Exclusive vector meson production with impact parameter dependent Balitsky-Kovchegov evolution equation [1]

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Motivation: Saturation of gluon densities

Figure: Diagram picturing the QCD evolution of the partonic structure of the proton.

Motivation: Saturation of gluon densities

- Exclusive production of a vector meson is very sensitive to gluon densities.
- An excellent probe of saturation effects.

Figure: Diagram picturing the QCD evolution of the partonic structure of the proton.


Figure: Exclusive production of a vector meson.
Vector meson production in the color dipole picture

- Exclusive cross section to produce the vector meson

\[
\frac{d\sigma}{d|t|}^{\gamma^* p \rightarrow VMp}_{T,L} = \left( R_g^{T,L} \right)^2 \left( 1 + \beta_{T,L}^2 \right) \frac{1}{16\pi} \left| \langle A_{T,L}^{\gamma^* p \rightarrow VMp} \rangle \right|^2.
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\]

- The scattering amplitude is given as a convolution of the photon-VM wave function with the dipole cross section:

\[
\mathcal{A}_{T,L}(x, Q^2, \Delta) = i \int d\bar{r} \int_0^1 \frac{dz}{4\pi} \int d\bar{b} |\Psi_{VM}^{*} \psi_{\gamma^*}|_{T,L} \exp \left[ -i \left( \bar{b} - (1 - z)\bar{r} \right) \Delta \right] \frac{d\sigma_{q\bar{q}}}{db}.
\]

**Figure:** Schematic picture of vector meson production within the color dipole approach.
Dipole-target cross section

- Carries information about the strong interaction between the dipole and the proton.
- The differential dipole-proton cross section is given by the dipole scattering amplitude

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  - Factorization of the impact parameter dependence \(^2\)
    \[ \frac{d\sigma_{q\bar{q}}}{db} \to \sigma_0 N(x, \vec{r}) T_p(\vec{b}). \]
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- Numerical solution of the Balitsky–Kovchegov (BK) equation with \( b \)-dependence.
  - Demanding on computational resources.
  - Until recently, the problem of Coulomb tails.

Balitsky–Kovchegov evolution equation

- Evolution with rapidity \( Y \sim \ln \left( \frac{1}{x} \right) \) of the scattering amplitude \( N(\vec{r}, \vec{b}, Y) \) of a q\bar{q} dipole with a hadronic target

\[
\frac{\partial N(\vec{r}, \vec{b}, Y)}{\partial Y} = \int d\vec{r}_1 K(r, r_1, r_2) \left( N(\vec{r}_1, \vec{b}_1, Y) + N(\vec{r}_2, \vec{b}_2, Y) - N(\vec{r}, \vec{b}, Y) - N(\vec{r}_1, \vec{b}_1, Y)N(\vec{r}_2, \vec{b}_2, Y) \right)
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\]

- Describes the dressing of a color dipole with the evolution to higher energies.

\[ q \quad \text{boost} \quad \bar{q} \quad x \]

\[ q \quad y \]

\[ \text{large } N_c = \]

\[ q \quad x + \bar{q} \quad y \]

\[ q \quad y \]

\[ \bar{q} \quad \bar{q} \quad x \]

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Evolution kernel

- Describes the probability of a gluon emission.
- Different approximations of the probability calculation lead to several forms of kernels.

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**Running coupling kernel** \[^{[3]}\]

\[
K^{rc}(r, r_1, r_2) = \frac{\alpha_s(r^2) N_C}{2\pi^2} \left[ \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]
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**Collinearly improved kernel** [4]

\[ K^{ci}(r, r_1, r_2) = \frac{\bar{\alpha}_S}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[ \frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm\bar{\alpha}_S A_1} K_{DLA} \left( \sqrt{L_{r_1} L_{r_2}} \right) \]

\[ K_{DLA}(\rho) = \frac{J_1(2\sqrt{\bar{\alpha}_S \rho^2})}{\sqrt{\bar{\alpha}_S \rho}}, \quad L_{r_i, r} = \ln \left( \frac{r_i^2}{r^2} \right) \]

---

Implementation of impact parameter dependence

- The equation is solved using a new initial condition that takes into account the location of the end-points of the dipole

\[ N(\vec{r}, \vec{b}, Y = 0) = 1 - \exp \left[ -\frac{1}{2} \frac{Q_s^2}{4} r^2 T(\vec{b}_{q_1}, \vec{b}_{q_2}) \right] \]

\[ T(\vec{b}_{q_1}, \vec{b}_{q_2}) = \exp \left( -\frac{b_{q_1}^2}{2B} \right) + \exp \left( -\frac{b_{q_2}^2}{2B} \right) \]
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\]

- The \( r \) behavior mimics the models for the \( b \)-independent dipole amplitude.

- The \( b \) behavior contains an exponential fall-off for dipole-ends far away from the target.
The problem of Coulomb tails

- Evolution with exponentially falling initial condition and $K^{rc}$ increases large dipoles into a power-like growth.
  - Destroys the predictive power of BK equation, doesn’t allow for phenomenological applications.
  - The growth violates the Martin-Froissart bound.

J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502
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- Evolution at high-$b$ can be suppressed by suppressing large daughter dipoles
  - By introducing a kernel cut-off $^a$
    - cut-off too strong
    - predictive power isn’t restored
  - Using a collinearly improved kernel $K^{ci}$
    - ressums single and double collinear logarithms
    - large daughter dipoles are suppressed.

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Impact parameter dependent dipole amplitude can be used to study several processes and observables.
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- Nuclear $F_2$
- Exclusive photo- and electroproduction of vector mesons (this talk)
Predictions and comparison with data for cross sections of $J/\psi$ meson

- Predictions compared to H1 and ALICE p–Pb data.

![Graph showing predictions and comparison with data for cross sections of $J/\psi$ meson](image)

Predictions and comparison with data for cross sections of $J/\psi$ meson

- Predictions compared to H1 and ALICE p–Pb data.

![Graph showing predictions for photoproduction and electroproduction](image-url)
Predictions and comparison with data for cross sections of $J/\psi$ meson

- Predictions compared to H1 and ALICE p–Pb data.

Predictions and comparison with data for cross sections of $\phi$ meson

- Predictions compared to H1 and ZEUS data.

![Graph showing predictions and data comparison for cross sections of $\phi$ meson](image)

Electroproduction, $\langle W \rangle = 75$ GeV
- ZEUS (2005) $Q^2 = 2.4$ GeV$^2$
- ZEUS (2005) $Q^2 = 3.6$ GeV$^2$
- ZEUS (2005) $Q^2 = 12.6$ GeV$^2$

Predictions and comparison with data for cross sections of $\phi$ meson

- Predictions compared to H1 and ZEUS data.

Predictions and comparison with data for cross sections of $\Upsilon$ meson

- Photoproduction predictions compared to H1, ZEUS, CMS and LHCb data.

Conclusions

- The collinearly improved kernel suppresses the Coulomb tails → allows for phenomenological applications.
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- Successful application of dipole amplitudes obtained by solving b-BK into exclusive vector meson production.
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- The collinearly improved kernel suppresses the Coulomb tails → allows for phenomenological applications.
- Successful application of dipole amplitudes obtained by solving b-BK into exclusive vector meson production.
- Good agreement of the predictions with data for $\phi$, $J/\psi$, and $\Upsilon(1S)$ mesons.
BACKUP SLIDES
Wave functions

- The overlap between the photon and vector meson wave functions:

\[
|\Psi_{VM}^* \Psi_{\gamma}^*|_T = \hat{e}_f e \frac{N_C}{\pi z(1 - z)} \left[ m_f^2 K_0(\epsilon r) \phi_T(r, z) - \left( z^2 + (1 - z)^2 \right) \epsilon K_1(\epsilon r) \partial_r \phi_T(r, z) \right],
\]

\[
|\Psi_{VM}^* \Psi_{\gamma}^*|_L = \hat{e}_f e \frac{N_C}{\pi} 2 Q z(1 - z) K_0(\epsilon r) \left[ M_{VM} \phi_L(r, z) + \delta \frac{m_f^2 - \nabla_r^2}{M_{VM} z(1 - z)} \phi_L(r, z) \right].
\]

\[
\epsilon = z(1 - z) Q^2 + m_f^2; \quad r \equiv |\vec{r}|
\]

- Scalar part of VM wave function from Boosted Gaussian model:

\[
\phi_{T, L}(r, z) = N_{T, L} z(1 - z) \exp \left[ - \frac{m_f^2 R^2}{8 z(1 - z)} - \frac{2z(1 - z) r^2}{R^2} + \frac{m_f^2 R^2}{2} \right].
\]

\[
\phi_{T, L}^{2S}(r, z) = \phi_{T, L}(r, z) \left( 1 + \alpha_{2S} \left( 2 + \frac{m_f^2 R^2}{4z(1 - z)} - \frac{4z(1 - z) r^2}{R^2} - m_f^2 R^2 \right) \right)
\]
Corrections

- Correction on the real part of the amplitude
  - $A_{T,L}$ is not purely imaginary, has a real part
    \[
    \lambda_{T,L} \equiv \frac{\partial \ln \left( A_{T,L}^{\gamma^*p \rightarrow VMp} \right)}{\partial \ln \left( \frac{1}{x} \right)}, \quad \beta_{T,L} = \tan \left( \frac{\pi \lambda_{T,L}}{2} \right) \tag{1}
    \]

- Skewedness correction
  - Takes into account the fact, that there are two $x$ involved in the dipole-target interaction
    \[
    R_g(\lambda_{T,L}) = \frac{2^{2\lambda_{T,L} + 3}}{\sqrt{\pi}} \frac{\Gamma \left( \lambda_{T,L} + \frac{5}{2} \right)}{\Gamma \left( \lambda_{T,L} + 4 \right)} \tag{2}
    \]
Influence of the individual collinear kernel terms on the Coulomb tails

\[ K_{ci}^1 = \frac{\bar{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2}; \quad K_{ci}^2 = \left[ \frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm \bar{\alpha}_s A_1}; \quad K_{ci}^3 = K_{DLA} \left( \sqrt{L_{r_1} L_{r_2}} \right) \]
DIS: Structure function $F_2(x, Q^2)$

- Cross section for $\gamma^* p$ scattering and structure function $F_2$

$$
\sigma_{T,L} = \sum_f \int d^2r \int dz |\Psi^* \Psi|^f_{T,L} \sigma_{q\bar{q}}(\bar{x}, r), \quad F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha} (\sigma_T + \sigma_L).
$$
References to experimental data

- **$J/\psi$ data**
  - B. B. Abelev et al. (ALICE Collaboration), Phys. Rev. Lett. 113 (2014) 232504

- **$\phi$ data**
  - F. D. Aaron et al. (H1 Collaboration), J. High Energy Phys. 05 (2010) 032

- **$\Upsilon(1S)$ data**