Exclusive vector meson production with impact parameter dependent Balitsky-Kovchegov evolution equation [1]

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[1] Based on: Phys. Rev. D 99, 051502 and Phys. Rev. D 100, 054015

Motivation: Saturation of gluon densities

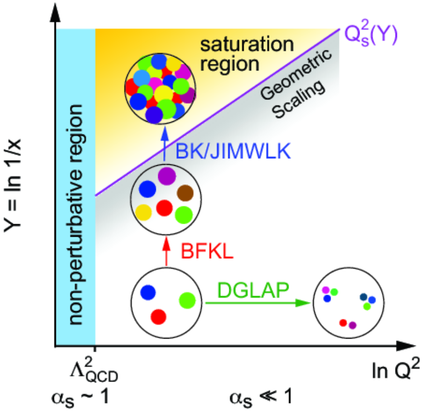

Figure: Diagram picturing the QCD evolution of the partonic structure of the proton.
C. Marquet, Nucl.Phys. A904-905 (2013) 294c-301c.

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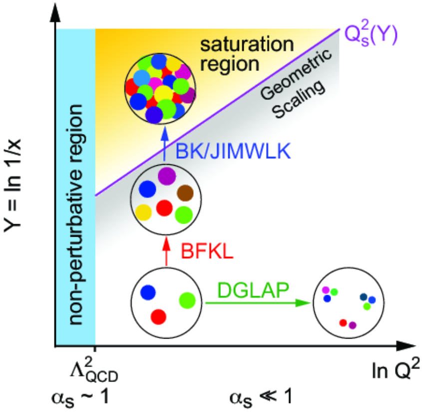

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- Exclusive production of a vector meson is very sensitive to gluon densities.
- An excellent probe of saturation effects.

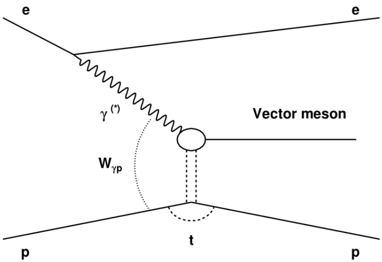

Figure: Exclusive production of a vector meson.

Vector meson production in the color dipole picture

- Exclusive cross section to produce the vector meson

$$\left.\frac{\mathrm{d}\sigma^{\gamma^* p\to VMp}}{\mathrm{d}|t|}\right|_{T,L} = \frac{\left(R_g^{T,L}\right)^2 (1+\beta_{T,L}^2)}{16\pi}\left|\langle\mathcal{A}_{T,L}^{\gamma^* p\to VMp}\rangle\right|^2.$$

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- The scattering amplitude is given as a convolution of the photon-VM wave function with the dipole cross section:

$$\mathcal{A}_{T,L}(x, Q^2, \vec{\Delta}) = i \int \mathrm{d}\vec{r} \int_0^1 \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}\vec{b} |\Psi_{VM}^* \Psi_{\gamma^*}|_{T,L} \exp\left[-i\left(\vec{b}-(1-z)\vec{r}\right)\vec{\Delta}\right] \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}\vec{b}}.$$

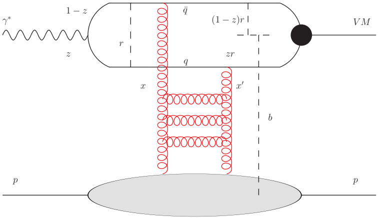

Figure: Schematic picture of vector meson production within the color dipole approach.

Dipole-target cross section

- Carries information about the strong interaction between the dipole and the proton.
- The differential dipole-proton cross section is given by the dipole scattering amplitude

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}\vec{b}} = 2N(x, \vec{r}, \vec{b}).$$

[2]e.g. Phys. Lett. B766 (2017) 186; Nucl. Phys. B934 (2018) 330; Phys. Rev. D99 (2019) 034025

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  - Factorization of the impact parameter dependence [2]

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}\vec{b}} \to \sigma_0 N(x, \vec{r}) T_p(\vec{b}).$$

    - ★ $T_p(\vec{b})$ – proton profile in the impact-parameter plane
    - ★ σ_0 – multiplicative factor obtained from fit to data or from normalization

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    - $T_p(\vec{b})$ – proton profile in the impact-parameter plane
    - σ_0 – multiplicative factor obtained from fit to data or from normalization
  - Numerical solution of the Balitsky–Kovchegov (BK) equation with b-dependence.
    - Demanding on computational resources.
    - Until recently, the problem of Coulomb tails.

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Balitsky–Kovchegov evolution equation

- Evolution with rapidity $Y \sim \ln\left(\frac{1}{x}\right)$ of the scattering amplitude $N(\vec{r}, \vec{b}, Y)$ of a $q\bar{q}$ dipole with a hadronic target

$$\frac{\partial N(\vec{r}, \vec{b}, Y)}{\partial Y} = \int \mathrm{d}\vec{r_1} K(r, r_1, r_2)\left(N(\vec{r_1}, \vec{b_1}, Y) + N(\vec{r_2}, \vec{b_2}, Y) - N(\vec{r}, \vec{b}, Y) - N(\vec{r_1}, \vec{b_1}, Y)N(\vec{r_2}, \vec{b_2}, Y)\right)$$

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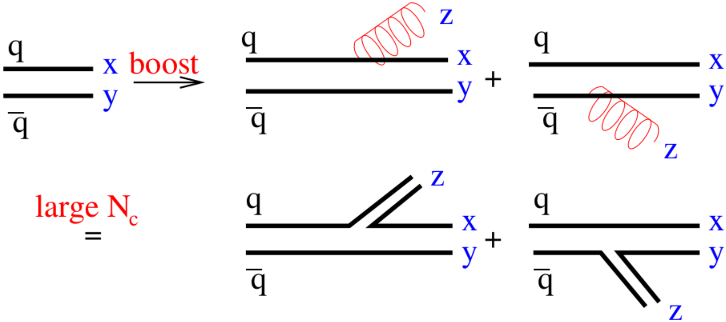
- Describes the dressing of a color dipole with the evolution to higher energies.



J. L. Albacete et al., Phys. Rev. D 71 (2005) 014003

Evolution kernel

- Describes the probability of a gluon emission.
- Different approximations of the probability calculation lead to several forms of kernels.

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[4] E. Iancu et al., Phys. Lett. B 750 (2015) 643; A. Sabio Vera, Nucl. Phys. B722 (2005) 65

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- **Running coupling kernel** [3]

$$K^{rc}(r, r_1, r_2) = \frac{\alpha_S(r^2)N_C}{2\pi^2}\left[\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_S(r_1^2)}{\alpha_S(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_S(r_2^2)}{\alpha_S(r_1^2)} - 1\right)\right]$$

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- **Collinearly improved kernel** [4]

$$K^{ci}(r, r_1, r_2) = \frac{\bar{\alpha}_S}{2\pi}\frac{r^2}{r_1^2 r_2^2}\left[\frac{r^2}{\min(r_1^2, r_2^2)}\right]^{\pm\bar{\alpha}_S A_1} K_{DLA}\left(\sqrt{L_{r_1 r} L_{r_2 r}}\right)$$

$$K_{DLA}(\rho) = \frac{J_1(2\sqrt{\bar{\alpha}_S \rho^2})}{\sqrt{\bar{\alpha}_S \rho}}, \qquad L_{r_i, r} = \ln\left(\frac{r_i^2}{r^2}\right)$$

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Implementation of impact parameter dependence

- The equation is solved using a new initial condition that takes into account the location of the end-points of the dipole

$$N(\vec{r}, \vec{b}, Y = 0) = 1 - \exp\left[-\frac{1}{2}\frac{Q_S^2}{4}r^2 T(\vec{b}_{q_1}, \vec{b}_{q_2})\right]$$

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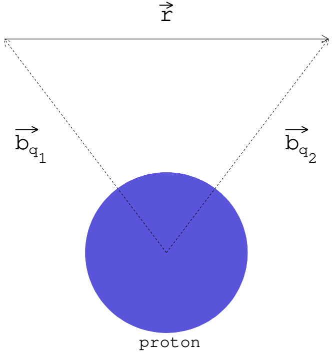
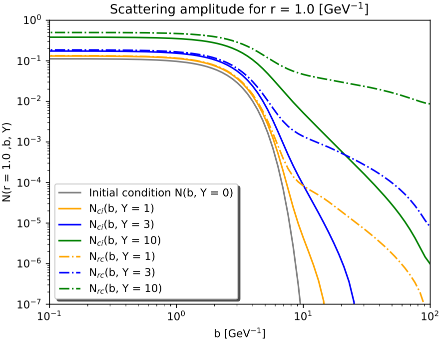


- The r behavior mimics the models for the b-independent dipole amplitude.
- The b behavior contains an exponential fall-off for dipole-ends far away from the target.

The problem of Coulomb tails

- Evolution with exponentially falling initial condition and K^{rc} increases large dipoles into a power-like growth.
  - Destroys the predictive power of BK equation, doesn't allow for phenomenological applications.
  - The growth violates the Martin-Froissart bound.



J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

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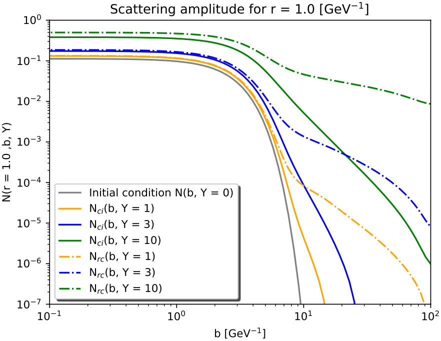

J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

- Evolution at high-b can be suppressed by suppressing large daughter dipoles
  - By introducing a kernel cut-off [a]
    - ★ cut-off too strong
    - ★ predictive power isn't restored
  - Using a collinearly improved kernel K^{ci}
    - ★ ressums single and double collinear logarithms
    - ★ large daughter dipoles are suppressed.

[a] J. Berger, A. M. Stasto, Phys. Rev. D 84 (2011) 094022

Phenomenology with b-BK

- Impact parameter dependent dipole amplitude can be used to study several processes and observables

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  - Deep Inelastic Scattering - structure functions F_2

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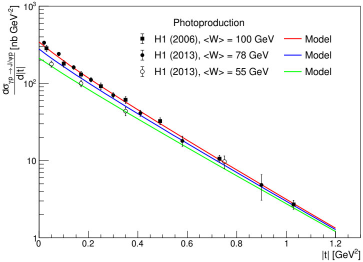
  - **Exclusive photo- and electroproduction of vector mesons** (this talk)

Predictions and comparison with data for cross sections of J/ψ meson

- Predictions compared to H1 and ALICE p–Pb data.



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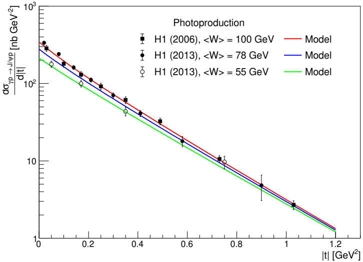

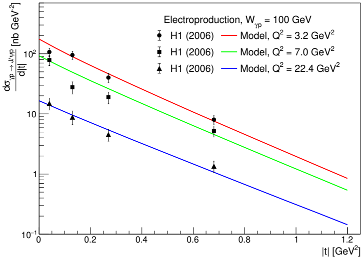

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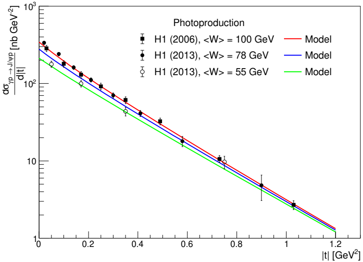

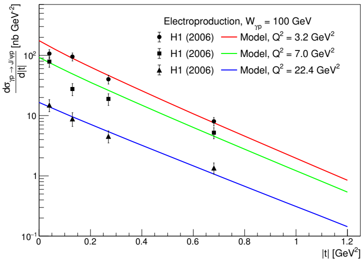

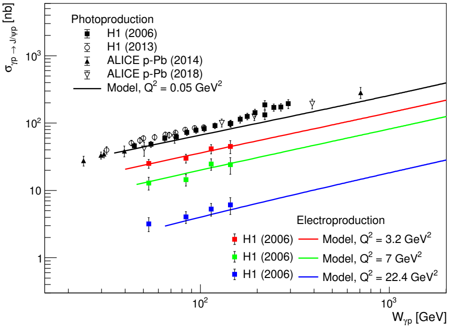

D. Bendova, J. Cepila, J. G. Contreras, M. Matas, Phys. Rev. D100 (2019) 054015; references to experimental data in backup

Predictions and comparison with data for cross sections of ϕ meson

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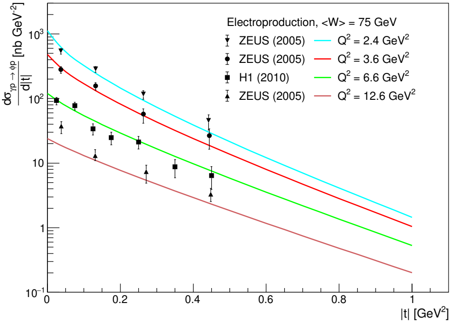

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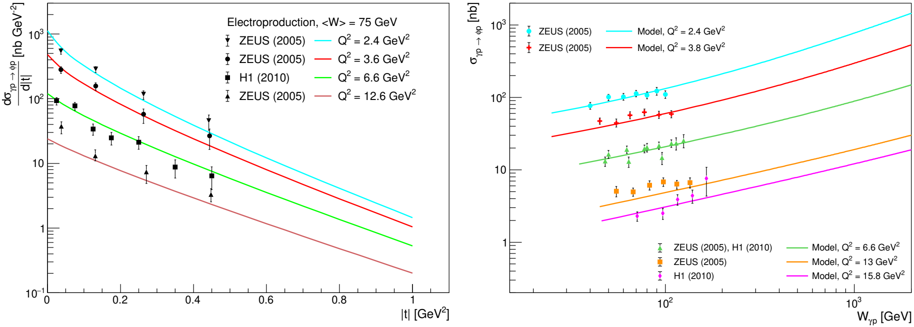

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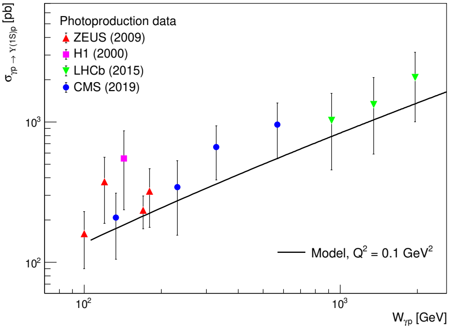
- Photoproduction predictions compared to H1, ZEUS, CMS and LHCb data.



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Conclusions

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- The collinearly improved kernel suppresses the Coulomb tails $\rightarrow$ allows for phenomenological applications.
- Successful application of dipole amplitudes obtained by solving b-BK into exclusive vector meson production.
- Good agreement of the predictions with data for ϕ, J/ψ, and $\Upsilon(1S)$ mesons.

BACKUP SLIDES

Wave functions

- The overlap between the photon and vector meson wave functions:

$$|\Psi^*_{VM}\Psi_{\gamma^*}|_T = \hat{e}_f e \frac{N_C}{\pi z(1-z)} \left[m_f^2 K_0(\epsilon r)\phi_T(r,z) - \left(z^2 + (1-z)^2\right) \epsilon K_1(\epsilon r)\partial_r \phi_T(r,z) \right],$$

$$|\Psi^*_{VM}\Psi_{\gamma^*}|_L = \hat{e}_f e \frac{N_C}{\pi} 2Qz(1-z) K_0(\epsilon r) \left[M_{VM}\phi_L(r,z) + \delta \frac{m_f^2 - \nabla_r^2}{M_{VM} z(1-z)} \phi_L(r,z) \right].$$

$$\epsilon = z(1-z)Q^2 + m_f^2; \qquad r \equiv |\vec{r}|$$

- Scalar part of VM wave function from Boosted Gaussian model:

$$\phi_{T,L}(r,z) = N_{T,L} z(1-z) \exp\left[-\frac{m_f^2 R^2}{8z(1-z)} - \frac{2z(1-z)r^2}{R^2} + \frac{m_f^2 R^2}{2} \right].$$

$$\phi^{2S}_{T,L}(r,z) = \phi_{T,L}(r,z) \left(1 + \alpha_{2S} \left(2 + \frac{m_f^2 R^2}{4z(1-z)} - \frac{4z(1-z)r^2}{R^2} - m_f^2 R^2 \right) \right)$$

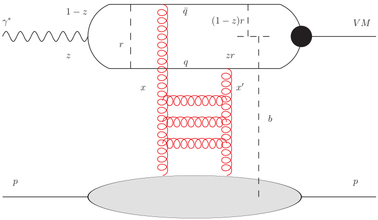
Corrections

- Correction on the real part of the amplitude
  - $\mathcal{A}_{T,L}$ is not purely imaginary, has a real part

$$\lambda_{T,L} \equiv \frac{\partial \ln \left(\mathcal{A}_{T,L}^{\gamma^* p \to VMp} \right)}{\partial \ln \left(\frac{1}{x} \right)}, \qquad \beta_{T,L} = \tan \left(\frac{\pi \lambda_{T,L}}{2} \right) \tag{1}$$

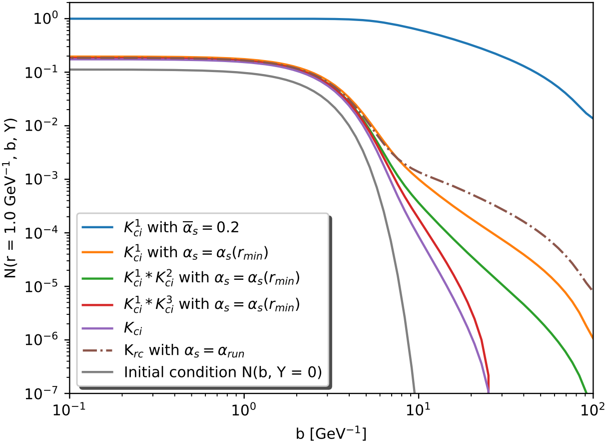
- Skewedness correction
  - Takes into account the fact, that there are two x involved in the dipole-target interaction

$$R_g(\lambda_{T,L}) = \frac{2^{2\lambda_{T,L}+3}}{\sqrt{\pi}} \frac{\Gamma\left(\lambda_{T,L} + \frac{5}{2}\right)}{\Gamma(\lambda_{T,L} + 4)} \tag{2}$$



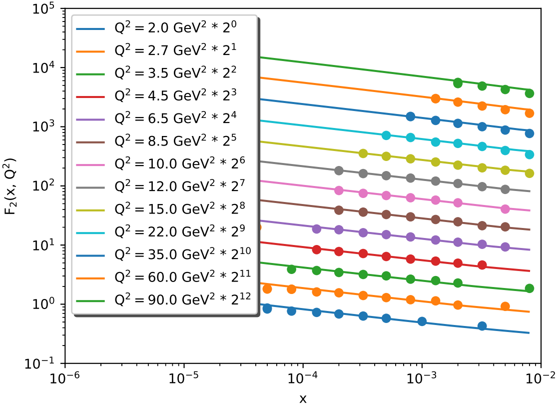
Influence of the individual collinear kernel terms on the Coulomb tails

$$K_{ci}^1 = \frac{\bar{\alpha}_S}{2\pi}\frac{r^2}{r_1^2 r_2^2}; \qquad K_{ci}^2 = \left[\frac{r^2}{\min(r_1^2, r_2^2)}\right]^{\pm\bar{\alpha}_S A_1}; \qquad K_{ci}^3 = K_{DLA}\left(\sqrt{L_{r_1 r} L_{r_2 r}}\right)$$



- Cross section for $\gamma^* p$ scattering and structure function F_2

$$\sigma_{T,L}^{\gamma^* p} = \sum_f \int \mathrm{d}^2 r \int \mathrm{d}z |\Psi^* \Psi|_{T,L}^f \sigma_{q\bar{q}}(\tilde{x}, r), \qquad F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha}(\sigma_T + \sigma_L).$$



References to experimental data

- J/ψ data
  - A. Aktas et al. (H1 Collaboration), Eur. Phys. J. C 46 (2006) 585
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  - S. Chekanov et al. (ZEUS Collaboration), Nucl. Phys. B718 (2005) 3
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