## Multiparton interactions in pp and pA interactions at the LHC

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## Outline

~ Intro - open questions in HE dynamics
Geometry of hard collisins - from on 1D to 2D and 3D

F- From geometry of multiple collisions to pQCD dynamics of double parton scattering (DPS)

- pA DPS collisions collisions, centrality and ultimate test of hard DPI dynamics

For a detailed review of MPI i see the our review in Adv.Ser.Direct.High Energy Phys. 29 (2018) 63-99

## Challenges for building realistic description of inelastic pp collisions (willhave time to discuss only I \& II

Challenge I - including realistic transverse parton distributions in modeling pp collisions
Challenge II - include parton - parton correlations to describe multiparton interactions with realistic single parton transverse densities

> Consistent evidence from analysis of HERA data and leading pion production in d- Au for black disk regime (BDR) for gluons up to transverse momenta I -- I.5 Gevic at x $10^{-4}$

Challenge III - realistic modeling effects of BDR at moderate transver
Challenge IV - accounting for diffractive part of inelastic cross section
Challenge $\boldsymbol{V}$ - studies of forward production at LHC - most sensitive to the BDR dynamics and in particular effective fractional energy losses

## High energy hard processes - dual role:

Understanding QCD dynamics in the ultra high energy limit when the rate of hard interactions per event is $>1$ leading to possibly to change of interplay between soft and hard QCD

## Exploring nucleon structure



One dimensional (ID) image of nucleon in momentum space distribution over $x$ - the fraction of the momentum carried by constituents ( $q^{2}$-dependent)

Naive model of proton $=2 \mathrm{u}$ quarks + I d-quark
At high resolution: proton $=$ valence quarks + sea quarks and antiquarks + a lot of gluons at $x<0.1$



QCD predicts evolution of quark

Determination of the longitudinal momentum distribution - parton [generic for quark and gluon] densities - QCD theorems for the inclusive cross sections - proofs based on closure was first step of the nucleon image reconstruction in QCD

Test: parton $\left(\mathrm{x}_{1}\right)+\operatorname{parton}\left(\mathrm{x}_{2}\right) \rightarrow$ parton' $\left(\mathrm{jet}_{1}\right)+\operatorname{parton"}\left(\mathrm{jet}_{2}\right)$


Cross section of dijet production


Dijet event at LHC, CMS

Important characteristic of high energy collisions is the impact parameter of collision. Well defined since angular momentum is conserved and $L=b p$ Different intensity of interactions for small and large impact parameters b


Peripheral Pp collisions

Small b lint large overlap of partons


Large probability of multiparton, soft/hard interactions

> Using realistic transverse parton distributions is critical for genuine understanding of final states in pp

Geometry of pp collision with production of dijet in the transverse plane


## For hard collision

$$
\vec{\rho}_{1}+\vec{b}-\vec{\rho}_{2} \propto 1 / p_{t j e t} \sim 0
$$

$$
\sigma_{h} \propto \int d^{2} b d^{2} \rho_{1} d^{2} \rho_{2} \delta\left(\rho_{1}+b-\rho_{2}\right) f_{1}\left(x_{1}, \rho_{1}\right) f_{2}\left(x_{2}, \rho_{2}\right) \sigma_{2 \rightarrow 2}
$$

$$
=\int d^{2} \rho_{1} d^{2} \rho_{2} f_{1}\left(x_{1}, \rho_{1}\right) f_{2}\left(x_{2}, \rho_{2}\right) \sigma_{2 \rightarrow 2}=f_{1}\left(x_{1}\right) f_{2}\left(x_{2}\right) \sigma_{2 \rightarrow 2}
$$

For inclusive cross section at high virtuality transverse size \& structure does not matter - convolution of parton densities.


However critical for understanding global structure of inelastic events

## If pQCD works at LHC at $\mathrm{p}_{\mathrm{t}}>4 \mathrm{GeV} / \mathrm{c}-$ a generic inelastic event should contain many mini dijets:

e jet $\sigma$ )/ $\sigma_{\text {inel (nondiffr) }}(p p)$



10 dijets with $\mathrm{pt}_{\mathrm{t}}>4 \mathrm{GeV} / \mathrm{c}$ per nondiffractive event !!!

Transverse size matters for distribution over fraction of events with $\mathbf{N}$ pairs of jets

$$
\sigma(p p \rightarrow N \text { dijets }+X) \propto(\text { transverse area of nucleon })^{1-N}
$$

## Onset of nonlinear regime and suppression of minijets in pp collisions

Observation of MC models - need to suppress production of minijets

PYTHIA - suppression factor $\quad R\left(p_{T}\right)=\left(\frac{p_{T}^{2}}{p_{T}^{2}+p_{0}^{2}(s)}\right)^{2} ; \quad p_{0}(\sqrt{s}=7 \mathrm{TeV}) \approx 3 \mathrm{GeV} / \mathrm{c}$

$$
R\left(p_{T}=4 \mathrm{GeV} / c\right)=0.4
$$

## HERWIG $\quad \theta\left(p_{T}-p_{0}^{\prime}(s)\right)$

$p_{0}(s) \propto s^{0.12}$

Is the need for modification of dynamics for minijet range ( $p_{0} \sim 10 \mathrm{GeV} / \mathrm{c}!$ ! at highest cosmic energies - near GZK cutoff) been an artifact of MC or signal for serious problems?

Is transverse size realistic?

## Transverse structure from exclusive processes.



Hard exclusive meson production Meson is produced in a small size $q$ bar q configuration

Parton form factors of nucleon universal (process independent)


- Transverse spatial distribution of gluons $x^{\prime}=x$ $G(x, \rho)=\int \frac{d^{2} \Delta_{T}}{(2 \pi)^{2}} e^{-i \rho \boldsymbol{\Delta}_{T}} \operatorname{GPD}(x, t) \quad 2 \mathrm{D}$ Fourier

Tomographic image of nucleon at fixed $x$, changes with $x$ and $Q^{2}$

## Transverse distributions: Gluons



## Two pairs of partons can collide in a single collision



Naive geometric picture - two independent parton - parton collisions --- rate depends on the nucleon size only. However this assumes absence of parton - parton correlations. Pairs collide at relative transverse distance $\sim 0.5 \mathrm{fm}$.

Experimentally one measures

$$
\frac{d \sigma(4 \rightarrow 4)}{d \Omega_{1} d \Omega_{2}}=\frac{1}{\sigma_{e f f}} \frac{d \sigma(2 \rightarrow 2)}{d \Omega_{1}} \frac{d \sigma(2 \rightarrow 2)}{d \Omega_{2}}
$$

where $f\left(x_{1}, x_{3}\right), f\left(x_{2}, x_{4}\right) \quad$ longitudinal light-cone double parton densities and $\sigma_{\text {eff }}$ is '"transverse correlation area". One selects kinematics where $2 \rightarrow 4$ (LT two partons into four partons) contribution is small
CDF observed the effect in a restricted x-range: two balanced jets, and jet + photon and found

$$
\sigma_{e f f}=14.5 \pm 1.7_{-2.3}^{+1.7} \mathrm{mb}
$$

No significant dependence of $\sigma_{\text {eff }}$ on $x_{i}$ was observed. A naive expectation (based on $\mathrm{r}_{\mathrm{N}}=0.8 \mathrm{fm}$ ) is $\sigma_{\text {eff }} \sim 55 \mathrm{mb}$ indicating high degree of correlations between partons in the nucleon in the transverse plane - next few more technical slides


Similar results from D0.

## Integral depends on convolution of functions

$$
\begin{aligned}
& f\left(x_{1}, x_{2}, \overrightarrow{\Delta \rho}\right) \stackrel{?}{=} \int d^{2} \rho_{1} d^{2} \rho_{2} \delta\left(\vec{\rho}_{1}-\vec{\rho}_{2}-\overrightarrow{\Delta \rho}\right) f\left(x_{1}, \vec{\rho}_{1}\right) f\left(x_{2}, \vec{\rho}_{2}\right) \\
& \sigma_{4}=\int d^{2} B d^{2} \rho d^{2} \rho_{1} d^{2} \rho_{2} d^{2} \rho_{3} d^{2} \rho_{4} D\left(x_{1}, x_{2}, \vec{\rho}_{1}, \vec{\rho}_{2}\right) \times D\left(x_{3}, x_{4}, \vec{\rho}_{3}, \vec{\rho}_{4}\right) \\
&=\int d^{2} B d^{2} \rho_{1} d^{2} \rho_{2} D\left(x_{1}, x_{2}, \vec{\rho}_{1}, \vec{\rho}_{2}\right) \times D\left(x_{3}, x_{4}, \vec{B}+\vec{\rho}_{1},-\vec{B}+\vec{\rho}_{2}\right) .
\end{aligned}
$$

Independent transverse distribution of partons - assumed in all MC models

$$
\sigma_{e f f}=\frac{28 \pi}{m_{g}^{2}} \sim 32 \mathrm{mb} .
$$

LF, MS, Weiss 03
MPI rate a factor of two smaller than experiment !!!

## Realistic model of MPI should include a factor $\sim 2$ contribution of correlations

To reproduce $\sigma_{\text {eff }}$ in the independent parton approximation one needs $\mathrm{m}_{\mathrm{g}}{ }^{2} \sim 2 \mathrm{GeV}^{2}$ (PYTHIA)
area occupied by gluons is at least a factor of two smaller than experiment

$\mathrm{mg}^{2} \sim 2 \mathrm{GeV}^{2}$ leads to t -dependence which is too weak to reproduce $\mathrm{J} / \Psi$ exclusive photoproduction

## FROM GEOMETRY TO QCD

Blok, Dokshitzer, LF, MS (BDFS) II-I2 derived geometric results from the first principles and developed pQCD theory of MPI. We also discovered new pQCD mechanism of MPI due to $p Q C D$ evolution


D's are double generalized parton distributions
parton with $\mathrm{x}_{1}$ receives transverse kick of $\Delta$, and $\mathrm{x}_{2}$ of $-\Delta$ and nucleon survives

$$
\frac{1}{\sigma_{e f f}}=\int \frac{d^{2} \vec{\Delta}}{(2 \pi)^{2}} \frac{D_{a}\left(x_{1}, x_{2},-\vec{\Delta}\right) D_{b}\left(x_{3}, x_{4}, \vec{\Delta}\right)}{D_{a}\left(x_{1}\right) D_{a}\left(x_{2}\right) D_{b}\left(x_{3}\right) D_{b}\left(x_{4}\right)}
$$

Independent particle approximation which could be reasonable for moderately small $\mathrm{x}_{1}, \mathrm{x}_{2}$

$$
D\left(x_{1}, x_{2}, p_{1}^{2}, p_{2}^{2}, \vec{\Delta}\right)=G\left(x_{1}, p_{1}^{2}, \vec{\Delta}\right) G\left(x_{2}, p_{2}^{2}, \vec{\Delta}\right)
$$

one line calculation $\quad F_{2 g}(x \sim 0.03, t)=\left(1-t / m_{g}^{2}\right)^{-2}, m_{g}^{2} \sim 1.1 \mathrm{GeV}^{2}$

$$
\sigma_{e f f}=\frac{28 \pi}{m_{g}^{2}} \sim 32 \mathrm{mb} . \text { Practically the same number with } \exp (\mathrm{Bt}) \text { fit. }
$$

Hence our result of 03 is pretty stable since $F_{2 g}{ }^{2}(\Delta)$ is measured directly.
A factor of at least 2 is missing !!!!

## Parton model expression for D

$$
\begin{aligned}
& D\left(x_{1}, x_{2}, p_{1}^{2}, p_{2}^{2}, \vec{\Delta}\right)=\sum_{n=3}^{\infty} \int \frac{d^{2} k_{1}}{(2 \pi)^{2}} \frac{d^{2} k_{2}}{(2 \pi)^{2}} \theta\left(p_{1}^{2}-k_{1}^{2}\right) \theta\left(p_{2}^{2}-k_{2}^{2}\right) \\
& \quad \times \int \prod_{i \neq 1,2} \frac{d^{2} k_{i}}{(2 \pi)^{2}} \int_{0}^{1} \prod_{i \neq 1,2} d x_{i}(2 \pi)^{3} \delta\left(\sum_{i=1}^{i=n} x_{i}-1\right) \delta\left(\sum_{i=1}^{i=n} \vec{k}_{i}\right)
\end{aligned}
$$

$$
\times \psi_{n}\left(x_{1}, \vec{k}_{1}, x_{2}, \vec{k}_{2}, ., \vec{k}_{i}, x_{i} . .\right) \psi_{n}^{+}\left(x_{1}, \overrightarrow{k_{1}}+\vec{\Delta}, x_{2}, \overrightarrow{k_{2}}-\vec{\Delta}, x_{3}, \vec{k}_{3}, \ldots\right) .
$$

## General case $\mathbf{N}$ of $\mathbf{N}$ hard collisions

$$
\begin{aligned}
\sigma_{2 N} & \propto \int \prod_{i=1}^{i=N} \frac{d \vec{\Delta}_{i}}{(2 \pi)^{2}} D_{a}\left(x_{1}, \ldots x_{N}, \vec{\Delta}_{1}, \ldots \vec{\Delta}_{N}\right) \\
& \times D_{b}\left(x_{1}^{\prime}, \ldots x_{N}^{\prime}, \vec{\Delta}_{1}, \ldots \vec{\Delta}_{N}\right) \delta\left(\sum_{i=1}^{i=N} \vec{\Delta}_{i}\right)
\end{aligned}
$$

Origin of correlations? Perturbative vs non-perturbative. In principle delicate interplay
Qualitatively pQCD mechanism is -- parton at $\mathrm{Q}_{0}$ scale can resolve into two or more partons at higher scale with all partons localized in transverse area $1 / \mathrm{Q}_{0}{ }^{2}$

$$
D_{h}^{1,2}\left(x_{1}, x_{2}, q_{1}^{2}, q_{2}^{2} ; \vec{\Delta}\right)={ }_{[2]} D_{h}\left(x_{1}, x_{2}, q_{1}^{2}, q_{2}^{2} ; \vec{\Delta}\right)+{ }_{[1]} D_{h}\left(x_{1}, x_{2}, q_{1}^{2}, q_{2}^{2} ; \vec{\Delta}\right)
$$

two partons from
the wave function

the two contributions do not enter the physical DPI cross section in arithmetic sum, driving one even farther from the familiar factorization picture based on universal(process independent) parton distributions.


A short evolution contributions to cross section

$$
R \equiv \frac{\sigma_{1 \otimes 2}}{\sigma_{2 \otimes 2}}=\frac{\sigma_{4}}{\sigma_{3}}
$$

For the effective interaction area,

$$
\sigma_{\mathrm{eff}}^{-1}=\sigma_{4}^{-1}+\sigma_{3}^{-1}
$$




The $1 \otimes 2 / 2 \otimes 2$ ratio,
in the CDF kinematics photon + 3 jets

Geometry and combinatorics enhance $1 \otimes 2$ by a factor of 5 in pp scattering

Lengthy eqs \& numerical calculations. Result: if pQCD evolution starts at low $\mathrm{Q}_{0}=$ $0.7 \div \mathrm{I} \mathrm{GeV}$ scale we explain a factor of $\sim 2$ enhancement $1 / \sigma_{\text {eff }}$ for large pt . $\sigma_{\text {eff }}$ grows with decrease of $\mathrm{P}_{\mathrm{t}}$ while in MC's it is assumed to be $\mathrm{Pt}_{\mathrm{t}}$ and process


$$
\begin{gathered}
\sigma_{\mathrm{eff}}=\frac{28 \pi}{m_{g}^{2}} \cdot \frac{1}{1+R} \simeq \frac{32 \mathrm{mb}}{1+R} \\
R \equiv \frac{\sigma_{1 \otimes 2}}{\sigma_{2 \otimes 2}} \sim 1
\end{gathered}
$$


$\sigma_{\text {eff }}$ for two dijets in DPS at the LHC

## Predictions for LHC

$$
\begin{array}{cl}
j j+j j: & \sigma_{\mathrm{eff}}=14.5 \div 20 \mathrm{mb} \\
W+j j: & \sigma_{\mathrm{eff}}=20 \div 23.5 \mathrm{mb} \\
W^{+} W^{+}: & \sigma_{\mathrm{eff}}=21.5 \div 25.4 \mathrm{mb} \\
\text { ZZ: } & \sigma_{\mathrm{eff}}(Z Z)=15.9 \div 18.5 \mathrm{mb}
\end{array}
$$

W+W+ predictions tend to be higher than the recent data, but errors are large - may require even more correlations than PQCD. Problem - subtraction of LT contribution, etc.

Small $x$ challenge: transverse size grows with decrease of $x$, leading to larger area and hence larger oeff

Double charm ( D-meson + D-meson) production do not show such a trend. - possible explanation is diffraction

${ }_{2} \mathrm{GPD}$ as a two Pomeron exchange

$2 \mathbb{P}$ contribution to ${ }_{2} \mathrm{D}$ and corresponding Reggeon diagrams


FIG. 13: $\sigma_{\text {eff }}$ as a function of the transverse scale $p_{\perp}$ for $Q_{0}^{2}=0.5$ (left), and $Q_{0}^{2}=1 \mathrm{GeV}^{2}$ (right) in the central kinematics. We present the mean field, the mean field plus $1 \otimes 2$ mechanism and total $\sigma_{\text {eff }}$ for $\sqrt{s}=$ 13 TeV .

A factor of two smaller $\sigma_{\text {eff }}$ than the mean field approximation

Consistent with LHCb data on DD production. Double J/psi much higher cross section.

Further studies are necessary to check the origin of the parton - parton correlations. Other mechanisms

* Correlations in the wave function at a low $Q$ scale Weiss \& MS - quark - antiquark correlations at moderate $x \sim 0.1$


## Pomeron splitting Correlations at $x \sim 10^{-3}$

Neglect of correlation term in MCs resulted in two versions of PYTHIA MC:
(a) $\sigma_{\text {eff }}=30 \mathrm{mb}$ for min-bias , (b) $\sigma_{\text {eff }}=15 \mathrm{mb}$ for high pt 4 jets

Dynamical approach to MPI four-jet production in Pythia: B. Blok , P. Gunnellini-(0215) build in dynamics: $\sigma_{\text {eff }}$ dropping with increase of $p_{t}$ of the jets -- describes both sets of data. Implemented now as one of the options of PYTHIA 8.242 1.7.2019). MPI are reweighs according to our GPDs and pQCD splitting

## Gluon GPDs and difference between transverse $s$ b distributions for four jet, dijet triggers and minimal bias collisions

The distribution of interactions over b for events with inclusive dijet trigger (Higgs production,...) is given by

$$
P_{2}(b)=\int d^{2} \rho_{1} \int d^{2} \rho_{2} \delta^{(2)}\left(\vec{b}-\vec{\rho}_{1}+\vec{\rho}_{2}\right) F_{g}\left(x_{1}, \rho_{1}\right) F_{g}\left(x_{2}, \rho_{2}\right)
$$

differential probability for dijet production to occur at given b
for $\quad F_{g}(x, t)=1 /\left(1-t / m_{g}(x)^{2}\right)$

$$
F_{g}(x, \rho)=\frac{m_{g}^{2}}{2 \pi}\left(\frac{m_{g} \rho}{2}\right) K_{1}\left(m_{g} \rho\right)
$$

$$
P_{2}(b)=\frac{m_{g}^{2}}{12 \pi}\left(\frac{m_{g} b}{2}\right)^{3} K_{3}\left(m_{g} b\right)
$$

Compare with b-distribution for minimal bias (generic) inelastic pp scattering

$$
P_{i n}(s, b)=\frac{2 R e \Gamma^{p p}(s, b)-\left|\Gamma^{p p}(s, b)\right|^{2}}{\sigma_{i n}(s)}
$$

where $\quad \Gamma_{h}(s, b)=\frac{1}{2 i s} \frac{1}{(2 \pi)^{2}} \int d^{2} \vec{q} e^{i \vec{q} \vec{b}} A_{h N}(s, t)$

$$
\Gamma(b)=1 \equiv \sigma_{\text {inel }}=\sigma_{e l} \quad \text { - black disk regime (BDR). }
$$

## Two -scale picture of strong interaction at the LHC (LF, MS, Weiss 2003



Impact parameter distrfButions of inelastic PP collisions at $\sqrt{ } s=7 \mathrm{TeV}$. Solid (dashed) line:
Distribution of events with a dijet trigger at zero rapidity, $\mathrm{y}_{1,2}=0, \mathrm{c}$, for $\mathrm{p}^{\mathrm{T}}=100(10) \mathrm{GeV}$.


Median impact pararpreter b(median) of events with a dijet trigger, as a function of the transverse momentum PT, cf. left plot. Solid line: Dijet at zero rapidity $y_{1,2}=0$.
Dashed line: Dijet with rapidities $y_{I, 2}= \pm 2.5$.

## Weak dependence of $\mathrm{P}_{2}(\mathrm{~b})$ on rapidity and $\mathrm{P}_{\mathrm{t}}$ of the dijet



Centrality strongly depends on the trigger. For DPS significantly smaller b's than for dijet trigger. Different structure of the underlying event -some features can be calculated (Azarkin, Dremin, MS)

## Conclusions I

~ Scenario: weak correlations / large oeff at low Q with pQCD generated correlations at large $Q Q$ allows to describe the data sensitive to moderate virtualities (underlying event),...

F Problem (?) - one uses pQCD perturbation theory starting at at low Q~1 GeV where fits require a strong suppression of the parton - parton interaction a compared to LT.,

A A stumbling block for deterring - uncertainties in modeling LT processes (2 $->4$ )

New ideas are necessary - one possibility is to use pA scattering

Centrality dependence of DPS in pA collisions - a window at minijet dynamics

M.Alvioli, M.Azarkin, B. Blok, M.Strikman

## Questions

How well does the picture of minijets works with $p_{t}$ down to $p_{t} \sim$ few GeV with momentum dependent suppression factor reflects underlying dynamics

Origin of $\sigma_{\text {eff }}=14$ (20?) mb which was till recently preferred by many MC models

How to separate DPS from contribution of $2 \rightarrow 3,2 \rightarrow 4, \ldots$ LT processes

MPI in nuclei: testing minijet dynamics probing parton


A $\mathbf{\sigma}_{1}$

$\sigma_{2}$

$$
R \equiv \frac{\sigma_{2}}{\sigma_{1} \cdot A} \approx \frac{(A-1)}{A^{2}} \cdot \sigma_{\text {eff }} \int T^{2}(b) d^{2} b \approx 0.68 \cdot\left(\frac{A}{12}\right)^{0.39} \begin{gathered}
\text { Nuclear pdf= } \\
\mathbf{A}^{*}(\text { nucleon pdf })
\end{gathered}
$$

$$
T(b)=\int_{-\infty}^{\infty} d z \rho_{A}(z, b), \int T(b) d^{2} b=A
$$

linear in $\sigma_{\text {eff }}!!$
"Antishadowing effect": For $A=200$, and $\sigma_{\text {eff }}=14 \mathrm{mb}$ [25 mb low Q]

$$
\begin{array}{lll}
\frac{\sigma_{p A}}{A \sigma_{p p}} \approx 3 & \frac{\sigma_{p A}}{\sigma_{p p}} \approx 3.8 & \frac{\sigma_{p A}}{A \sigma_{p p}} \approx 4.5 \\
\sigma_{\text {eff }}=14 \mathrm{mb} & \sigma_{\text {eff }}=20 \mathrm{mb} & \sigma_{\text {eff }}=25 \mathrm{mb}
\end{array}
$$

LHC large Q
low Q

Measurement of $R=\sigma_{2} / A \sigma_{1}$ allows to separate longitudinal and transverse correlations of partons as it measures

$$
R \propto \frac{f\left(x_{1}, x_{2}\right)}{f\left(x_{1}\right) f\left(x_{2}\right)}
$$

## Idea in nutshell

if no 2\to 3, 4 and and impact parameter $b$ is known for each event

$$
R_{D P S / L T}(b)=\sigma_{e f f} T(b)
$$

also

$$
\frac{d \sigma_{p A}^{(D P S)}}{d^{2} b}=\sigma_{p N} T_{A}(b)+\sigma_{1} \sigma_{2} T^{2}(b)
$$

model independent prediction for $\$ \mathrm{~b} \$$ - dependence of DPS in terms of the elementary (DPS+LT ) pp cross section, $\sigma_{1}, \sigma_{2}$ and $T(b)$.

No need to separate LT!!!

Nuclear pdf / nucleon pdf $=1$ for high p_t kinematics for leading pair of jets

## Problems

DPS/Total

1) DPS usually a rather small fraction of the cross section in a given kinematics. Difficult to use pp data even the ones in rather similar kinematics

2) Accuracy of optical approximation
3) How to measure b? Impossible - try geometrical (Glauber) model
4) can be addressed by subtracting large b cross section using small nuclear modification of pdf of nucleus.

## pQCD theory of MPI in pA

MS \& Treleani treatment of pA was based on partonic model. Since in QCD in addition to
$2 \otimes 2$ there is an important $\mathrm{I} \otimes 2$ contribution - need to perform a new analysis, to calculate ${ }_{2}$ GPD of nuclei, estimate uncertainties of the nuclear effects.

Summary
Blok, MS ,Weidemann 2012

Impulse approximation - two partons (or one parton in the case of $3 \otimes 4$ ) are taken from the same nucleon. For $x>0.03$ when no $G_{A}^{1 N}\left(x_{1}, x_{2}, \vec{\Delta}\right)=A G^{1 N}\left(x_{1}, x_{2}, \vec{\Delta}\right)\left(1+O\left(\int(\alpha-1)^{2} \rho_{A}^{N}\left(\alpha, p_{t}\right) \frac{d \alpha}{\alpha} d^{2} p_{t}\right)\right)$
light-cone nucleon density matrix

## Use of light cone nucleus wave functions with nonrelativistic reduction FS 76-81

Double nucleon interaction: diagonal case

$$
G_{A}^{2 N}\left(x_{1}, x_{2}, \vec{\Delta}\right)=A(A-1) \int \frac{1}{\alpha_{1} \alpha_{2}} \prod_{i=1}^{i=A} \frac{d \alpha_{i} d^{2} p_{t i}}{\alpha_{i}} \delta\left(\sum_{i} \alpha_{i}-A\right) \delta^{(2)}\left(\sum_{i} \mathbf{p}_{t i}\right) \psi_{A}^{*}\left(\alpha_{1}, \alpha_{2}, p_{t 1}, .\right.
$$

Derived expressions allow very compact calculations both for heavy and light nuclei

$$
\text { Since }<\Delta^{2}>r N^{2} \ll 1
$$

## Correction:

$$
G_{A}^{2 N}\left(x_{1}, x_{2}, \vec{\Delta}\right) \rightarrow f_{N}\left(x_{1}\right) f_{N}\left(x_{2}\right) F_{A}^{\text {double }}(\vec{\Delta},-\vec{\Delta}) \quad O\left(r_{N}^{2} / r_{A}^{2}\right)
$$

$$
\frac{\sigma_{4}\left(x_{1}^{\prime}, x_{2}^{\prime}, x_{1}, x_{2}\right)}{d \hat{t}_{1} d \hat{t}_{2}}=\frac{f_{p}\left(x_{1}^{\prime}, x_{2}^{\prime}\right)}{f_{p}\left(x_{1}^{\prime}\right) f_{p}\left(x_{2}^{\prime}\right)} \frac{d \sigma_{2 \text { jet }}\left(x_{1}^{\prime}, x_{1}\right)}{d \hat{t}_{1}} \frac{d \sigma_{2 \text { jet }}\left(x_{2}^{\prime}, x_{2}\right)}{d \hat{t}_{2}} \frac{(A-1)}{A} \underbrace{\int T^{2}(b) d^{2} b}_{\alpha A^{4 / 3}}
$$

measures longitudinal correlations
$I+R / 5$ in $p Q C D$ where $R$ is calculated from $p Q C D$ evolution $\sigma_{\text {eff }}=\sigma_{\text {meanf }} /(I+R)$


To take into account finite nucleon size, short range NN correlations in nuclei, generate number of wounded nucleons in addition to the ones involved in DPS we build a new MC for pA which extends MC M. Alvioli and MS build for single hard collision.
(a) Generate a A - nucleon configuration
(b) Generate impact parameter $b$ of the nucleon relative to the center of the nucleus
(c) Generate transverse positions $\rho_{1}, \rho_{2}$ of two partons in the nucleon involved in DPS
(d) Calculate parton density along the paths of the two partons

$$
\begin{array}{r}
\int d^{2} b T^{2}(b) \rightarrow T_{2}=\int d^{2} b d^{2} \rho_{1} d^{2} \rho_{2} g_{N}\left(\rho_{1}, \rho_{2}\right) \psi_{A}^{2}\left(r_{t}^{(k)}, z_{i}\right)\left[\sum_{k=1}^{A} g_{N}\left(\rho_{1}+b-r_{t}^{(k)}, \rho_{2}+b-r_{t}^{(i)}\right)\right. \\
\\
\left.+\sum_{i=1}^{A} f_{N}^{(i)}\left(\rho_{1}+b-r_{t}^{(i)}\right) \cdot \sum_{j=1, j \neq i}^{A} f_{N}^{(k)}\left(\rho_{2}+b-r_{t}^{(i)}\right)\right]
\end{array}
$$

where $f_{N}(\rho)$ are the ratios of diagonal generalized parton distributions GPDs and single parton densities and $g_{N}\left(\rho_{1}, \rho_{2}\right)$ are the ratios of the diagonal double generalized parton distributions (GPD) and corresponding single parton densities

$$
\begin{aligned}
D^{a n y 2}(b) & =\int d \boldsymbol{\rho}_{1} d \boldsymbol{\rho}_{2} f_{p}\left(\rho_{1}\right) f_{p}\left(\rho_{2}\right) \sum_{i=1}^{A} f_{N}\left(\left|\boldsymbol{\rho}_{1}+\boldsymbol{b}-\boldsymbol{r}_{t}^{(i)}\right|\right) \sum_{k=1}^{A} f_{N}\left(\left|\boldsymbol{\rho}_{2}+\boldsymbol{b}-\boldsymbol{r}_{t}^{(k)}\right|\right) \\
D^{2 o n 1}(b) & =\int d \boldsymbol{\rho}_{1} d \boldsymbol{\rho}_{2} f_{p}\left(\rho_{1}\right) f_{p}\left(\rho_{2}\right) \sum_{i=1}^{A} f_{N}\left(\left|\boldsymbol{\rho}_{1}+\boldsymbol{b}-\boldsymbol{r}_{t}^{(i)}\right|\right) f_{N}\left(\left|\boldsymbol{\rho}_{2}+\boldsymbol{b}-\boldsymbol{r}_{t}^{(i)}\right|\right), \\
D^{2 o n 2}(b) & =\int d \boldsymbol{\rho}_{1} d \boldsymbol{\rho}_{2} f_{p}\left(\rho_{1}\right) f_{p}\left(\rho_{2}\right) \sum_{i=1}^{A} f_{N}\left(\left|\boldsymbol{\rho}_{1}+\boldsymbol{b}-\boldsymbol{r}_{t}^{(i)}\right|\right) \sum_{k \neq i}^{A} f_{N}\left(\left|\boldsymbol{\rho}_{2}+\boldsymbol{b}-\boldsymbol{r}_{t}^{(k)}\right|\right) \\
& =D^{a n y 2}(b)-D^{2 o n 1}(b),
\end{aligned}
$$

(a1) we assign hard process due to the interaction of the parton of the proton with transverse position \$1rho\$ to a particular nucleon with probability

$$
P_{j}=\frac{g_{N}^{(j)}(\rho)}{\sum_{k=1}^{A} g_{N}^{(j)}(\rho)}
$$

generate wounded nucleons in Glauber like procedure and exclude (don't count twice )the nucleons which were involved in hard collisions

Next Numerics



Transverse parton distribution

For heavy nuclei optical approximation is a good approximation: for $\mathbf{p - P b}$

$$
\frac{\int d^{2} b T_{2}(b)}{\int d^{2} b T^{2}(b)}=0.94 \text { accounting for finite size, skin, NN correlations }
$$



Distribution over the number of wounded nucleons $v$ - strong suppression of double scattering for small $v$

## $\Sigma \mathrm{E}^{\mathrm{Pb}}$ distribution as a function of v : modeling by ATLAS at large negative rapidities $-3>\eta>-5$

Transverse energy distributions in $\mathrm{p}^{+} \mathrm{p}$ collisions are typically well described by gamma distributions

$$
\operatorname{gamma}(x ; k, \theta)=\frac{1}{\Gamma(k)} \frac{1}{\theta}\left(\frac{x}{\theta}\right)^{k-1} e^{-x / \theta}
$$

gamma distribution has convolution property:

$$
\begin{aligned}
& k\left(N_{\text {part }}\right)=k_{0}+k_{1}\left(N_{\text {part }}-2\right), \\
& \theta\left(N_{\text {part }}\right)=\theta_{0}+\theta_{1} \log \left(N_{\text {part }}-1\right) .
\end{aligned}
$$

N -fold conv. of $\operatorname{gamma}(\mathbf{x}, \mathrm{k}, \boldsymbol{\theta})=\operatorname{gamma}(x, k, \theta) \equiv \frac{1}{\Gamma(N k)} \frac{1}{\theta}\left(\frac{x}{\theta}\right)^{N k-1} e^{-x / \theta}$
Note: for $\mathrm{k}=\mathrm{I}$, gamma distribution is exponential, $\mathrm{k}<\mathrm{I}$ is "superexponential"

## Glauber and Glauber-Gribov analysis



-With Glauber-Gribov $N_{\text {part }}$ distribution, the best fits become more WN-like

From
-e.g. for $\Omega=0.55, k_{1}=0.9\left(0.64 k_{0}\right), \theta_{1}=0.07$
B.Cole
$\Rightarrow$ Glauber-Gribov smooths out the knee in the $N_{\text {part }}$

## Procedure to observe DPI/ MPI in pA

Three contributions to the final state:
(i) the leading twist contribution
(ii) DPS due to the interaction with one nucleon
(iii) DPS due to the interaction with two nucleons.
(i) \& (ii) $\propto \mathrm{T}(\mathrm{b})$
(iii) $\propto \mathrm{T}^{2}(\mathrm{~b})$
$R^{\text {double } / \text { inclusive }}=N($ dijet + pion $/$ jet $) / N($ dijet $)=c_{1}+c_{2} T(b)$

No need to model LT contribution

## A bit more advanced procedure

Define $\mathrm{N}_{\mathrm{i}}=$ multiplicity of second pair of jets / high $\mathrm{p}_{\mathrm{t}}$ pion in the event belonging to a bin of $\Sigma \mathrm{E}_{\mathrm{T}}$

$$
R_{i}=\frac{N_{i}-N_{2}}{N_{3}-N_{2}}
$$

Main requirement for using a particular process: process is due to LT plus DPS that is Dependence of $\mathbf{N}_{\mathrm{i}}$ on centrality is due to hard scatterings. $\sigma_{\text {eff }}$ cancels in the ratio R. For different pt of jets, pions universal function. We use $\mathbf{N}_{2}$ to avoid super peripheral collisions where diffraction, etc maybe important. We avoid using information from pp - but can compare $\mathrm{N}_{1}, \mathrm{~N}_{2}$ with pp.


Centrality dependence of DPS multiplicity enhancement as a function of $\sum \mathrm{E}_{\mathrm{T}}$ measured in-3.2 $\geq \eta \geq-4.9$ (along the nucleus direction) which corresponds centrality bins denoted in the plot.

## Conclusions II

pA dijets with centrality trigger provide new direction for study of 3D structure of the nucleon as well as study of nuclear parton structure at high densities

In the 4 jet LHC kinematics MPI vs LT $2 \rightarrow 4$ processes are enhanced in pA by a factor of 3 as compared to pp case. Minijet enhancement is significantly larger.

We developed a MC for DPS process which takes into account transverse geometry of hard and soft collisions which allows to generate DPS events for different centrality bins.

New observable for observing DPS in PA is proposed which allows to observe DPS in the kinematics where DPS << LT

Hope to see the data analysis soon

Theory: i) different models for centrality determination, ii) study of LT nuclear shadowing effects.

