Negative pressure in nucleons?
A white (bland) spot in HEP

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Geometrical scaling (GS), saturation and unitarity

1. On-shell (hadronic) reactions \((s,t, Q^2=m^2)\);
   
   \[ t \leftrightarrow b \text{ transformation: } h(s, b) = \int_0^\infty d\sqrt{-t} \sqrt{-t} A(s, t) \]

   and dictionary:
DEEP INELASTIC SCATTERING (DIS) – a microscope

\[
\begin{align*}
\text{e} & \rightarrow \text{e}' \\
\text{k} & \rightarrow \text{k}' \\
\alpha^{1/2} & \rightarrow \gamma \\
-q^2 = Q^2 & \\
p & \rightarrow X
\end{align*}
\]

V. Gribov, L. Lipatov; Alterelli, Parisi; Dokshitzer = DGLAP
Structure of the Proton

3 “valence quarks”

“sea” of gluons and short lived $qq$ pairs
A diagram illustrating a holographic setup with labeled components. The setup includes a laser, splitter, lens, mirror, object, reference beam, holographic plate, and other relevant optics.
From GPD to GFF $d_1(t)$ to $p(r)$

**DVCS**

![Diagrams showing DVCS process]

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**BH**

![Diagrams showing BH process]

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Polarized beam, unpolarized target:

$$\Delta \sigma_{LU} \sim \sin \phi \text{ Im}\{F_1 \mathcal{H} + ..\}$$

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Bessel Integral relates $d_1(t)$ to the radial pressure $p(r)$.

$$d_1(t) \propto \int d^3r \frac{j_0(r \sqrt{-t})}{2t} p(r)$$
GPDs cannot be measured directly, instead they appear as convolution integrals, difficult to be inverted!

\[ A(\xi, \eta, t) \sim \int_{-1}^{1} dx \frac{GPD(x, \eta, t)}{x - \xi + i \epsilon} \]

We need clues from phenomenological models - Regge behaviour, t-factorization etc.

\[ \sigma_{tot} \sim \mathcal{I}mA, \quad \frac{d\sigma}{dt} \sim |A|^2 \]
**Fundamental global properties of the proton**

The structure of strongly interacting particles can be probed by means of the other fundamental forces: **electromagnetic**, **weak**, and **gravity**.

<table>
<thead>
<tr>
<th>Force</th>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>em:</strong></td>
<td>$\partial_{\mu} J_{\text{em}}^\mu = 0$</td>
<td>$Q_{\text{prot}} = 1.602176487(40) \times 10^{-19} \text{C}$</td>
</tr>
<tr>
<td></td>
<td>$\langle N'</td>
<td>J_{\text{em}}^\mu</td>
</tr>
<tr>
<td><strong>weak:</strong></td>
<td>PCAC</td>
<td>$g_A = 1.2694(28)$</td>
</tr>
<tr>
<td></td>
<td>$\langle N'</td>
<td>J_{\text{weak}}^\mu</td>
</tr>
<tr>
<td><strong>gravity:</strong></td>
<td>$\partial_{\mu} T_{\text{grav}}^{\mu\nu} = 0$</td>
<td>$M_{\text{prot}} = 938.272013(23) \text{MeV}/c^2$</td>
</tr>
<tr>
<td></td>
<td>$\langle N'</td>
<td>T_{\text{grav}}^{\mu\nu}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$D = ?$</td>
</tr>
</tbody>
</table>


The D-term is the “last unknown global property” of the nucleon.
Probing structure of the proton

- **Electromagnetic properties**: probed with photons
  - **Charge**: electromagnetic form factors, inelastic structure functions, proton charge radius, charge and current densities.
  - **Magnetic moment**: helicity densities

- **Gravitational properties**: probed with gravitons
  - **Mass**: energy and mass densities
  - **Spin**: angular momentum distribution
  - **D-term**: dynamical stability, normal and shear forces, pressure distribution

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2018 Review of Particle Physics.

<table>
<thead>
<tr>
<th>GAUGE AND Higgs Bosons</th>
<th></th>
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<tr>
<td>graviton</td>
<td>$J = 2$</td>
</tr>
</tbody>
</table>

| graviton MASS          | $< 6 \times 10^{-21}$ eV |
GPDs – GFFs Relations

Nucleon matrix element of the Energy-Momentum Tensor contains three scalar form factors and can be written as:

\[ \langle p_2 | \hat{T}^{q}_{\mu \nu} | p_1 \rangle = \bar{U}(p_2) \left[ M_2^q(t) \frac{P_{\mu} P_{\nu}}{M} + J^q(t) \frac{1}{2M} (P_{\mu} \sigma_{\nu \rho} + P_{\nu} \sigma_{\mu \rho}) \Delta^\rho + d_1^q(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu \nu} \Delta^2}{5M} \right] U(p_1) \]

\[ M_2(t) : \text{Mass/energy distribution inside the nucleon} \]
\[ J(t) : \text{Angular momentum distribution} \]
\[ d_1(t) : \text{Forces and pressure distribution} \]

\[ \begin{align*}
GPDs & \quad \leftrightarrow \quad GFFs \\
\int dx \, x \left[ H(x, \xi, t) + E(x, \xi, t) \right] &= 2J(t) \\
\int dx \, x H(x, \xi, t) &= M_2(t) + \frac{4}{5} \xi^2 d_1(t),
\end{align*} \]

• Bibliography:


• S. Weinberg, Gravitation and Cosmology, John Wiley and Sons, 1972, Sec. 9.7.

• **Russia, Dubna:** M. Polyakov, A. Efremov, A. Radyushcin, O. Teryaev.

2. Metasable supercooling @ NICA

*EOS at high energies (temperatures)*:

\[ \mu = 0; \quad p(T), \quad s(T) = p'(T); \]

\[ \epsilon(T) = p'(T)T - p(T) = s(T)T - p(T). \]
Collective properties of the nuclear matter vs. the $S$ matrices, or how can the EOS (equation of state) be inferred from the scattering amplitude (data)?


$$\beta(\Omega-\Omega_0) = -\frac{1}{4\pi} \sum_{n=2}^{\infty} z^n \int_{nm}^{\infty} dE e^{-\beta E} (T_{\tau n A S^{\frac{1}{dE}} S}),$$

where $\Omega$ is the thermodynamical potential, $z = e^{\beta \mu}$, $\beta = 1/T$.

The $S$ matrix can be saturated either by experimental data points or by a model for the scattering amplitude.

For the latter a direct-channel resonance model was used by P. Fre and L. Sertorio (Nuovo Cim. 28A (1975) 538; 31A (1076) 365).

$$p_q((T)) = a_q T^4 - AT, \quad p_h(T) = a_h T^4;$$

$$\epsilon_q = 3 a_q T^4, \quad \epsilon_h = 3 a_h T^4;$$

$$s_q(T) = 4 a_q T^3 - A, \quad s_h(T) = 4 a_h T^3,$$

where $A = (a_q - a_h)T_c^3$.

This system of bag equations of state can be written in one line:

$$s(T) = p'(T) = \frac{2}{45} \pi^2 T^3 \left(g_h(1 - \Theta(T - T_c)) + g_q \Theta(T - T_c)\right).$$


$$\Theta(x) \to (1/2) [1 + th(\frac{x}{\Delta T})],$$
Modified bag EOS
Metastability in the bag EOS
(Jenkovszky, Kaemfer, Sysosev)