Radius of convergence in lattice QCD at finite μ_B

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arXiv:1911.00043 [hep-lat]

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19. Zimányi School
2019.12.03.

Introduction

Standard statistical mechanical systems in a finite volume

- Partition function of a standard lattice model is a finite polynomial of fugacity (up to a non-vanishing factor)

$$Z = \sum_{n=-kV}^{n=+kV} Z_n \left(\mathrm{e}^{\mu/T}\right)^n$$

- Since it is a polynomial it has $2kV$ roots in the fugacity ($z = \mathrm{e}^{\mu/T}$): **Lee-Yang zeros**
- $Z_n \in \mathbf{R}$, thus if z is a Lee-Yang zero so is z^*
- Due to CP symmetry Z has $z \to 1/z$ symmetry
- The leading singularity of the pressure ($p \sim \ln Z$) is the closest root of Z to the origin in μ which gives the **radius of convergence** of the Taylor-expansion of the pressure in μ

Introduction

- Due to the sign problem configurations cannot be generated at finite μ
- Our goal is to determine the radius of convergence of the pressure in $2+1$ flavor lattice QCD
- If the leading singularity of the pressure in the thermodynamic limit is not on the real axis $\rightarrow$ **lower bound on the critical endpoint (CEP) location**
- If it is on the real axis $\rightarrow$ **phase transition**
- The equation of state as calculated from the Taylor expansion is only reliable up to the radius of convergence

Rooted staggered fermions I.

Partition function of QCD

$$Z = \int \mathcal{D}U \det \mathrm{M}(U;\mu) e^{-S_G[U]} \quad (1)$$

- The determinant can be extracted with the eigenvalues (ξ_i) of a reduced matrix

Fermion determinant at finite lattice spacing

$$\mathrm{detM}(a\mu) = \mathrm{e}^{-3N_s^3 N_t a\mu} \prod_{i=1}^{6N_s^3} \left(\mathrm{e}^{N_t a\mu} - \xi_i\right) \quad (2)$$

- A single determinant describes four quark flavors in the continuum and it is a polynomial in fugacity in a finite lattice

Rooted staggered fermions II.

- The physical expectation is that quartets appear in the spectrum of the reduced matrix
- The standard solution (in case of $N_f = 2$) is taking the square root of the determinant

Standard staggered rooting

$$Z = \int \mathcal{D}U \ \left(\det \mathrm{M}(U;\mu)\right)^{N_f/4} \mathrm{e}^{-S_G[U]} \tag{3}$$

- The determinant and thus the partition function are **no more polynomials** in the fugacity
- Taking the square root at finite lattice spacing introduces extra singularities that decrease the radius of convergence

Rooting by geometric matching

- We suggest a **new definition** of the "rooted" staggered determinant, which ensures that the partition function is a polynomial at a finite lattice spacing

Geometric matching

We group the eigenvalues of the reduced matrix and take the geometric mean:

$$\xi_{\mathrm{rooted}} = \sqrt{\xi_1 \xi_2}$$

- After this type of "rooting" the rooted determinant is again a polynomial:

The new "rooted" determinant

$$\det\mathrm{M}(a\mu) = \mathrm{e}^{-\frac{3}{2}N_s^3 N_t a\mu} \prod_{i=1}^{3N_s^3} \left(\mathrm{e}^{N_t a\mu} - \xi_i^{\mathrm{rooted}}\right) \tag{4}$$

Geometric matching

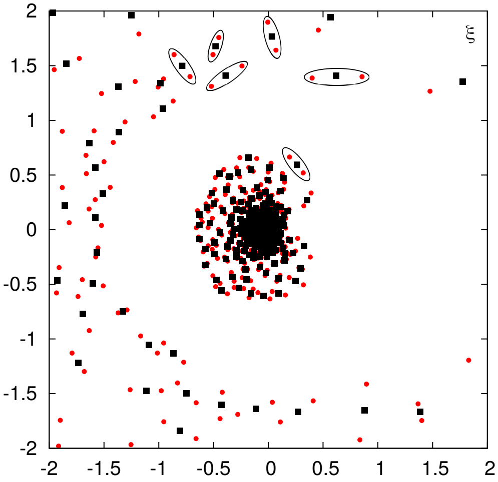

Figure: Eigenvalues of the reduced matrix at $N_s = 12$ and the geometric matching denoted with circles.

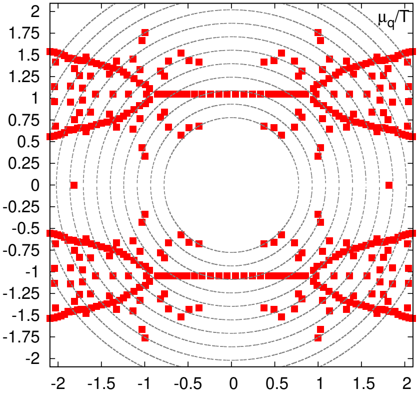

Figure: The roots in complex μ_q of the partition function at $N_s = 12$. The absolute value of the zero closest to the origin gives the radius of convergence

Radius of convergence

Infinite volume extrapolations

Lattice setup: $N_f = 2 + 1$, 2-stout improved staggered action, $N_s = 8,10,12$; $N_t = 4$; $T \sim T_c$

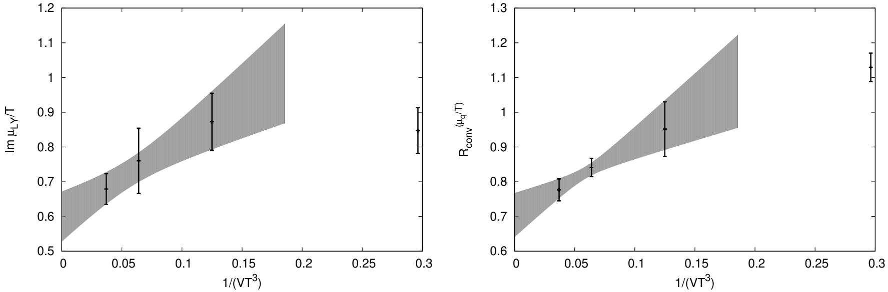

Figure: Infinite volume extrapolation of the imaginary part of μ_{LY} (on the left side) and the radius of convergence (on the right side)

Radius of convergence
Temperature dependence

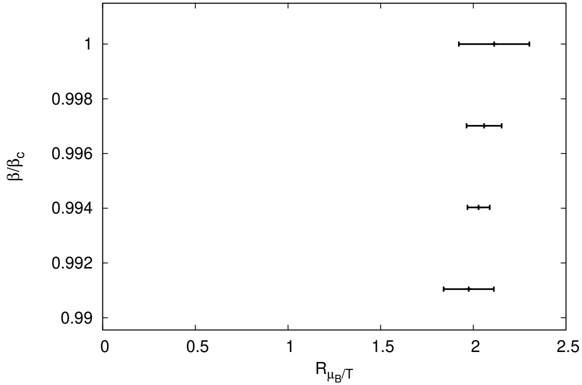

Figure: Infinite volume extrapolations of the radius of convergence for various temperatures (with linear fit on $8^3 4, 10^3 4, 12^3 4$ lattices)

Summary

- The radius of convergence gives a lower bound on the location of CEP or predicts a phase transition
- The standard staggered rooting ruins the polynomial behavior of the partition function
- The so called **geometric matching** gives the 'rooted' determinant in a way that the partition function remains a polynomial
- From the roots of the polynomial the **radius of convergence of the pressure can be obtained directly**