Deuteron (and cluster) production

Radka Sochorová
FNSPE CTU in Prague
Zimányi School '19, Budapest

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Get introduced: the deuteron

Wave functions

- **spherical harmonic oscillator** \((d = 3.2 \text{ fm})\)

  \[
  \phi_d(r) = (\pi d^2)^{-3/4} \exp\left(-\frac{r^2}{2d^2}\right)
  \]

- **Hulthen form** \((\alpha = 0.23 \text{ fm}^{-1}, \beta = 1.61 \text{ fm}^{-1})\)

  \[
  \phi_d(r) = \sqrt{\frac{\alpha \beta (\alpha + \beta)}{2\pi (\alpha - \beta)^2}} \frac{e^{-\alpha r} - e^{-\beta r}}{r}
  \]

  \(r\) ... distance between proton and neutron
  rms radius: 1.96 fm
  binding energy: 2.2 MeV
Clusters and statistical model

Universal description with the statistical model

Is this well-known feature, or is this a result of fine-tuning (transport model)?

What does it actually tell us?

Clusters actually carry \textit{femtoscopic} information about the freeze-out → motivation for studying clusters

Thermal model vs. coalescence model

**Thermal model**

- at $T_{ch} \rightarrow$ chemical equilibrium, inelastic collisions are ineffective to change abundances of individual species
- below $T_{ch} \rightarrow$ yields unchanged, elastic scattering until kinetic freeze-out

**Coalescence model**

- light nuclei $\rightarrow$ formed only after the breakup of the fireball $\rightarrow$ by recombination of protons and neutrons
Thermal model vs. coalescence model

- for Pb-Pb collisions at the LHC - thermal model describes yields of all hadron species with $T \approx 156$ MeV

- light nuclei seem to behave like ALL other hadrons

- VERY SURPRISING - it is hard to imagine that loosely bound sizeable nuclei can exist in the HOT and DENSE hadron gas

- both models $\rightarrow$ similar deuteron yields

- possible difference for elliptic flow
Production mechanism: coalescence

Projection of the deuteron density matrix onto two-nucleon density matrix

Deuteron spectrum:

\[ E_d \frac{dN_d}{d^3 P_d} = \frac{3}{8(2\pi)^3} \int \frac{P_d \cdot d \Sigma_f(R_d) f_p(R_d, P_d/2) f_n(R_d, P_d/2) C_d(R_d, P_d)}{f^2(R_d, P_d/2)} |\phi_d(\vec{r})|^2 \]

QM correction factor

\[ C_d(R_d, P_d) \approx \int d^3 r \frac{f(R_+, P_d/2) f(R_-, P_d/2)}{f^2(R_d, P_d/2)} |\phi_d(\vec{r})|^2 \]

\( r \) ... relative position
\( R_+, R_- \) ... positions of nucleons

approximation: narrow width of deuteron Wigner function in momentum
DRAGON - Monte Carlo generator

- blast-wave model
- 277 particles included - stable and resonances

- Cooper-Frye formula ($p_T$ spectrum):

\[
E_p \frac{d^3 N}{dp^3} = \frac{d^3 N}{p_T dp_T dy d\phi} = \int d\Sigma_\mu(x) \cdot p^\mu f_{eq}(p^\alpha \cdot u_\alpha(x))
\]

\[
f_{eq}(p^\alpha \cdot u_\alpha) = \frac{1}{(2\pi)^3} \left[ \exp\left( \frac{p^\alpha \cdot u_\alpha - \mu}{T} \right) - \epsilon \right]^{-1}
\]

transverse expansion

\[
\nu_t = \tanh(\eta_t)
\]
\( p_t \) spectrum can be developed into Fourier expansion

\[
E \frac{d^3}{dp^3} = \frac{1}{2\pi} \frac{d^2N}{p_t dp_t dy} \left( 1 + 2 \sum_{n=1}^{\infty} \nu_n \cos(n(\phi - \Theta_n)) \right) \tag{1}
\]

where anisotropic flow \( \nu_n \)

\[
\nu_n = \frac{\int_0^{2\pi} d\phi \cos(n(\phi - \Theta_n)) \frac{d^3 N_d}{p_T dp_T dy d\phi}}{\int_0^{2\pi} d\phi \frac{d^3 N_d}{p_T dp_T dy d\phi}} \tag{2}
\]

- \( \nu_1 \) ... directed flow coefficient
- \( \nu_2 \) ... elliptic flow coefficient
- expansion anisotropy \( \rho(\bar{r}, \Theta_b) = \bar{r} \eta_f (1 + \sum_{n=2}^{\infty} 2\rho_2 \cos(2(\Theta_b - \Theta_2))) \)
- spatial anisotropy \( R(\Theta) = R_0 (1 - \sum_{n=2}^{\infty} a_2 \cos(2(\Theta - \Theta_2))) \)
protons and neutrons generated by Monte Carlo generator

each p-n pair $\rightarrow$ momentum and position of $p$ and $n$ boosted to the 2-particle rest-frame

particle that decoupled earlier $\rightarrow$ propagated to the later time of the other particle

relative momenta $\Delta p = |\vec{p}_1 - \vec{p}_2|$ and relative distances $\Delta r = |\vec{x}_1 - \vec{x}_2|$
Firstly, we fitted proton spectra and elliptic flow - then we used these parameters for describing deuterons spectra and $v_2$

- 0 – 5% centrality: $T = 80$ MeV, $\eta_f = 1.4$
- 30 – 40% centrality: $T = 100$ MeV, $\eta_f = 1.3$
0 − 5% centrality: $T = 80$ MeV, $\eta_f = 1.4$, $R_b = 10$ fm, $\Delta p \leq 0.200$ GeV/c, $\Delta r \leq 2.1$ fm

30 − 40% centrality: $T = 100$ MeV, $\eta_f = 1.3$, $R_b = 6$ fm, $\Delta p \leq 0.200$ GeV/c, $\Delta r \leq 2.1$ fm
Conclusions

- we added coalescence to Monte Carlo generator DRAGON (blast-wave model) - this is only pilot study
- we found that this model is not very good (problems with fitting $v_2$) → it needs improvements

OUTLOOK:

- $v_2$ for other types of particles, for other clusters, ...
- comparison with direct deuteron production
- we could add also other parameters: baryochemical potentials, ...
Backup slides
- 0 – 5% centrality: $T = 80$ MeV, $\eta_f = 1.4$, $R_b = 10$ fm
- 30 – 40% centrality: $T = 100$ MeV, $\eta_f = 1.3$, $R_b = 6$ fm
0 − 5% centrality: \( T = 80 \text{ MeV}, \eta_f = 1.4, R_b = 10 \text{ fm} \)

30 − 40% centrality: \( T = 100 \text{ MeV}, \eta_f = 1.3, R_b = 6 \text{ fm} \)