Radius of convergence in lattice QCD at finite $\mu_B$

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arXiv:1911.00043 [hep-lat]

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19. Zimányi School
2019.12.03.
Introduction
Standard statistical mechanical systems in a finite volume

- Partition function of a standard lattice model is a finite polynomial of fugacity (up to a non-vanishing factor)

\[ Z = \sum_{n=-kV}^{n=+kV} Z_n \left( e^{\mu/T} \right)^n \]

- Since it is a polynomial it has \( 2kV \) roots in the fugacity (\( z = e^{\mu/T} \)): Lee-Yang zeros
- \( Z_n \in \mathbb{R} \), thus if \( z \) is a Lee-Yang zero so is \( z^* \)
- Due to CP symmetry \( Z \) has \( z \to 1/z \) symmetry
- The leading singularity of the pressure (\( p \sim \ln Z \)) is the closest root of \( Z \) to the origin in \( \mu \) which gives the radius of convergence of the Taylor-expansion of the pressure in \( \mu \).
Due to the sign problem configurations cannot be generated at finite $\mu$

Our goal is to determine the radius of convergence of the pressure in $2 + 1$ flavor lattice QCD

If the leading singularity of the pressure in the thermodynamic limit is not on the real axis $\rightarrow$ lower bound on the critical endpoint (CEP) location

If it is on the real axis $\rightarrow$ phase transition

The equation of state as calculated from the Taylor expansion is only reliable up to the radius of convergence
Rooted staggered fermions I.

Partition function of QCD

\[ Z = \int \mathcal{D}U \det M(U; \mu) e^{-S_G[U]} \]  \hspace{1cm} (1)

- The determinant can be extracted with the eigenvalues \( (\xi_i) \) of a reduced matrix

Fermion determinant at finite lattice spacing

\[ \det M(a\mu) = e^{-3N_s^3N_t a \mu} \prod_{i=1}^{6N_s^3} (e^{N_t a \mu} - \xi_i) \]  \hspace{1cm} (2)

- A single determinant describes four quark flavors in the continuum and it is a polynomial in fugacity in a finite lattice
The physical expectation is that quartets appear in the spectrum of the reduced matrix.

The standard solution (in case of $N_f = 2$) is taking the square root of the determinant.

**Standard staggered rooting**

$$Z = \int \mathcal{D}U \ (\det M(U; \mu))^{N_f/4} e^{-S_G[U]} \quad (3)$$

The determinant and thus the partition function are **no more polynomials** in the fugacity.

Taking the square root at finite lattice spacing introduces extra singularities that decrease the radius of convergence.
Rooting by geometric matching

- We suggest a **new definition** of the "rooted" staggered determinant, which ensures that the partition function is a polynomial at a finite lattice spacing.

**Geometric matching**

We group the eigenvalues of the reduced matrix and take the geometric mean:

\[ \xi_{\text{rooted}} = \sqrt{\xi_1 \xi_2} \]

- After this type of "rooting" the rooted determinant is again a polynomial:

**The new "rooted" determinant**

\[
\det M(a\mu) = e^{-\frac{3}{2} N_s^3 N_t a\mu} \prod_{i=1}^{3N_s^3} \left( e^{N_t a\mu} - \xi_{\text{rooted}}^i \right) \quad (4)
\]
Figure: Eigenvalues of the reduced matrix at $N_s = 12$ and the geometric matching denoted with circles.
Figure: The roots in complex $\mu_q$ of the partition function at $N_s = 12$. The absolute value of the zero closest to the origin gives the radius of convergence.
Radius of convergence
Infinite volume extrapolations

Lattice setup: $N_f = 2 + 1$, 2-stout improved staggered action, $N_s = 8, 10, 12$; $N_t = 4$; $T \sim T_c$

Figure: Infinite volume extrapolation of the imaginary part of $\mu_{LY}$ (on the left side) and the radius of convergence (on the right side)
Figure: Infinite volume extrapolations of the radius of convergence for various temperatures (with linear fit on $8^34,10^34,12^34$ lattices)
Summary

- The radius of convergence gives a lower bound on the location of CEP or predicts a phase transition.

- The standard staggered rooting ruins the polynomial behavior of the partition function.

- The so-called geometric matching gives the ’rooted’ determinant in a way that the partition function remains a polynomial.

- From the roots of the polynomial the radius of convergence of the pressure can be obtained directly.