Lattice QCD at $T > 0$
A short introduction and selected topics

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4 Summary
Different means for studying hot nuclear matter

**EXPERIMENT**

- Heavy-ion collision experiments @ RHIC, LHC, FAIR, NICA
- Create a primordial fireball which is **out of equilibrium**
- Measure **real-time processes** within the primordial fireball

**SIMULATION**

- Numerical simulations of QCD on a grid with imaginary time
- Simulate **high-temperature medium in equilibrium**
- Compute **imaginary-time in-medium correlation functions**

Connect heavy-ion collisions and lattice QCD simulations by interfacing with model or weak-coupling calculations
QCD in a nutshell

QCD action encodes gauge symmetry and other classical symmetries

\[ S_{\text{QCD}}[A, \bar{\psi}, \psi] = \int d^4x \sum_{f=1}^{N_f} \bar{\psi}^f(x) (\mathcal{D}[A(x)] + m_f) \psi^f(x) \quad \text{quark action} \]

\[ - \int d^4x \frac{1}{2} \text{tr} \ F_{\mu\nu}(x)F_{\mu\nu}(x) \quad \text{gauge action} \]

\[ D_\mu[A(x)]\psi^f(x) = [\partial_\mu + igA_\mu(x)] \psi^f(x) \quad \text{covariant derivative} \]

\[ F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + ig[A_\mu(x), A_\nu(x)] \quad \text{field strength} \]

Calculate observables via path integral and compare to experiment

\[ \langle O \rangle_{\text{QCD}} = \frac{1}{Z} \int \mathcal{D}A \; O[A] \prod_{f=1}^{N_f} \det (\mathcal{D}[A] + m_f) \exp (-S_g[A]), \quad Z = \langle 1 \rangle_{\text{QCD}} \]
**QCD on a lattice**

\[ S_{QCD}[U, \bar{\psi}, \psi] = a^4 \sum_x \sum_{f=1}^{N_f} \bar{\psi}^f(x) \left( \Phi[U(x)] + m_f \right) \psi^f(x) \]

\[ - a^4 \sum_x \sum_{\mu < \nu} \frac{2}{g_0^2} \text{Re} \text{ tr} \left\{ 1 - U_{\mu \nu}(x) + \mathcal{O}(a^2) \right\} \]

\[ D_\mu[U_\mu(x)]\psi^f(x) = \frac{U_\mu(x)\psi^f(x + a\hat{\mu}) - U_\mu^\dagger(x - a\hat{\mu})\psi^f(x - a\hat{\mu})}{2a} + \mathcal{O}(a^2) \]

\[ U_\mu(x) = \exp[i g_0 A_\mu(x)] \quad \text{gauge link} \]

\[ U_{\mu \nu}(x) = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^\dagger(x + a\hat{\nu}) U_\nu^\dagger(x) \quad \text{plaquette} \]
# Lattice QCD simulations in a box on a computer

Stochastically sample the (Euclidean) QCD path integral

$$\langle O \rangle_{\text{QCD}} = \frac{1}{Z} \frac{1}{N} \sum_{\{U\}}^{N} O[U] \prod_{f=1}^{N_{f}} \det (\not{D}[U] + m_{f}) \exp (-S_{g}[U]) + \mathcal{O} \left( \frac{1}{\sqrt{N}} \right)$$

using MCMC algorithm with importance sampling

QCD on a lattice with spacing $a$ in a box of $N_{\sigma}^3 \times N_{\tau}$ points

- scale setting: lattice spacing $a$ is determined a posteriori
  - control the approach to the continuum limit $a \to 0$
- time (Euclidean): periodic for gluons, antiperiodic for quarks
- space: periodic for gluons and quarks
  - always at finite temperature and in finite volume $aN_{\tau} = 1/T$ (volumes only must be large enough)
- quark masses: light quarks at the physical point are expensive
  - control the quark mass dependence through $\chi$PT
- quark flavors: usually $N_{f} = 2 + 1$ or $N_{f} = 2 + 1 + 1$, or $N_{f} = 0$
Wide ranges of lattice spacings from super coarse to super fine

\[ f_Q^{\text{bare}} = - \log \langle P \rangle_T \]

Two different volumes and two quark masses
\[ \left( r^2 \frac{\partial V}{\partial r} \right)_{r=r_1} = 1 \]

Same parametrization \( \frac{r_1}{a}(\beta) \) for all ensembles

- \( N_T = 4 - 16 \): 12 - 40 ens. each, \( 5.9 \leq \beta = \frac{10}{g_0} \leq 9.67 \), \( a = 0.0085 - 0.25 \) fm
- HISQ action, errors: \( \mathcal{O}(\alpha_s a^2, a^4) \); lattice artifacts are reduced
- \( T = 0 \) ens. for \( m_\pi \approx 160 \text{ MeV}; a \geq 0.04 \) fm & \( m_\pi \approx 320 \text{ MeV}; a \geq 0.025 \) fm
- \( r_1 \) scale for \( \beta > 8.4 \) from nonperturbative \( \beta \) function

Continuum limit with realistic sea quark content
\[ N_T a(\beta) = \frac{1}{T(N_T, \beta)} \]
\[ T \in [135, 2325] \text{ MeV} \]
with at least four \( N_T \)
Earliest beginnings

- First perturbative EoS calculation\(^1\) (left)
- First lattice pure gauge \(SU(2)\) EoS calculation\(^2\) (right)

\(^1\)Kapusta (1979)
\(^2\)Engels et al. (1981)
Trace anomaly and pressure

- The QCD partition function

\[ Z = \int \mathcal{D}U \prod_{f=1}^{N_f} \det (\mathcal{D}[U] + m_f) \exp (-S_g[u]) \]

- The pressure is obtained from the trace anomaly via integral method

\[ \Theta^{\mu\mu} \equiv \epsilon - 3p = -\frac{T}{V} \frac{d \ln Z}{d \ln a} \Rightarrow \frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\epsilon - 3p}{T'^5} \]

- Subtraction of UV divergences (subtract divergent vacuum contribution evaluated at the same values of the gauge coupling):

\[ \frac{\epsilon - 3p}{T^4} = R_{\beta}[\langle S_g \rangle_0 - \langle S_g \rangle_T] \]

\[ - R_{\beta} R_m[2m_\ell(\langle \bar{\ell}\ell \rangle_0 - \langle \bar{\ell}\ell \rangle_T) + m_s(\langle \bar{s}s \rangle_0 - \langle \bar{s}s \rangle_T)] \]

\[ R_{\beta}(\beta) = -a \frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m} \frac{dm}{d\beta}, \quad \beta = \frac{10}{g_0^2} \]
Low temperatures

- We have improved the low-temperature region\(^3\) by adding $T = 123$ MeV at $N_\tau = 10$ and $T = 133, 140$ MeV at $N_\tau = 12$
- interpolated lattice data (bands) vs hadron resonance gas (lines)
- $\Theta^{\mu\mu}$: continuum HRG model with PDG or Quark Model spectrum
- $p/T^4$: lattice HRG model with spectrum distorted by HISQ artifacts
- Cutoff effects for $T \lesssim T_c$: no trend in $\Theta^{\mu\mu}$ vs clear pattern in $p/T^4$

\(^3\)Bazavov et al. [TUMQCD] (2016)
• At high $T$ ($> 2T_c$) the HISQ cutoff dependence resembles the free theory, i.e. the continuum is approached from below ($p4$ from above)\textsuperscript{4}

\textsuperscript{4}Cheng et al. [HotQCD] (2008); Bazavov et al. [MILC, HotQCD] (2009)

\textsuperscript{5}Bazavov et al. [HotQCD] (2013)
Pressure at higher temperatures

- At high $T$ ($> 2T_c$) the HISQ cutoff dependence resembles the free theory, i.e. the continuum is approached from below ($p_4$ from above)\(^4\)
- Cutoff dependence similar in pressure & quark number susceptibilities\(^5\)

\[
\chi^q_{2n} = \frac{\partial^{2n} p(T, \mu_q)}{\partial \mu_q^{2n}}, \quad n = 1, 2, \quad q = \ell, s, \quad \frac{p^q(T, N_\tau)}{p^q(T)} \simeq \frac{\chi^I_2(T, N_\tau)}{\chi^I_2(T)}
\]

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At high $T > 2T_c$ the HISQ cutoff dependence resembles the free theory, i.e. the continuum is approached from below ($p_4$ from above)\(^4\)

Cutoff dependence similar in pressure & quark number susceptibilities\(^5\)

$$\chi_{2n}^q = \frac{\partial^{2n} p(T, \mu_q)}{\partial \mu_q^{2n}}, \quad n = 1, 2, \quad q = \ell, s, \quad \frac{p^q(T, N_{\tau})}{p^q(T)} \approx \frac{\chi_{2}^l(T, N_{\tau})}{\chi_{2}^l(T)}$$

Estimate corrections from QNS and pressure in ideal gas limit as

$$p(T) \simeq p(T, N_{\tau}) + p^{q, id-15\%}(T) \left(1 - \frac{\chi_{2}^l(T, N_{\tau})}{\chi_{2}^l(T)}\right)$$

\(^4\)Cheng et al. [HotQCD] (2008); Bazavov et al. [MILC, HotQCD] (2009)

\(^5\)Bazavov et al. [HotQCD] (2013)
QCD Equation of State on the lattice and at weak coupling

- 2018: The **trace anomaly** (left) and **pressure** (right)\(^6\) compared with HTL\(^7\) and electrostatic QCD (EQCD)\(^8\) calculations @ \((\mathcal{O}(g^6))\)

  - The red/green lines are the HTL/EQCD calculations with \(\mu = 2\pi T\)

  \[ \Rightarrow \text{ Lattice and weak-coupling approaches compatible for } T \gtrsim 300 \text{ MeV} \]

  - Uncertainties in weak-coupling results still \(\sim 8\%\) at \(T = 2 \text{ GeV}\)

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\(^6\) Bazavov et al. (2017)
\(^7\) Haque et al. (2014)
\(^8\) Laine and Schröder (2006)
How is color screening related to in-medium quarkonium?

Quarkonium suppression\textsuperscript{a}

\textsuperscript{a}CMS collaboration, 2012

\textsuperscript{b}USQCD whitepaper, 2018

- Hypothesis\textsuperscript{9}: color screening → sequential melting → thermometer
- Not the full story – dissociative real-time dynamics is more important

\textsuperscript{9}Matsui and Satz, 1986
Relevant scales for **in-medium** quarkonium

- Non-relativistic EFTs with non-relativistic hierarchy of scales

\[
\begin{align*}
M_{\text{hard}} & \gg Mv \sim p \sim \frac{1}{r} \gg Mv^2 \sim E \sim \frac{\alpha_s}{r} \\
\text{soft} & \quad \text{ultrasoft}
\end{align*}
\]

- Successively integrate out heavy scales \( \Rightarrow \) NRQCD and pNRQCD
- In a thermal bath, the thermal scales become relevant, too

\[
\begin{align*}
T \text{ or } \pi T & \gg gT \sim m_D \gg g^2 T \\
\text{thermal} & \quad \text{electric} \quad \text{magnetic}
\end{align*}
\]

- Rich interplay between different sets of hierarchically ordered scales
Polyakov loop correlators, $Q\bar{Q}$ and singlet free energies

- The **free energy** $F_{Q\bar{Q}}$ of a $Q\bar{Q}$ pair (also: color-averaged potential)

  $$C_P(r, T) = \langle P(0) P^\dagger(r) \rangle_T^{\text{ren}} = e^{-\frac{F_{Q\bar{Q}}(r, T)}{T}} = \frac{1}{9} e^{-\frac{F_S(r, T)}{T}} + \frac{8}{9} e^{-\frac{F_A(r, T)}{T}}$$

- The **singlet free energy** is related to the gauge-fixed static meson correlator at $\tau/a = N_\tau$ in Coulomb gauge

  $$C_S(r, T) = \frac{1}{3} \left\langle \sum_{a=1}^{3} W_a(\tau, 0) W_a^\dagger(\tau, r) \right\rangle_T^{\text{ren}} = e^{-\frac{F_S(r, T)}{T}}$$
The effective coupling $\alpha_{Q\bar{Q}}(r, T)$ is a suitable proxy for the force between the $Q\bar{Q}$ pair and for the QCD coupling $\alpha_s$ running with $1/r$

$$\alpha_{Q\bar{Q}}(r) = \frac{r^2}{C_F} \frac{\partial E(r)}{\partial r}$$

Generalize: $\alpha_{Q\bar{Q}}(r, T)$ for singlet free energy $F_S(r, T)$ instead of $E(r)$
Effective running coupling: vacuum-like and screening regimes

\[ \alpha_{Q\bar{Q}}(r,T) \]

- **Running coupling (Coulomb-like)**
- **Running coupling (string tension-like)**

\[ T \ll 1/r \quad \frac{1}{rT} \ll 0.3 \]
\[ T \gtrsim 1/r \quad \frac{1}{rT} \gtrsim 0.3 \]
\[ \max(\alpha_{Q\bar{Q}}) \quad r_{\text{max}} \sim 0.4 \]

- \( \max(\alpha_{Q\bar{Q}}) \) is proxy for the maximal force defines \( r_{\text{max}} \sim 0.4/T \)

\[ \Rightarrow \text{Weak coupling may work for } T \gtrsim 300 \text{ MeV: } \max(\alpha_{Q\bar{Q}}) \lesssim 0.5 \]
Effective running coupling: vacuum-like and screening regimes

- $F_{Q\bar{Q}}$ has for $rT \ll 1$ two distinct regimes $\alpha_s/r \gg T$ and $T \gg \alpha_s/r$

<table>
<thead>
<tr>
<th>Singlet-dominance</th>
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Effective running coupling: vacuum-like and screening regimes

- $F_{Q\bar{Q}}$ has for $rT \ll 1$ two distinct regimes $\alpha_s/r \gg T$ and $T \gg \alpha_s/r$
  - Singlet-dominance: $rT \lesssim 0.05, \ldots, 0.15$
  - Thermal dissociation: $rT \lesssim 0.3$
  - Screening regime: $rT \gtrsim 0.3$

- Can define a running coupling $\alpha_{Q\bar{Q}}(r, T) \sim \alpha_s(1/r)$ in regime $\alpha_s/r \gg T$
The vacuum-like regimes

- The vacuum-like regime is defined in terms of \( rT \ll 1 \)
- \( r \ll 1/T \) is suitable for *multipole expansion*, i.e. *pNRQCD*
- The vacuum-like regime has two physically distinct sub-regimes
  \[
  \frac{\alpha_s}{r} \ll T \quad \text{and} \quad \frac{\alpha_s}{r} \gg T
  \]
- For \( \frac{\alpha_s}{r} \ll T \) weak-coupling calculations are available\(^\text{10}\)
  - up to \( \mathcal{O}(g^7) \) for \( F_{Q\bar{Q}} \)
  - up to \( \mathcal{O}(g^5) \) for \( F_S \); *matching term* \( g^6 T \) & *ultrasoft term* \( g^6 \ln(rT)/r \)
- For \( \frac{\alpha_s}{r} \gg T \) weak-coupling calculations are unavailable, but deviation from vacuum physics is exponentially suppressed\(^\text{11}\) due to
  \[
  e^{-(V_0-V_s)/T} \sim e^{-\alpha_s/rT}
  \]

\(^{10}\)Berwein et al. (2016, 2017)
\(^{11}\)Brambilla et al. (2008)
Direct comparison to the weak-coupling calculation\textsuperscript{12} using pNRQCD

\[ \frac{[V_S(r) - F_S(r,T)]}{T} \]

\[ T = 500 \text{ MeV} \]

\[ O(g^5) + \text{const} \]

- \( E(r) - F_S(r, T) \): thermal effects small, almost constant for \( rT \lesssim 0.3 \)
- \( \alpha_s/r \ll T \): \( E - F_S \sim -g^4 \left\{ \#_1 R - \#_2 R^2 + \#_3 R^3 \right\} - g^5 \#_4 R^2, \ R = rT \)
- Accidental compensation of non-static gluons/quarks by static gluons

\textsuperscript{12}Berwein et al. (2017)
Direct comparison to the weak-coupling calculation\textsuperscript{12} using pNRQCD

- $E(r) - F_S(r, T)$: thermal effects small, almost constant for $rT \lesssim 0.3$
- $\alpha_s/r \ll T$: $E - F_S \sim -g^4 \left\{ \#_1 R - \#_2 R^2 + \#_3 R^3 \right\} - g^5 \#_4 R^2$, $R = rT$
- Accidental compensation of non-static gluons/quarks by static gluons
- $F_{QQ}^{\text{sub}} = F_{Q\bar{Q}} - 2F_Q$ – large uncertainty of $O(g^7)$ even at $T \gtrsim 10 T_c$
- Hierarchy $\alpha_s/r \ll T$ breaks down for $rT \ll 0.2$

\textsuperscript{12} Berwein et al. (2017)
**α_s from the singlet free energy at T > 0**

- Consistent and competitive with state-of-the-art $T = 0$ determination\(^{13}\)
- Use only $N_r = 12$ data with $r/a \leq 2$ or 3, i.e., $r \leq 0.17/T$ or $0.25/T$
- Restrict to tiny distances $r \leq 0.03$ fm to reduce the perturbative error

\(^{13}\)Bazavov et al. [TUMQCD] (2019)
Polyakov loop correlator in pNRQCD

\[ C_P(r, T) = e^{-F_{Q\bar{Q}}(r, T)} = \frac{1}{N_c^2} e^{-\nu_s(r)/T} + \frac{N_c^2 - 1}{N_c^2} L_A(T) e^{-\nu_o(r)/T} + O(g^6(rT)^0) \]

- Low \( T = 172 \text{ MeV} \): reconstruct \( C_P \) from only color-singlet, i.e., \( V_S \) (no sensitivity to the color octet states, the hierarchy is \( \alpha_s/r \gg T \))
- High \( T = 666 \text{ MeV} \): cancellation between color-singlet and octet leads to \( 1/r^2 \) behavior in \( F_{Q\bar{Q}} \). Data satisfy the hierarchy \( \alpha_s/r \ll T \)
- Must include Casimir scaling violation \(^{14}\) \( 8 V_o + V_s = 3 \frac{\alpha_s^3}{r} [\frac{\pi^2}{4} - 3] \)

\(^{14}\)Kniehl et al. (2005)
Electric or asymptotic screening regimes

- The screening regime is defined in terms of \( r \gtrsim 1/m_D \), is most conveniently studied in terms of \( F_{\text{sub}}(r, T) = F_{\text{...}}(r, T) - 2F_Q \)
- Hierarchy \( r \gtrsim 1/m_D \) automatically built into dimensionally-reduced EFT ⇒ the appropriate EFT is the electrostatic QCD

- The screening regime has two distinct sub-regimes:

  \[
  r \sim 1/m_D \quad \text{and} \quad r \gg 1/m_D
  \]

- In the electric screening regime, \( r \sim 1/m_D \), chromo-electric fields are dominant. Weak-coupling calculations are available up to \( \mathcal{O}(g^5) \)\(^{15}\)
- In the asymptotic screening regime, \( r \gg 1/m_D \), chromo-magnetic fields are important, and non-perturbative methods are required\(^{16}\)
- Asymptotic screening masses can be calculated as the bound states in EQCD ⇒ bound states including electric \( A_0 \) gluons for \( T \lesssim \text{TeV} \)\(^{17}\)

\(^{15}\)Nadkarni (1986); Laine et al. (2007); Berwein et al. (2017)
\(^{16}\)Arnold and Yaffe (1995); Laine and Vepsalainen (2009)
\(^{17}\)Hart et al. (2000)
Direct comparison to the weak-coupling calculation using EQCD

\[ F_{S}^{\text{sub}} |_{\text{LO}} = -C_F \left[ \frac{\alpha_s e^{-r m D}}{r T} \right] T \]

- NLO singlet free energy (two-gluon exchange is deferred to $O(g^6)$)
  \[ F_{S}^{\text{sub}} |_{\text{NLO}} = F_{S}^{\text{sub}} |_{\text{LO}} \left( 1 + \alpha_s N_c r T \left[ 2 - \ln(x) - \gamma_E + e^x E_1(x) \right] \right), \quad x = 2 rm_D \]
- Correction due to field renormalization \[ \delta F_{S}^{\text{sub}} = F_{S}^{\text{sub}} |_{\text{LO}} \left( 1 - \frac{r m D}{2} \delta Z_1 \right) \]

\[ ^{18}\text{Berwein et al. (2017)} \]
Direct comparison to the weak-coupling calculation using EQCD

\[ F_{S}^{\text{sub}}|_{\text{LO}} = -C_{F} \left[ \frac{\alpha_{s} e^{-rmD}}{rT} \right] T \]

\[ F_{Q\bar{Q}}^{\text{sub}}|_{\text{LO}} = -\frac{N_{c}^{2}-1}{8N_{c}^{2}} \left[ \frac{\alpha_{s} e^{-rmD}}{rT} \right]^{2} T \]

- **NLO singlet free energy** (two-gluon exchange is deferred to \( O(g^{6}) \))
  \[ F_{S}^{\text{sub}}|_{\text{NLO}} = F_{S}^{\text{sub}}|_{\text{LO}} (1+\alpha_{s}N_{c}rT[2-\ln(x)-\gamma_{E}+e^{x}E_{1}(x)]) , \quad x = 2rmD \]

- Correction due to field renormalization \( \delta F_{S}^{\text{sub}} = F_{S}^{\text{sub}}|_{\text{LO}} \left(1-\frac{rmD}{2} \delta Z_{1}\right) \)

- **\( F_{Q\bar{Q}}^{\text{sub}} \)**: severe **signal-to-noise** problem \( \rightarrow \) high statistics, larger volumes

- Use data for \( N_{\tau} = 4 \) with (estimated) correction for cutoff effects

- **\( F_{Q\bar{Q}}^{\text{sub}} \)** shows reasonable compatibility with the full \( O(g^{5}) \) result

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\(^{18}\) Berwein et al. (2017)
Asymptotic screening regime

\[ \frac{F_{SS}^{\text{sub}}}{T} \sim \left[ \frac{\exp(-rm_S)}{rT} \right] \]

\[ \frac{F_{QQ}^{\text{sub}}}{T} \sim \left[ \frac{\exp(-rm_{QQ})}{rt} \right] \]

- Screening masses are related to the EQCD spectrum \((E \ll 2\pi T)\)
- \(F_{QQ}^{\text{sub}}\): severe signal-to-noise problem \(\rightarrow\) no continuum limit
- Poor results for \(T \lesssim 200\,\text{MeV}\) \(\Rightarrow\) need noise suppression techniques\(^{19}\)
- Screening mass only slightly larger than \(2m_D\) for \(F_{QQ}\)
- Good agreement with results from direct EQCD simulations\(^{20}\)

\(^{19}\) Steinbeißer et al \textit{in preparation}

\(^{20}\) Hart et al. (2000)
Truly asymptotic regime: static quark free energy and entropy shift

\[ S_Q(T) = -\frac{dF_Q(T)}{dT} \]

- \( S_Q(T, N_\tau) \) peaks at the same \( T_c(N_\tau) \) as chiral susceptibility \( \chi_2(T, N_\tau) \)
- HRG model of Polyakov loop breaks down early below 130 MeV

\(^{21}\) Berwein et al. (2016)
Truly asymptotic regime: static quark free energy and entropy shift

\[ S_Q(T) = -\frac{dF_Q(T)}{dT} \]

\[ F_Q|_{LO} = -\frac{C_F \alpha_s m_D}{2} m_D \sim g^T \Rightarrow S_Q|_{LO} \sim g^3 \]

- \( S_Q(T, N_\tau) \) peaks at the same \( T_c(N_\tau) \) as chiral susceptibility \( \chi_2(T, N_\tau) \)
- HRG model of Polyakov loop breaks down early below 130 MeV
- Free energy and entropy shift are known\(^{21}\) at \( O(g^5) \)
- Poor convergence of expansion – NLO in \( g \) is still only LO in \( \alpha_s \)
- Lattice and \( O(g^5) \) agree only for \( T \gtrsim 10 T_c \): static mode dominates \( F_Q \)

\(^{21}\)Berwein et al. (2016)
We obtained the (2+1)-flavor QCD equation of state up to $T \sim 2$ GeV, between EQCD and HTL, suggests underestimated PQCD uncertainty. We study color screening and deconfinement using the renormalized Polyakov loop correlator and related observables. We extract the continuum limit of static quark correlators at $\tau/a = N_\tau$ in (2+1)-flavor QCD up to $T \sim 2$ GeV and down to $r \sim 0.01$ fm. Vacuum-like regime at $rT \sim 0.3$, pNRQCD suitable at $T > 300$ MeV, total $T > 0$ effects small due to various cancellations. For $F_{Q\bar{Q}}$ evidence of the distinctive sub-regimes singlet dominance, $\alpha_s/r \gg T$, and thermal dissociation, $\alpha_s/r \lesssim T$. $\alpha_s(M_Z, N_f = 5) = 0.11638^{+95}_{-87}$ from $F_S(r, T)$ agrees with $T = 0$ result. Evidence of electric screening regime controlled by the perturbative Debye mass at $0.3 \lesssim rT \lesssim 0.6$, described by EQCD at $T > 300$ MeV. Static quark entropy shift $S_Q = -dF_Q/dT$ probes the deconfinement, delivers $T_c$ surprisingly (?) similar to chiral observables (→ Petreczky).