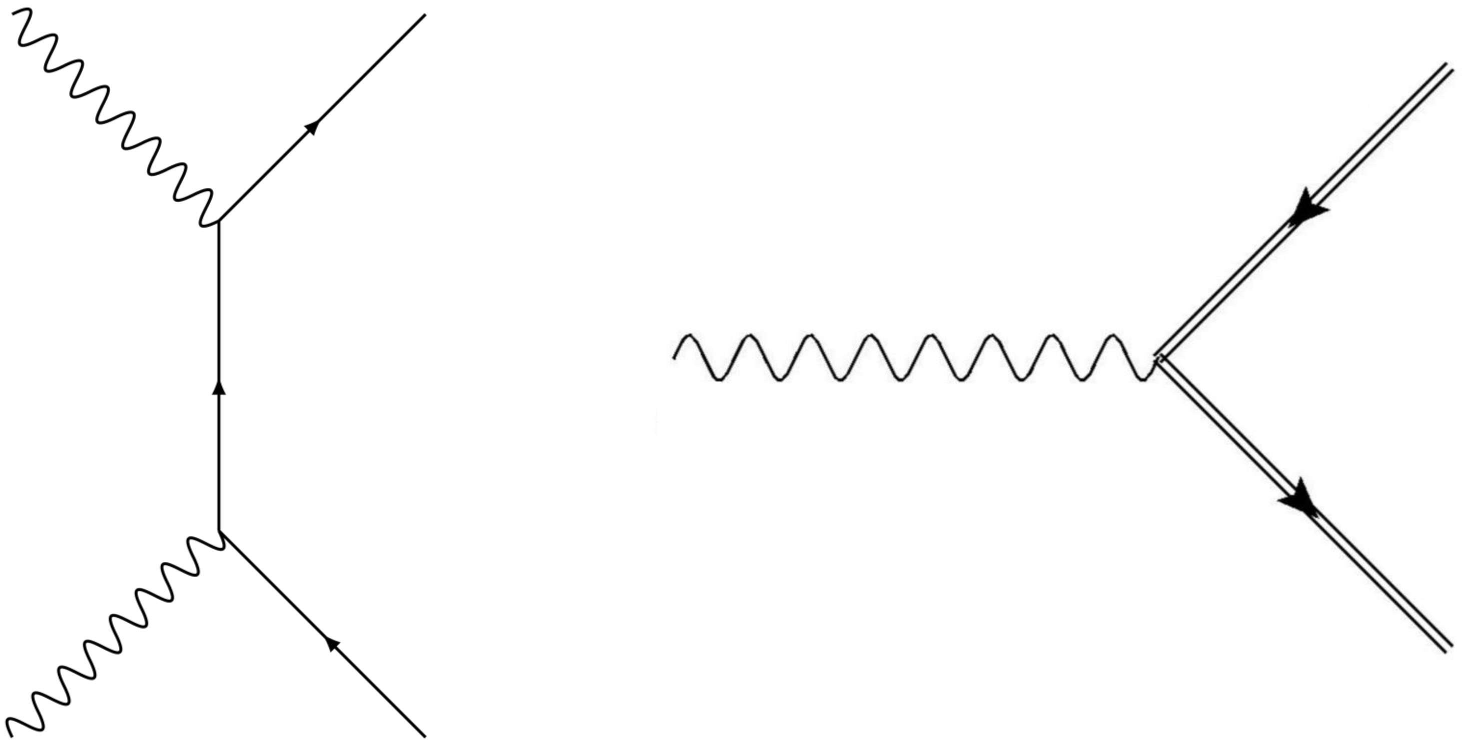
Numerical Modelling of Breit-Wheeler Detection Experiments

Robbie Watt

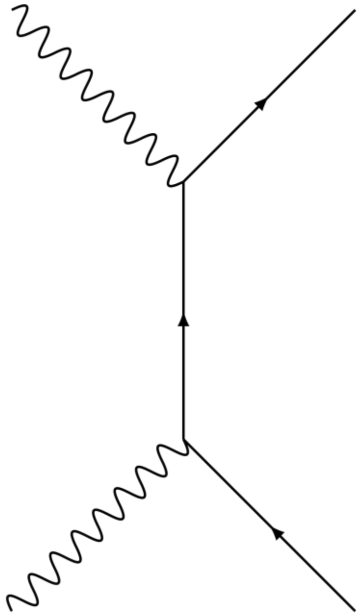


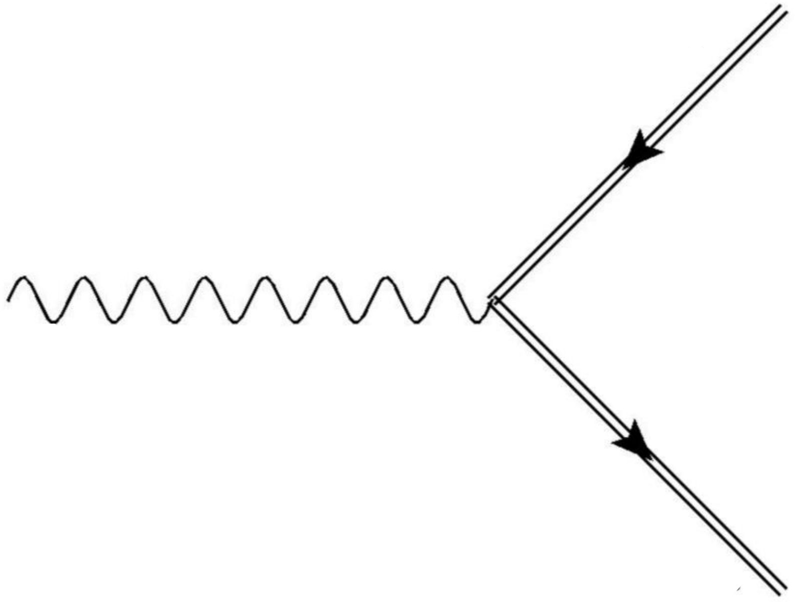
Imperial College London

Supervisors: Dr Stuart Mangles, Prof Steven Rose

Outline

- What is the Breit-Wheeler process?
- Why is it important to study?
- Previous Breit-Wheeler detection experiments.
- Developing linear and nonlinear Breit-Wheeler Geant4 modules.

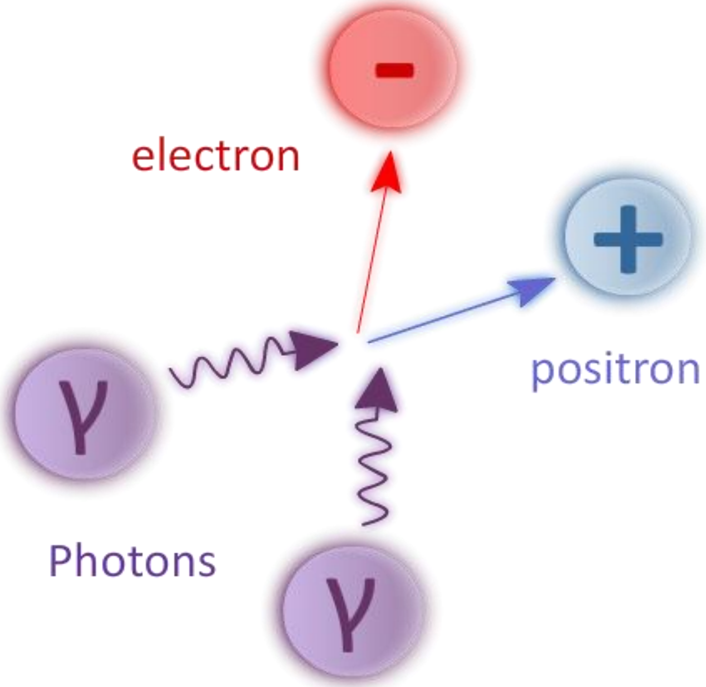




What is the Breit-Wheeler process?

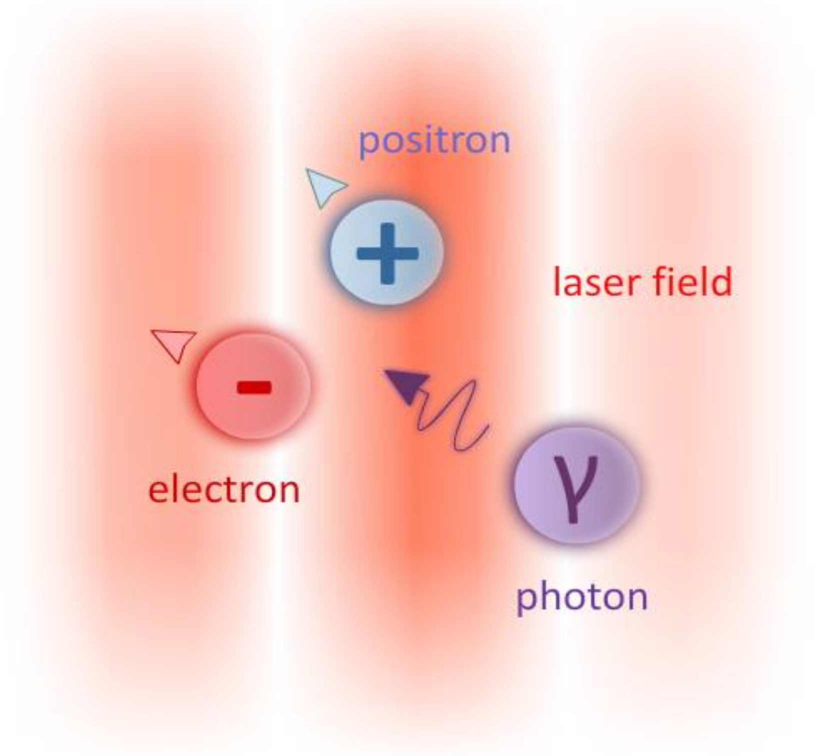
The Breit-Wheeler process is the annihilation of two or more photons to produce an electron-positron pair.

Linear Breit-Wheeler



Never observed in lab

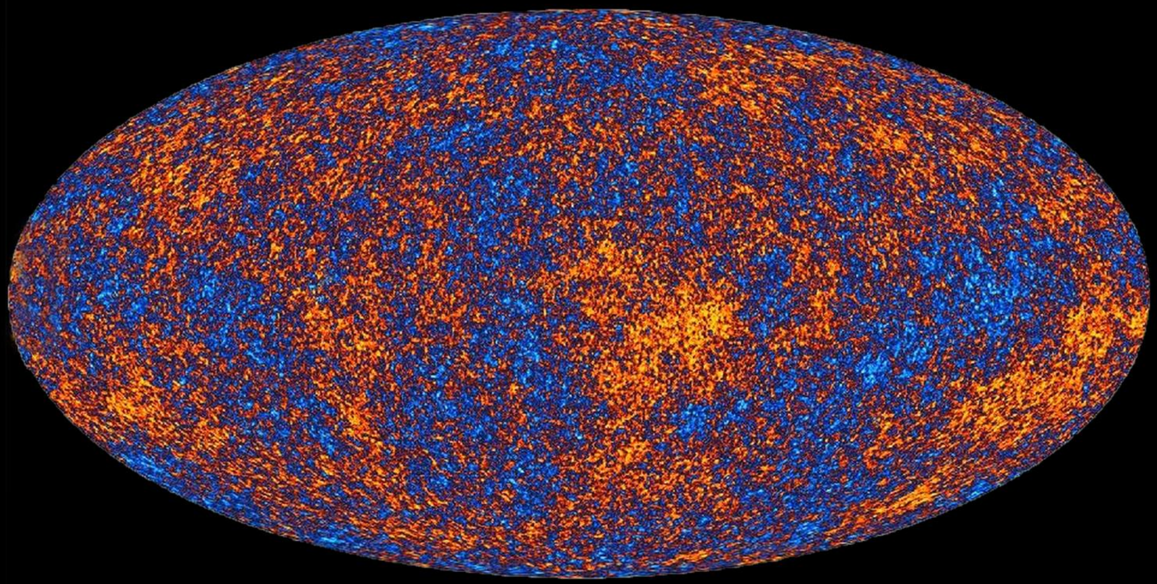
Nonlinear Breit-Wheeler



Observed in lab 1997

Where is the Breit-Wheeler process important?

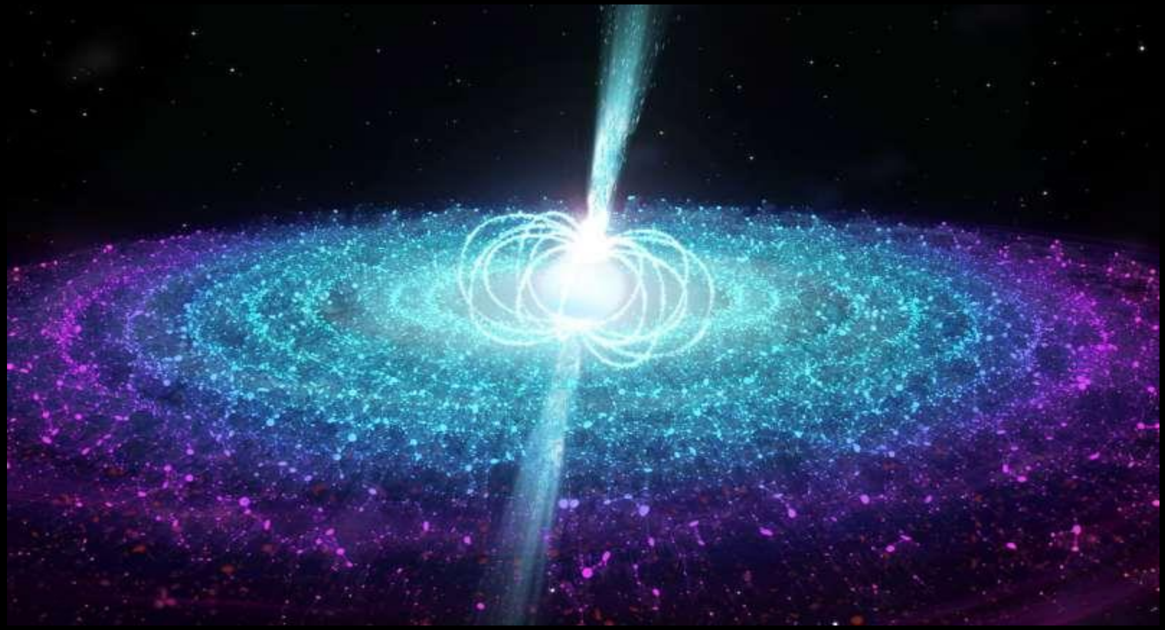
The cosmic gamma spectrum



The Universe is opaque to high energy gamma rays due to annihilation with the cosmic microwave background.

Pulsar Magnetosphere

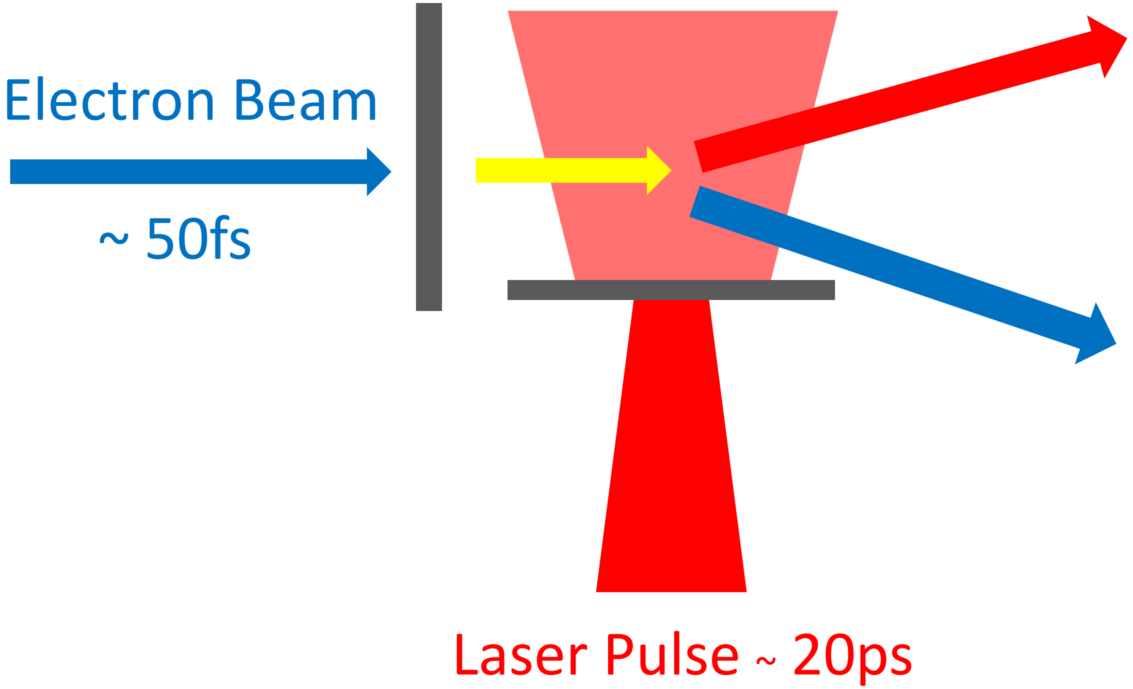
Energy is released from a pulsar magnetosphere through multiple Compton scattering and Breit-Wheeler events known as a QED cascade



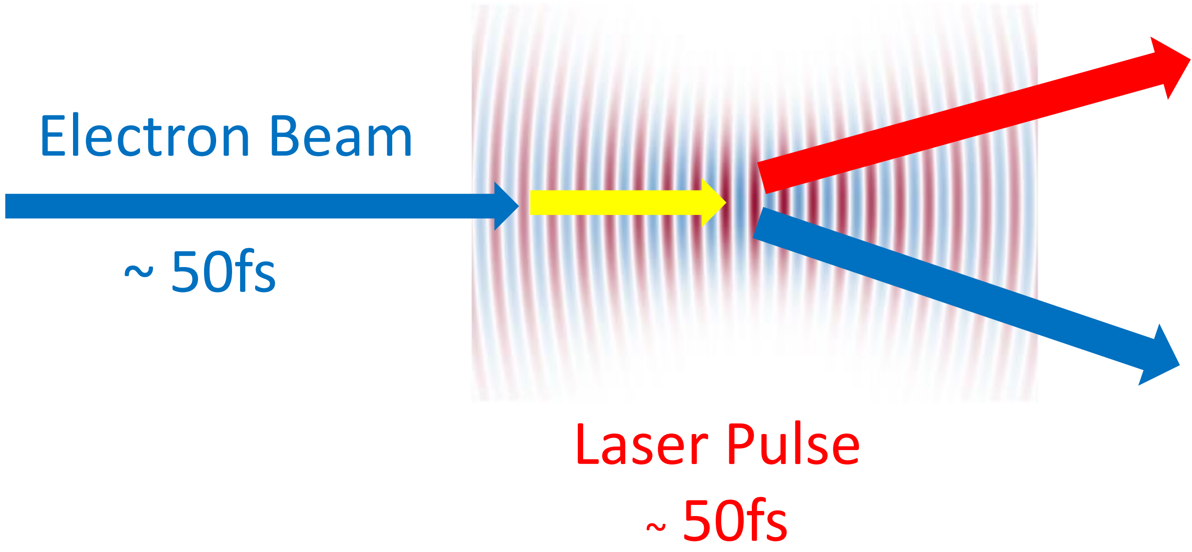
Breit-Wheeler Detection Experiments

We have carried out both linear Breit-Wheeler and nonlinear strong field QED cascade experiments using the Gemini laser facility.

Linear Breit-Wheeler

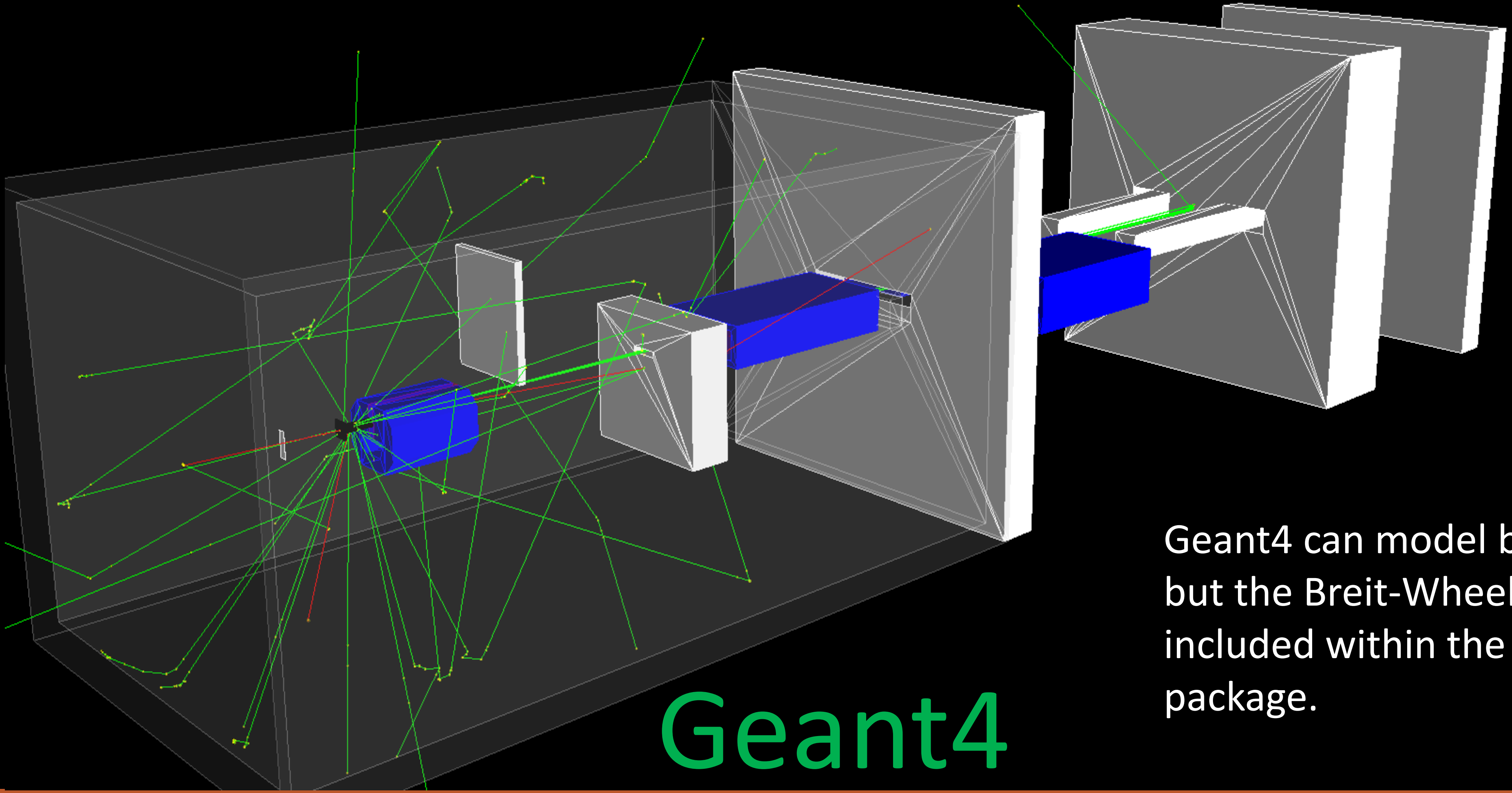


Strong Field QED Cascade



Modelling Signal-to-Noise

Breit-Wheeler experiments have low signal to noise ratios. Therefore, detailed numerical modelling is vital to increase chance of detection.



Geant4

Geant4 can model background noise but the Breit-Wheeler process is not included within the standard physics package.

Modelling the Linear Breit-Wheeler process in Geant4

The linear Breit-Wheeler process is modelled as a particle matter interaction by treating one photon source as a static field.

Step 1 Mean free path calculation

$$\lambda = \left(\frac{E}{m^2\,c^4}\right)^2 \left[\int_0^{2\pi} d\phi \int_{m^2c^4/E}^{\infty} d\epsilon, \int_1^{\epsilon E/m^2c^4} ds \; \epsilon^{-2}\, n(\phi,\epsilon,s)\,\sigma(s)s\right]^{-1}$$

$$\sigma = \frac{\pi}{2} r_e^2 (1-\beta^2)\left[-2\beta(2-\beta^2) + (3-\beta^4)\ln\frac{1+\beta}{1-\beta}\right]$$

E Dynamic photon energy

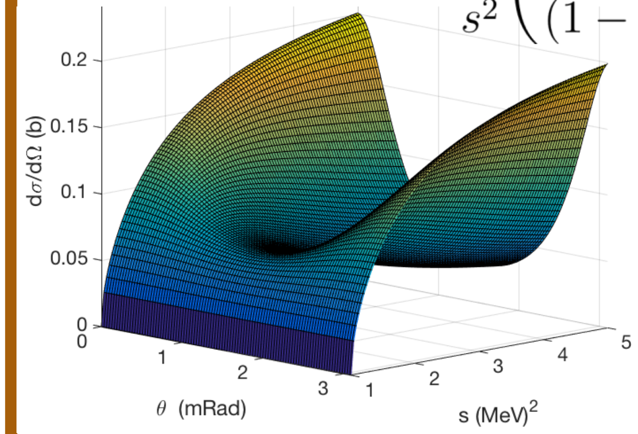
ϵ Photon field energy

s Centre of mass energy squared

Step 2 e+/e- properties calculation

- Sample interacting photon properties
- Sample scattering angle from differential cross-section

$$\frac{d\sigma_{BW}}{d\Omega} = \frac{r_e^2\beta}{s}\left[-1 + \frac{3-\beta^4}{2}\left(\frac{1}{1-\beta\cos\theta} + \frac{1}{1+\beta\cos\theta}\right) - \frac{2}{s^2}\left(\frac{1}{(1-\beta\cos\theta)^2} + \frac{1}{(1+\beta\cos\theta)^2}\right)\right]$$



Remove dynamic photon from simulation and replace with e^+/e^- pair.

Increasing Speed with Gaussian Process Regression

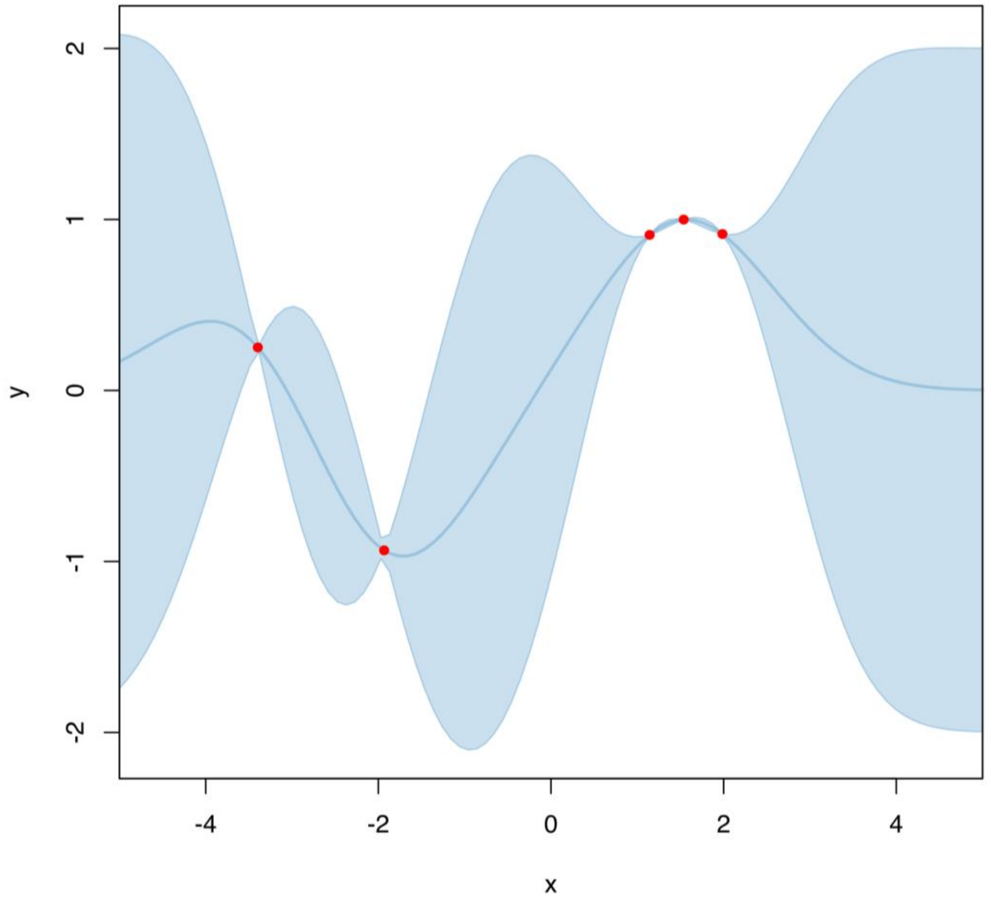
The calculation of λ is computationally expensive and must be calculated for every dynamic photon. Is there a faster way?

$$\lambda = f(E, \theta, \phi)$$

f is an expensive function mapping E, ϑ and φ to λ and can be replaced with Gaussian process regression.

A Gaussian process can be used for non-parametric Bayesian regression, giving a smooth function through the data with uncertainty quantification.

Gaussian Process Regression



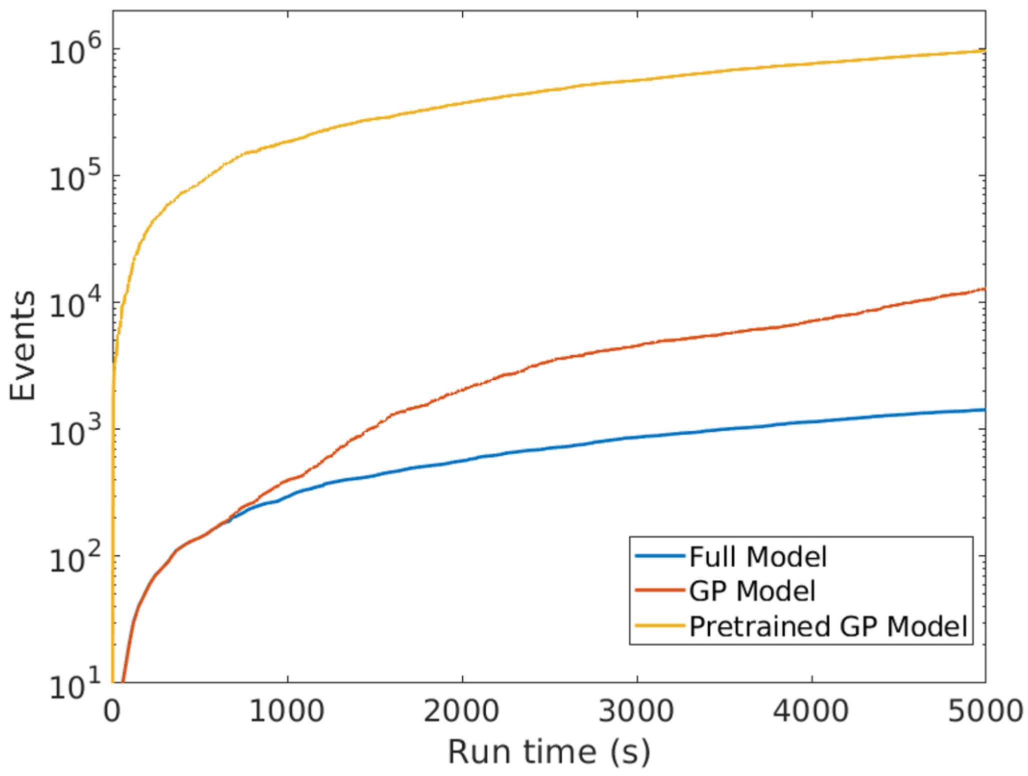
Increasing Speed with Gaussian Process Regression

Gaussian process implementation:

1. Data accumulation stage, MFP is solved in full and results are saved.
2. Training stage, after n events the GP regression model is trained.
3. Acceleration stage, GP regression model variance (v) is calculated. If v is less than predefined value, MFP is calculated from GP regression. If v is greater than predefined value, MFP is calculated in full and result added to training data.

As the simulation progresses, more data is added to the training set, reducing the variance and increasing the speed.

Simulation Event Calculation Rate



Modelling Strong Field QED Interactions

Nonlinear processes cannot be treated as particle matter interaction. Instead, a separate nonlinear QED Monte Carlo code has been developed and integrated into Geant4.

Code based on:

Monte Carlo calculations of pair production in high-intensity laser–plasma interactions,
R. Duclous, J. G. Kirk and A. R. Bell

Code summary:

Leptons / photons are tracked through high intensity field, generating more leptons / photons through nonlinear Compton scattering and nonlinear Breit-Wheeler process.

Modelling Strong Field QED Interactions

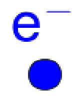
Nonlinear processes cannot be treated as particle matter interaction. Instead, a separate nonlinear QED Monte Carlo code has been developed and integrated into Geant4.

Code based on:

Monte Carlo calculations of pair production in high-intensity laser–plasma interactions,
R. Duclous, J. G. Kirk and A. R. Bell

Code summary:

Leptons / photons are tracked through high intensity field, generating more leptons / photons through nonlinear Compton scattering and nonlinear Breit-Wheeler process.



Nonlinear Monte Carlo code is computationally expensive. Is there a faster alternative?

Increasing Model Efficiency

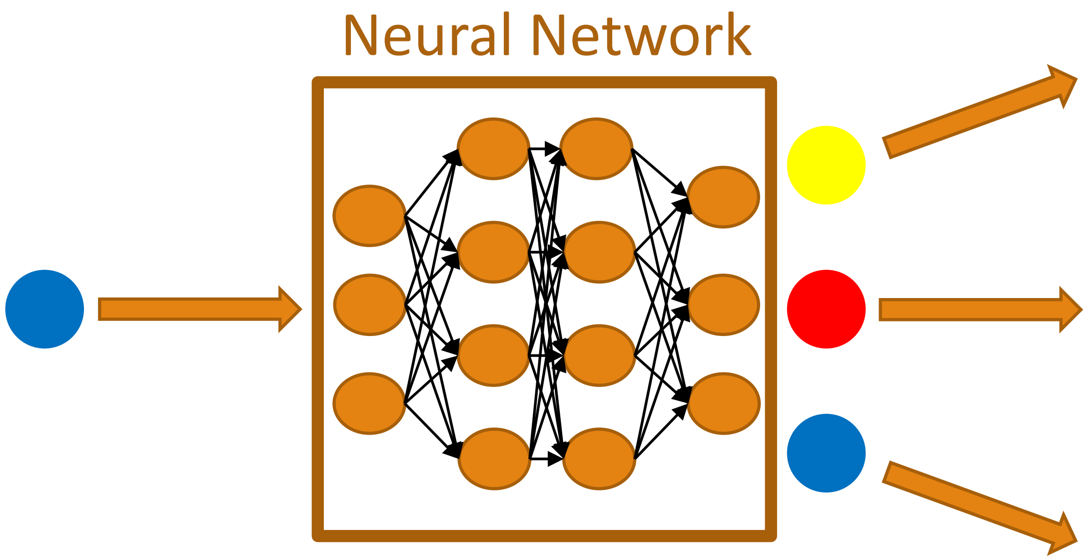
Fully solving strong field QED is computationally expensive. Is there a faster alternative?

Input:

- Energy
- Momentum
- Position

Outputs:

- Energy
- Momentum
- Position



Input:

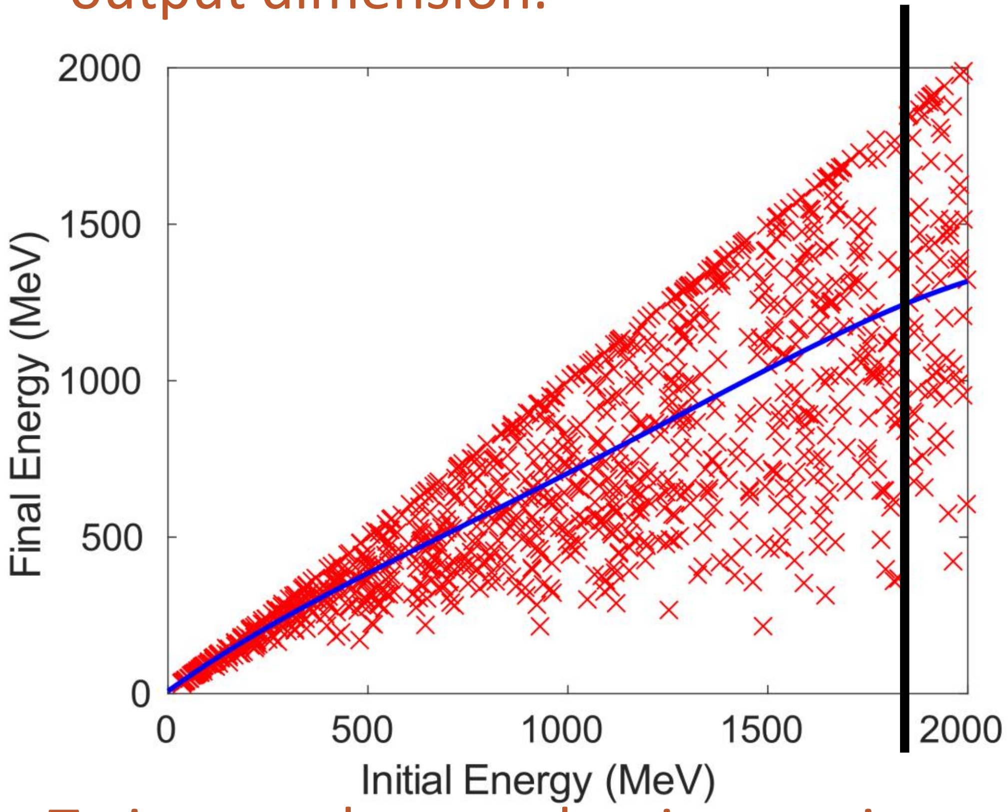
- Energy
- Momentum
- Position

Outputs:

- Energy
- Momentum
- Position

Regression With Neural Network

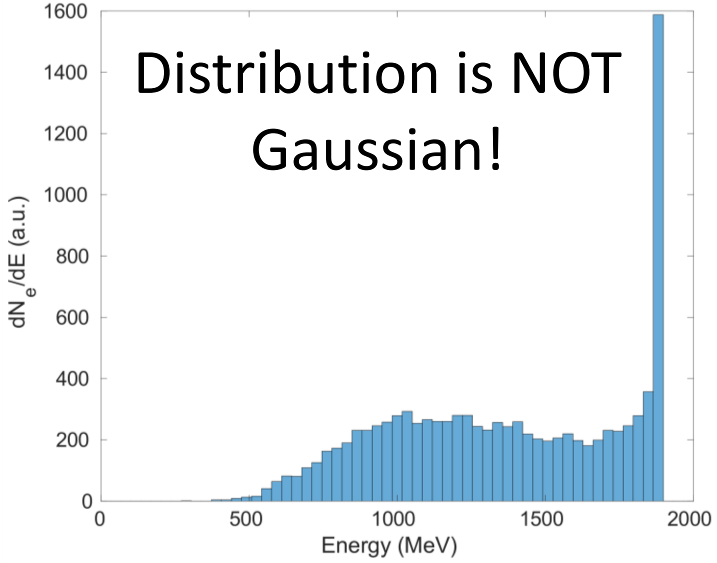
To explore how the NN is used, lets examine some simulation data reduced to one input and one output dimension.



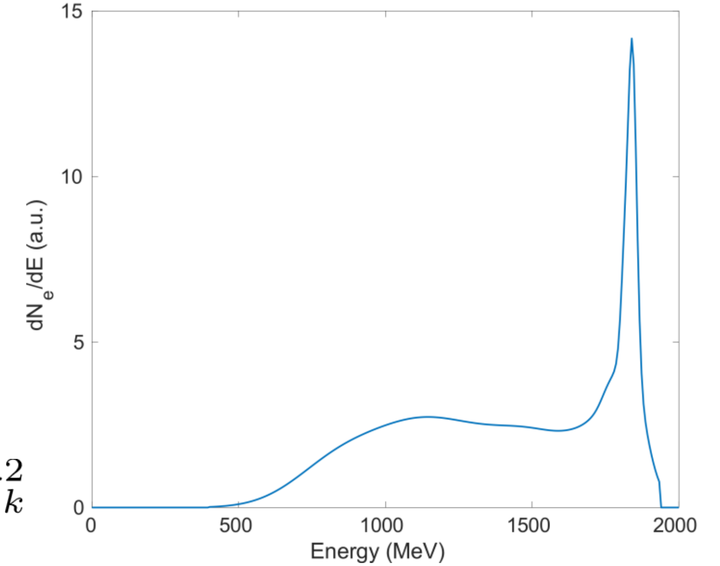
Lineout

$$P(E_F/E_I = 1800\,\text{MeV})$$

Simulation Data



NN Output Density



We can learn a Gaussian mixture model to approximate any distribution.

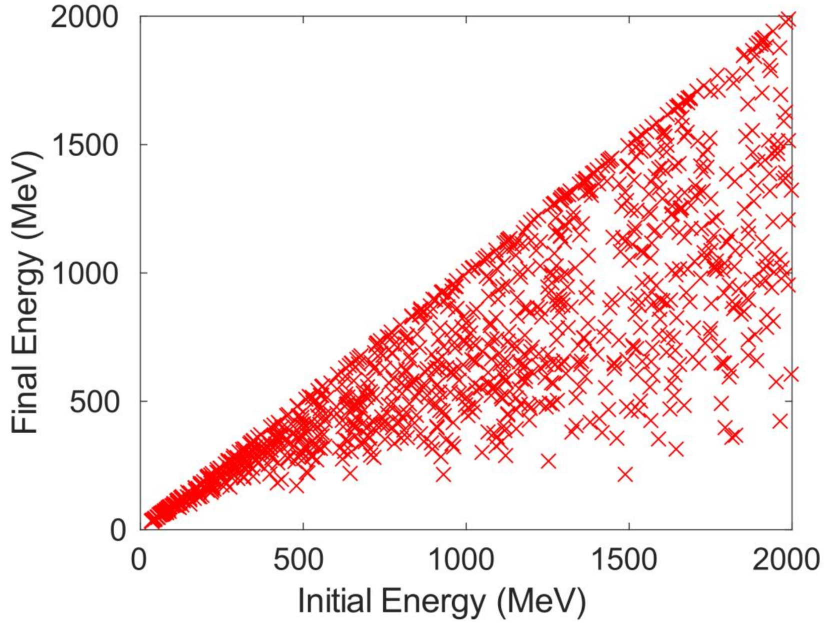
Train neural network using maximum likelihood: $\widehat{\theta} \in \{\arg\max_{\theta \in \Theta} \mathcal{L}(\theta\,; x)\}$

$$P(E_F/E_I) = \Sigma_k\, a_k\, e^{(\mu_k - x)^2/\sigma_k^2}$$

Generating New Data From NN

For new input electrons we can use the neural network to predict the output density, then sample from to generate new data.

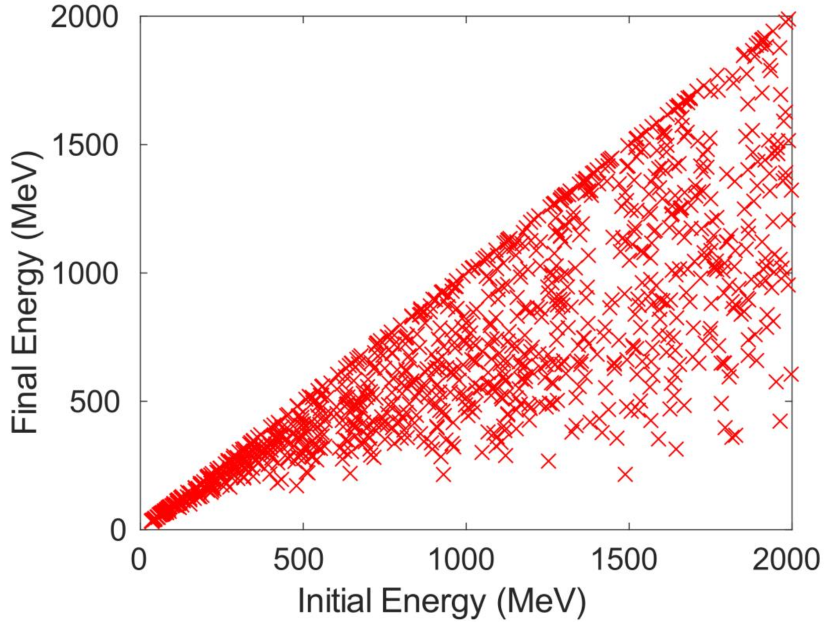
Full Model Data



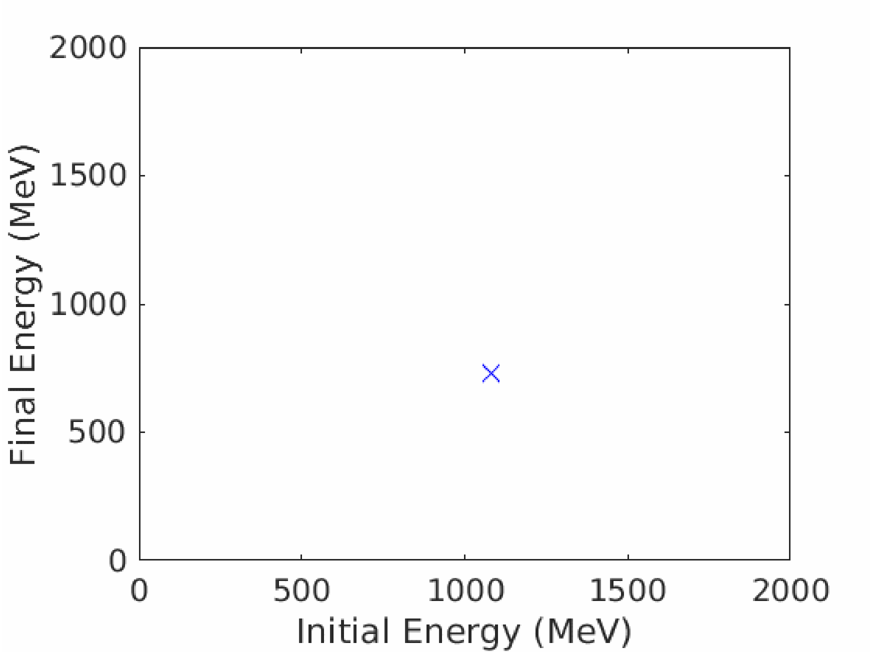
Generating New Data From NN

For new input electrons we can use the neural network to predict the output density, then sample from to generate new data.

Full Model Data



NN Sampled Data



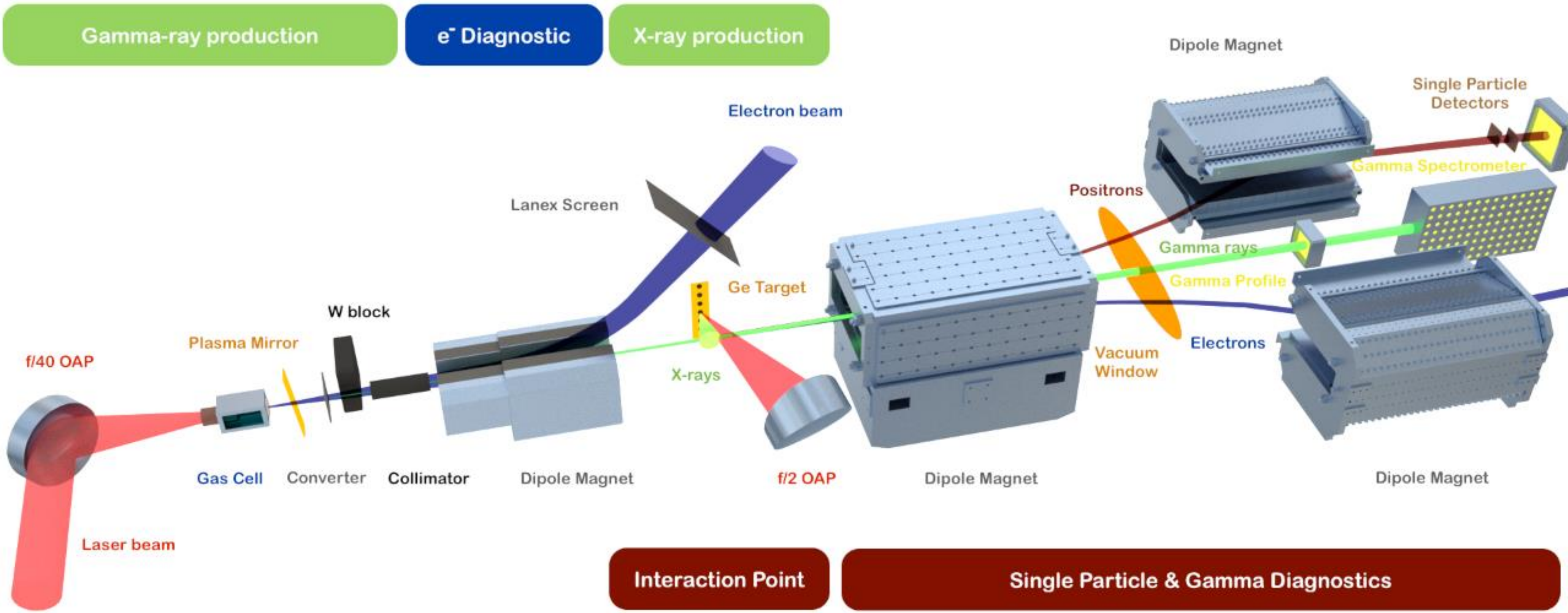
Sampling from NN gives increase in event rate by factor of ~20.

	Full Model	NN Model
Event Rate	160 s^{-1}	3100 s^{-1}

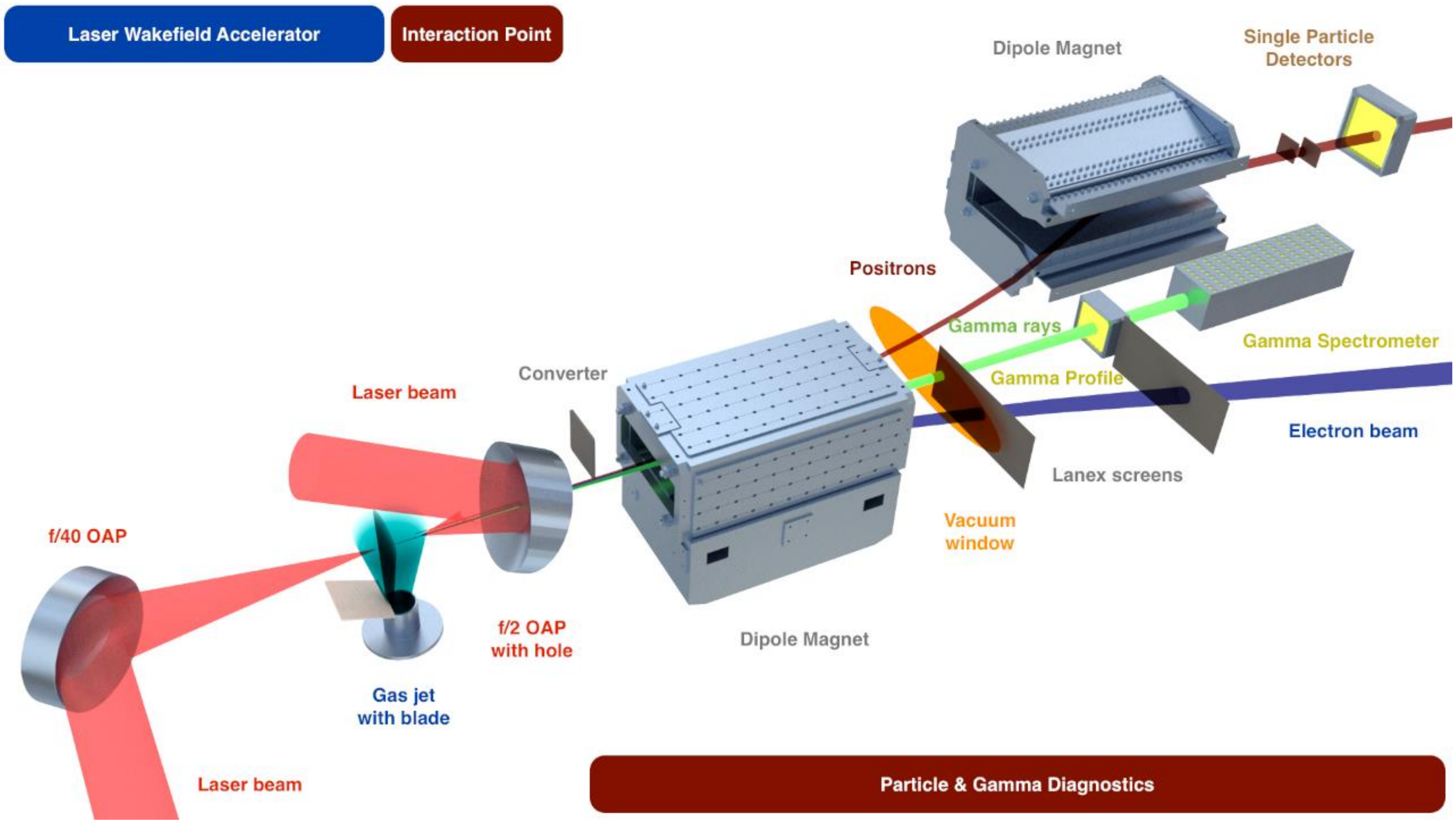
Summary

- The Breit-Wheeler process is the annihilation of photons to produce and electron positron pair and is important in many astrophysical environments.
- We have performed Breit-Wheeler detection experiment and analysis is still ongoing.
- I have developed the capability to perform signal to noise ratio calculations of Breit-Wheeler detection experiments within a single framework.
- Machine learning algorithms can be used to greatly increase the efficiency of these calculations.

Linear Breit-Wheeler Detection Experiments



Nonlinear Breit-Wheeler Detection Experiments



Modelling Strong Field QED Interactions

Strong field processes cannot be treated as particle matter interaction. Instead, a separate nonlinear QED Monte Carlo code has been developed and integrated into Geant4.

Algorithm summary:

- Photons / leptons are tracked through laser field.
- Particles are assigned initial optical depth (τ) and is updated by solving:

Nonlinear Compton

$$\frac{d\tau_\gamma}{dt} = \int_0^{\eta/2} \frac{d^2 N_C}{d\chi dt} d\chi$$

Nonlinear BW

$$\frac{d\tau_e}{dt} = \int_0^{\chi} \frac{d^2 N_{BW}}{d\eta\, dt} d\eta$$

- When τ=0, particle interacts (nonlieanr BW / nonlinear Compton) and new photon / e^+e^- pair are added to simulation.

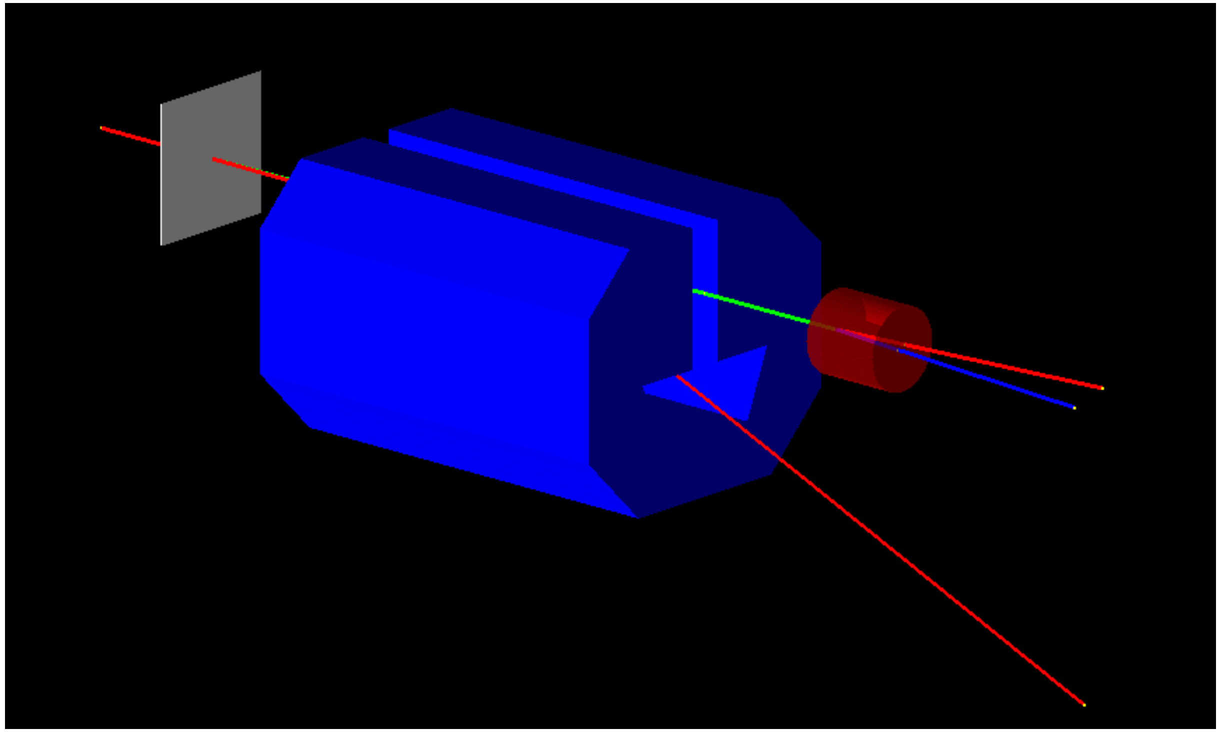
Code based on:

Monte Carlo calculations of pair production in high-intensity laser–plasma interactions, R. Duclous, J. G. Kirk and A. R. Bell

Linear Breit-Wheeler Module Demonstration

We can now use the Breit-Wheeler module to design Experiments.

Visualisation of Geant4 Breit-Wheeler Experiment



Positron Energy / Angular Spectrum