Coulomb correction for Lévy-type sources

Máté Csanád, Márton Nagy and Sándor Lökös

Eötvös University, Budapest and EFOP-3.6.1-16-2016-00001, Gyöngyös

Zimányi Winter School, 2019, Budapest, Hungary

Outline

- Bose-Einstein correlations
- Lévy-type sources
- Motivation from experimental results
- Final state interactions and distortion effects
- Different methods for Coulomb-correction
- Summary

Bose-Einstein correlations – basic definitions

- $S(x,p)$ source function, the two-particle wavefunction is symmetrized
- Momentum distribution can be calculated:
  - $N_1(p) = \int S(x,p)|\Psi^{(1)}(x)|^2 d^4x$
  - $N_2(p) = \int S(x_1,p_1)S(x_2,p_2)|\Psi^{(2)}(x_1,x_2)|^2 d^4x_1 d^4x_2$
- Correlation function from one- and two-particle momentum distributions:

$$C_2(p_1,p_2)=\frac{N_2(p_1,p_2)}{N_1(p_1)N_1(p_2)} \rightarrow \Psi \text{ is planewave} \rightarrow C_2^{(0)}(q,K)\approx 1+\frac{|\tilde{S}(q,K)|^2}{|\tilde{S}(q=0,K)|^2}$$

  where $q = p_1 - p_2$ and $K = (p_1 + p_2)/2$
- $^{(0)}$ means no final state interaction
- Introducing the spatial correlation as

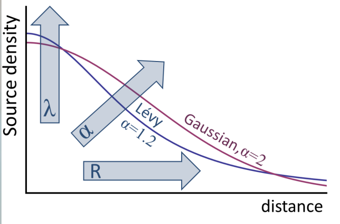
$$D(r,K) = \int S(\rho+\frac{r}{2},K)S(\rho-\frac{r}{2},K)d\rho \rightarrow C_2^{(0)}(q,K) \approx 1 + \frac{\tilde{D}(q,K)}{\tilde{D}(q=0,K)}$$

- **The Bose-Einstein correlation measures the spatial correlation!**

Lévy-type of source

- Expanding medium, increasing mean free path: anomalous diffusion
- Lévy-distribution from generalized central limit theorem could be valid

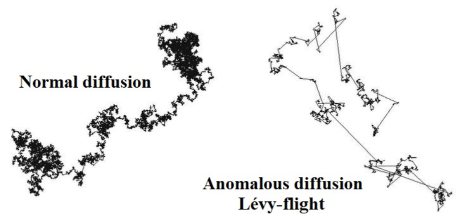
$$S(x,p) = \frac{1}{(2\pi)^3}\int d^3q e^{i\mathbf{qx}} e^{-\frac{1}{2}|\mathbf{x}R|^\alpha}$$



- Correlation function with Lévy source:

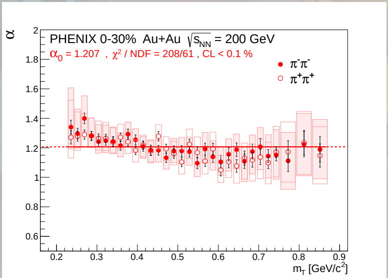
$$C_2^{(0)}(Q) = 1 + \lambda \cdot e^{-(RQ)^\alpha}$$

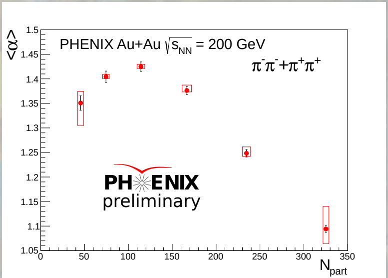
- $\alpha = 2$ Gaussian (normal diffusion)
- $\alpha < 2$ Lévy (anomalous diffusion)
- Change in $\alpha_{\text{Lévy}} \leftrightarrow$ proximity of CEP?
- Non-static system, finite size effects...

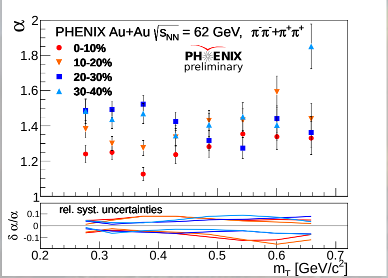


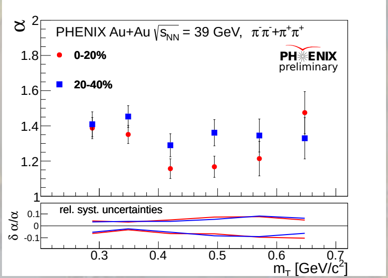
Experimental results – motivation

PHENIX results from different centralities and energies are available









Bose-Einstein correlations – distortion effects

There are several effects which could distort this simple picture

- Strong final state interaction (see: Daniel's presentation)
- Resonance effects $\rightarrow$ core-halo model
- Partially coherent particle production
- Squeezed states
- Effect of random phase shifts (see: Ayon's presentation)
- **Coulomb correction**, since we measure (like-)charged pions

The Coulomb final state effect

- Generally the correlation function:

$$C_2(q) = \frac{\int d^3\mathbf{r} D(\mathbf{r},\mathbf{K}) |\psi_{\mathbf{q}}^{(2)}(\mathbf{r})|^2}{\int d^3\mathbf{r} D(\mathbf{r},\mathbf{K})}$$

- No final state interaction $\rightarrow \psi_{\mathbf{q}}^{(2)}(\mathbf{r}) = 1 + \cos(\mathbf{qr})$ symmetrized $\rightarrow C_2^{(0)}(q)$
- Two-particle Coulomb interacting pair-wave function in the final state:

$$\psi_{\mathbf{q}}^{(2)}(\mathbf{r}) = \frac{1}{\sqrt{2}} \frac{\Gamma(1+i\eta)}{e^{\pi\eta/2}} \{ e^{i\mathbf{kr}} F\left(-i\eta, 1, i(kr - \mathbf{kr})\right) + [\mathbf{r} \leftrightarrow -\mathbf{r}] \}$$

- For pointlike source: Gamow correction $G(q) = |\psi_{\mathbf{q}}^{(2)}(0)|^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1}$ with the Sommerfeld parameter: $\eta = \alpha_{EM} \frac{m_\pi c}{q}$
- Generally: evaluate the integral $\rightarrow$ Coulomb-correction: $K(q) = \frac{C_2(q)}{C_2^{(0)}(q)}$
- Complicated integral $\rightarrow$ numerical approaches!

How to handle the Coulomb effect?

- Perform the integral numerically, fill it into a look-up table
- Numerical fluctuations $\rightarrow$ iterative fit (like in SPS):
  1. Fit with the $C_2^{(0)}$ Lévy-type of correlation function $\Rightarrow \lambda_0, R_0, \alpha_0$
  2. Fit with $C_2(q) = C_2^{(0)}(\lambda, R, \alpha; Q)\frac{C_2(\lambda_0, R_0, \alpha_0; Q)}{C_2^{(0)}(\lambda_0, R_0, \alpha_0; Q)} \Rightarrow \lambda_1, R_1, \alpha_1$
  3. Repeat while λ_0, R_0, α_0 and λ_1, R_1, α_1 differ less then 1%
- This known method was used in PRC97 (2018) 064911 succesfully!
- Reading a huge binary table and do an iterative fit is a bit slow
- Alternative: use a parametrization
  1. Get the table and parametrize the Q dependence for different α and R values $\rightarrow$ several α and R dependent parameters
  2. Parametrize the α and R dependence of the parameters!
  3. Take care of the out-of-range behavior (smoothings and cut-offs)!
- $\alpha = 1$ parametrization from CMS: PRC 97 (2018) 064912
- For arbitrary α: arXiv:1910.02231

Numerical table

- Evaluate the integral numerically and fill the values into a binary table
- Use $R_{cc} = 2^{1/\alpha} R$
- Resolutions:
    - $\alpha \in [0.5, 2]$, $\Delta\alpha = 0.00625 \rightarrow N_\alpha = 240$
    - $Q \in [10, 200]$ MeV/c, $\Delta Q = 0.5$ MeV/c $\rightarrow N_Q = 380$
    - $R_{cc} \in [2, 30]$ fm, $\Delta R_{cc} = 0.01$ fm $\rightarrow N_{R_{cc}} = 2800$
- Number of point: $240 \times 380 \times 2800 = 2.5 \cdot 10^8$
- Size of table ~ 1 GB
- Interpolation is done via linear interpolation
- For $Q > 200$ MeV gives back the evaluation of $C_2^{(0)}(Q, \alpha, R)$

Parametrizations

„With six parameters, you can fit even an octopus."
Unknown physicist

- Parametrization of the Q-dependence with:

$$K_{Cauchy}(q,\alpha=1,R) = K_{Gamow}(q) \times K_{CMS}(q,\alpha=1,R)$$

- The correction for the Gamow correction

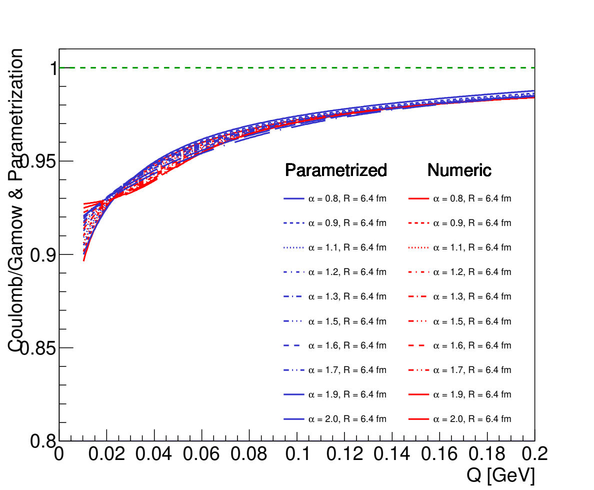
$$K_{CMS}(q,\alpha=1,R) = 1 + \frac{\alpha_{EM}\pi m_\pi R}{1.26\hbar c + qR}$$

- Advantage: simple, has only one numerical parameter
- Disadvantage: only good for $\alpha \approx 1$
- Looking for an improved $K_{CMS} \to K_{mod}$ parametrization ...
  - ... which can follow the weak α dependence
  - ... which can restore the „CMS formula" in case of $\alpha = 1$

$$K_{mod}(q,\alpha,R) = 1 + \frac{A(\alpha,R)\frac{\alpha_{EM}\pi m_\pi R}{\alpha\hbar c}}{1 + B(\alpha,R)\frac{qR}{\alpha\hbar c} + C(\alpha,R)\left(\frac{qR}{\alpha\hbar c}\right)^2 + D(\alpha,R)\left(\frac{qR}{\alpha\hbar c}\right)^4}$$

The correction in the Lévy case – Q dependence

$$K_{mod}(q,\alpha,R) = 1 + \frac{A(\alpha,R)\frac{\alpha_{EM}\pi m_\pi R}{\alpha\hbar c}}{1 + B(\alpha,R)\frac{qR}{\alpha\hbar c} + C(\alpha,R)\left(\frac{qR}{\alpha\hbar c}\right)^2 + D(\alpha,R)\left(\frac{qR}{\alpha\hbar c}\right)^4}$$



Parameter functions

Four 2D empirical parameter functions:

$$A(\alpha, R) = (a_A\alpha + a_B)^2 + (a_C R + a_D)^2 + a_E(\alpha R + 1)^2$$

$$B(\alpha, R) = \frac{1 + b_A R^{b_B} - \alpha^{b_C}}{\alpha^2 R(\alpha^{b_D} + b_E R^{b_F})}$$

$$C(\alpha, R) = \frac{c_A + \alpha^{c_B} + c_C R^{c_D}}{c_E} \left(\frac{\alpha}{R}\right)^{c_F}$$

$$D(\alpha, R) = d_A + \frac{R^{d_B} + d_C \alpha^{d_F}}{R^{d_D} \alpha^{d_E}}$$

- These can describe the α and the R dependencies
- Quite complicated but suitable
- $a_A, a_B, ..., d_F$ are numbers (see our paper: `arXiv:1910.02231`)

Out-of-range regularization

- We fit to $q \in [0.01, 0.2]$ GeV/c
- Beyond the fitted range, the function cannot be used to extrapolate
- Exponential type function parametrized based on the intermediate $q \in [0.1, 0.2]$ GeV/c fits to the numerical table

$$E(q) = 1 + A(\alpha, R)e^{-B(\alpha,R)q}$$

with

$$A(\alpha, R) = A_a + A_b\alpha + A_c R + A_d\alpha R + A_e R^2 + A_f(\alpha R)^2,$$
$$B(\alpha, R) = B_a + B_b\alpha + B_c R + B_d\alpha R + B_e R^2 + B_f(\alpha R)^2$$

where $A_a, A_b, ..., B_f$ are numbers (see our paper: `arXiv:1910.02231`)

Smoothing on the edge

- Exponential damping factor should be „joined" to the parametrization
- Can be done with a Wood-Saxon-type of cut-off

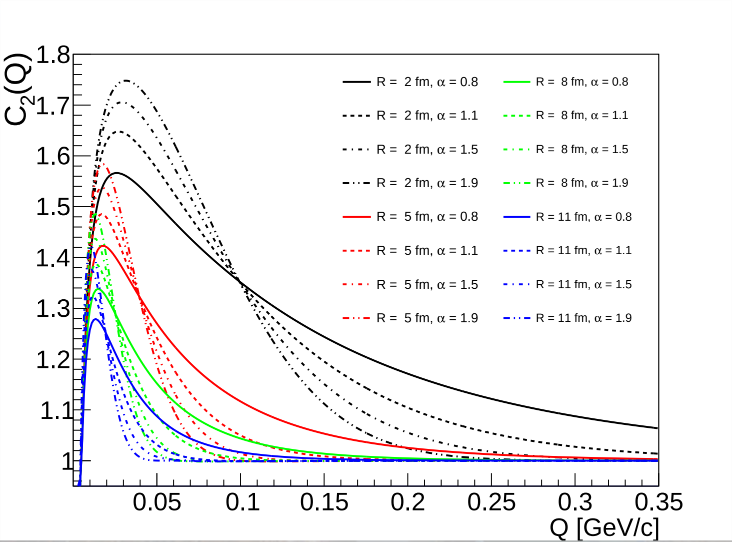
$$F(q) = \frac{1}{1 + \left(\frac{q}{q_0}\right)^n}$$

where $q_0 = 0.07$ GeV/c and $n = 20$.

- Previous work has different cut-off functions: $\left(1 + \exp\left[\frac{q - q_0}{D_q}\right]\right)^{-1}$
- The results are quite independent from the choice.
- New $F(q)$ has better behavior.

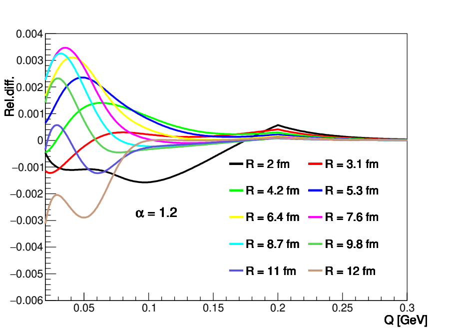
Example Coulomb corrected Lévy-type of C_2

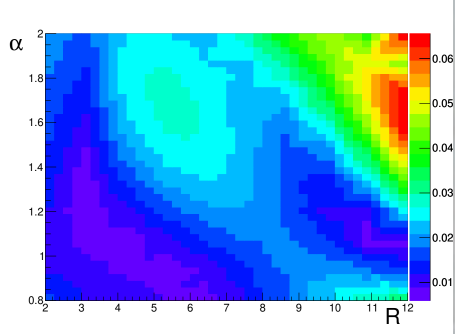
α control the shape of the function and R control the width



The validity of the parametrization

- $\alpha \in [0.8, 2.0]$ and $R \in [2, 12]$ fm
- In these range of the parameters the relative difference of the parametrization and the table is less then 0.06%





Bowler–Sinyukov-method

- Core-halo model: $S = \sqrt{\lambda}S_c + (1-\sqrt{\lambda})S_h^{R_h}$
- With two-particle source function:

$$D(\mathbf{r},\mathbf{K}) = \lambda D_{cc}(\mathbf{r},\mathbf{K}) + (1-\lambda)D_{(h)}(\mathbf{r},\mathbf{K})$$

- $D_{(h)}(\mathbf{r},\mathbf{K})$ contains $core - halo$ and $halo - halo$ parts

$$C_2(\mathbf{r},\mathbf{K}) \approx \lambda \int d^3\mathbf{r} D_{cc}(\mathbf{r},\mathbf{K})|\psi_{\mathbf{q}}^{(2)}(\mathbf{r})|^2 + (1-\lambda)\int d^3\mathbf{r} D_{(h)}(\mathbf{r},\mathbf{K})|\psi_{\mathbf{q}}^{(2)}(\mathbf{r})|^2$$

- If we take the $R_h \to \infty$ limit (the size of the halo is infinite):

$$C_2(\mathbf{r},\mathbf{K}) = 1 - \lambda + \lambda \int d^3\mathbf{r} D_{cc}(\mathbf{r},\mathbf{K})|\psi_{\mathbf{q}}^{(2)}(\mathbf{r})|^2$$

- Bowler,PLB270 69(1991) and Sinyukov et al.PLB432 248(1998)

Final form of the correlation function

- Parametrization of the numerical table: $K(q) = K_{Gamow} \times K_{mod}$
- Exponential function for large q-values: $E(q)$
- Smoothing to „joined" $E(q)$ and $K(q)$: $F(q)$

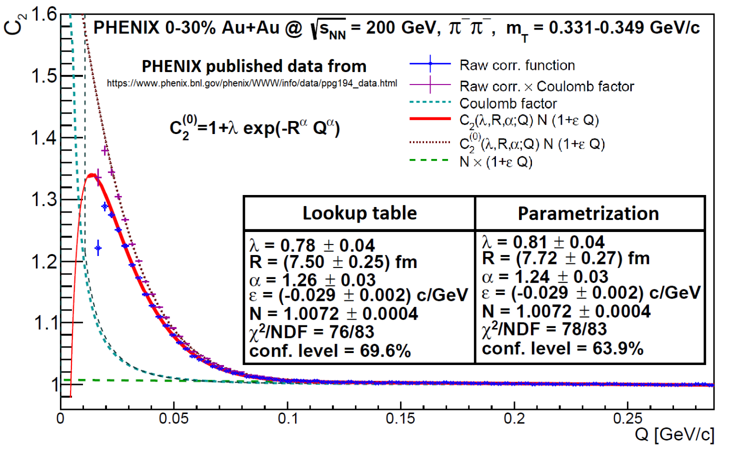
$$K(q,\alpha,R)^{-1} = F(q) \times K_{Gamow}^{-1}(q) \times K_{mod}^{-1}(q;\alpha,R) + (1-F(q)) \times E(q)$$

- Coulomb corrected correlation function which could be fitted to data with the Bowler–Sinyukov-method:

$$C_2(q;\alpha,R) = [1 - \lambda + K(q;\alpha,R)\lambda\,(1 + \exp[|qR|^\alpha])] \cdot (\text{assumed background})$$

Compare with previous fit results

- Test it on data previously fitted with the numerical table
- Good agreement



Summary

- Coulomb final state interaction is important in correlation measurements
- For extended source, exact and analytic form is not known
- Experimetal results motivate to investigate the Lévy-case
- Numerical techniques can be employed $\rightarrow$ numerical table
- Parametrization based on the table is more convenient to use
- Parametrization for Lévy sources in $\alpha \in [0.8, 2.0]$ and $R \in [2, 12]$ fm
- Paper from our recent results: `arXiv:1910.02231`
- Code: `https://github.com/csanadm/coulcorrlevyparam`

Thank you for your attention!