Embedded method = 2nd 'line' to estimate error

$f_1 = F(x_0 , y_0)$

$f_2 = F(x_0 + a_2 h , y_0 + h b_{21} f_1)$

$f_3 = F(x_0 + a_3 h , y_0 + h b_{31} f_1 + h b_{32} f_2)$

a_i | b_{ij}

0			
$\frac{1}{2}$	$\frac{1}{2}$		
$\frac{3}{4}$	0	$\frac{3}{4}$	
1	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{4}{9}$

c_j

$y_{RBS3}=2f_1/9 + f_2/3 + 4f_3/9$

c'_j | $\frac{7}{24}$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{8}$ |

$y'(x_0 +h) = \Sigma_i c'_i f_i$

$\Delta y = \Sigma_i (c'_i - c_i) f_i$

Alternative Runge-Kutta steppers – some context

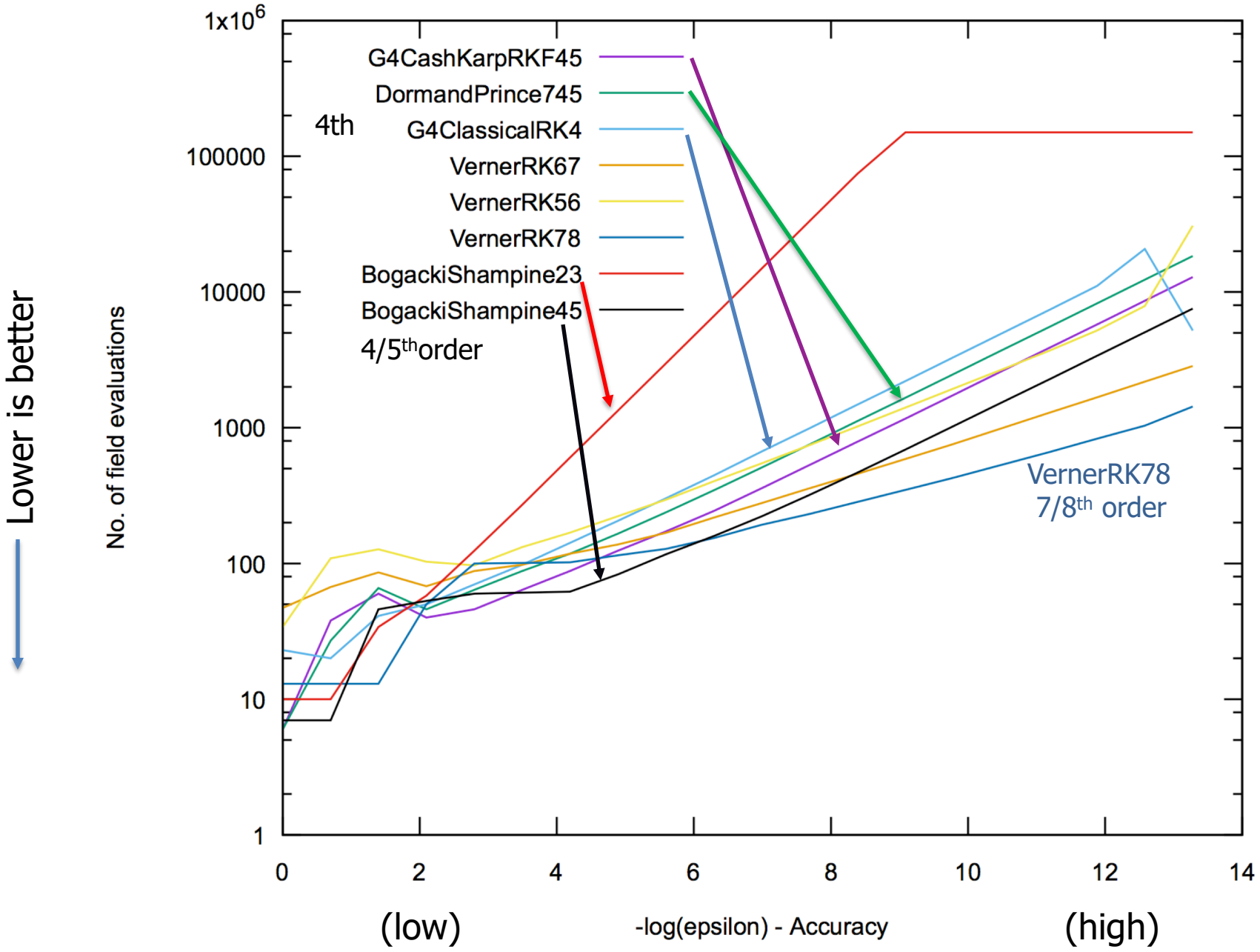
- Embedded methods (invented by Felhberg) provide built-in error estimates by comparing estimators of 2 different orders (that share evaluation points).
  - Dormand Prince (DoPri5), the most widely used method provides stability and performance. It's the default also in MATLAB, GNU Octave ("ode45")
    - It evaluates the field value and derivative at the final point (with p_{final}), called FSAL. So they are ready for use in next step (=> 1 less eval.)
  - Cash-Karp (1990) - uses difference of 5th & 4th order method, six stages = 6 evaluations of derivative
- AtlasRK4/NystromRK4: 3 field evaluations + evaluation of error using numerical estimate of 4th derivative - restricted to B-fields
- Helix (G4ExactHelix) - for constant field
- 'Classical RK4' = original 4th order method, 4 stages (i.e. evaluations)
  - Error estimate from breaking step in two => 1(initial)+10 evaluations
  - General and very robust, but costly (due to 10 field evaluations per step.)
- Lower order RK methods for short steps, and/or lower accuracy

FSAL = First Same As Last

Choices of Runge-Kutta methods (steppers)

- The default is the general purpose G4DormandPrinceRKF45 an embedded 4th-5th order RK stepper. (Embedded = compares 4th & 5th order to estimate error.)
  - It typically uses 6 field evaluates per field step, and provides the derivative .
  - Note that earlier Geant4 versions (<10.4) had G4ClassicalRK4 as default – it is a good robust backup stepper.
- If the field is very smooth, you may consider higher order steppers
  - of most potential interest in large volumes filed with gas or vacuum.
- If the field is rough, 3rd order steppers may obtain the results of same quality using fewer computing cycles
  - 3rd order stepper(s): G4SimpleHeum, G4BogackiShampine23 (FSAL)
- For reasonably smooth (or not very rough) fields, the choice between 3rd , 4th or 5th order steppers should be made by trial and error.
- In a field calculated from a 'rough' field map, a lower order stepper can be more robust.
  - 2nd order G4SimpleRunge and G4ImplicitEuler
- The less smooth the field is, we would expect that a lower the order of the stepper would be more robust (but no lower than 2nd order.) However the relative performance may again depend on many factors, and trial/error is recommended to identify the best performing stepper.

Thanks to Somanth Banerjee (Google Summer of Code student 2015)



Customizing field integration

- Trying a few different types of steppers for a particular field or application is suggested, if maximum performance is a goal.
  - What is the most performant option may also be different in different regions – e.g. depending on whether the field is varying greatly.
- Specialized steppers for pure magnetic fields are available. Some assume that a local trajectory in a slowly varying field will resemble a helix.
  - Combining this in with a variation, the Runge-Kutta method can provide higher accuracy at lower computational cost when large steps are possible
  - Suggested are G4HelixSimpleRunge and G4HelixSimpleHeum
- To change the stepper

```
theChordFinder
   ->GetIntegrationDriver()
   ->RenewStepperAndAdjust( newStepper );
```

- Further details are described in Section 4.3 (Electromagnetic Field) of the Application Developers Manual.

Other types of field

- For pure electric field, Geant4 has G4ElectricField (base) and the simple G4UniformElectricField (concrete) classes.
- G4ElectroMagneticField is the base class for combined electro-magnetic fields.
- The *equation of motion* class for electromagnetic field is G4MagElectricField.
- An example:

```
G4ElectricField* EMfield
    = new G4UniformElectricField( G4ThreeVector(0., 100000.*kilovolt/cm, 0.) );
auto equation = new G4EqMagElectricField(EMfield);
auto stepper = new G4CashKarpRKF45( equation, nvar );
G4FieldManager* pFieldManager
    = G4TransportationManager::GetTransportationManager()-> GetFieldManager();
pFieldManager->SetDetectorField( EMfield );
auto integrDriver= new G4MagInt_Driver( fMinStep, stepper,
                                        stepper->GetNumberOfVariables() );
G4ChordFinder* fChordFinder = new G4ChordFinder(integrDriver);
```

- *Notes:*
  1. *A user-defined field type can be time-dependent (its value can change with time).*
  2. *A user can create their own type of field, inheriting from G4VField, and an associated Equation of Motion class (that inherits from G4EqRhs).*

LATEST DEVELOPMENTS

Latest developments - Interpolation

The Bulirsch Stoer method is a multi-step method, alternative to Runge-Kutta:

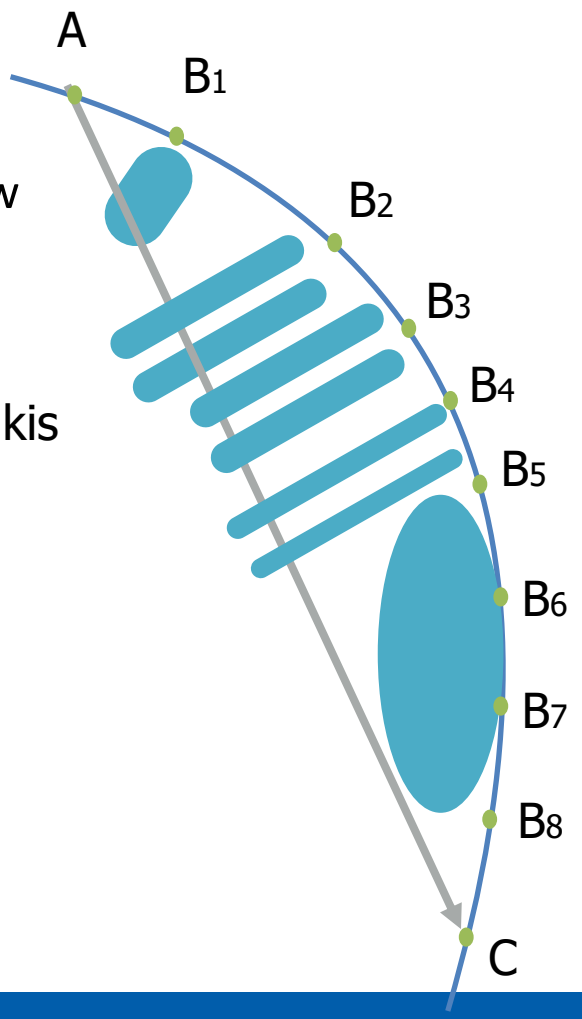
```
G4BulirschStoer* pBSstepper =
        new G4BulirschStoer( fEquation, nVar, epsilon );
auto pDriver = new G4IntegrationDriver<G4BulirschStoer>( stepMinimum,
                                                         pBSstepper, nVar );
```

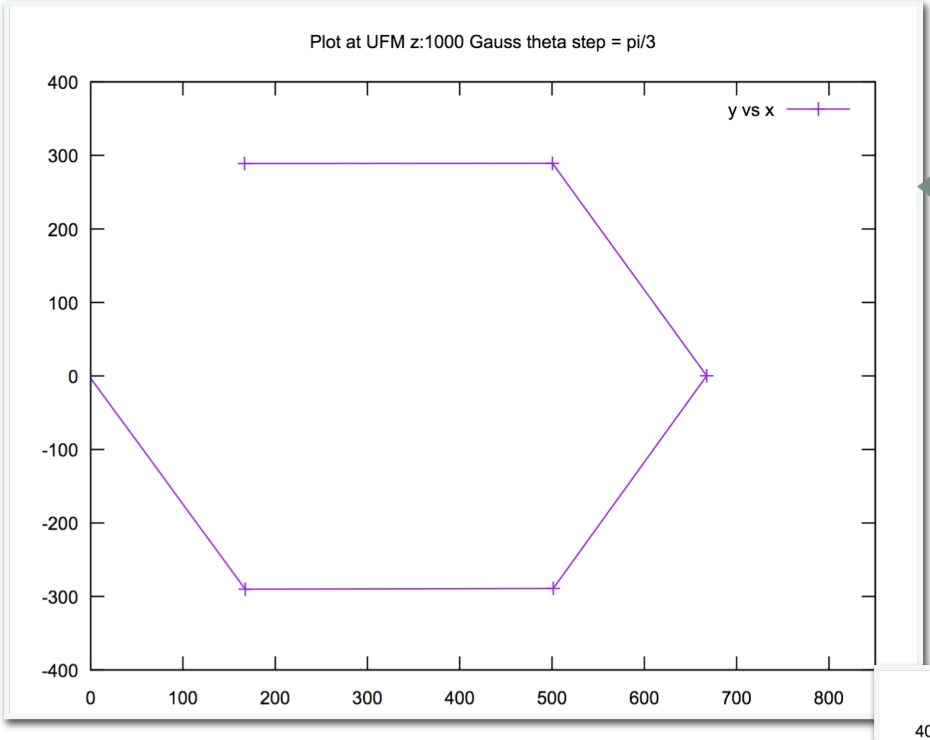
A variety of promising Runge-Kutta methods, seeking improvements in

- efficiency - accuracy of error estimation (fewer 'bad' steps)
- accuracy of solution - extending it further (larger steps)
- speed - reduce number of evaluations of derivative/field, e.g. reuse derivatives
- **interpolation**: obtain the state (x,p) at any intermediate point

New RK methods - Interpolation

- Selected RK methods offer capability of estimating any intermediate point given its 'distance' along the curve
  - One-time cost of a few extra field evaluations
- Reduced cost of evaluating intermediate points (vs new integration)
  - Enable faster location of intersection point with surface boundary
- Enabled using a new type of IntegrationDriver:
  - auto interpStepper= new G4DormandPrince745(equation, nvar);
  - auto * integrDriver= new G4InterpolationDriver(fMinStep, interpStepper, interpStepper->GetNumberOfVariables());

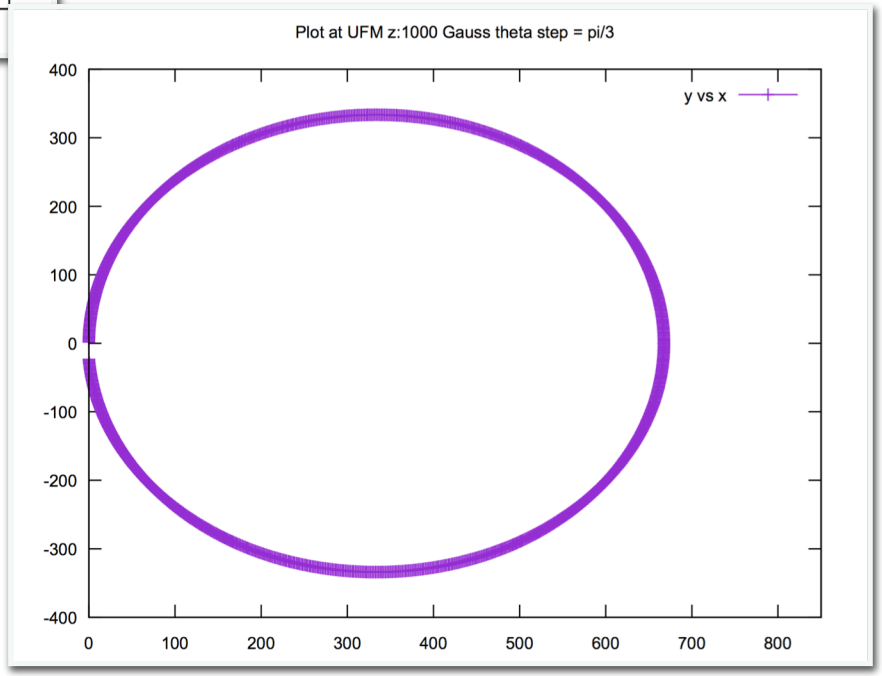




6 computed points in a circular trajectory

Calling 100 times between each pair of points, to give "Dense" output

Interpolated result



Using the latest developments – FSAL and Interpolation

These promising Runge-Kutta methods are currently optional, and can be tried out as follows:

- Interpolation via the G4InterpolationDriver class and a compatible stepper:

```
#include “G4InterpolationDriver.hh”
…
auto stepper= new G4DormandPrince745( equation, nvar );
auto interpolationDriver= new G4InterpolationDriver(fMinStep, stepper, nvar );
```

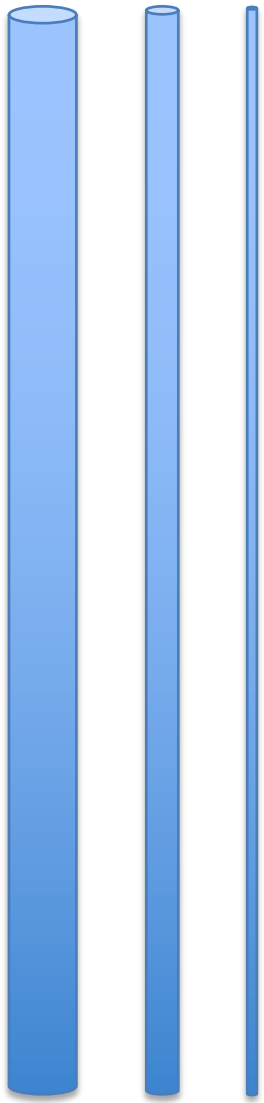
- The FSAL property of a stepper is available for a few selected steppers. To use it we also use an alternative ‘Driver’ class:

(Both thanks to the work of Dmitry Sorokin, GSoC 2016 & 2017)

```
#include “G4RK547FEq1.hh”
#include “G4IntegrationDriver.hh”
..
auto stepper1 = new G4RK547FEq1( equation );
auto fsalDriver = new G4FSALIntegrationDriver<G4RK547FEq1>( fMinStep, stepper1 );
```

Dealing with looping particles (1/3)

- In volumes with magnetic field and vacuum or gas as material some tracks can need a large (>10^4) number of integration steps
  - This can be a major sink of CPU time
- So within a physics step there is a limit of 1,000 integration steps (tunable.)
  - When a track reaches this limit, it is marked as 'looping'. It is now a candidate for being killed.
- Geant4 will kill particles found to be looping:
  - If $E < E_{warning}$ a track is killed immediately without warning
  - If $E_{warning} < E < E_{important}$ the track is killed, and a small warning is printed (to cout in Geant4 versions < 10.5)
  - If $E > E_{important}$ the track is given an extra number of chances (by default 10) before being killed.
- Their values can be changed using methods of G4Transportation. Default values are (chosen for collider HEP experiments):
  - $E_{warning}$ = 100 MeV
  - $E_{important}$ =250 MeV



Dealing with looping particles (2/3)

- These values can be changed using G4Transportation's methods:

```
SetThresholdWarningEnergy( G4double );
SetThresholdImportantEnergy( G4double );
SetThresholdTrials( G4int maxTrials );
```

First you must find the G4Transportation process

```
#include "G4Proton.hh"
#include "G4ParticleDefinition.hh"
#include "G4Transportation.hh"

G4ParticleDefinition particleDef= G4Proton::G4Proton();
G4VProcess procTr = particleDef->GetProcessManager()
                    ->GetProcess("Transportation");
G4Transportation* protonTransport =
   dynamic_cast<G4Transportation*>(procTr);
```

Then you can change its properties:

```
if( protonTransport )
  protonTransport->SetThresholdWarningEnergy(10.0*CLHEP::keV);
```

Dealing with looping particles (3/3)

- In Geant4 10.5 several changes were implemented:
  - only stable particles are killed
  - each particle with energy above the warning energy which is killed **generates a detailed warning** (using G4Exception) with location, volume, material, particle momentum and energy.
  - for the first 5 tracks killed a detailed description is printed that describes the criteria and parameters used to decide what tracks are killed, and guidance.
- Guidance regarding how to 'save' tracks:
  - by changing the values of thresholds or
  - by adopting different integration methods.

Takeaways

- Runge-Kutta (RK) integration is used to compute the motion of a charged track in a any type of field: magnetic, electric, combined EM, gravitational or a mix.
  - Many general steppers are available applicable to any equation / field
  - A few specialised steppers can be used only for pure magnetic fields.
- Default in G4 is the general purpose G4DormandPrinceRKF45 an embedded 4^{th}-5^{th} order RK stepper
  - It typically uses 6 field evaluates per field step & provides the end derivative .
- If the field has very rough or smooth, consider lower or higher order steppers
  - Expect same quality using fewer computing cycles,
  - Try a different stepper (or two) to see whether it improves CPU time.
- RK steppers with interpolation will reduce the number of field calls for each intersection boundary
  - Currently optional, with plans to introduce them as default in 2019.
- Different types of fields available, and user can create their own
  - A field must be accompanied by its corresponding equation of motion.