Using the IBEX Paul trap to test nonlinear integrable optics

Lucy Martin

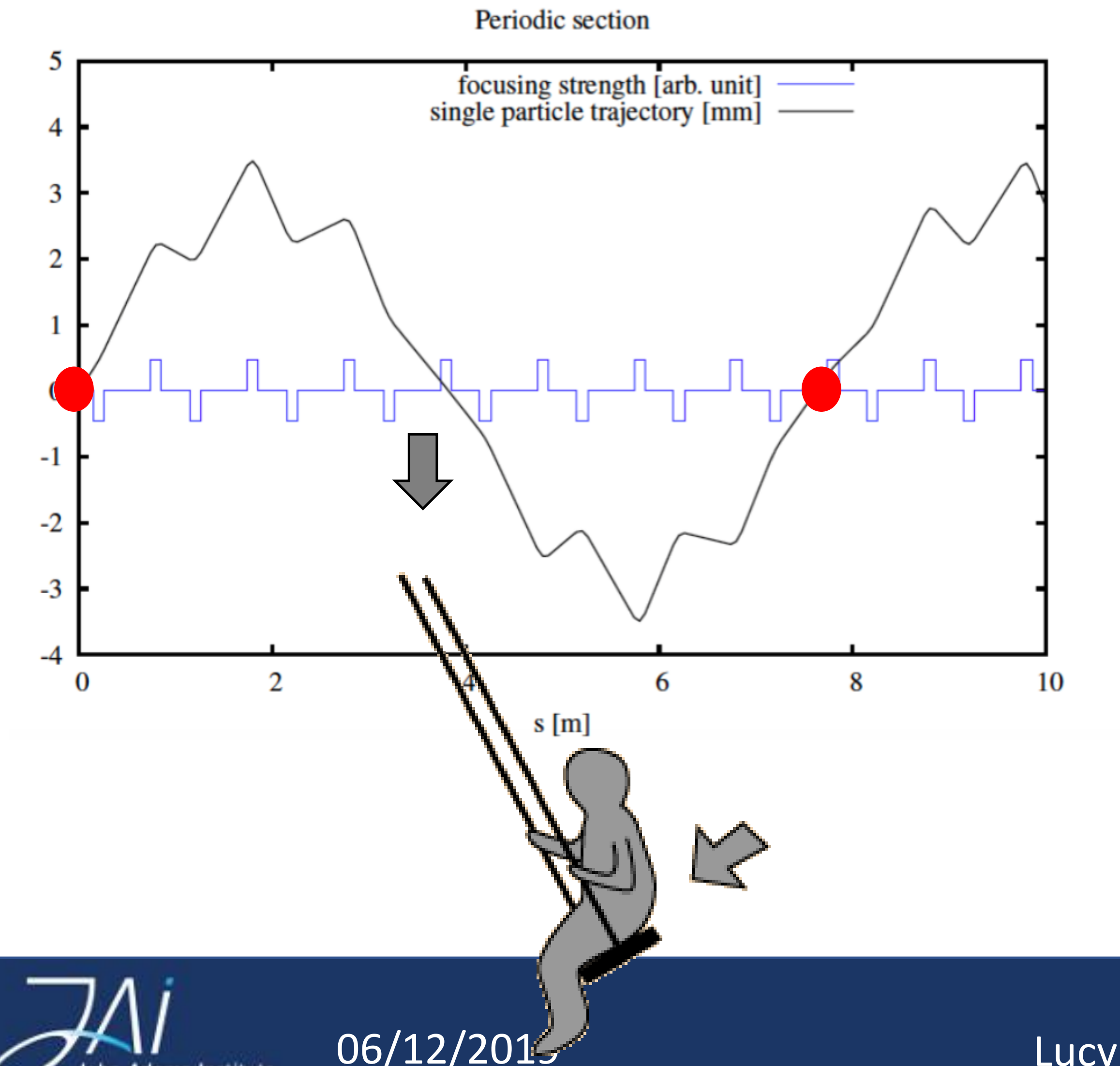
06/12/19

Introduction

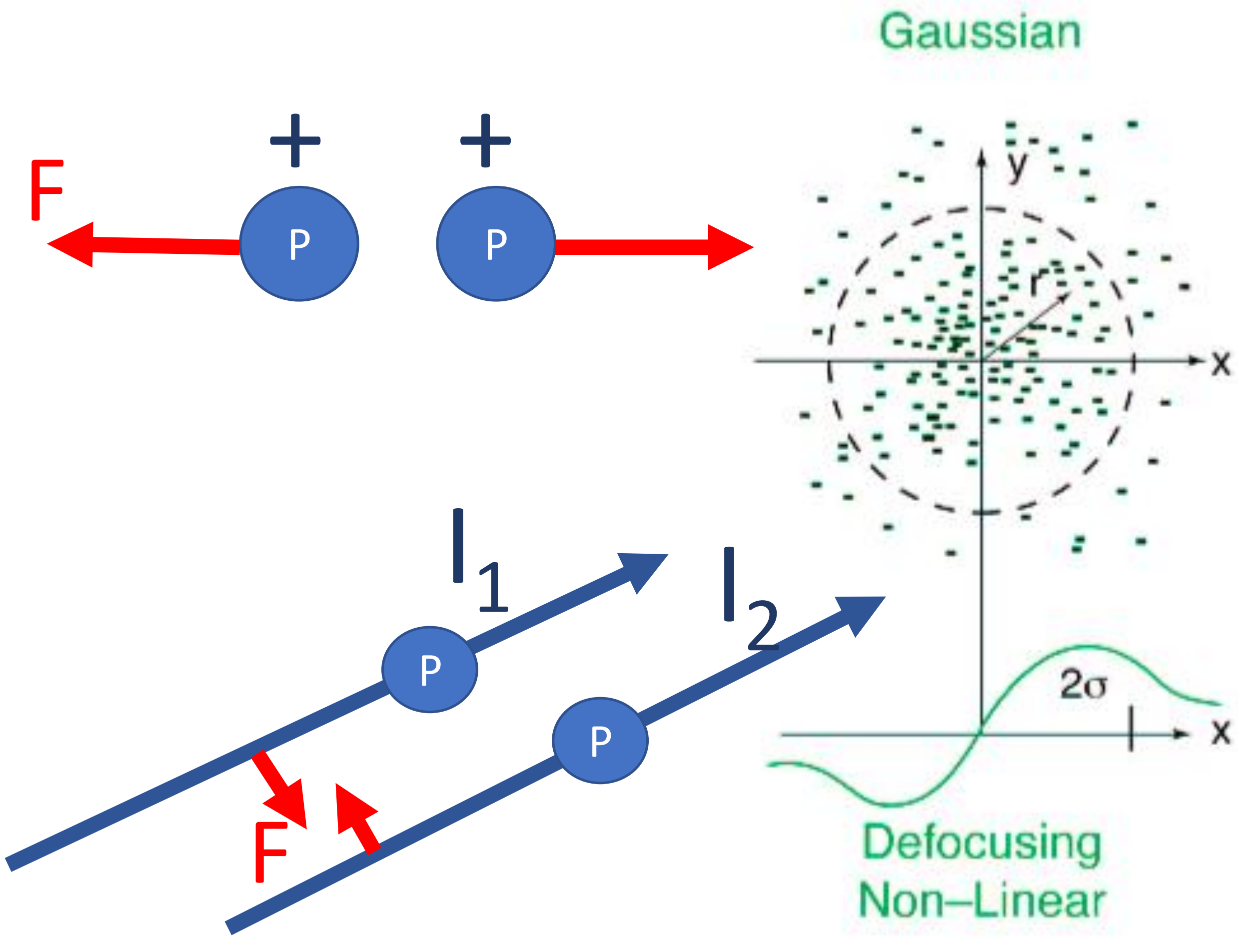
- How is a LPT useful for accelerator physics?
- What is a linear Paul trap (LPT) and how do they work?
- Which accelerator physics questions can we answer with a Paul trap?

- Recap of tune, resonance and intense beams
- Nonlinear integrable optics (NIO) - an interesting idea for high intensity
- Can it be tested on a Paul trap?
- Experimental progress towards NIO
- Future work

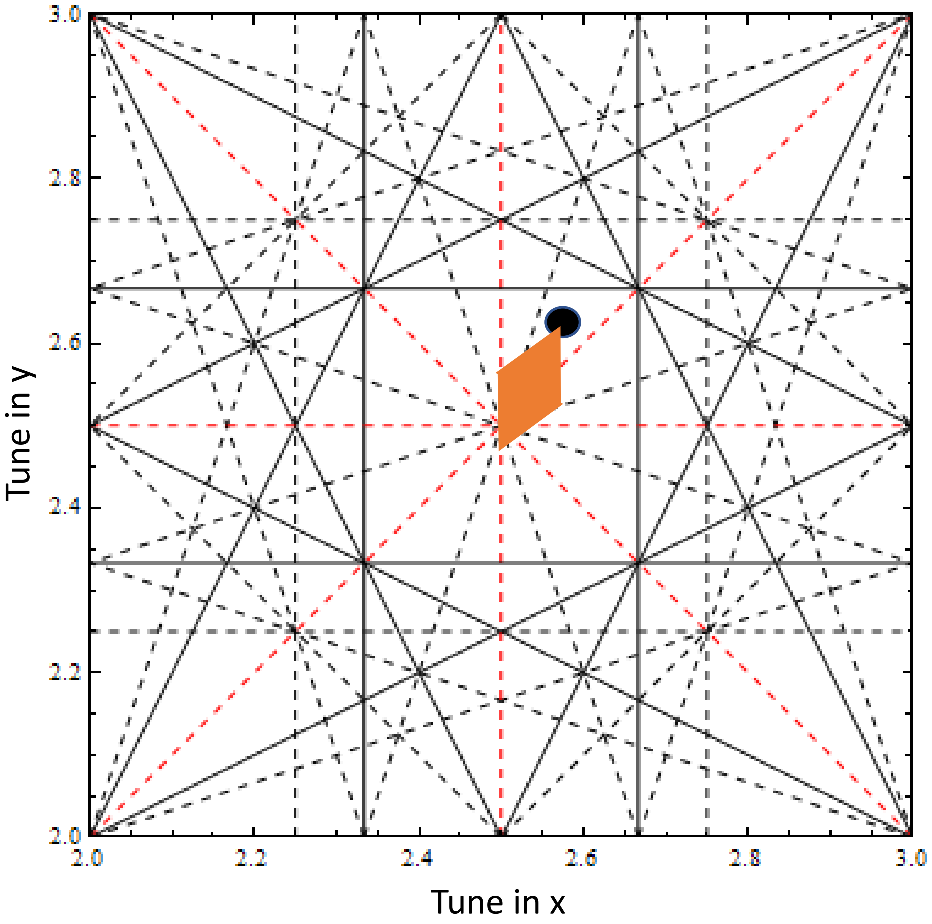
Tune and resonance



High intensity



Why is achieving high intensity difficult?

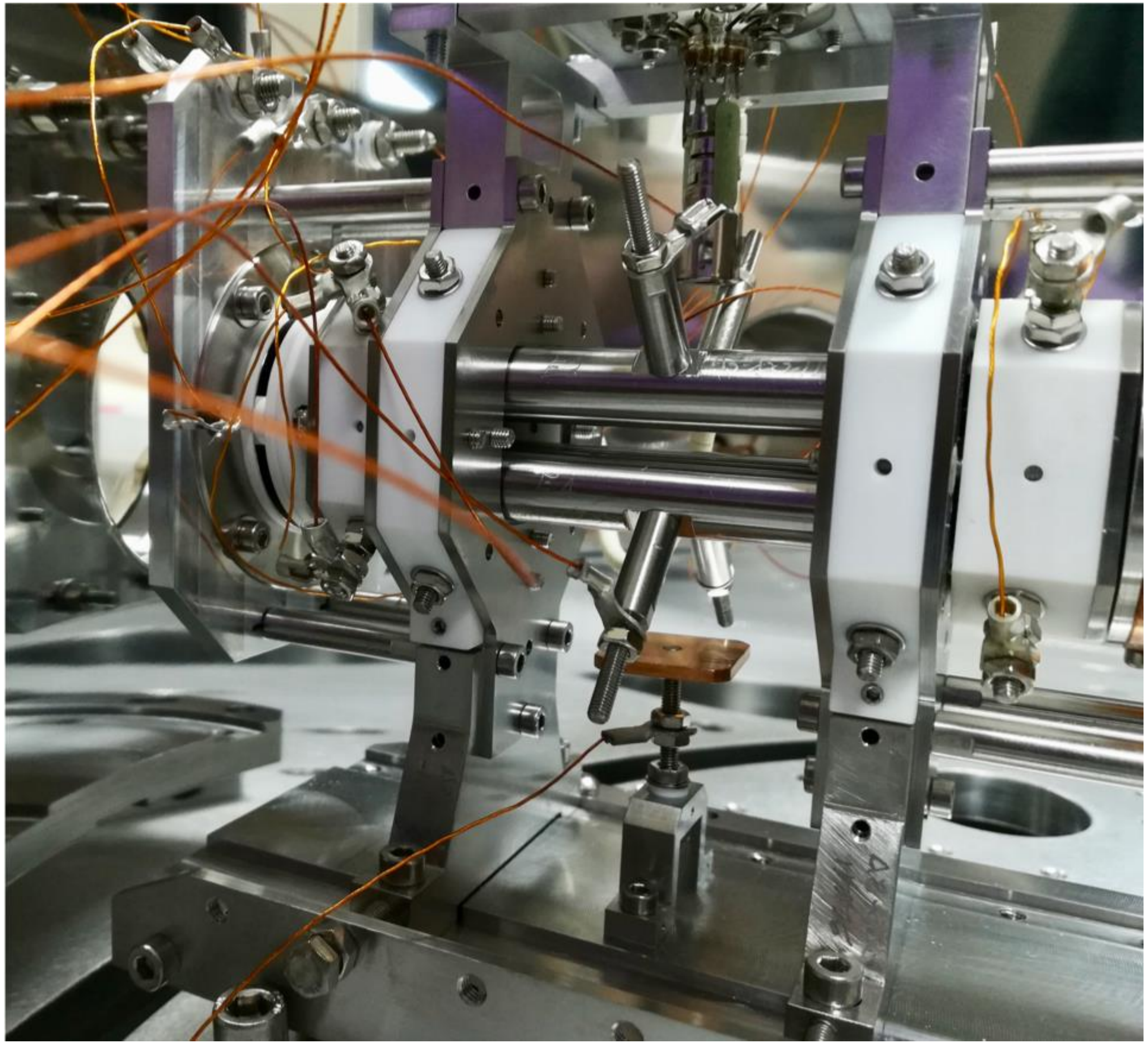


Why study these effects?

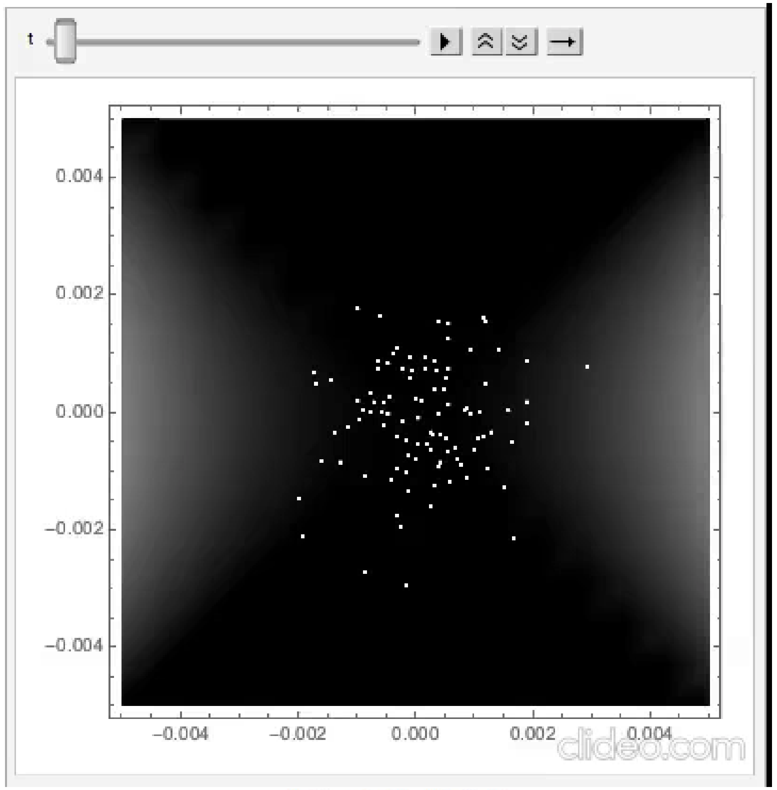
Why are they hard to understand?

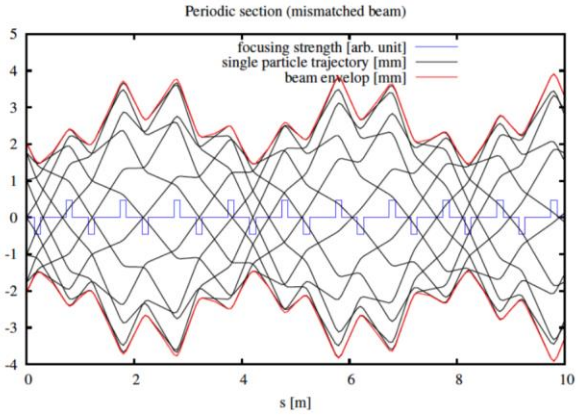
Why not just use simulation?

Why not study resonances with an accelerator?



The intense Beam Experiment (IBEX) at the Rutherford Appleton Lab, Oxfordshire





Motion in a Paul trap

- Hamiltonian of a Paul trap :

$$H_{paul} = \frac{(p_x^2 + p_y^2)}{2} + \frac{1}{2}K_p(\tau)(x^2 - y^2) + \frac{q}{mc^2}(\phi_{sc})$$

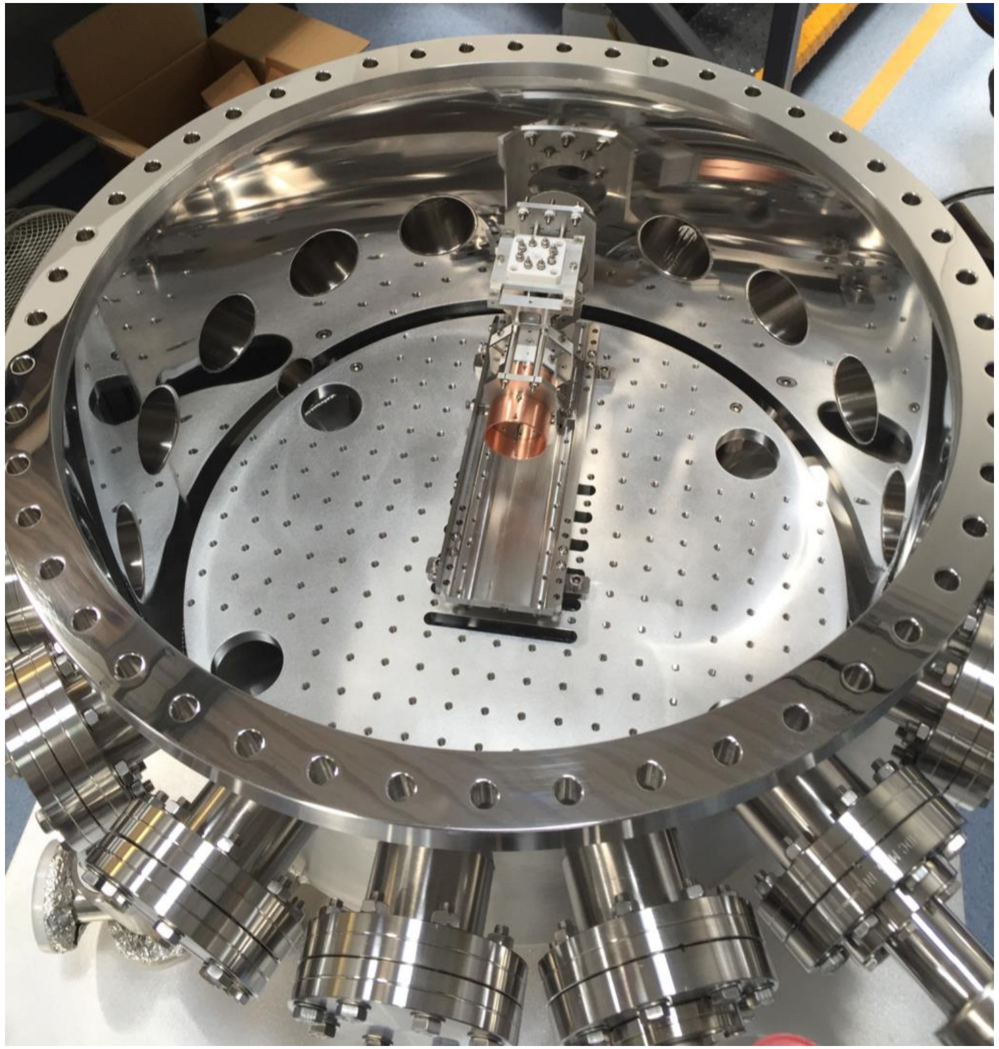
where $K_p(\tau) = \dfrac{2qV_Q(\tau)}{mc^2 r_0^2}$

- Hamiltonian of a conventional accelerator:

$$H_{beam} = \frac{(p_x^2 + p_y^2)}{2} + \frac{1}{2}K(s)(x^2 - y^2) + \frac{q}{p_0\beta_0 c\gamma_0^2}\phi$$

where $K(s) = \dfrac{-q}{p_0}\dfrac{dB_z}{dx} = -\dfrac{-1}{B\rho}\dfrac{dB_z}{dx}$

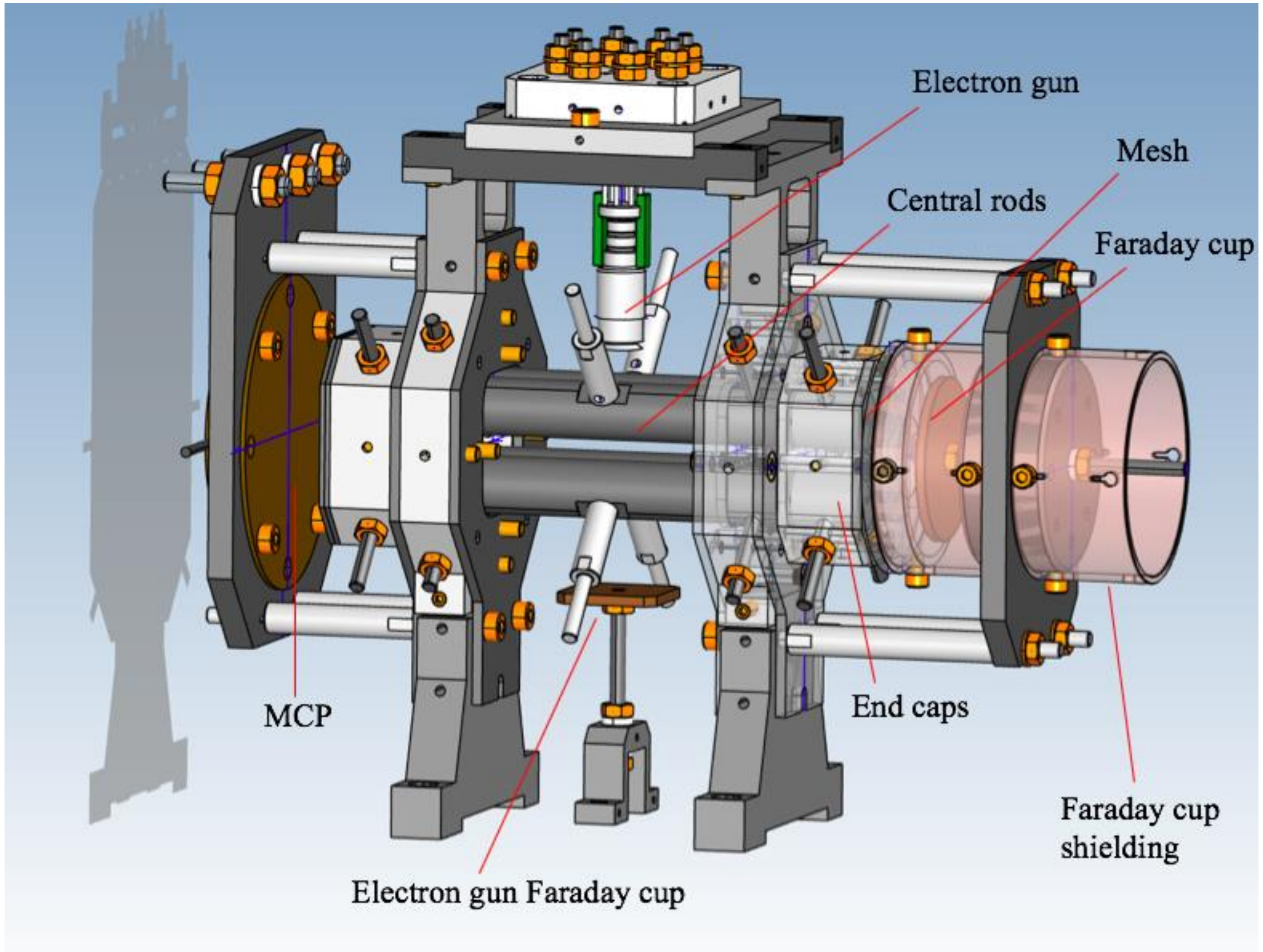
IBEX and SPOD: Linear Paul Traps



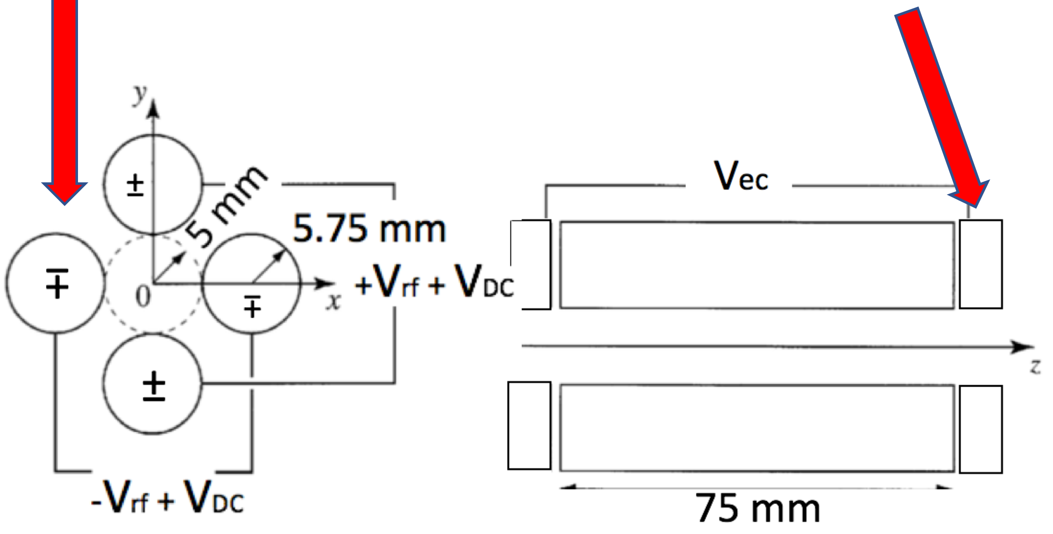
IBEX at the Rutherford Appleton Lab, UK
S-POD at Hiroshima University, Japan
PTSX at Princeton Plasma Physics lab, US

- What are the advantages of a LPT?
- What are the limitations?
  - No longitudinal effects = coasting beam

Linear Paul Trap



1. Argon gas introduced to vessel at ~10^{-7} mbar
2. Electron gun ionises Ar in trapping region
3. Ions confined transversely via 4 cylindrical rods
4. Ions confined longitudinally with end caps



What can we discover using IBEX?

1. We want to know the location of (and understand!) dangerous resonances to help with the understanding and operation of current machines

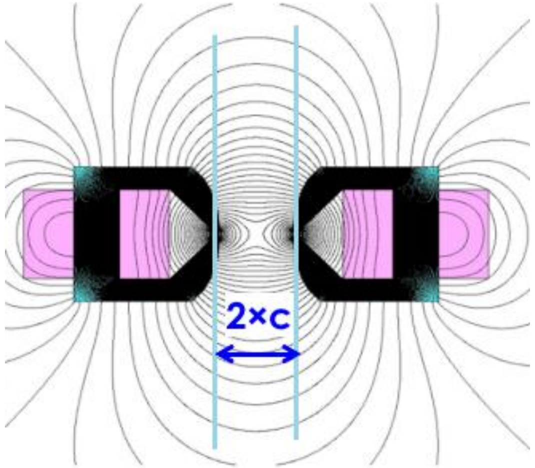
2. We want to investigate novel schemes for creating high intensity beams

Future accelerators (Nonlinear Integrable Optics)

- In linear accelerators the motion is integrable – it is known to be bounded
- This is exactly what we want, the beam won't be lost!
- Susceptible to resonances
- Realistically an accelerator can never be totally linear (errors + space charge)
- Nonlinearities -> no longer integrable due to coupling between x and y

- Require integrable system where small perturbations are allowed
- At Fermilab they found such a system (Danilov & Nagaitsev 2010)
- Unfortunately it requires a complicated potential

$$U(x,y) \approx \frac{t}{c^2}\,\mathrm{Im}\left((x+iy)^2 + \boxed{\frac{2}{3c^2}(x+iy)^4} + \frac{8}{15c^4}(x+iy)^6 + \frac{16}{35c^6}(x+iy)^8 + \ldots\right)$$



Non-linear integrable optics

- Quasi-integrable version involves only octupoles
- Octupole must vary in strength proportional to $1/\beta^3$

$$V(x,y,s) = \frac{k}{\beta(s)^3}\left(\frac{x^4}{4} + \frac{y^4}{4} - \frac{3x^2y^2}{2}\right)$$

- Can’t just have an octupole, need linear focusing
- Requires round beams and $n\pi$ phase advance
  - This is called a “T-insert”

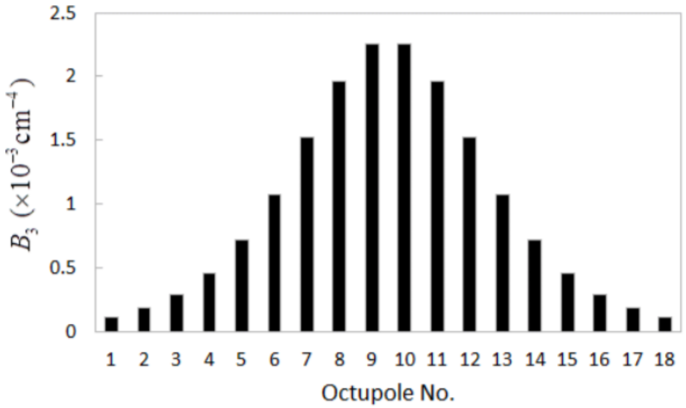


Image from [12]

To test in a Paul trap we require 2 things:

1. To be able to create a good enough T-insert with the Paul trap
2. Create correct octupole potential independent of linear focusing

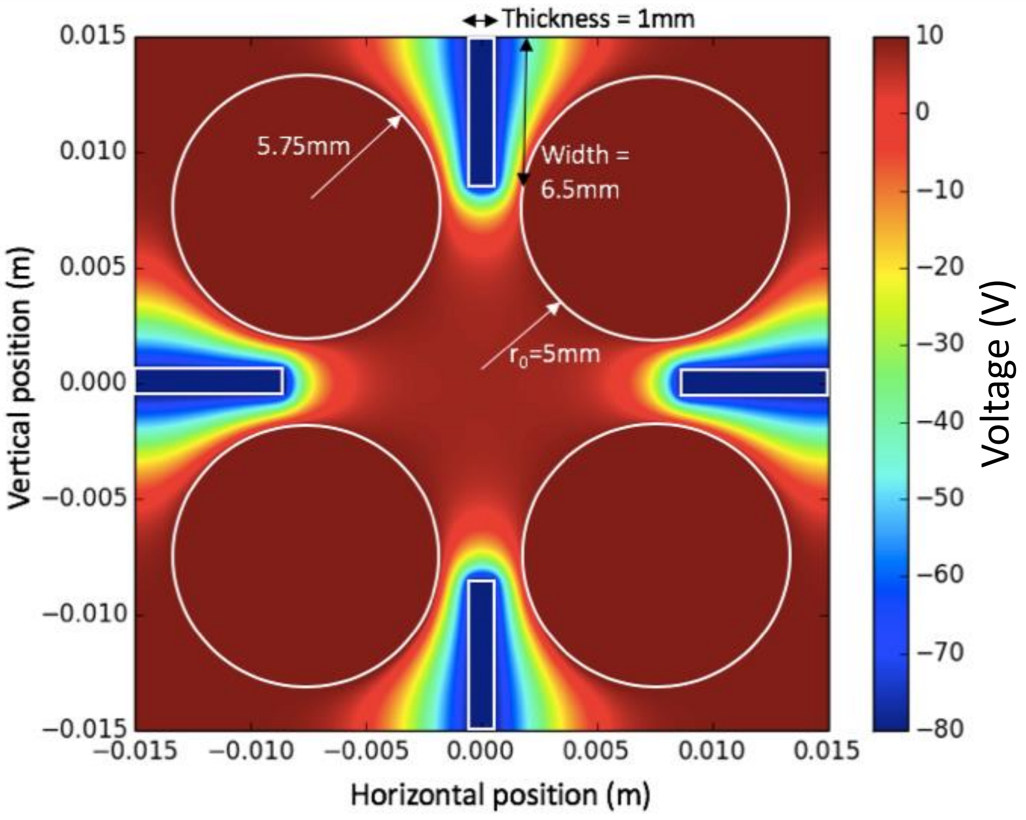
Why bother testing NIO in a Paul trap?

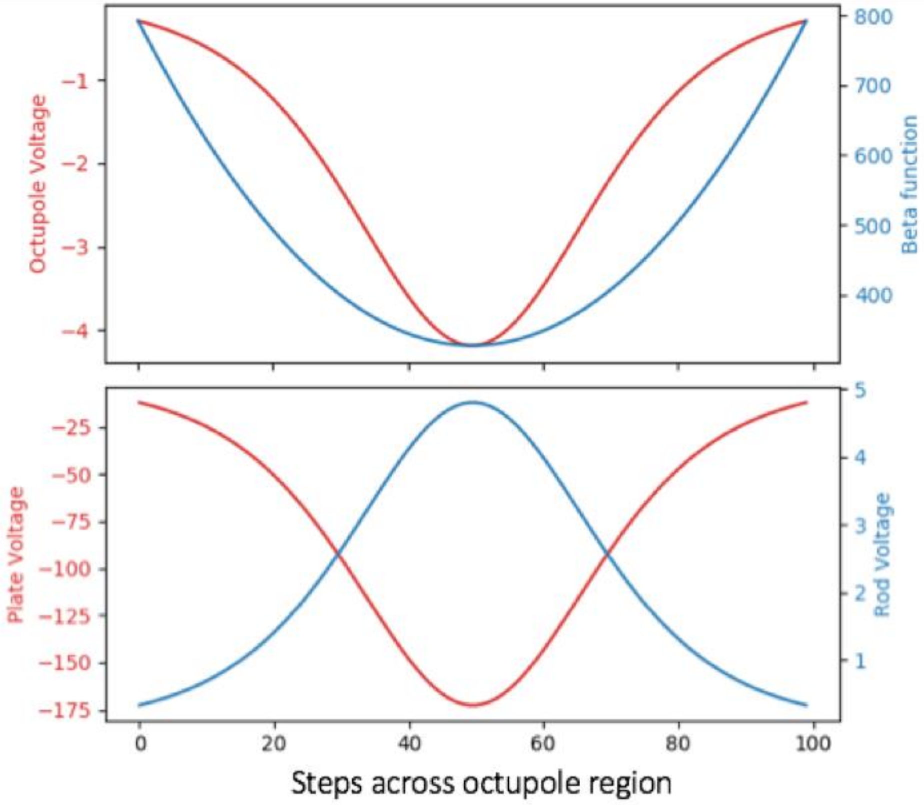
- No dispersion
- No chromatic effects
- T-insert parameters are easily variable
- Space charge effects easily included
- Can sit on resonance to study stability and excite resonances at arbitrary frequencies
- Already testing at IOTA and UMER – each facility has different advantages

Problem: Paul traps are not usually operated in this way!

1. Quadrupole + Octupole trap

- Building on work from Hiroshima University

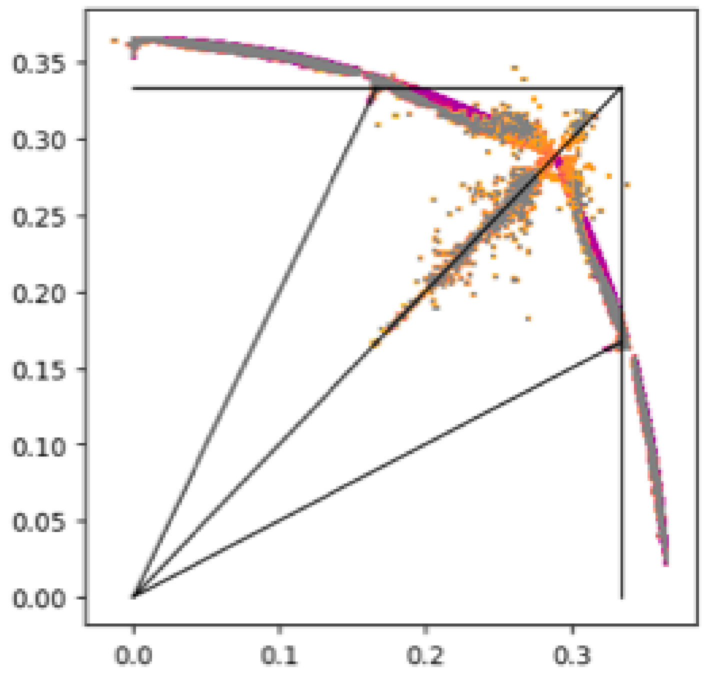




2. Design a T-insert

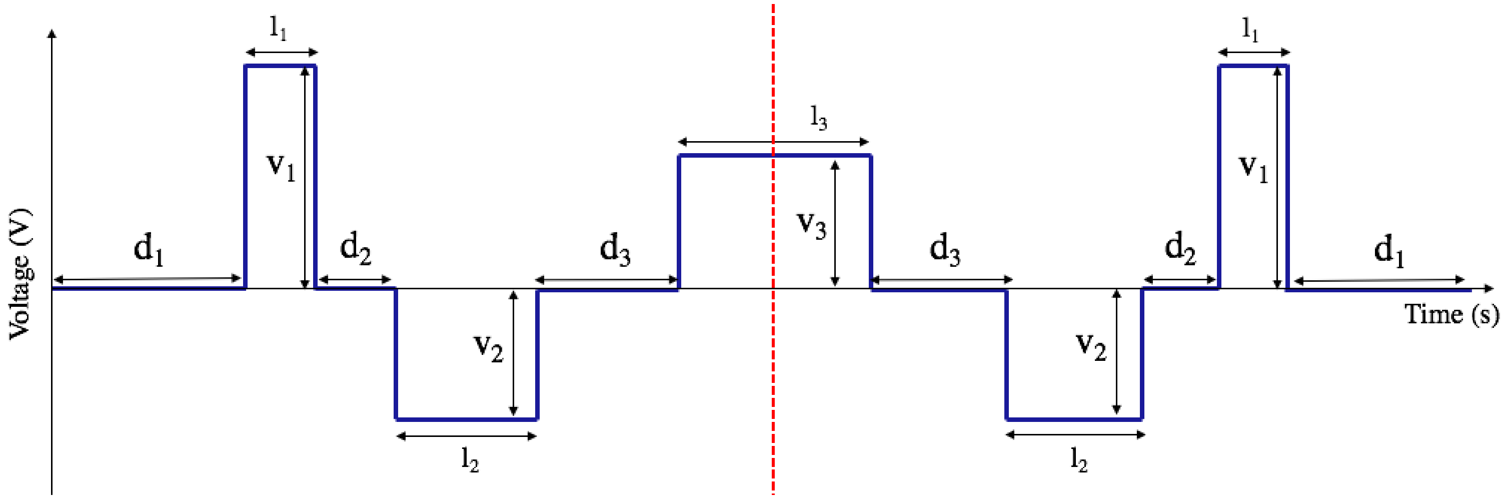
Constraints:

- $n\pi$ phase advance in x and y
- $\beta_x = \beta_y$ in centre of drift
- $\alpha_x = \alpha_y = 0$ in centre of drift



- Max β_x and β_y
- Phase advance close to 0.3 over drift
- Bandwidth of amplifiers – pulses can't be too short or too large
- Try to avoid too much asymmetry – more susceptible to resonance at high intensity

3. T-insert testing



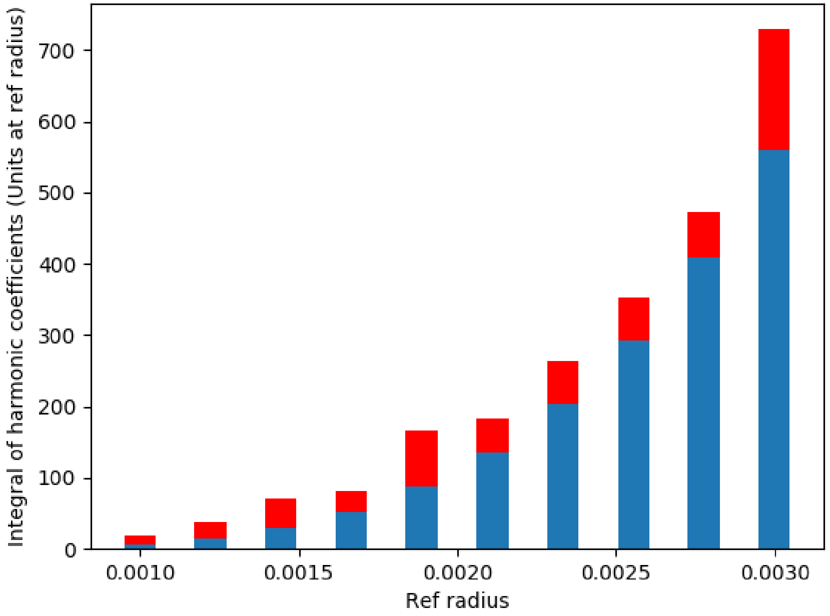
$$\text{Length}_2 = \text{Length}_1 * \frac{1}{\sqrt{k}}$$

$$\text{Voltage}_2 = \text{Voltage}_1 * \kappa$$

Lattice parameter	Element number		
	1	2	3
d (drift length (m))	150	90	180
l (quadrupole length (m))	80	180	180
v (quadrupole voltage (V))	26.83	-16.70	13.64

- We varied pulse strength and length
- Created a short T-insert that was easier to correct
- Looked at 3 different T-inserts scaling voltage and length
- To minimise beta function operated at lower tune (0.5 + 0.135)
    - UMER also looking to operate at this phase advance

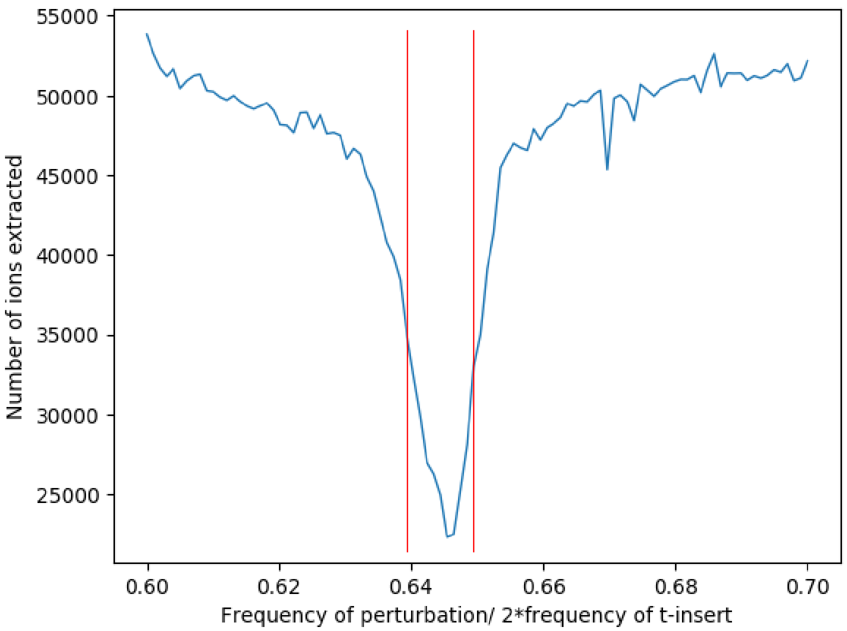
4. Quality control



Blue = perfect rod and plate placement

Red = example with 20μm error on rod and plate location

R_{meas} = 3mm (from simulation)



See tune control of ~10^{-2}

- For octupole only desired phase advance tolerance in IOTA is 10^{-2} (reduction in DA of 10%).
- Magnet tolerance is stated to be an integrated strength of harmonics relative to octupole field is less than 1% (100 units).

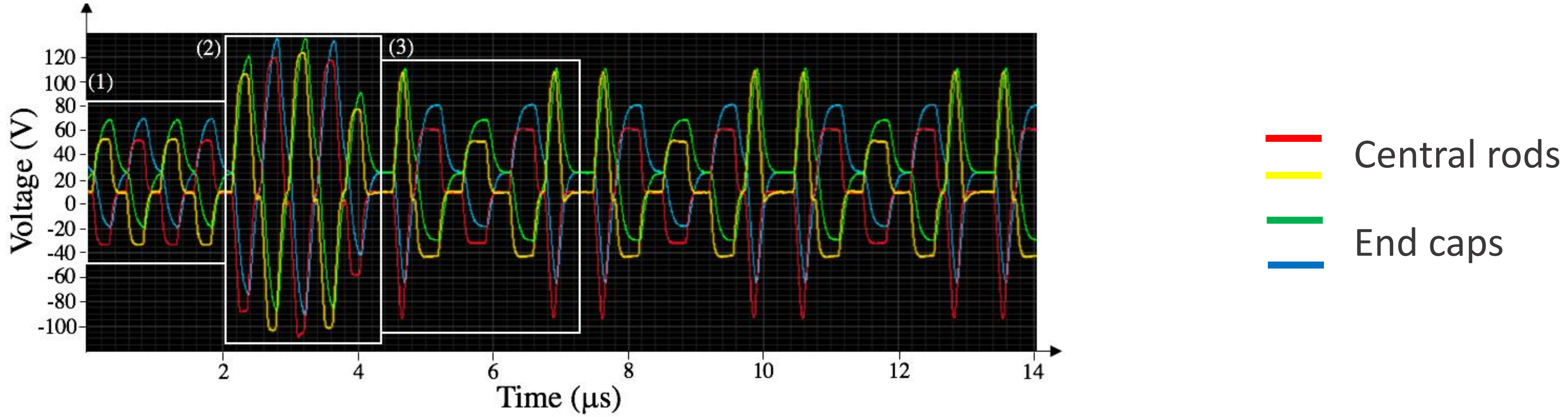
Conclusions and further work

- IBEX is a useful tool for accelerator physics studies
- Commission of IBEX has ended and we’re now capable of a range of interesting physics
- To test quasi-NIO a number of often competing constraints should be met
- However, it should be possible.

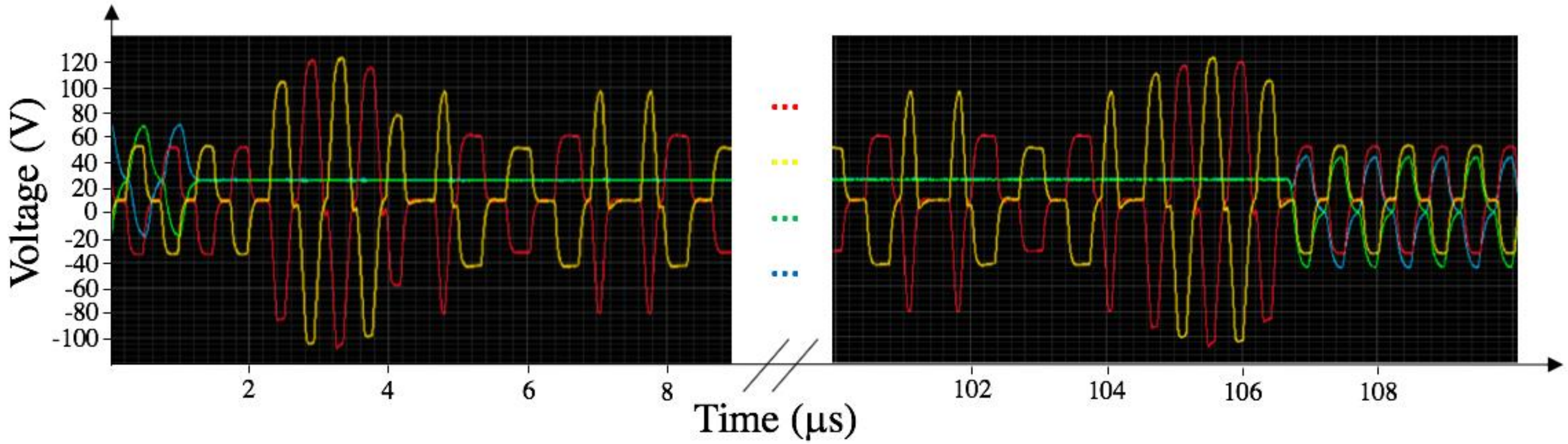
- Further simulations of the new T-insert required
  - Space charge
- Octupole upgrade to the trap needed – must be well designed
- Experimental testing of quasi integrable NIO
- Potential to vary a wide range of experimental parameters.

Thanks!

- For the waveforms with larger voltages ran into problems with the amplifiers, especially those on the endcaps, with the lower band width:



- Decided to not apply the alternating voltage to the end caps and to match back to original waveform before extracting:



Warp simulations

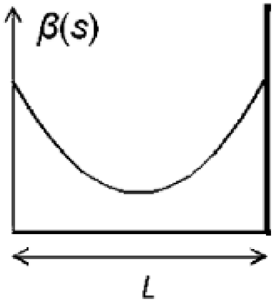
- Looked at dynamic aperture and tune spread over 1000 periods.
- The T-insert is applied using a single matrix transformation.

Simulation 1

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -k & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k & 1 \end{pmatrix}$$

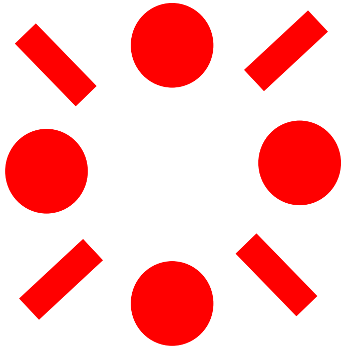
Pure octupole field

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -k & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k & 1 \end{pmatrix} \cdots$$



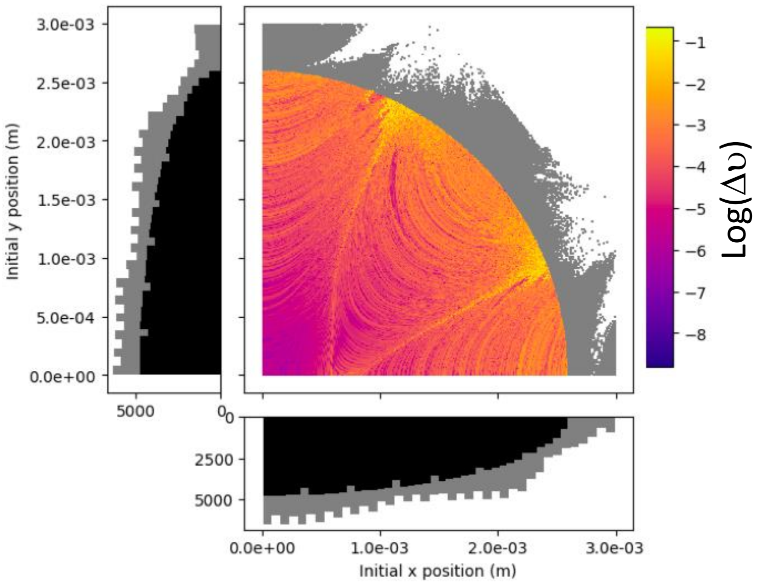
Simulation 2

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -k & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -k & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -k & 1 \end{pmatrix} \cdots$$

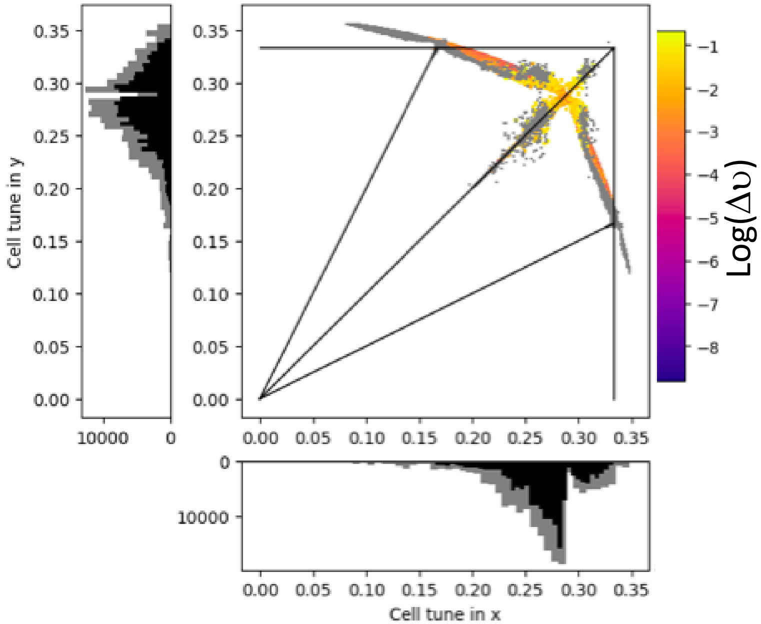
- The colour bar shows the log of the change in tune over the simulation for particles within the dynamic aperture.
- Grey regions are particles that survived but are outside of the dynamic aperture[10].
- Dynamic aperture is defined as the radius of the maximum circle in physical space, in which all particle survive the 1000 periods.

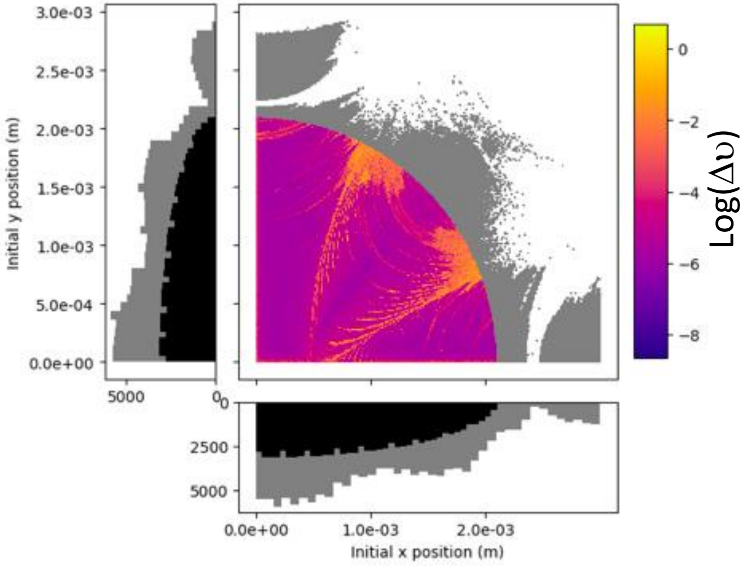


Simulation 1

Perfect octupole

Dynamic Ap = 2.6 mm





Simulation 2

Realistic Octupole

Dynamic Ap = 2.1 mm