Levy analysis of three particle correlation functions at PHENIX

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Different collision energies
7.7-200 GeV in $\sqrt{s_{NN}}$
20-400 MeV in $\mu_B$

Different collision systems
$p+p$, $p+A$, $A+A$
Levy-type source assumption

Correlation function: \( C_2(q) \approx 1 + | \int S(r) e^{iqr} |^2 \)

Assumption for source function \( \rightarrow \) Levy-type source

\[
S(r) = \mathcal{L}(\alpha, R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha}
\]

- Generalized centr. lim. theorem
- Levy-exponent: \( \alpha \) (Gaussian \( \alpha = 2 \), Cauchy \( \alpha = 1 \))
- Levy-scale parameter: \( R \)

Probable reasons:
- Anomalous diffusion
- QCD jets
- Critical phenomena
- \ldots
The connection of Lévy-index and the critical point

- Looking for critical behavior with critical exponents
- Critical spatial correlation: \( \sim r^{-(d-2+\eta)} \)
- Lévy source: \( \sim r^{-(d-2+\alpha)} \rightarrow \eta \iff \alpha \) ?


- QCD universality class \( \iff \) 3D Ising

- Critical point:
  - Random field 3D Ising: \( \eta = 0.50 \pm 0.05 \)
  - 3D Ising: \( \eta = 0.03631(3) \)

- Motivation for precise Lévy HBT!
- Finite size, non-equilibrium effects
  - What does the power-law tail mean?
Levy exponent from two particle correlations

**PHENIX 0-30% Au+Au \( \sqrt{s_{NN}} = 200 \text{ GeV} \)**

\[ \alpha_0 = 1.207, \chi^2 / \text{NDF} = 208/61, \text{CL} < 0.1\% \]

- **Lévy-exponent vs. \( m_T \)**
  - A. Adare et al. PRC97 (2018) 064911
  - Far from Gaussian (\( \alpha = 2 \))
  - Inconsistent with Cauchy (\( \alpha = 1 \))
  - Far from CEP (\( \alpha \leq 0.5 \))
  - Constant within systematics
Centrality and energy dependence

- Non-monotonic behavior of $\alpha$ vs. $N_{coll}$ (arXiv:1801.08827)
  - No clear interpretation
- $\alpha$ is far from CEP value for these energies (arXiv:1811.08311)
  - $m_T$ bin width dependent values
Cross check with 3D analysis

- Similar to 1D results
  - arXiv:1809.09392
- Still far from Gaussian ($\alpha = 2$)
- Differences at low $m_T$
  - Due to non-spherical source?
Lévy scale parameter \( \neq \) RMS

- Same behavior as for Gaussian HBT radii
- Hydro scaling observable: \( 1/R^2 = A m_T + B \)
Correlation strength, core-halo independent parameter

- Two component source: \( f_c = \frac{N_{\text{core}}}{N_{\text{total}}} \)
- If there are pions emitted coherently: \( p_c = \frac{N_{\text{coherent}}}{N_{\text{core}}} \)
- Partial coherence + Core-Halo:
  \[
  \lambda_2 = f_c^2 \left[ (1 - p_c)^2 + 2p_c(1 - p_c) \right]
  \lambda_3 = 2f_c^3 \left[ (1 - p_c)^3 + 3p_c(1 - p_c)^2 \right] + 3f_c^2 \left[ (1 - p_c)^2 + 2p_c(1 - p_c) \right]
  \]
- Experimentally: \( \lambda_n = C_n(q_1 = q_2 \cdots = 0) - 1 \)
- Core-halo \( (f_c) \) independent parameter: \( \kappa_3 = 0.5(\lambda_3 - 3\lambda_2) / \lambda_2^{3/2} \)
Two particle correlation strength from previous analysis

- Hole in $\lambda_2$ at low $m_T$
- In medium $\eta'$ mass modification?
- Decreased $\eta'$ mass $\rightarrow \eta'$ enhancement
- $\eta' \rightarrow \pi\pi\pi\pi$
- Halo enhancement $\rightarrow$ lower $\lambda_2$

J. I. Kapusta et al., PRD53 (1996) 5028
S. E. Vance et al., PRL81 (1998) 2205
T. Csörgő et al., PRL105 (2010) 182301

- Results aren’t incompatible
- From $\lambda_2$, $\lambda_3$ calculate $\kappa_3$
Three particle correlation strength

- From Core-Halo model: $0 < \lambda_3 < 5$
Core-Halo independent parameter

- Independent from $f_c$
- Expectation for fully chaotic, core+halo type source: $\kappa_3 = 1$

$$\kappa_3 = 0.5(\lambda_3 - 3\lambda_2)/\lambda_2^{3/2} \text{ vs. } m_T$$

**PHENIX 0-30% Au+Au @ $\sqrt{s_{NN}} = 200$ GeV**
Summary

Results:
- Levy sources ($\alpha < 2$) seen in 1D, 3D, $2\pi$, $3\pi$ analyses
- $\kappa_3 = 1$ consistent with chaotic emission
- $0 < \lambda_3 < 5$ within errors

Thank you for your attention!

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Three particle correlation functions

- Correlation function:

\[
C_3(k_1, k_2, k_3) = \frac{N_3(k_1, k_2, k_3)}{N_1(k_1)N_1(k_2)N_1(k_3)}
\]

- Single particle momentum distribution:

\[
N_1(k) = \int S(k, r)|\psi_k(r)|^2 d^4 r
\]

- Three particle momentum distribution:

\[
N_3(k_1, k_2, k_3) = \int |\psi_{k_1,k_2,k_3}(r_1, r_2, r_3)|^2 \prod_{i=0}^{3} S(k_i, r_i) d^4 r_i
\]
Core-Halo model

- Two component source: \( S = S_c + S_h \)
  - Core: thermalized medium, expanding source
  - Halo: long lived resonances \((\tau > 10 \text{ fm/c})\) → experimentally unresolvable

- Fraction of core: \( f_c = \frac{N_{\text{core}}}{(N_{\text{core}} + N_{\text{halo}})} \)

- Two particle correlation strength: \( \lambda_2 = f_c^2 \)

- Three particle correlation strength: \( \lambda_3 = 2f_c^3 + 3f_c^2 \)

- Core-Halo independent parameter: \( \kappa_3 = 0.5(\lambda_3 - 3\lambda_2)/\lambda_2^{3/2} = 1 \)
Partial coherence

If there are pions emitted coherently:

\[ S_c = S_c^{\text{coherent}} + S_c^{\text{incoherent}} \]

Fraction of pions emitted coherently:

\[ p_c = \frac{N_{\text{coherent}}}{N_{\text{coherent}} + N_{\text{incoherent}}} \]

Partial coherence + Core-Halo:


\[ \lambda_2 = f_c^2 \left[ (1 - p_c)^2 + 2p_c(1 - p_c) \right] \]
\[ \lambda_3 = 2f_c^3 \left[ (1 - p_c)^3 + 3p_c(1 - p_c)^2 \right] + 3f_c^2 \left[ (1 - p_c)^2 + 2p_c(1 - p_c) \right] \]
Three pion Levy HBT

The correlation function (without final Coulomb-interaction):

\[ C_3^{(0)}(q_{12}, q_{13}, q_{23}) = 1 + \ell_3 e^{-0.5(|q_{12}R|^\alpha + |q_{13}R|^\alpha + |q_{23}R|^\alpha)} \]

\[ + \ell_2 \left( e^{q_{12}R^\alpha} + e^{q_{13}R^\alpha} + e^{q_{23}R^\alpha} \right) \]

Parameters:
- Already known from two particle measurements: \( \alpha, R \)
- Now measured: \( \ell_2, \ell_3 \)

We are looking for three-particle correlation strength: \( \lambda_3 \)

\[ \lambda_3 = C_3(q_{12} = q_{13} = q_{23} \to 0) - 1 = \ell_3 + 3\ell_2 \]
Analysis details

- 200 GeV Au+Au collisions
- 29 $m_T$ bins
- Correlation functions of identified, same charged pion triplets
- Cuts:
  - Event selection: $z$-vertex, 0-30% Centrality
  - Particle selection: $2\sigma$ cuts for PID
  - Single track cuts: $2\sigma$ matching
  - Pair cuts: customary shaped cuts for $\Delta z - \Delta \varphi$ distributions
Fit function:

\[ C_3^{(fit)} = N(1 + \varepsilon q_{12})(1 + \varepsilon q_{13})(1 + \varepsilon q_{23})K_3 C_3^{(0)} \]

Background and normalisation: \( \varepsilon, N \)

Coulomb-correction:

- Generalized Riverside method:

\[ K_3(q_{12}, q_{13}, q_{23}) \approx K_1(q_{12})K_1(q_{13})K_1(q_{23}) \]

- Detailed numerical table for \( K_1(q, \alpha, R) \)

Fit parameters: \( \ell_2, \ell_3, N, \varepsilon \)

Already known from 2-particle correlations: \( \alpha, R \)
Appendix

Example fit

- Diagonal visualization of 3D correlation function

\[ C_3(k_{12}, k_{13}, k_{23}) \]

**PHENIX 0-30 % Au+Au @ \( \sqrt{s_{NN}} = 200 \text{ GeV} \), \( \pi^+\pi^+\pi^+ \), \( p_T = 0.48-0.5 \text{ GeV/c} \)**

- \( R = 7.2 \text{ fm}, \text{ fixed} \)
- \( \alpha = 1.1, \text{ fixed} \)
- \( N = 1.004 \pm 0.0003 \)
- \( \varepsilon = -0.022 \pm 0.0005 \)
- \( l_2 = 0.98 \pm 0.004 \)
- \( l_3 = 2.9 \pm 0.3 \)
- \( \chi^2/\text{NDF} = 8764/8365 \)
- \( \text{conf. level} = 0.117\% \)

**PHENIX preliminary**
MinBias vs 0-30% Centrality: $\lambda_3$

- 0-30% Centrality
- Density & size dependence
- Similar to MinBias
- Within Core-Halo range
MinBias vs 0-30% Centrality: $\kappa_3$

$\kappa_3 = 0.5(\lambda_3 - 3\lambda_2)/\lambda_2^{3/2}$ vs. $m_T$

- 0-30% Centrality
- Density & size dependence
- Similar to MinBias
- 0-30% compatible with 1