Hydrodynamization, hydrodynamic attractors and all that — an overview of the field

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1610.02023: lecture notes emphasizing the holographic component

1707.02282: review emphasizing the hydrodynamic component

state of the field: **www.birs.ca/events/2019/5-day-workshops/19w5048**

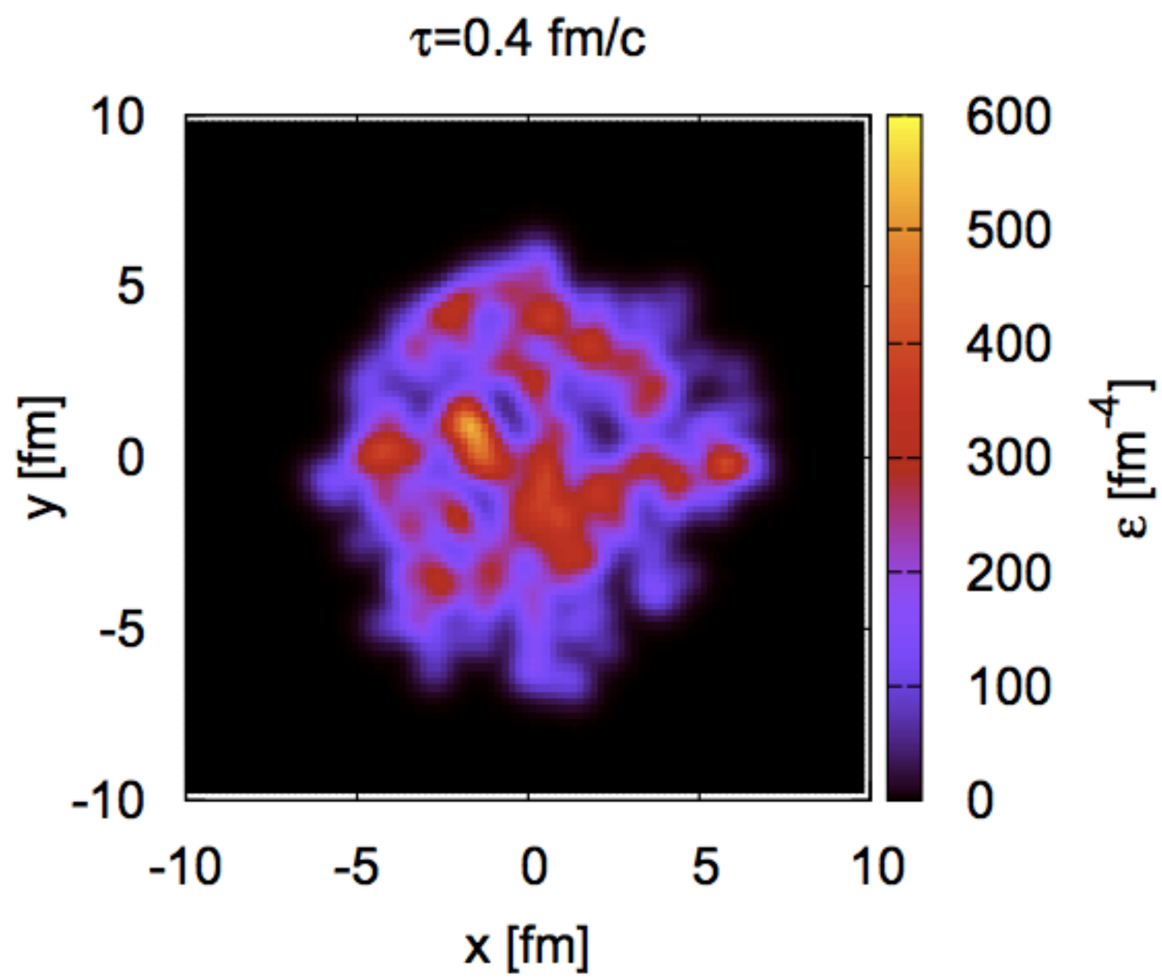
Introduction & motivation

// in this lecture the vast majority of results concern conformally-invariant theories without a set of dim-less scales (~ strong coupling) //

Motivation: hydrodynamic simulations for HIC

Generic energy density at the moment hydrodynamics simulation starts:

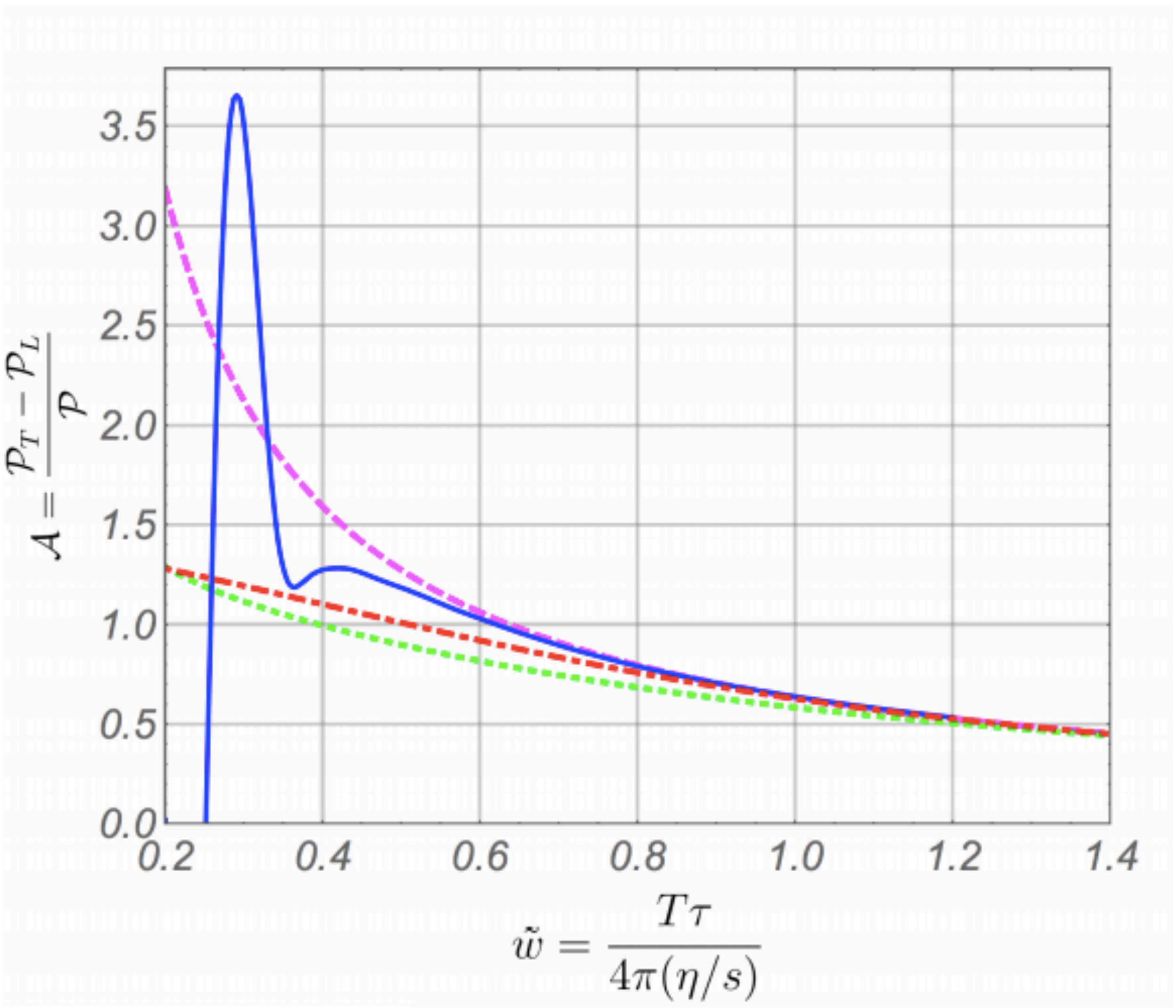
1009.3244 by Schenke, Jeon & Gale



Simulations like this match the data, however, they apply hydrodynamics in the regime of large gradients: $\frac{\Delta\mathcal{E}}{\Delta d} \times \left(\frac{\mathcal{E}_{\rm av}}{(\frac{1}{\mathcal{E}_{\rm av}/10})^{1/4}}\right)^{-1} \approx 1$. Does it even make sense?

Hydrodynamization

Ab initio studies in holography and later studies in other models show that viscous hydro can work even when deviations from local equilibrium are large:
0906.4426, 1011.3562 by Chesler & Yaffe; **1103.3452** with Janik & Witaszczyk



sample n-eq states in:

N=4 SYM

EKT with $\eta/s = 0.624$

RTA with $\eta/s = 0.624$

viscous hydro prediction:

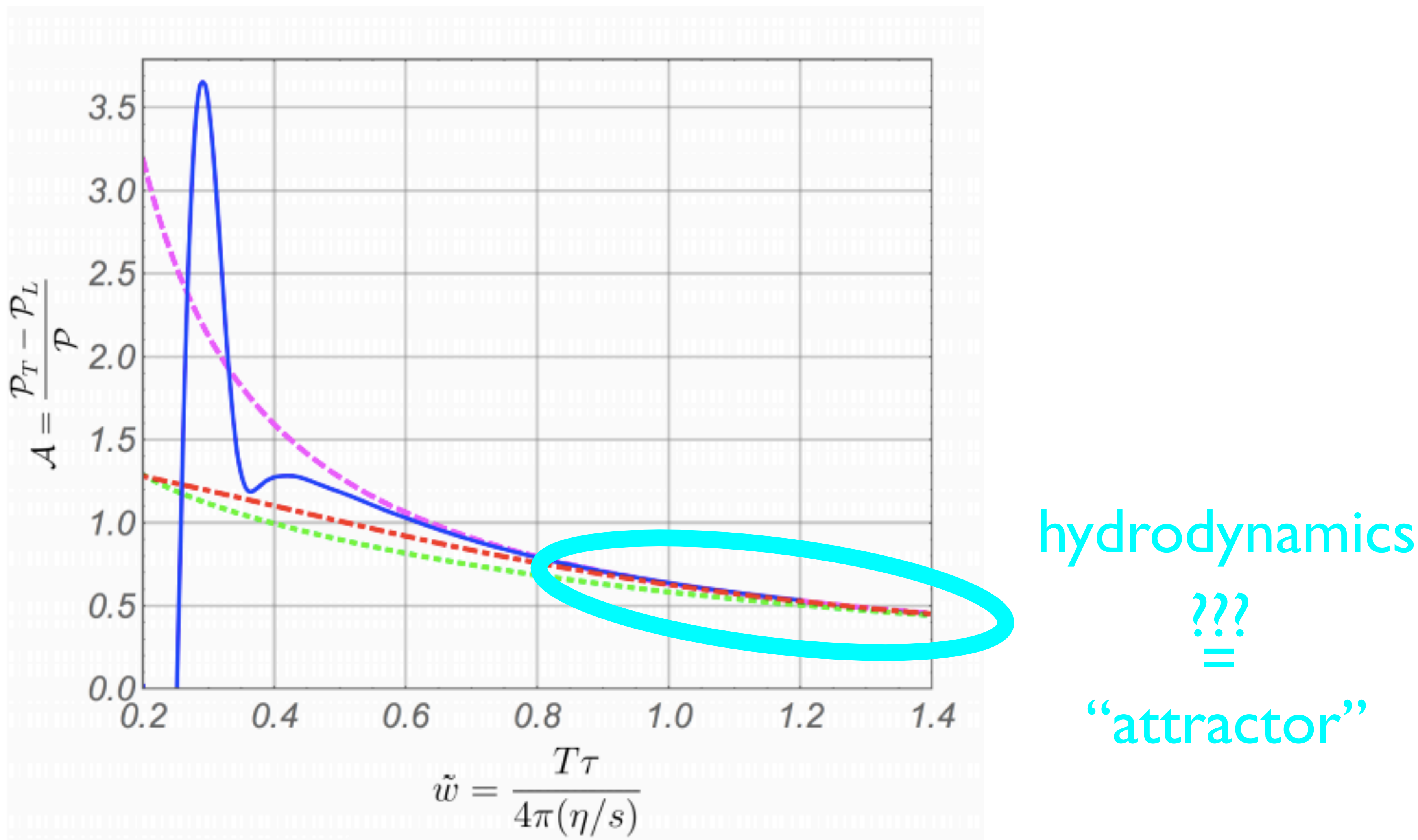
$$\frac{\Delta\mathcal{P}}{\mathcal{E}/3} = \frac{2}{\pi}\tilde{w}^{-1}$$

plot from **1609.04803v2** with Kurkela, Spalinski & Svensson

Preview: hydrodynamic attractors

Viscous hydrodynamics works* despite huge gradients in the system:

0906.4426, 1011.3562 by Chesler & Yaffe; **1103.3452** with Janik & Witaszczyk



plot from **1609.04803v2** with Kurkela, Spalinski & Svensson

Relativistic hydrodynamics - textbook definition

hydrodynamics is an EFT of the slow (?) evolution of conserved currents in collective media close to equilibrium (?)

DOFs: always local energy density ϵ and local flow velocity u^μ $\;(u_\nu u^\nu = -1)$

EOMs: conservation eqns $\nabla_\mu \langle T^{\mu\nu}\rangle = 0$ for $\langle T^{\mu\nu}\rangle$ <u>expanded in gradients</u>

$$\langle T^{\mu\nu}\rangle = \epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu\,\} \overbrace{- \eta(\epsilon)\,\sigma^{\mu\nu} - \zeta(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu\,\}(\nabla\cdot u) + \ldots}^{\pi^{\mu\nu}}$$

microscopic input:

EoS $\left(P(\epsilon) = \frac{1}{3}\epsilon\right.$ for CFTs)

shear viscosity contribution

$$\frac{\Delta\mathcal{P}}{\mathcal{E}/3} = \frac{2}{\pi}\tilde{w}^{-1}$$

bulk viscosity (vanishes for CFTs)

Here we assume Landau frame, c.f. 1907.08191 by Kovtun / 1708.06255 by Bemfica et al.

Hydrodynamic & transient modes

Theories of viscous hydrodynamics

The crucial subtlety: $\nabla_\mu\Big(\,\epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu\,\} - \eta(\epsilon)\,\sigma^{\mu\nu} \,+ \ldots\Big) = 0$ does not have a well-posed initial value problem $\longrightarrow$ hydrodynamic theories

(there is a brand new approach that deals with the problem somewhat differently, see 1907.08191 by Kovtun and 1708.06255 by Bemfica, Disconzi & Noronha)

Overall idea: make $\pi^{\mu\nu}$ obey an independent PDE ensuring its $\searrow$ to $-\eta\,\sigma^{\mu\nu}$

$$(\tau_\pi\, u^\alpha \mathcal{D}_\alpha + 1)\ [\pi^{\mu\nu} - (-\eta\,\sigma^{\mu\nu})] = 0 \longrightarrow \pi^{\mu\nu} = -\eta\,\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha\, \pi^{\mu\nu} - \cancel{\tau_\pi\, u^\alpha \mathcal{D}_\alpha\, (\eta\,\sigma^{\mu\nu})}$$

decay timescale

Müller 1967, Israel 1976, Israel & Stewart 1976

New incarnation: **0712.2451** by Baier, Romatschke, Son, Starinets & Stephanov

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} + \lambda_1 \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha}$$

Modes in BRSSS theory

Mode = solution of linearized equations of finite-T state without any sources

Technical issue: tensor perturbs.$\longrightarrow$ channels (**here and later sound channel**):

Assuming momentum along x³ direction $e^{-i\,\omega\,x^0 + i\,k\,x^3}$: δT, δu^3 & $\delta\pi^{33}$

conservation

$+$

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} + \lambda_1 \pi^{\langle\mu}{}_{\alpha}\,{}^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_{\alpha}\Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_{\alpha}\Omega^{\nu\rangle\alpha}$$

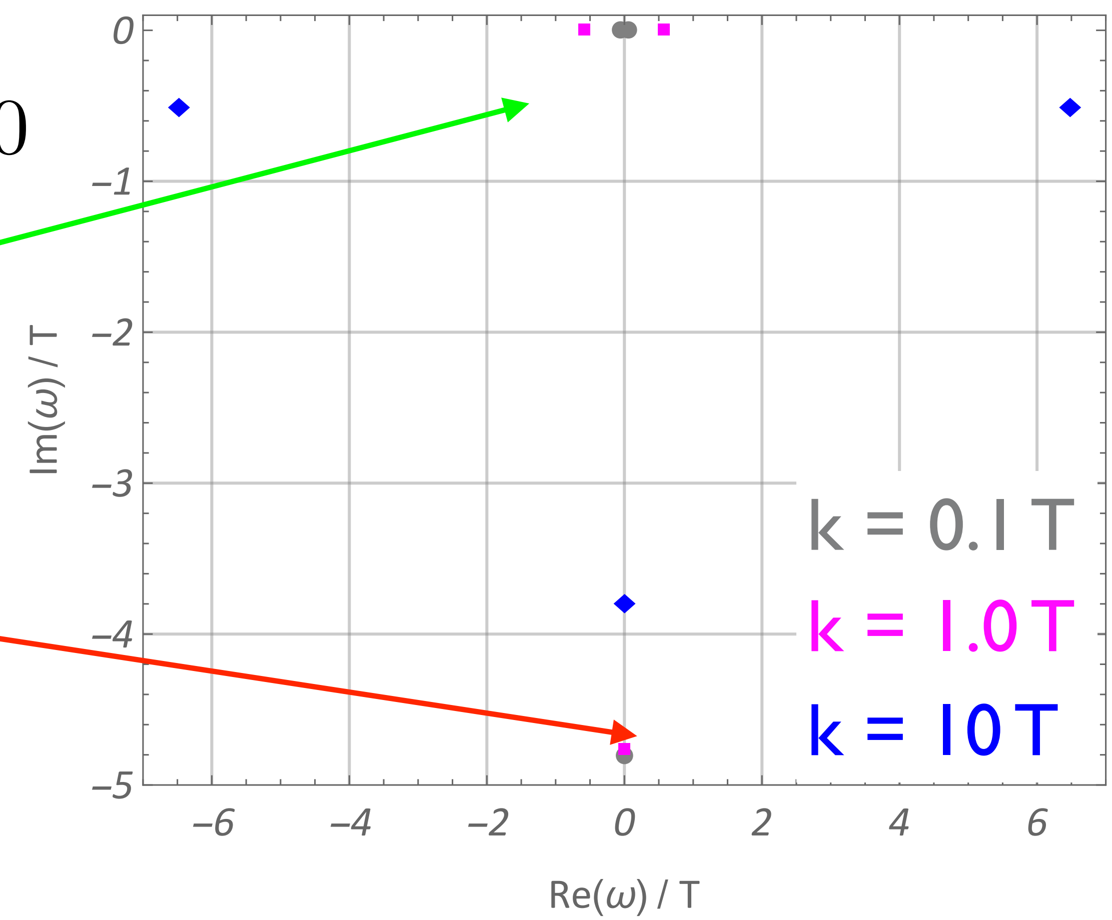
$$\omega^3 + (\ldots)\,\omega^2 + (\ldots)\omega + (\ldots) = 0$$

two modes:

hydro (sound wave)

transient (pure decay):

$$\omega\Big|_{k=0} = -i\,\frac{1}{\tau_\pi}$$



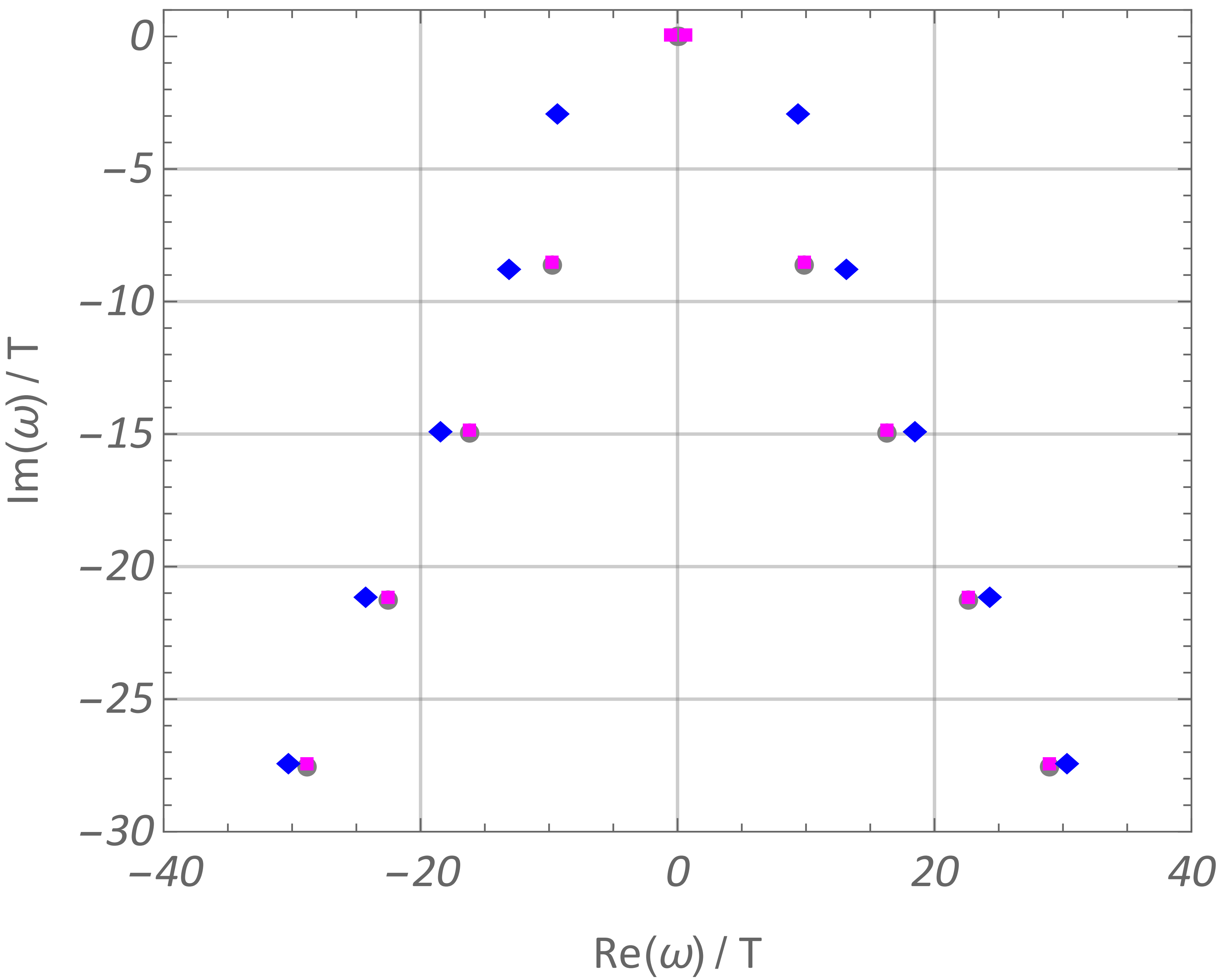
Modes in Einstein-Hilbert holography = QNMs

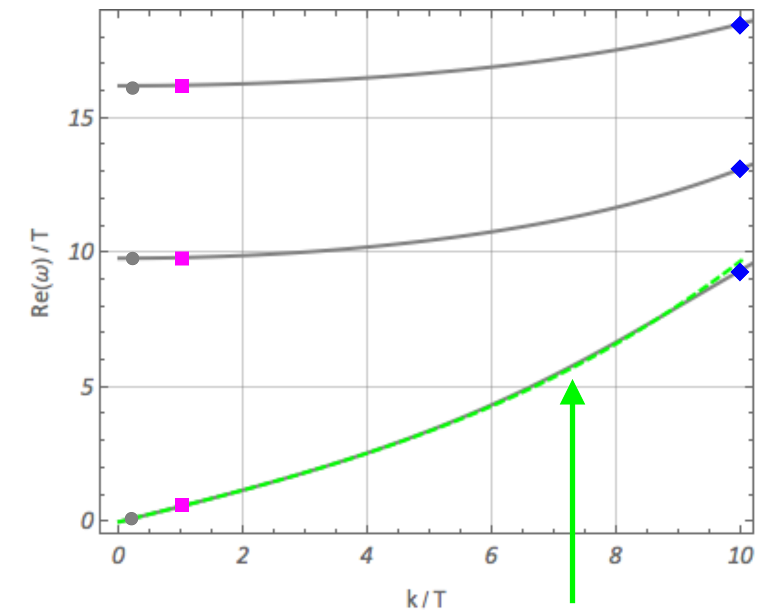
hep-th/0506184 by Kovtun & Starinets

vanishing at the boundary

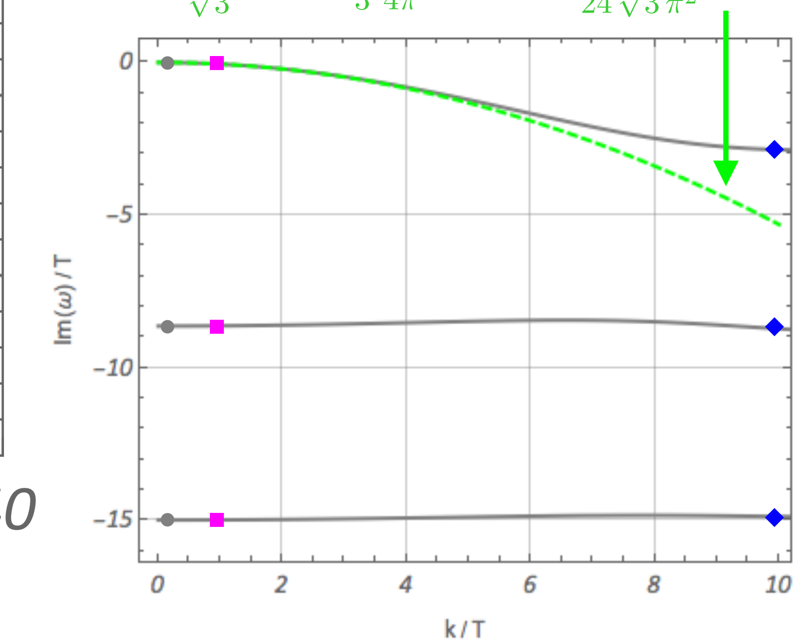
$$ds^2 = \frac{L^2}{u^2}\left\{-2dx^0du - (1-\pi^4T^4u^4)\left(x^0\right)^2 + d\vec{x}^2\right\} + \delta g_{ab}(u)\, e^{-i\,\omega\, x^0 + i\, k\, x^3}$$

ingoing (regular) at the horizon





$$\omega/T \approx \pm\frac{1}{\sqrt{3}}\,k/T - i\,\frac{2}{3}\,\frac{1}{4\pi}\,(k/T)^2 \pm \frac{3-2\log 2}{24\sqrt{3}\,\pi^2}\,(k/T)^3$$

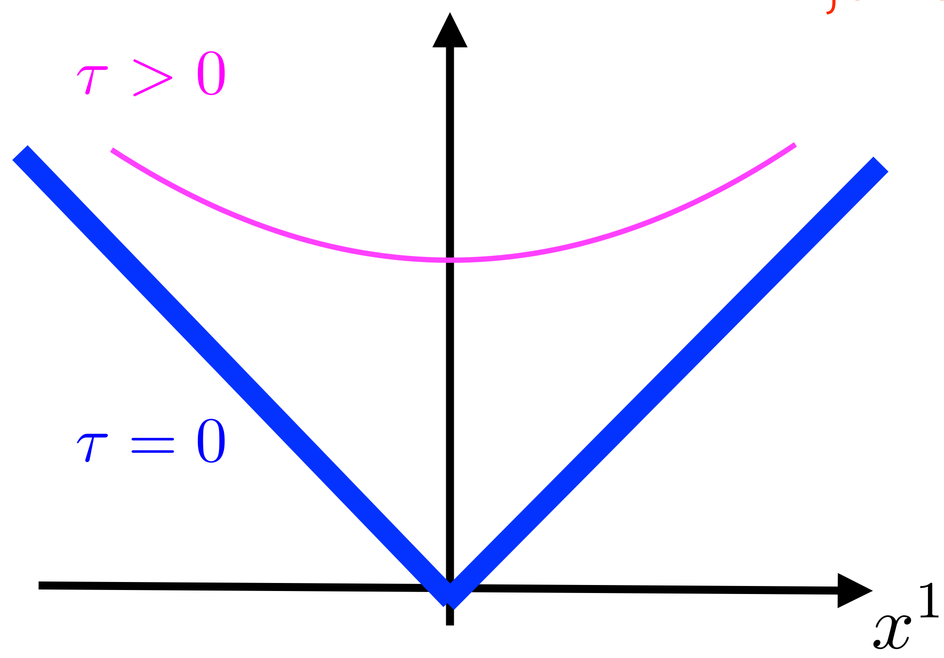


Hydrodynamics at large orders

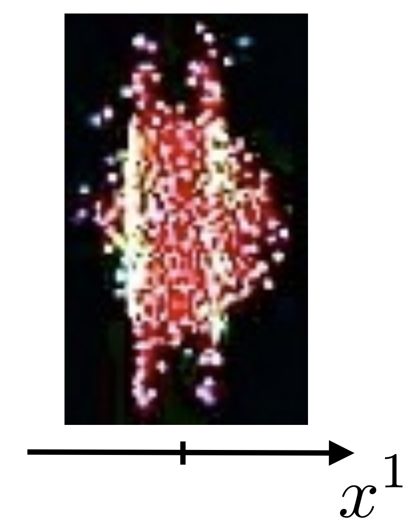
here focus on **1503.07514** with Spalinski

see also **1302.0697** with Janik & Witaszczyk for an earlier work

Boost-invariant flow Bjorken 1982



const x^0 slice:



Boost-invariance: in $\left(\tau \equiv \sqrt{x_0^2 - x_1^2},\quad y \equiv \operatorname{arctanh}\frac{x_1}{x_0},\ x_2,\ x_3\right)$ coords no y-dep

In a CFT: $\langle T^\mu_{\ \nu}\rangle = \mathrm{diag}\left\{-\mathcal{E}(\tau), -\mathcal{E} - \tau\,\dot{\mathcal{E}},\ \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}},\ \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}}\right\}$

$$\langle T^2_{\ 2}\rangle - \langle T^y_{\ y}\rangle \equiv \Delta\mathcal{P} \qquad\qquad \left(\frac{\mathcal{E}(\tau)}{\frac{3}{8}\pi^2 N_c^2}\right)^{1/4} \equiv T$$

and via scale-invariance $\dfrac{\Delta\mathcal{P}}{\mathcal{E}/3} \equiv \boxed{\mathcal{A}}$ is a function of $\boxed{w} \equiv \tau\, T$

Gradient expansion: series in $\dfrac{1}{w}$ **1103.3452** with Janik & Witaszczyk

Large order gradient expansion: BRSSS

1503.07514 with Spalinski

conservation (always the same) $\longrightarrow$ $\dfrac{\tau}{w}\dfrac{dw}{d\tau} = \dfrac{2}{3} + \dfrac{1}{18}\mathcal{A}$

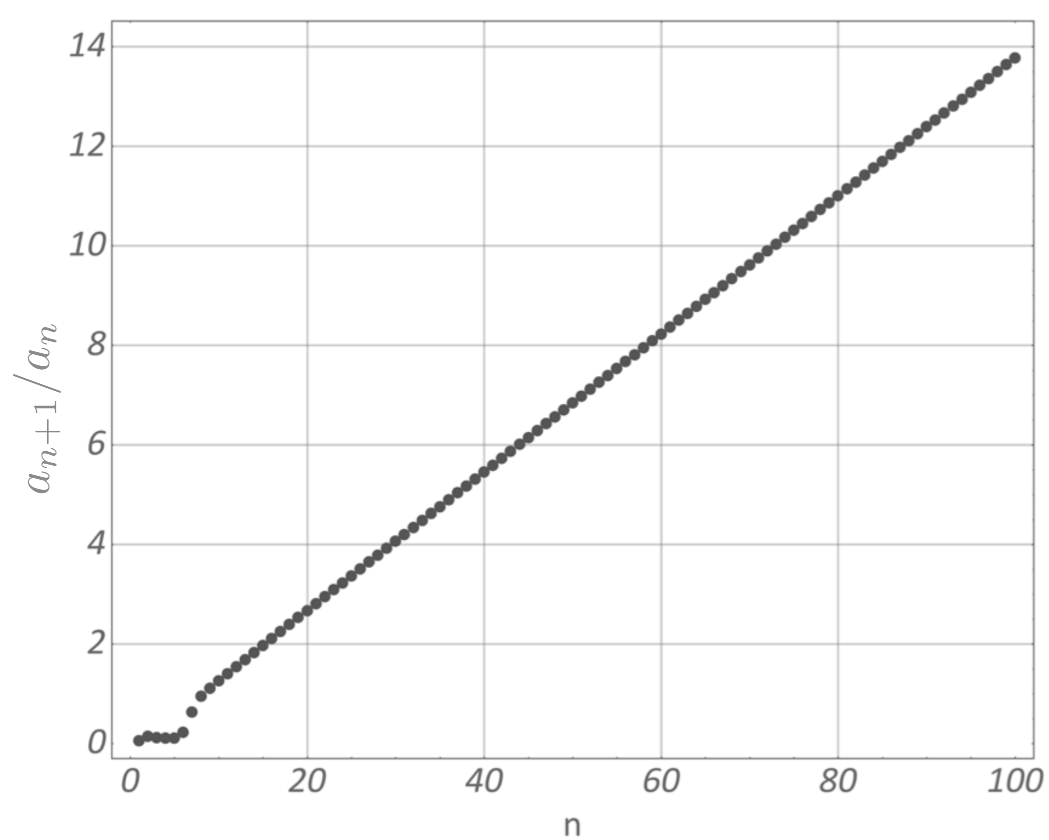
$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi\, u^\alpha \mathcal{D}_\alpha \pi^{\mu\nu} +$$
$$+\,\lambda_1 \pi^{\langle\mu}{}_\alpha \pi^{\nu\rangle\alpha} + \lambda_2 \pi^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha} + \lambda_3 \Omega^{\langle\mu}{}_\alpha \Omega^{\nu\rangle\alpha}$$

$$\rightarrow \boxed{C_{\tau_\pi}\, w\left(1 + \frac{1}{12}\mathcal{A}\right)\mathcal{A}' + \left(\frac{1}{3}C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right)\mathcal{A}^2 + \frac{3}{2}\,w\,\mathcal{A} - 12\,C_\eta = 0}$$

$$\left(\eta = \underbrace{C_\eta}_{=\frac{1}{4\pi}}\,\mathcal{S}, \qquad \tau_\pi = \frac{\overbrace{C_{\tau_\pi}}^{=\frac{2-\log 2}{2\pi}}}{T}, \qquad \lambda_1 = \underbrace{C_{\lambda_1}}_{=\frac{1}{2\pi}}\frac{\eta}{T}\right)$$

$$\mathcal{A}(w) \approx \sum_{n=1}^{\infty} \frac{a_n}{w^n} = 8\,C_\eta\,\frac{1}{w} + \frac{16}{3}C_\eta\left(C_{\tau_\pi} - C_{\lambda_1}\right)\frac{1}{w^2}\boxed{+\ldots} \rightarrow$$

(note that a_n's do not depend on ini. cond.)



Divergent series: $a_n \sim n!$

Transseries and resurgence 1503.07514 with Spalinski

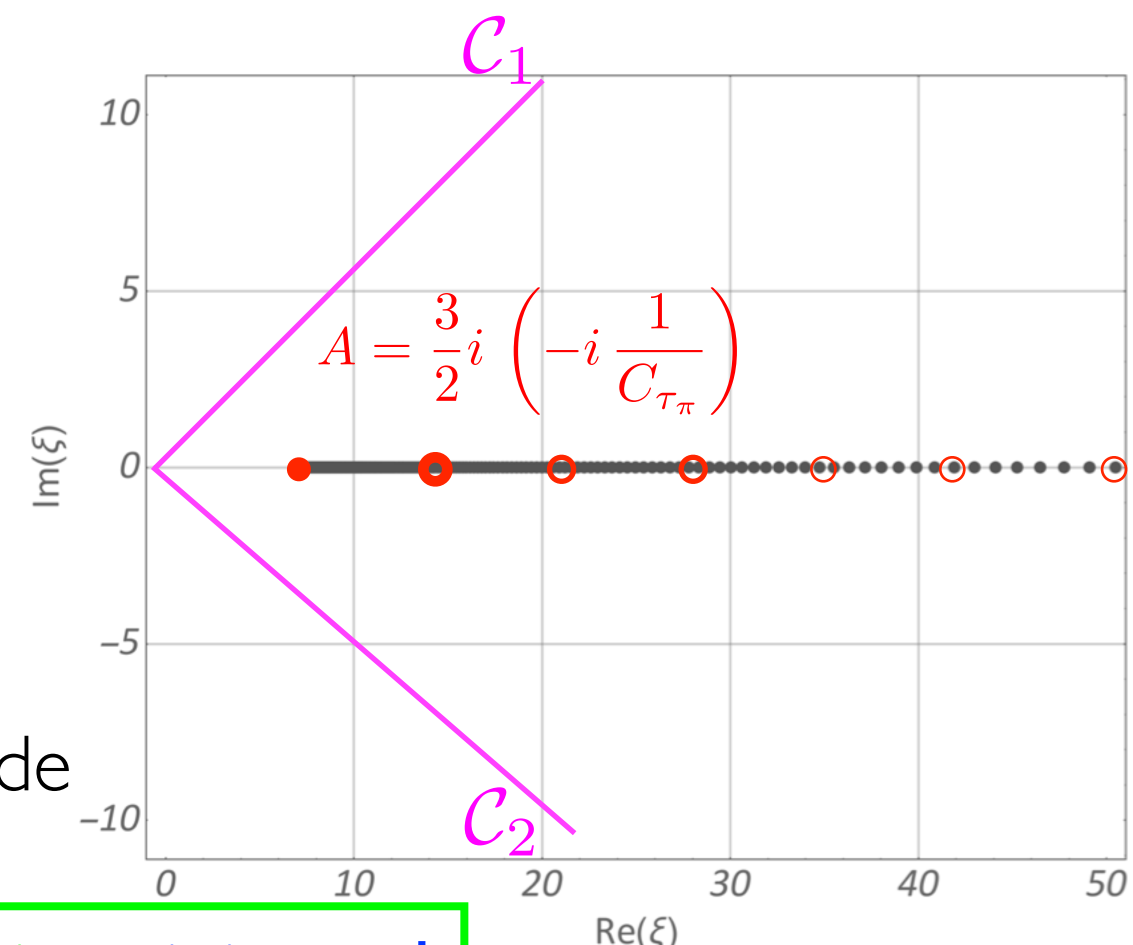
$$\mathcal{A}(w) \approx \sum_{n=1}^{\infty} \frac{a_n}{w^n} \xrightarrow[\text{Borel trafo.}]{} B\mathcal{A}(\xi) = \sum_{n=1}^{\infty} \frac{a_n}{n!}\,\xi^n \approx \frac{b_0 + \ldots + b_{100}\,\xi^{100}}{c_0 + \ldots + c_{100}\,\xi^{100}}$$

Borel (re)summation:

$$\left(\int_{\mathcal{C}_1} d\xi - \int_{\mathcal{C}_2} d\xi\right) \left[w\, e^{-w\,\xi} B\mathcal{A}(\xi)\right]$$

$$\sim e^{-\overbrace{\frac{3}{2}\,\frac{1}{C_{\tau_\pi}}}^{\equiv A} w}\, w^{\overbrace{\frac{C_\eta - 2\,C_{\lambda_1}}{C_{\tau_\pi}}}^{\equiv \beta}} \ldots$$

Ambiguity in resummation ~ transient mode



nonlinear effects

~ resum. ambig. + ini. cond.

Transseries: $\mathcal{A}(w) = \sum_{j=0}^{\infty} \sigma^j\, e^{-j\,A\,w}\, w^{j\,\beta}\, \Phi_{(j)}(w)$

~ 1/w expansions

Resurgence: transseries yields an unambiguous answer up to 1 real int. const.

Attractors

1503.07514 with Spalinski

1704.08699 by Romatschke

… ~15 other distinct studies of attractors …

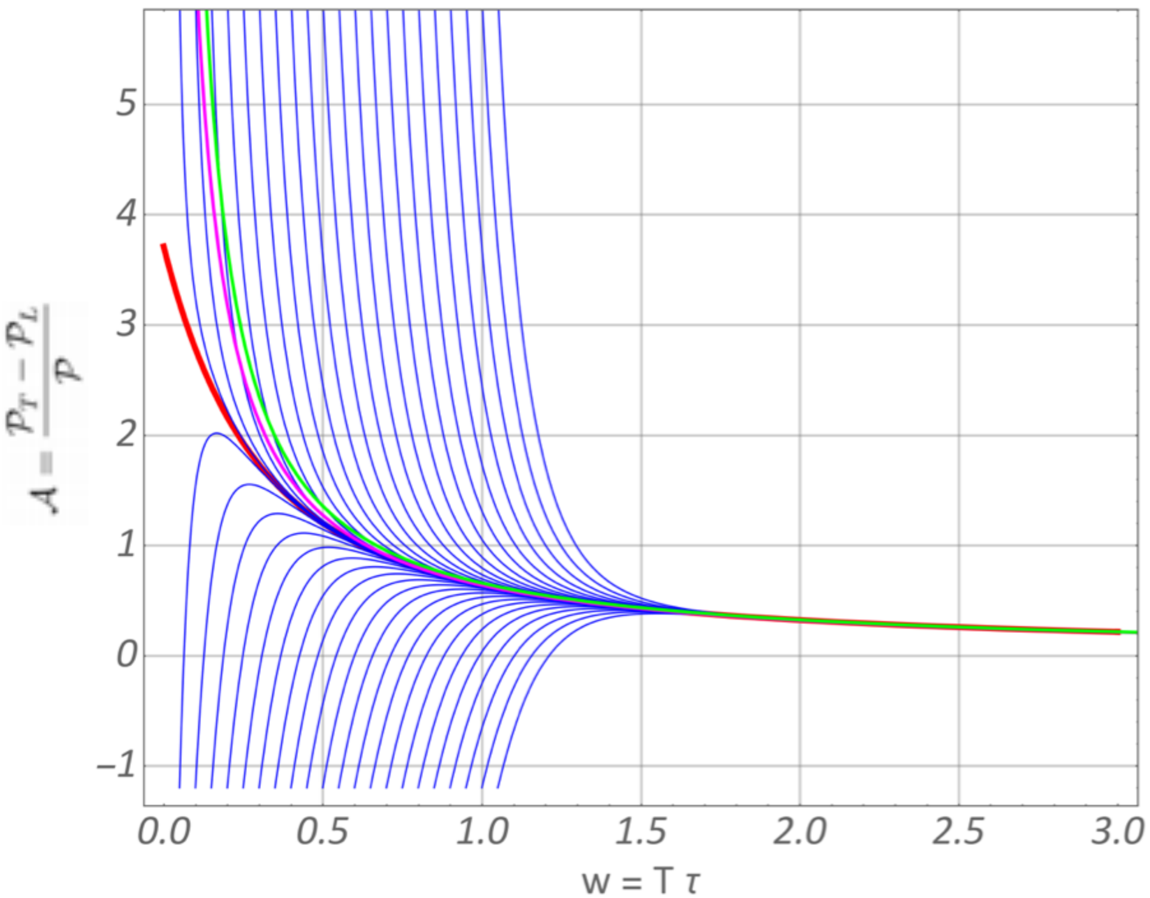
1907.08101 by Kurkela, van der Schee, Wiedemann & Wu

1912.xxxxx with Jefferson, Spalinski & Svensson, see **https://bit.ly/2DCt0bl**

(BRSSS) resummed hydrodynamics

1503.07514 with Spalinski

Idea: resummed / far-from-equilibrium hydrodynamics = attractor solutions



BRSSS:

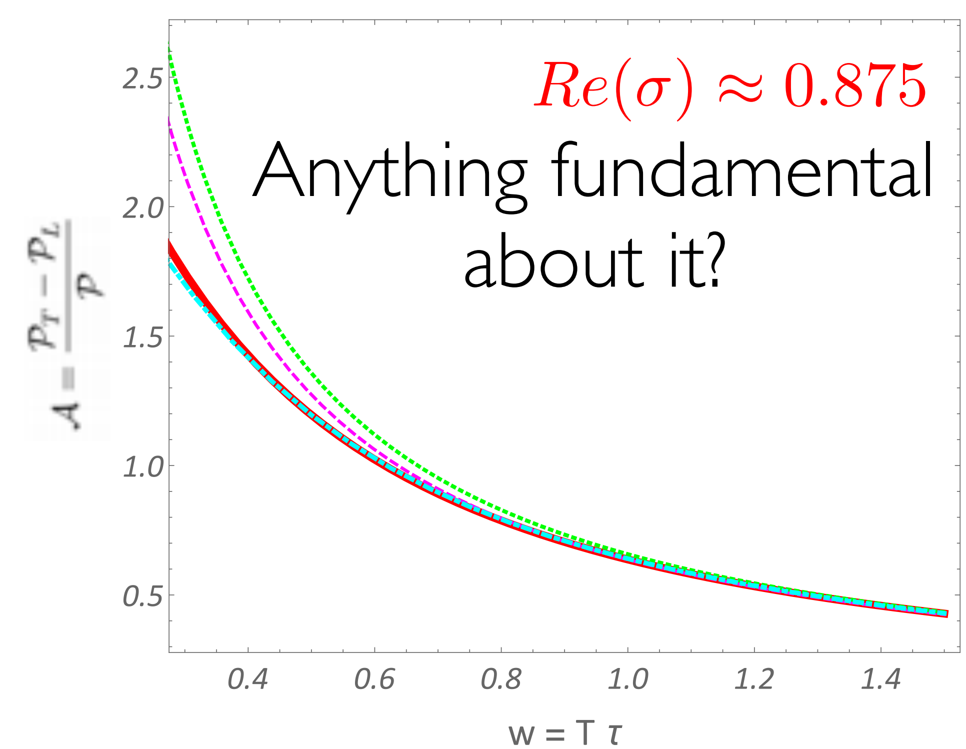
$$\cancel{C_{\tau_\pi}\, w \,(1 + \frac{1}{12}\mathcal{A})\,\mathcal{A}'} + \left(\frac{1}{3} C_{\tau_\pi} + \frac{1}{8}\frac{C_{\lambda_1}}{C_\eta} w\right)\mathcal{A}^2 + \frac{3}{2}\, w\, \mathcal{A} - 12\, C_\eta = 0$$

„slow roll'' approximation
reveals an attractor solution

One can also approx. resum transseries:

$$\mathcal{A}(w) \approx \sum_{j=0}^{2} \sigma^j\, e^{-j\, A\, w}\, w^{j\,\beta}\, \Phi_{(j)}(w)$$

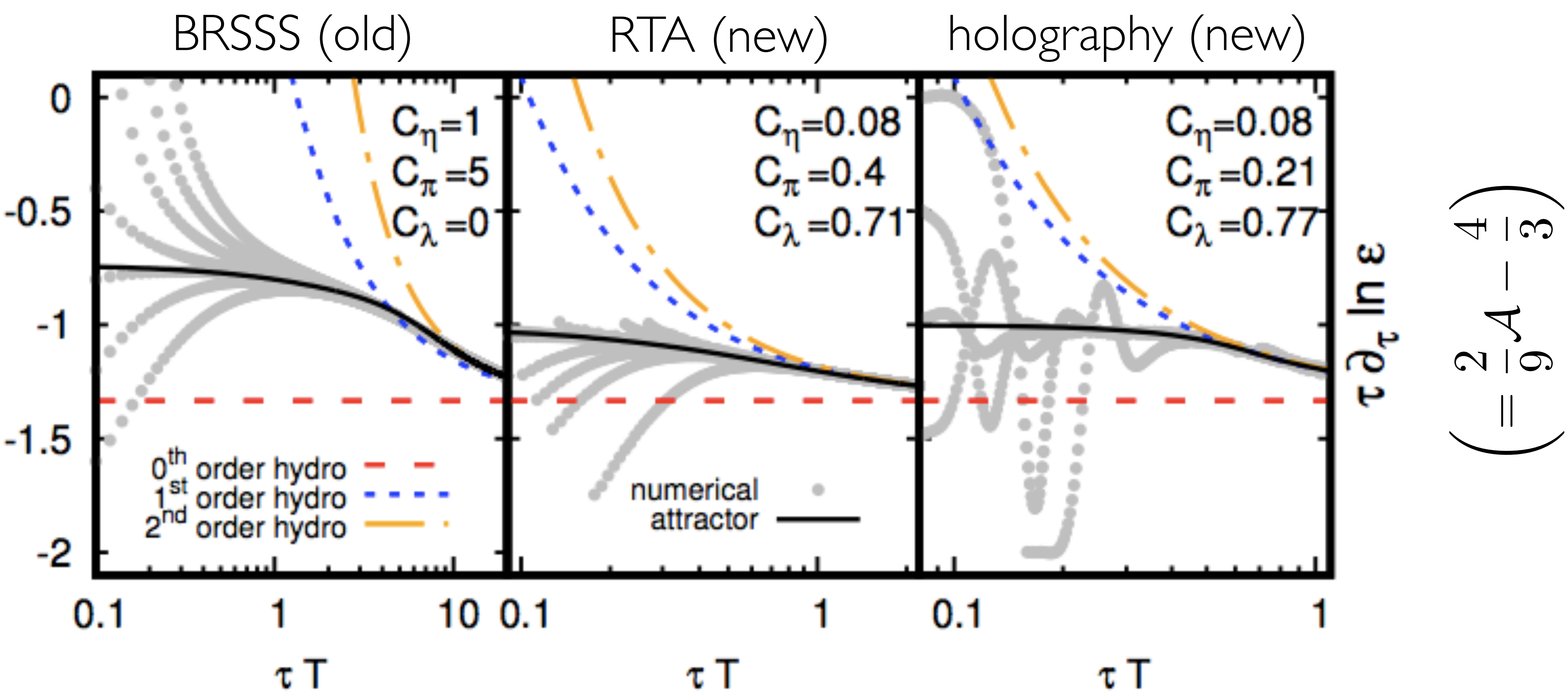
Requires 3 Borel summations



Attractor in kinetic theory and holography

1704.08699 by Romatschke (figure imported from the arXiv ver)

Idea: use the slow roll approximation to generate attractors in other theories



Longitudinal expansion important for the initial memory loss of solutions

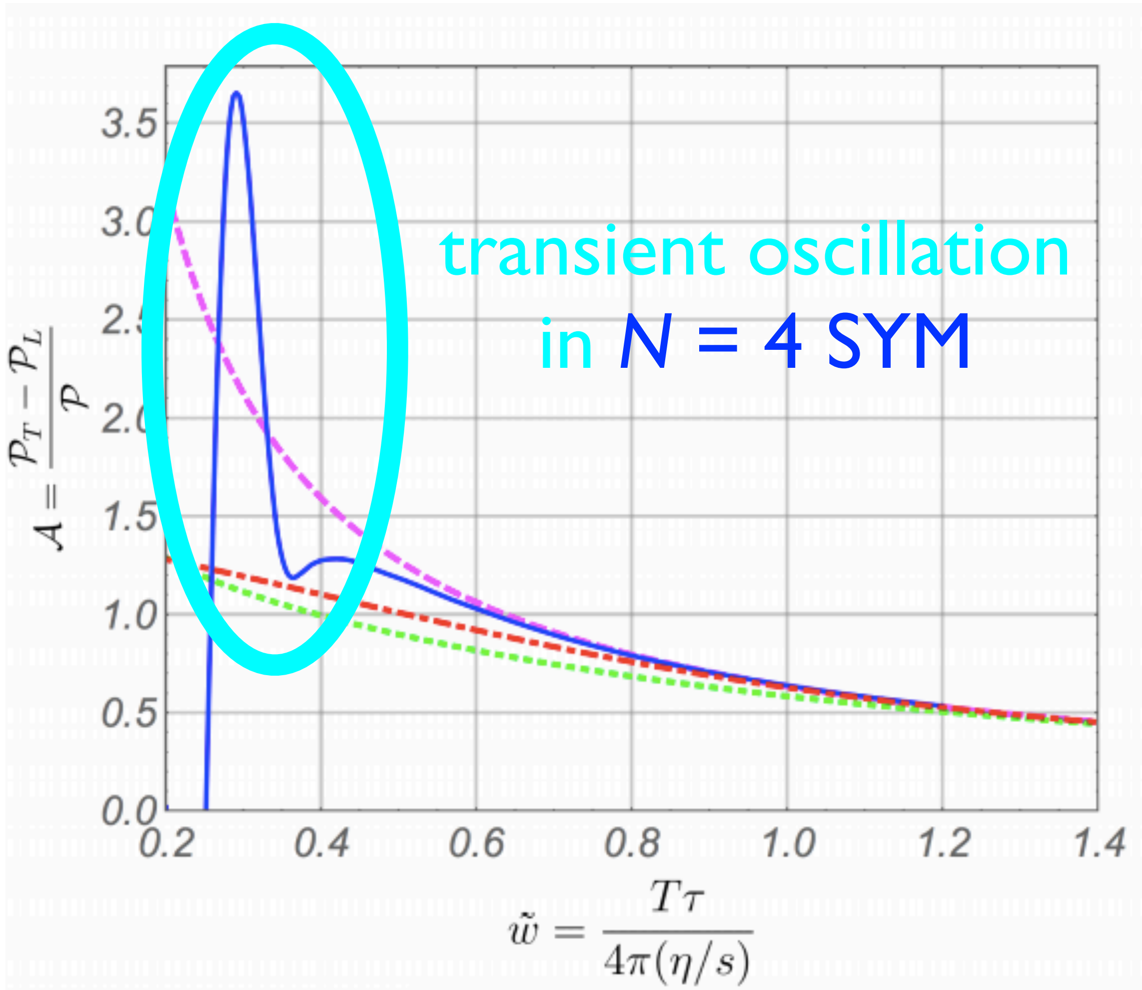
1907.08101 by Kurkela, van der Schee, Wiedemann & Wu

Note: centre and right are projections from infinitely-dimensional phase space

Executive summary

Executive summary

What seems to control the applicability of hydrodynamics is not the gradient expansion itself, but what comes on top of it — expansion & transients



As a result, applying hydro to HIC early on is not a priori crazy

Hydrodynamic attractors = intermediate-to-late time universalities