Lattice QCD at $T > 0$

A short introduction and selected topics

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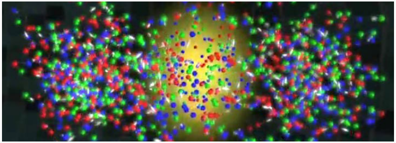
PRD 93 no. 11, 114502 (2016); PRD 97 no. 1, 014510 (2017); PRD 98 no. 5, 054511 (2018); *arXiv:1907.11747*;

Outline

Different means for studying hot nuclear matter

EXPERIMENT



- Heavy-ion collision experiments @ RHIC, LHC, FAIR, NICA
- Create a primordial fireball which is **out of equilibrium**
- Measure **real-time processes** within the primordial fireball

SIMULATION

SUPERMUC @ LRZ, Munich

- Numerical simulations of QCD on a grid with imaginary time
- Simulate high-temperature medium **in equilibrium**
- Compute **imaginary-time** in-medium correlation functions

Connect heavy-ion collisions and lattice QCD simulations by interfacing with model or weak-coupling calculations

QCD in a nutshell

QCD action encodes gauge symmetry and other classical symmetries

$$S_{QCD}[A,\bar{\psi},\psi] = \int d^4x \sum_{f=1}^{N_f} \bar{\psi}^f(x)\left(\not{D}[A(x)] + m_f\right)\psi^f(x) \qquad \text{quark action}$$

$$- \int d^4x \frac{1}{2}\mathrm{tr}\; F^{\mu\nu}(x)F_{\mu\nu}(x) \qquad \text{gauge action}$$

$$D_\mu[A(x)]\psi^f(x) = \left[\partial_\mu + \mathrm{i}gA_\mu(x)\right]\psi^f(x) \qquad \text{covariant derivative}$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + \mathrm{i}g[A_\mu(x), A_\nu(x)] \qquad \text{field strength}$$

Calculate observables via path integral and compare to experiment

$$\langle O\rangle_{\mathrm{QCD}} = \frac{1}{Z}\int \mathcal{D}A\; O[A] \prod_{f=1}^{N_f} \det\left(\not{D}[A] + m_f\right)\exp\left(-S_g[A]\right), \quad Z = \langle 1\rangle_{\mathrm{QCD}}$$

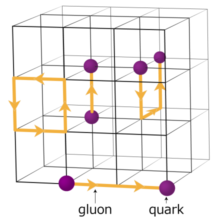
QCD on a lattice

$$S_{QCD}[U,\bar{\psi},\psi] = a^4 \sum_x \sum_{f=1}^{N_f} \bar{\psi}^f(x)\left(\not{D}[U(x)] + m_f\right)\psi^f(x)$$
$$- a^4 \sum_x \sum_{\mu<\nu} \frac{2}{g_0^2} \mathrm{Re\ tr}\left\{1 - U_{\mu\nu}(x) + \mathcal{O}(a^2)\right\}$$

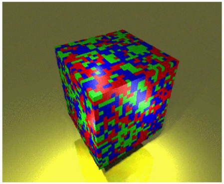
$$D_\mu[U_\mu(x)]\psi^f(x) = \frac{U_\mu(x)\psi^f(x+a\hat{\mu}) - U_\mu^\dagger(x-a\hat{\mu})\psi^f(x-a\hat{\mu})}{2a} + \mathcal{O}(a^2)$$

$$U_\mu(x) = \exp[ig_0 A_\mu(x)] \qquad \text{gauge link}$$

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x) \qquad \text{plaquette}$$



$\xrightarrow{\text{HPC}}$



Lattice QCD simulations in a box on a computer

Stochastically sample the (Euclidean) QCD path integral

$$\langle O\rangle_{\mathrm{QCD}} = \frac{1}{Z}\frac{1}{N}\sum_{\{U\}}^{N} O[U]\prod_{f=1}^{N_f}\det\left(\not{D}[U]+m_f\right)\exp\left(-S_g[U]\right)+\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

using MCMC algorithm with importance sampling

QCD on a lattice with spacing a in a box of $N_\sigma^3 \times N_\tau$ points

- scale setting: lattice spacing a is determined a posteriori
  control the approach to the continuum limit $a \to 0$
- time (Euclidean): periodic for gluons, antiperiodic for quarks
- space: periodic for gluons and quarks
  always at finite temperature and in finite volume
  $aN_\tau = 1/T$ **(volumes only must be large enough)**
- quark masses: light quarks at the physical point are expensive
  control the quark mass dependence through χPT
- quark flavors: usually $N_f = 2+1$ or $N_f = 2+1+1$, or $N_f = 0$

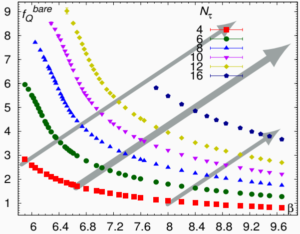
Wide ranges of lattice spacings from super coarse to super fine

$f_Q^{\mathrm{bare}} = -\log \langle P \rangle_T$

Two different volumes and two quark masses

$$\left(r^2 \partial V / \partial r\right)_{r=r_1} = 1$$

Same parametrization $\frac{r_1}{a}(\beta)$ for all ensembles



Continuum limit with **realistic** sea quark content

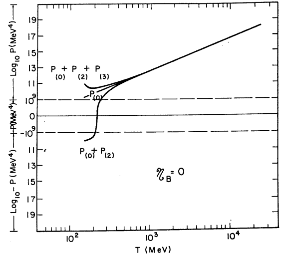
$N_\tau a(\beta) = 1/T(N_\tau, \beta)$

$T \in [135, 2325]\,\mathrm{MeV}$

with at least four N_τ

- $N_\tau = 4-16$: $12-40$ ens. each, $5.9 \leq \beta = \frac{10}{g_0^2} \leq 9.67$, $a = 0.0085-0.25\,\mathrm{fm}$
- HISQ action, errors: $\mathcal{O}(\alpha_s a^2, a^4)$; lattice artifacts are reduced
- $T = 0$ ens. for $m_\pi \approx 160\,\mathrm{MeV}$; $a \geq 0.04\,\mathrm{fm}$ & $m_\pi \approx 320\,\mathrm{MeV}$; $a \geq 0.025\,\mathrm{fm}$
- r_1 scale for $\beta > 8.4$ from nonperturbative β function

Earliest beginnings



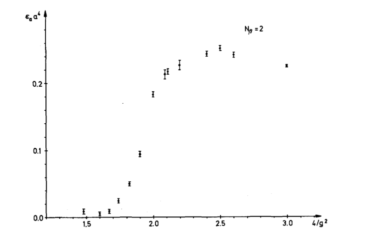


Fig. 3. Energy density of gluon matter versus $4/g^2$, at fixed lattice size $N_\beta = 2$, after about 500 iterations.

- First perturbative EoS calculation[1] (left)
- First lattice pure gauge $SU(2)$ EoS calculation[2] (right)

[1] Kapusta (1979)
[2] Engels et al. (1981)

Trace anomaly and pressure

- The QCD partition function

$$Z = \int \mathcal{D}U \prod_{f=1}^{N_f} \det\left(\not{D}[U] + m_f\right) \exp\left(-S_g[u]\right)$$

- The pressure is obtained from the trace anomaly via integral method

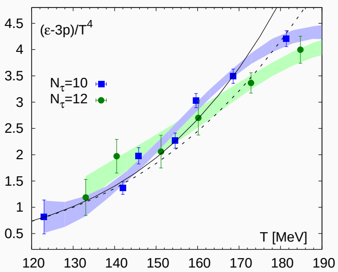
$$\Theta^{\mu\mu} \equiv \varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a} \quad \Rightarrow \quad \frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}$$

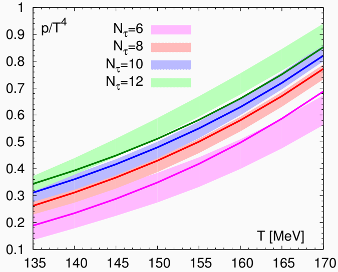
- Subtraction of UV divergences (subtract divergent vacuum contribution evaluated at the same values of the gauge coupling):

$$\begin{aligned} \frac{\varepsilon - 3p}{T^4} &= R_\beta[\langle S_g\rangle_0 - \langle S_g\rangle_T] \\ &- R_\beta R_m[2m_\ell(\langle\bar{\ell}\ell\rangle_0 - \langle\bar{\ell}\ell\rangle_T) + m_s(\langle\bar{s}s\rangle_0 - \langle\bar{s}s\rangle_T)] \end{aligned}$$

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m}\frac{dm}{d\beta}, \quad \beta = \frac{10}{g_0^2}$$

Low temperatures

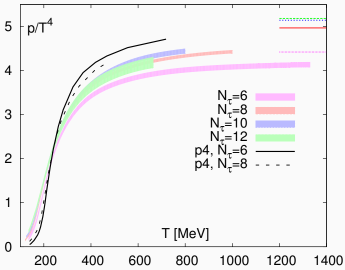




- We have improved the low-temperature region[3] by adding $T = 123$ MeV at $N_\tau = 10$ and $T = 133, 140$ MeV at $N_\tau = 12$
- interpolated lattice data (bands) vs hadron resonance gas (lines)
- $\Theta^{\mu\mu}$: continuum HRG model with PDG or Quark Model spectrum
- p/T^4: lattice HRG model with spectrum distorted by HISQ artifacts
- Cutoff effects for $T \lesssim T_c$: no trend in $\Theta^{\mu\mu}$ vs clear pattern in p/T^4

[3] Bazavov et al. [TUMQCD] (2016)

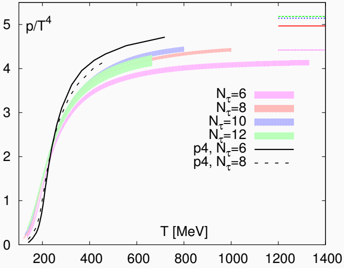
Pressure at higher temperatures

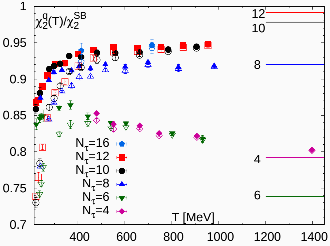


- At high T ($> 2T_c$) the HISQ cutoff dependence resembles the free theory, i.e. the continuum is approached from below (p4 from above)[4]

[4] Cheng et al. [HotQCD] (2008); Bazavov et al. [MILC, HotQCD] (2009)
[5] Bazavov et al. [HotQCD] (2013)

Pressure at higher temperatures





- At high T ($> 2T_c$) the HISQ cutoff dependence resembles the free theory, i.e. the continuum is approached from below (p4 from above)[4]
- Cutoff dependence similar in pressure & quark number susceptibilities[5]

$$\chi_{2n}^q = \frac{\partial^{2n} p(T,\mu_q)}{\partial \mu_q^{2n}},\ n = 1,2,\ q = \ell, s, \qquad \frac{p^q(T,N_\tau)}{p^q(T)} \simeq \frac{\chi_2^l(T,N_\tau)}{\chi_2^l(T)}$$

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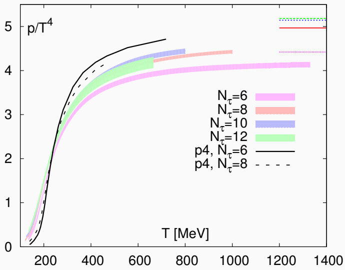
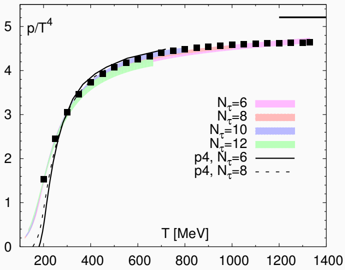
Pressure at higher temperatures





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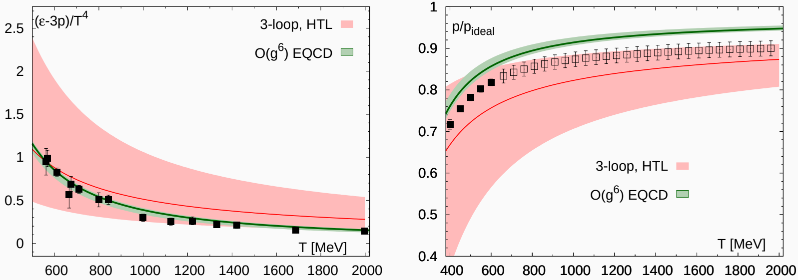
- Estimate corrections from QNS and pressure in ideal gas limit as

$$p(T) \simeq p(T,N_\tau) + p^{q,id-15\%}(T)\left(1 - \frac{\chi^l_2(T,N_\tau)}{\chi^l_2(T)}\right)$$

[4] Cheng et al. [HotQCD] (2008); Bazavov et al. [MILC, HotQCD] (2009)
[5] Bazavov et al. [HotQCD] (2013)

QCD Equation of State on the lattice and at weak coupling



- 2018: The **trace anomaly** (left) and **pressure** (right)[6] compared with HTL[7] and electrostatic QCD (EQCD)[8] calculations @ ($\mathcal{O}(g^6)$)
- The red/green lines are the HTL/EQCD calculations with $\mu = 2\pi T$

$\Rightarrow$ Lattice and weak-coupling approaches **compatible for** $T \gtrsim 300\,\text{MeV}$

- Uncertainties in weak-coupling results still $\sim 8\%$ at $T = 2$ GeV

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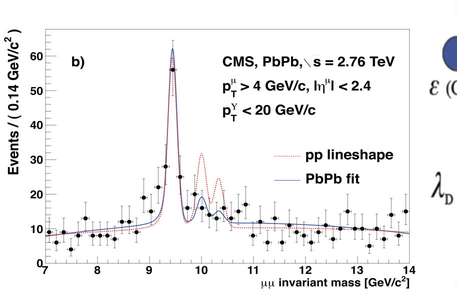
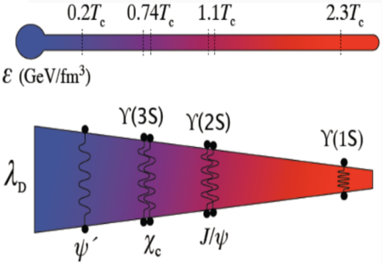
[6] Bazavov et al. (2017)
[7] Haque et al. (2014)
[8] Laine and Schröder (2006)

How is color screening related to in-medium quarkonium?



Quarkonium suppression[a]



Sequential melting picture[b]

[a] CMS collaboration, 2012
[b] USQCD whitepaper, 2018

- Hypothesis:[9] color screening $\rightarrow$ sequential melting $\rightarrow$ thermometer
- Not the full story – dissociative real-time dynamics is more important

[9] Matsui and Satz, 1986

Relevant scales for in-medium quarkonium

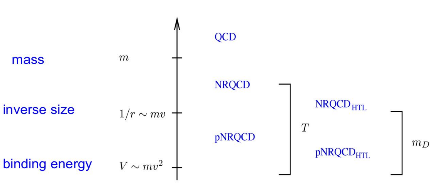
- Non-relativistic EFTs with non-relativistic hierarchy of scales

$$\underbrace{M}_{\text{hard}} \gg \underbrace{Mv \sim p \sim \frac{1}{r}}_{\text{soft}} \gg \underbrace{Mv^2 \sim E \sim \frac{\alpha_s}{r}}_{\text{ultrasoft}}$$

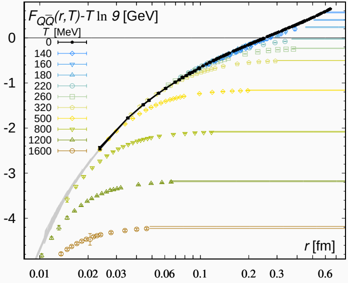
- Successively integrate out heavy scales ⇒ NRQCD and pNRQCD
- In a thermal bath, the thermal scales become relevant, too

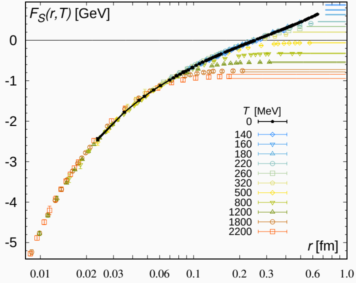
$$\underbrace{T \text{ or } \pi T}_{\text{thermal}} \gg \underbrace{gT \sim m_D}_{\text{electric}} \gg \underbrace{g^2 T}_{\text{magnetic}}$$

- Rich interplay between different sets of hierarchically ordered scales



Polyakov loop correlators, $Q\bar{Q}$ and singlet free energies





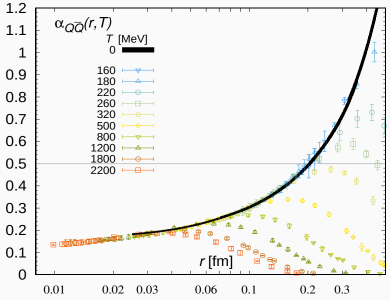
- The **free energy** $F_{Q\bar{Q}}$ of a $Q\bar{Q}$ pair (also: *color-averaged potential*)

$$C_P(r,T) = \langle P(0) P^\dagger(\boldsymbol{r}) \rangle_T^{\mathrm{ren}} = e^{-\frac{F_{Q\bar{Q}}(r,T)}{T}} = \frac{1}{9} e^{-\frac{F_S(r,T)}{T}} + \frac{8}{9} e^{-\frac{F_A(r,T)}{T}}$$

- The **singlet free energy** is related to the *gauge-fixed* static meson correlator at $\tau/a = N_\tau$ in *Coulomb gauge*

$$C_S(r,T) = \frac{1}{3} \left\langle \sum_{a=1}^{3} W_a(\tau,0) W_a^\dagger(\tau,\boldsymbol{r}) \right\rangle_T^{\mathrm{ren}} = e^{-F_S(r,T)/T}$$

Effective running coupling: vacuum-like and screening regimes



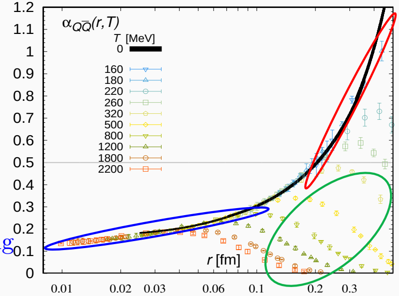
- The **effective coupling** $\alpha_{Q\bar{Q}}(r,T)$ is a suitable proxy for the **force** between the $Q\bar{Q}$ pair and for the QCD coupling α_s running with $1/r$

$$\alpha_{Q\bar{Q}}(r) = \frac{r^2}{C_F}\frac{\partial E(r)}{\partial r}$$

- Generalize: $\alpha_{Q\bar{Q}}(r,T)$ for **singlet free energy** $F_S(r,T)$ instead of $E(r)$

Effective running coupling: vacuum-like and screening regimes

Running coupling (Coulomb-like)



Running coupling (string tension-like)

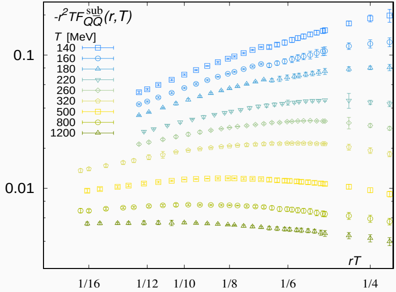
Effective 3d theory (running of $\alpha_{Q\bar{Q}}$ stalls)

$T \ll 1/r$	$T \gtrsim 1/r$	$\max(\alpha_{Q\bar{Q}})$
$rT \ll 0.3$	$rT \gtrsim 0.3$	$r_{\max} T \sim 0.4$

- $\max(\alpha_{Q\bar{Q}})$ is proxy for the **maximal force** defines $r_{\max} \sim 0.4/T$

$\Rightarrow$ Weak coupling may work for $T \gtrsim 300\,\mathrm{MeV}$: $\max(\alpha_{Q\bar{Q}}) \lesssim 0.5$

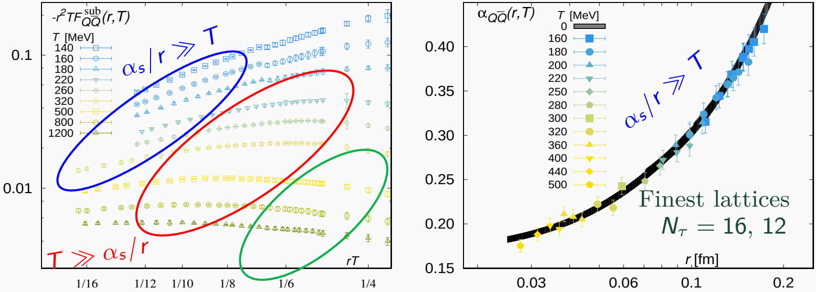
Effective running coupling: vacuum-like and screening regimes



- $F_{Q\bar{Q}}$ has for $rT \ll 1$ two distinct regimes $\alpha_s/r \gg T$ and $T \gg \alpha_s/r$

Singlet-dominance	Thermal dissociation	Screening regime
$rT \lesssim 0.05, \ldots, 0.15$	$rT \lesssim 0.3$	$rT \gtrsim 0.3$

Effective running coupling: vacuum-like and screening regimes



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Singlet-dominance	Thermal dissociation	Screening regime
$rT \lesssim 0.05, \ldots, 0.15$	$rT \lesssim 0.3$	$rT \gtrsim 0.3$

- Can define a running coupling $\alpha_{Q\bar{Q}}(r,T) \sim \alpha_s(1/r)$ in regime $\alpha_s/r \gg T$

The vacuum-like regimes

- The vacuum-like regime is defined in terms of $rT \ll 1$
- $r \ll 1/T$ is suitable for *multipole expansion*, i.e. *pNRQCD*
- The vacuum-like regime has two physically distinct sub-regimes

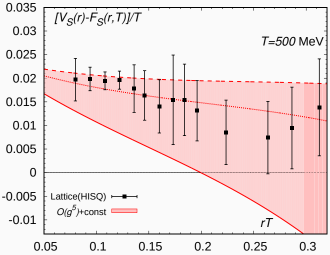
$$\alpha_s/r \ll T \quad \text{and} \quad \alpha_s/r \gg T$$

- For $\alpha_s/r \ll T$ weak-coupling calculations are available[10]
  - up to $\mathcal{O}(g^7)$ for $F_{Q\bar{Q}}$
  - up to $\mathcal{O}(g^5)$ for F_S; **matching term** $g^6 T$ **& ultrasoft term** $g^6 \ln(rT)/r$
- For $\alpha_s/r \gg T$ weak-coupling calculations are unavailable, but deviation from vacuum physics is exponentially suppressed[11] due to

$$e^{-(V_o - V_s)/T} \sim e^{-\alpha_s/rT}$$

[10] Berwein et al. (2016, 2017)
[11] Brambilla et al. (2008)

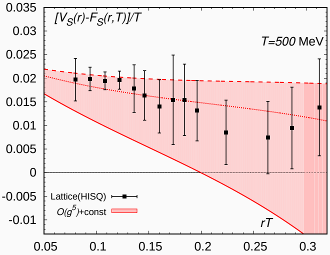
Direct comparison to the weak-coupling calculation[12] using pNRQCD

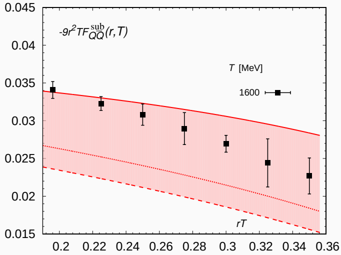


- $E(r) - F_S(r, T)$: thermal effects small, almost constant for $rT \lesssim 0.3$
- $\alpha_s/r \ll T$: $E - F_S \sim -g^4 \left\{\#_1 R - \#_2 R^2 + \#_3 R^3\right\} - g^5 \#_4 R^2$, $R = rT$
- Accidental compensation of non-static gluons/quarks by static gluons

[12] Berwein et al. (2017)

Direct comparison to the weak-coupling calculation[12] using pNRQCD

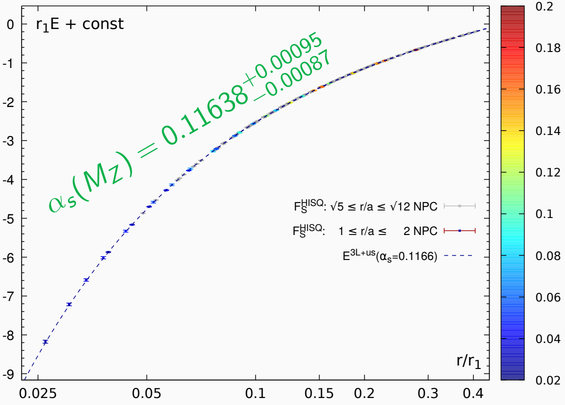




- $E(r) - F_S(r, T)$: thermal effects small, almost constant for $rT \lesssim 0.3$
- $\alpha_s/r \ll T$: $E - F_S \sim -g^4 \left\{\#_1 R - \#_2 R^2 + \#_3 R^3\right\} - g^5 \#_4 R^2$, $R = rT$
- Accidental compensation of non-static gluons/quarks by static gluons
- $F^{\mathrm{sub}}_{Q\bar{Q}} = F_{Q\bar{Q}} - 2F_Q$ – large uncertainty of $\mathcal{O}(g^7)$ even at $T \gtrsim 10\,T_c$
- Hierarchy $\alpha_s/r \ll T$ breaks down for $rT \ll 0.2$

[12] Berwein et al. (2017)

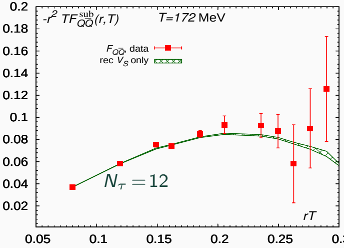
α_s from the singlet free energy at $T > 0$

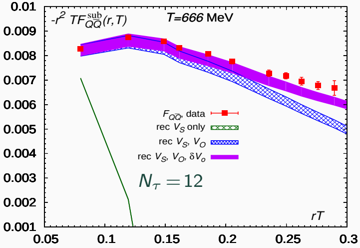


- Consistent and competitive with state-of-the-art $T = 0$ determination[13]
- Use only $N_\tau = 12$ data with $r/a \leq 2$ or 3, i.e., $r \leq 0.17/T$ or $0.25/T$
- Restrict to tiny distances $r \leq 0.03\,\text{fm}$ to reduce the perturbative error

[13] Bazavov et al. [TUMQCD] (2019)

Polyakov loop correlator in pNRQCD





$$C_P(r,T) = e^{-F_{Q\bar{Q}}(r,T)} = \frac{1}{N_c^2} e^{-V_s(r)/T} + \frac{N_c^2-1}{N_c^2} L_A(T) e^{-V_o(r)/T} + \mathcal{O}(g^6 (rT)^0)$$

- Low $T = 172\,\text{MeV}$: reconstruct C_P from only **color-singlet, i.e., V_S** (no sensitivity to the **color octet** states, the hierarchy is $\alpha_s/r \gg T$)
- High $T = 666\,\text{MeV}$: cancellation between **color-singlet** and **-octet** leads to $1/r^2$ behavior in $F_{Q\bar{Q}}$. Data satisfy the hierarchy $\alpha_s/r \ll T$
- Must include **Casimir scaling violation**[14] $8V_o + V_s = 3\frac{\alpha_s^3}{r}[\frac{\pi^2}{4} - 3]$

[14] Kniehl et al. (2005)

Electric or asymptotic screening regimes

- The screening regime is defined in terms of $r \gtrsim 1/m_D$, is most conveniently studied in terms of $F^{\rm sub}_{\dots}(r,T) = F_{\dots}(r,T) - 2F_Q$
- Hierarchy $r \gtrsim 1/m_D$ automatically built into dimensionally-reduced EFT $\Rightarrow$ the appropriate EFT is the **electrostatic QCD**

- The screening regime has two distinct sub-regimes:

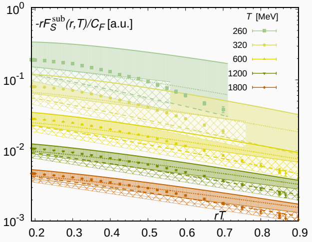
$$r \sim 1/m_D \quad \text{and} \quad r \gg 1/m_D$$

- In the electric screening regime, $r \sim 1/m_D$, chromo-electric fields are dominant. Weak-coupling calculations are available up to $\mathcal{O}(g^5)$[15]
- In the asymptotic screening regime, $r \gg 1/m_D$, chromo-magnetic fields are important, and non-perturbative methods are required[16]
- Asymptotic screening masses can be calculated as the bound states in EQCD $\Rightarrow$ bound states including electric A_0 gluons for $T \lesssim$ TeV[17]

[15] Nadkarni (1986); Laine et al. (2007); Berwein et al. (2017)
[16] Arnold and Yaffe (1995); Laine and Vepsalainen (2009)
[17] Hart et al. (2000)

Direct comparison to the weak-coupling calculation[18] using EQCD

$$F_S^{\rm sub}|_{\rm LO} = -C_F \left[\frac{\alpha_s e^{-rm_D}}{rT}\right] T$$

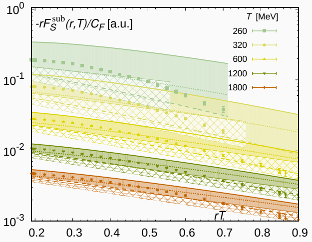


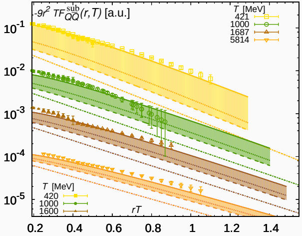
- NLO **singlet free energy** (two-gluon exchange is deferred to $\mathcal{O}(g^6)$)
  $$F_S^{\rm sub}|_{\rm NLO} = F_S^{\rm sub}|_{\rm LO}\left(1+\alpha_s N_c rT[2-\ln(x)-\gamma_E+e^x E_1(x)]\right), \quad x = 2rm_D$$
- Correction due to field renormalization $\delta F_S^{\rm sub} = F_S^{\rm sub}|_{\rm LO}\left(1-\frac{rm_D}{2}\delta Z_1\right)$

[18] Berwein et al. (2017)

Direct comparison to the weak-coupling calculation[18] using EQCD

$$F_S^{\rm sub}|_{\rm LO} = -C_F \left[\frac{\alpha_s e^{-rm_D}}{rT}\right] T \qquad F_{Q\bar{Q}}^{\rm sub}|_{\rm LO} = -\frac{N_c^2-1}{8N_c^2}\left[\frac{\alpha_s e^{-rm_D}}{rT}\right]^2 T$$





- NLO **singlet free energy** (two-gluon exchange is deferred to $\mathcal{O}(g^6)$)
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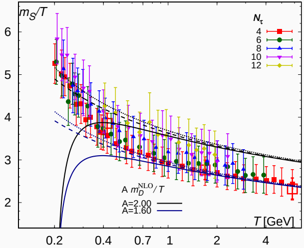
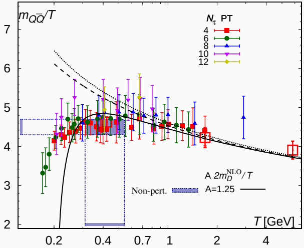
- Correction due to field renormalization $\delta F_S^{\rm sub} = F_S^{\rm sub}|_{\rm LO}\left(1-\frac{rm_D}{2}\delta Z_1\right)$
- $F_{Q\bar{Q}}^{\rm sub}$: severe **signal-to-noise** problem → high statistics, larger volumes
- Use data for $N_\tau = 4$ with (estimated) correction for cutoff effects
- $F_{Q\bar{Q}}^{\rm sub}$ shows reasonable compatibility with the full $\mathcal{O}(g^5)$ result

[18] Berwein et al. (2017)

Asymptotic screening regime

$$\frac{F_S^{\rm sub}}{T} \sim \left[\frac{\exp(-rm_S)}{rT}\right] \qquad \frac{F_{Q\bar{Q}}^{\rm sub}}{T} \sim \left[\frac{\exp(-rm_{Q\bar{Q}})}{rt}\right]$$



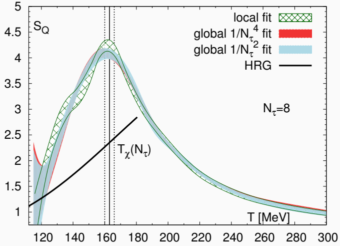


- Screening masses are related to the EQCD spectrum ($E \ll 2\pi T$)
- $F_{Q\bar{Q}}^{\rm sub}$: severe **signal-to-noise** problem → no continuum limit
- Poor results for $T \lesssim 200$MeV ⇒ need noise suppression techniques[19]
- Screening mass only slightly larger than $2m_D$ for $F_{Q\bar{Q}}$
- Good agreement with results from direct EQCD simulations[20]

[19] Steinbeißer et al *in preparation*
[20] Hart et al. (2000)

Truly asymptotic regime: static quark free energy and entropy shift

$$S_Q(T) = -\frac{dF_Q(T)}{dT}$$

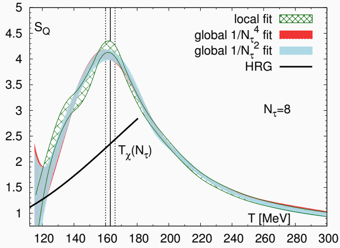


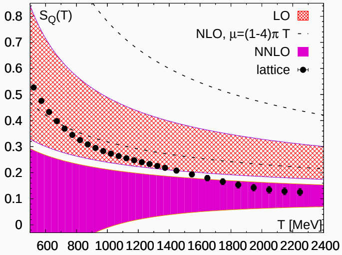
- $S_Q(T, N_\tau)$ peaks at the same $T_c(N_\tau)$ as chiral susceptibility $\chi_2(T, N_\tau)$
- HRG model of Polyakov loop breaks down early below 130 MeV

[21] Berwein et al. (2016)

Truly asymptotic regime: static quark free energy and entropy shift

$$S_Q(T) = -\frac{dF_Q(T)}{dT} \qquad F_Q|_{\mathrm{LO}} = -\frac{C_F\alpha_s m_D}{2} \overset{m_D\sim gT}{\Rightarrow} S_Q|_{\mathrm{LO}} \sim g^3$$





- $S_Q(T, N_\tau)$ peaks at the same $T_c(N_\tau)$ as chiral susceptibility $\chi_2(T, N_\tau)$
- HRG model of Polyakov loop breaks down early below 130 MeV
- Free energy and entropy shift are known[21] at $\mathcal{O}(g^5)$
- Poor convergence of expansion – NLO in g is still only LO in α_s
- Lattice and $\mathcal{O}(g^5)$ agree only for $T \gtrsim 10\,T_c$: static mode dominates F_Q

[21] Berwein et al. (2016)

Summary

- We obtained the (2+1)-flavor QCD equation of state up to $T \sim 2\,\mathrm{GeV}$, between EQCD and HTL, suggests underestimated PQCD uncertainty
- We study color screening and deconfinement using the renormalized Polyakov loop correlator and related observables
- We extract the continuum limit of static quark correlators at $\tau/a = N_\tau$ in (2+1)-flavor QCD up to $T \sim 2\,\mathrm{GeV}$ and down to $r \sim 0.01\,\mathrm{fm}$
- **Vacuum-like regime** at $rT \lesssim 0.3$, pNRQCD suitable at $T > 300\,\mathrm{MeV}$, total $T > 0$ effects small due to various cancellations
- For $F_{Q\bar{Q}}$ evidence of the distinctive sub-regimes **singlet dominance,** $\alpha_s/r \gg T$, and **thermal dissociation,** $\alpha_s/r \lesssim T$
- $\alpha_s(M_Z, N_f = 5) = 0.11638^{+95}_{-87}$ from $F_S(r, T)$ agrees with $T = 0$ result
- Evidence of **electric screening regime** controlled by the perturbative Debye mass at $0.3 \lesssim rT \lesssim 0.6$, described by EQCD at $T > 300\,\mathrm{MeV}$
- Static quark entropy shift $S_Q = -dF_Q/dT$ probes the deconfinement, delivers T_c surprisingly (?) similar to chiral observables ($\rightarrow$ Petreczky)