Lorentz invariance of collision criteria

Damjan Mitrovic

mitrovic@fias.uni-frankfurt.de

In Cooperation with:
Hannah Elfner, Justin Mohs & the SMASH Group

FIAS Frankfurt Institute for Advanced Studies

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

GSI

smash

19. Zimányi School

Budapest

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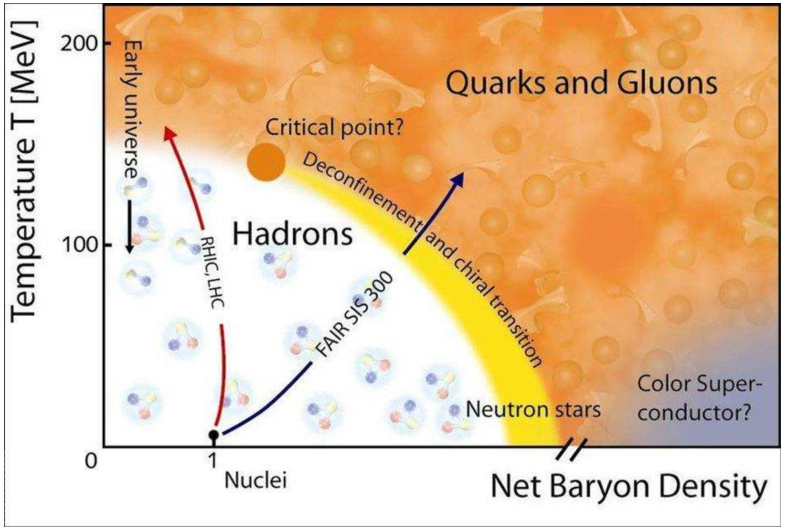
Agenda

- **Motivation**
- Collision Criteria
- First Results
- Wrap up

Why are we studying it?

Motivation

Understand
QCD-Phase Diagram

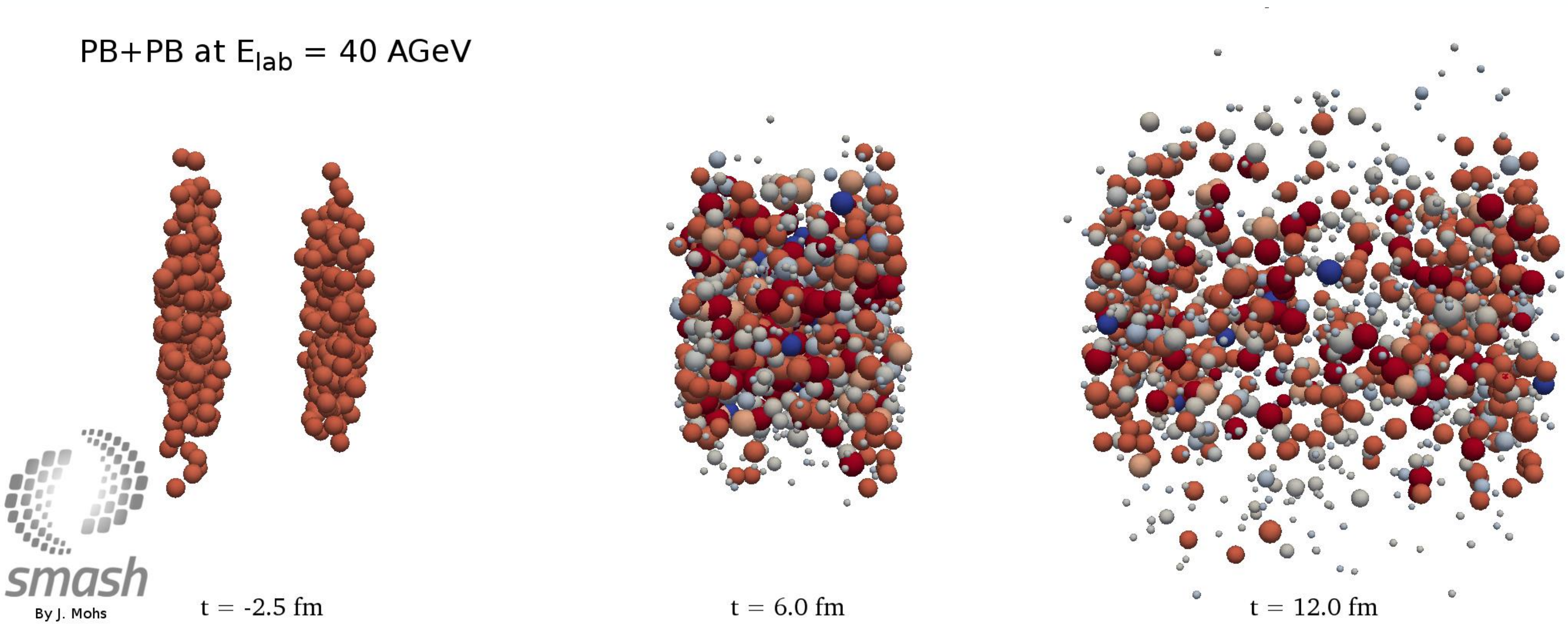


- Signatures of the QGP modified by the hadronic phase
- Understanding the hadronic interactions important

➡ **Hadronic Transport Approaches**

Hadronic Transport Approaches

Motivation



SMASH[1]

Motivation

- Hadronic Transport Approach
- Applicable at low energies or as afterburner
- Established Hadrons from PDG up to $m \approx 2$ GeV
- Open Source C++ Code
- More information on the homepage:

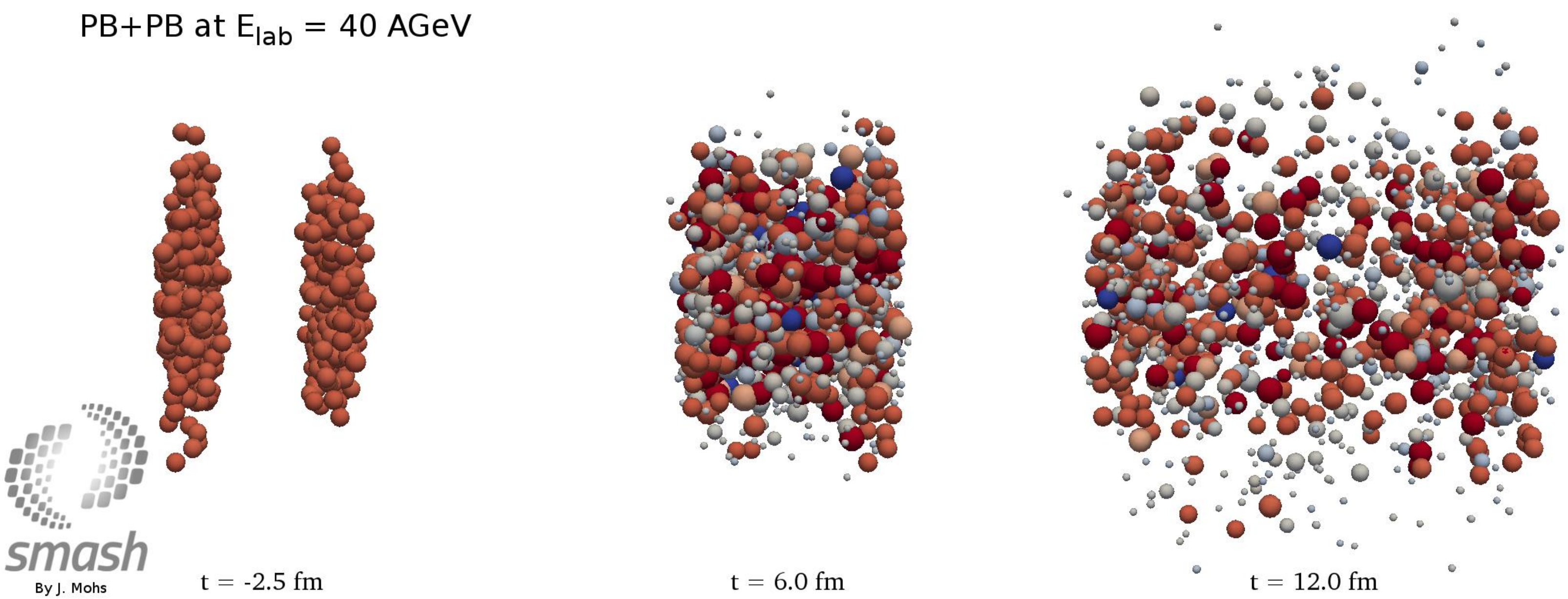
https://smash-transport.github.io/

1 J. Weil, et al., Phys. Rev. C **94**, 054905 (2016)

Hadronic Transport Approaches

Motivation

Collision Criteria?



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Geometric Criterion[1]

Collision Criteria

DEFAULT

- Particles collide if: $d_{trans} < d_{int} = \sqrt{\frac{\sigma_{tot}}{\pi}}$

with $$d_{trans} = \left(\vec{r}_i - \vec{r}_j\right)^2 - \frac{\left(\left(\vec{r}_i - \vec{r}_j\right) \cdot \left(\vec{p}_i - \vec{p}_j\right)\right)^2}{\left(\vec{p}_i - \vec{p}_j\right)^2}$$

- Impact parameter obtained in binary center of mass frame

- Collision time is taken as the time of the closest approach in observational frame:

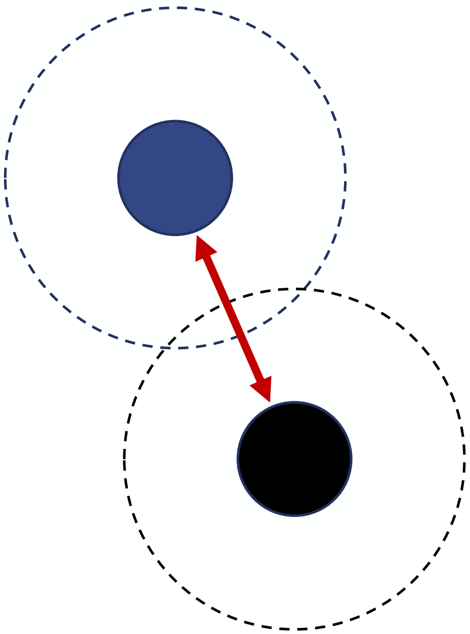
$$t_{coll} = -\frac{\left(\vec{r}_i - \vec{r}_j\right) \cdot \left(\frac{\vec{p}_i}{E_i} - \frac{\vec{p}_j}{E_j}\right)}{\left(\frac{\vec{p}_i}{E_i} - \frac{\vec{p}_j}{E_j}\right)^2}$$

1 S.A. Bass, et al., Prog.Part.Nucl.Phys. **41**, 255-369 (1998)

Finite Range Interactions

Collision Criteria

- Instantaneous interactions over a finite distance
- Information transferred faster than the speed of light
- Gives rise to causality violations



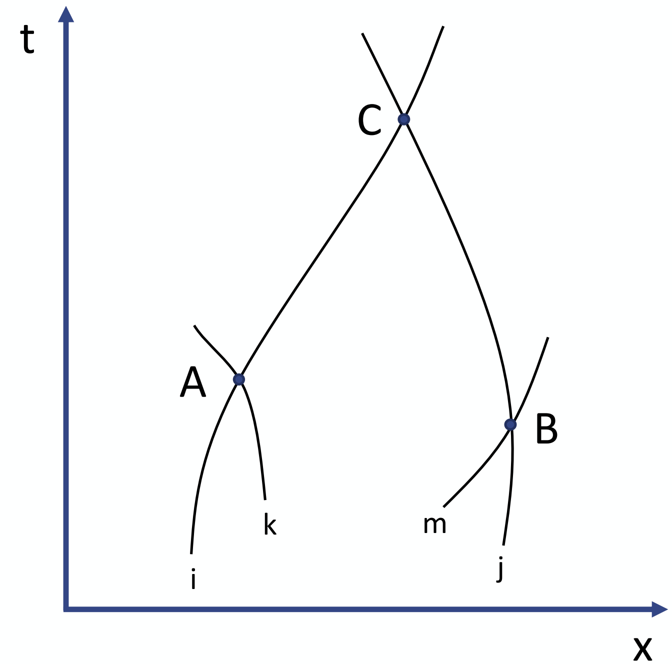
➡ **Full Lorentz invariance very difficult**

Causality

Collision Criteria

Invariant time ordering

- Impossible for spacelike events
- But for timelike events imposed by causality



Ref: T. Kodama, et al., Phys. Rev. C **29**, 2146 (1984)

Try to conserve time ordering for timelike events

Kodama Criterion[1]

Collision Criteria

WORK IN PROGRESS

- Covariant distance criterion:

$$b_{ij}^2 = -(x_i - x_j)^2 + \frac{\left[(p_i + p_j)\cdot(x_i - x_j)\right]^2}{s_{ij}} - \frac{\left[(p_i - p_j)\cdot(x_i - x_j)\right]^2}{s_{ij} - 4m^2}$$

again: $b_{ij} < \sqrt{\frac{\sigma_{tot}}{\pi}}$

- Proper time intervals between 2 collisions: $\delta\tau_i(j) = \tau_{c_j} - \tau_i$

- Causality imposes: $\delta\tau_i(j), \delta\tau_j(i) > 0$
- Later on compared:

$\delta\tau_i > 0$ | All $\delta\tau_i$

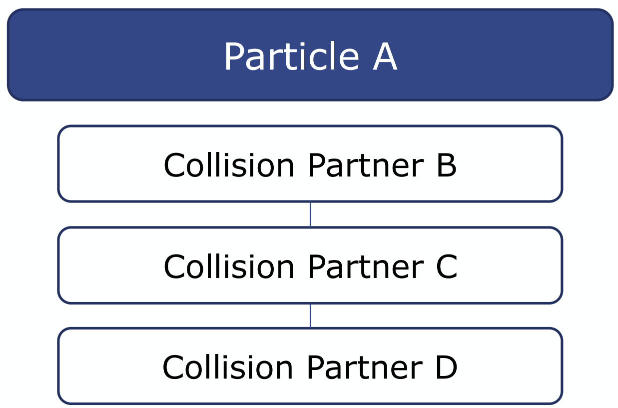
1 T. Kodama, et al., Phys. Rev. C **29**, 2146 (1984)

Kodama Criterion[1]

Collision Criteria

WORK IN PROGRESS

- Evaluate collision times in particles rest frame
- Perform only collisions next in line for both particles

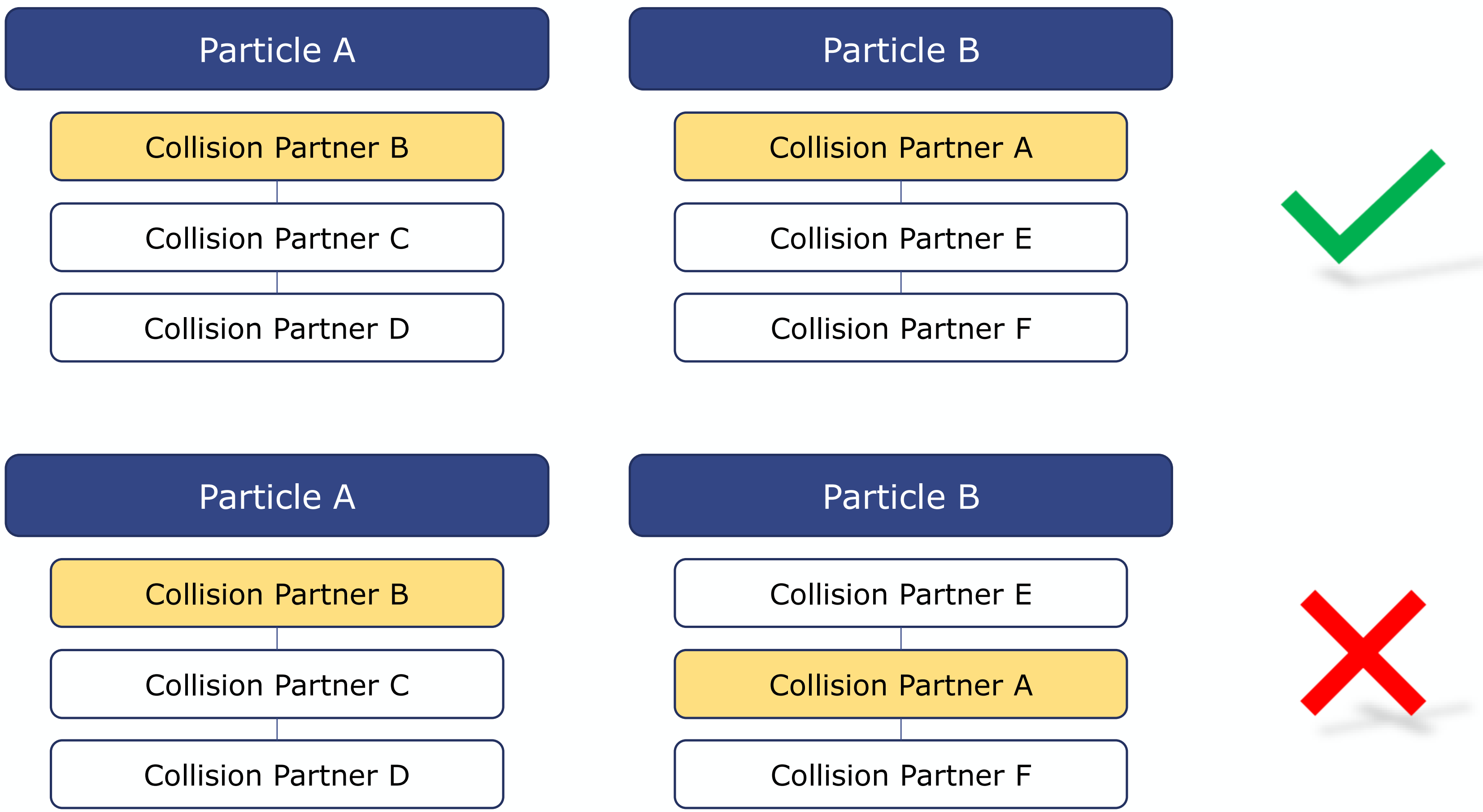


1 T. Kodama, et al., Phys. Rev. C **29**, 2146 (1984)

Kodama Criterion[1]

Collision Criteria

WORK IN PROGRESS



1 T. Kodama, et al., Phys. Rev. C **29**, 2146 (1984)

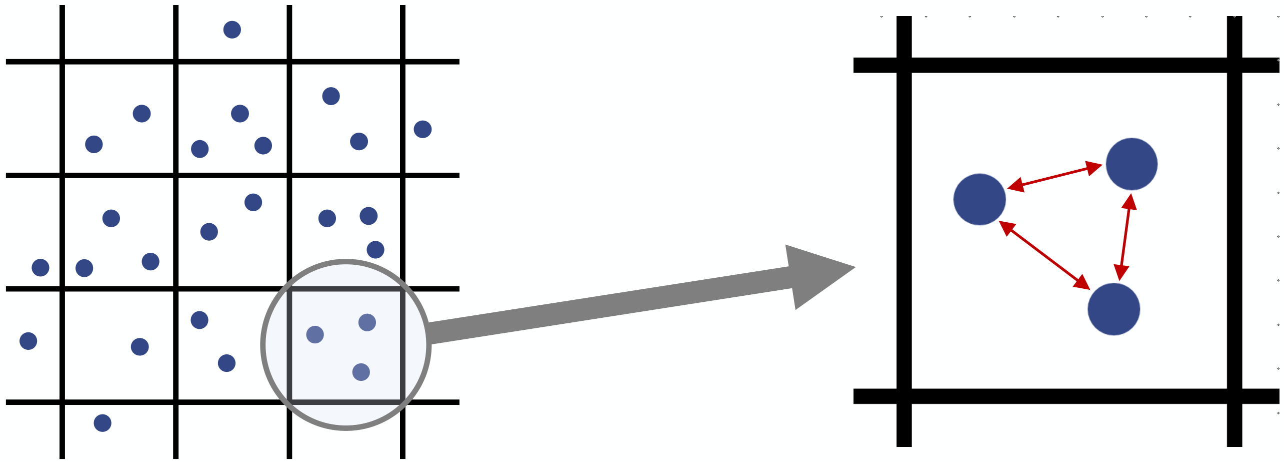
Stochastic Criterion[1]

Collision Criteria

ELASTIC COLLISIONS

- Divide space into cells
- Calculate probabilities for collisions between particles in one cell
- Monte-Carlo Method

$$P_{22} = \frac{\sigma}{N_{test}} v_{rel} \frac{\Delta t}{\Delta^3 x}$$



1 A. Lang, et al., J. Comp. Phys. **106**, 391 (1993)

Agenda

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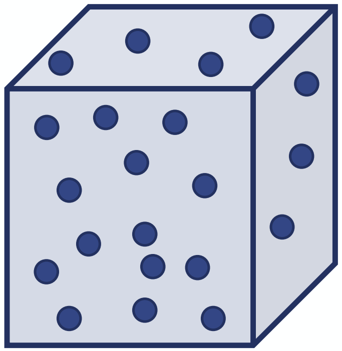
Elastic Box

First results

- Elastic Pion Box
  - Periodic Boundary Conditions
- Isotropic Cross Section
- Comparing Scattering Rates to analytical estimate:

$$N_{coll}^{analytic} = \frac{N}{2}\, \rho\, N_{test}\, \sigma\, \langle v \rangle\, t$$



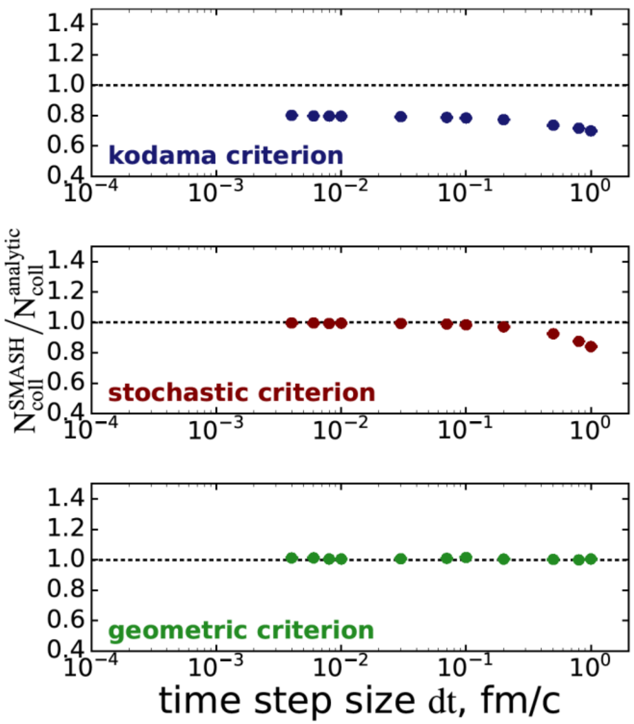
Left-hand side	Right-hand side
$\delta\tau_i > 0$	All $\delta\tau_i$

Elastic Box

First Results

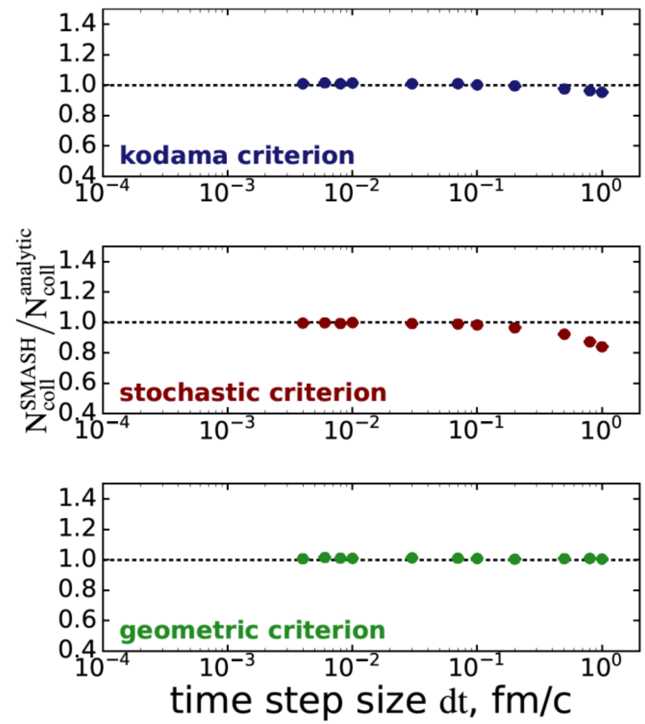
WORK IN PROGRESS

$\delta\tau_i > 0$



V = 1000 fm
σ = 1 fm^2
N = 200
T = 200 MeV
t_{tot} = 55 fm/c
N_{test} = 1

All $\delta\tau_i$

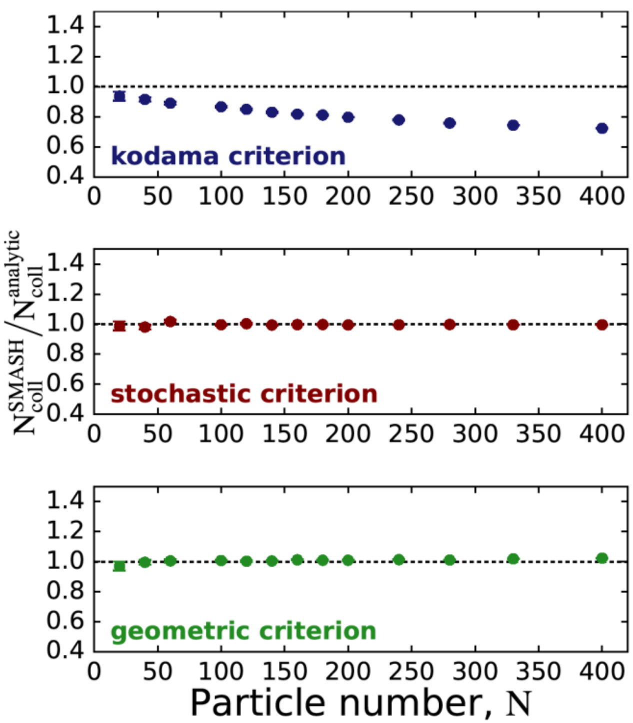


Elastic Box

First Results

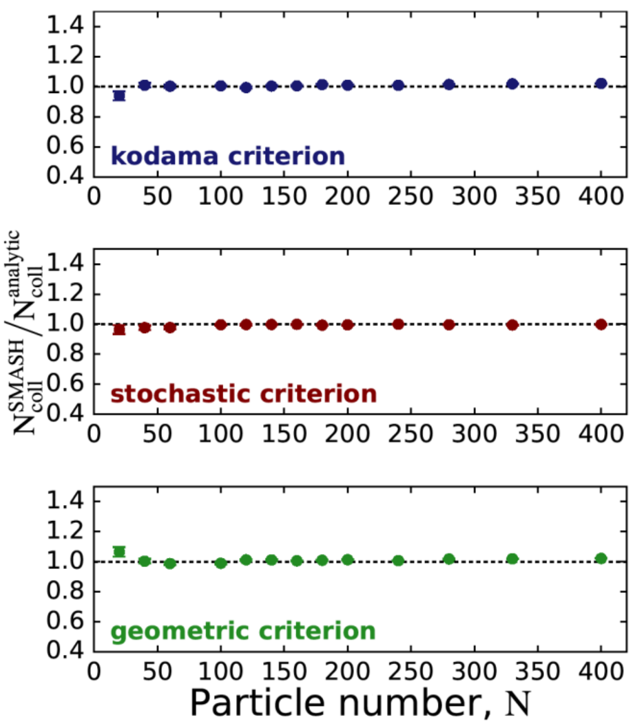
WORK IN PROGRESS

$\delta\tau_i > 0$



V = 1000 fm
σ = 1 fm²
T = 200 MeV
dt = 0.01 fm/c
t_{tot} = 55 fm/c
N_{test} = 1

All $\delta\tau_i$

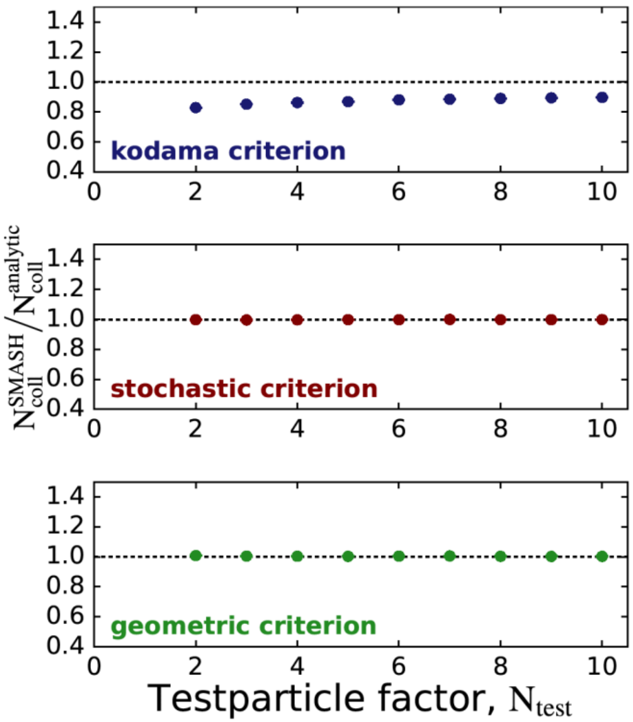


Elastic Box

First Results

WORK IN PROGRESS

$\delta\tau_i > 0$



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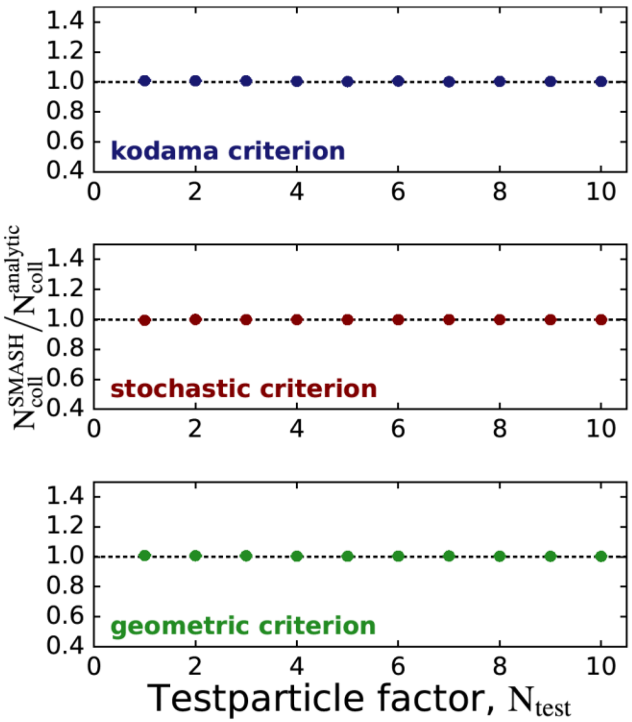
All $\delta\tau_i$

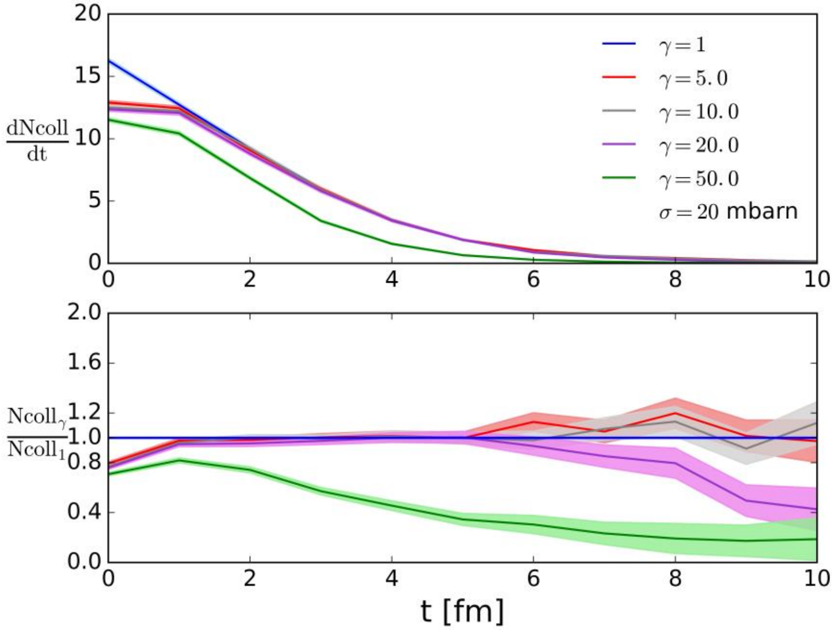


Boosted Sphere

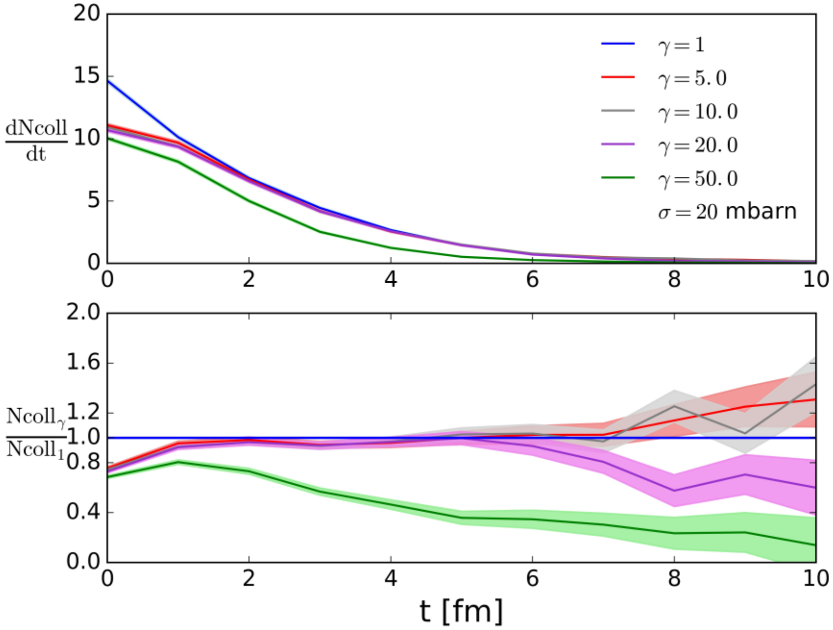
First Results

WORK IN PROGRESS

Geometric Criterion



Kodama Criterion



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Summary & Outlook

Wrap up

Summary

- Implemented first version of Kodama Criterion in SMASH
  - Ongoing work

- Causality criterion $\Delta\tau > 0$ reduces scattering rate significantly

- Time ordering effect visible for large timesteps

Outlook

- Create a test for causality
- Look into Collision Time determination
- Boost checks without timesteps

Thank you for your attention!

Damjan Mitrovic

Master Student Physics

mitrovic@fias.uni-frankfurt.de

Summary & Outlook

Wrap up

Summary

- Implemented first version of Kodama Criterion in SMASH
  - Ongoing work
- Causality criterion $\Delta\tau > 0$ reduces scattering rate significantly
- Time ordering effect visible for large timesteps

Outlook

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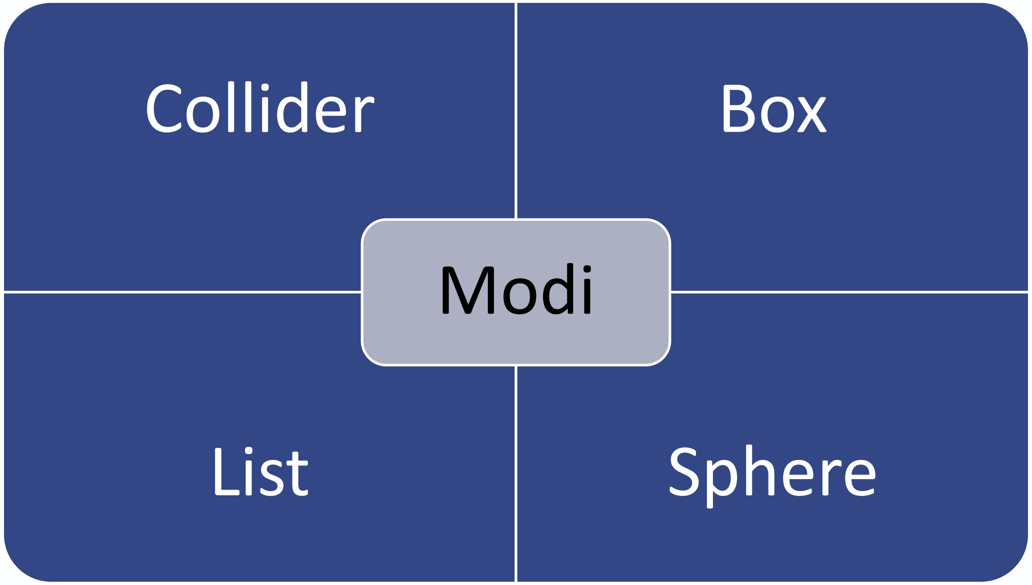
BACKUP

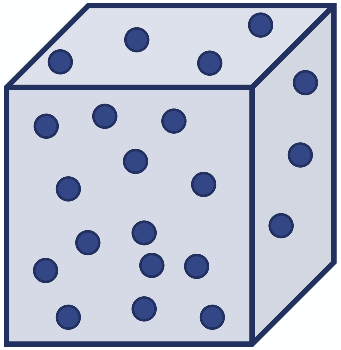
SMASH Modi

BACKUP



Scattering rate

BACKUP



- Scattering rate for ideal gas approximation
- Mean free path: $\lambda = \frac{1}{\rho\sigma}$ ⟹ $t_\lambda = \frac{1}{\rho\sigma\langle v\rangle}$
- Collisions per particle: $N_{coll_1} = \frac{t}{t_\lambda} = \rho\sigma\langle v\rangle t$
- Taking into account Testparticles: $N_{coll}^{analytic} = \frac{N}{2}\rho N_{test}\sigma\langle v\rangle t$
- Ratio: $\frac{N_{coll}^{SMASH}}{N_{coll}^{analytic}} = \frac{2}{N\rho\, N_{test}\, \sigma\, \langle v\rangle\, t} \frac{N_{coll}}{N_{events}}$

Kodama Formulas

BACKUP

$$\delta\tau_{j_m}(i_m) = Min\{\delta_{l_m}(l) > 0, l = 1, \dots, A; l \neq j_m\}$$

$$\delta\tau_{i_m}(j_m) = Min\{\delta_{l_m}(l) > 0, l = 1, \dots, A; l \neq i_m\}$$

$$\delta\tau_i(k) = \tau_c - \tau_i$$

$$\tau_c = \tau_i - \frac{\vec{v}_{ij}^* \cdot \vec{r}_{ij}^*}{\left|\vec{v}_{ij}^*\right|^2}$$

$$\delta\tau_i(j), \delta\tau_j(i) > 0$$