Proton number fluctuations due to mundane effects

Boris Tomášik

FNSPE, České vysoké učení technické, Praha, Czech Republic
and Univerzita Mateja Bela, Banská Bystrica, Slovakia

boris.tomasik@umb.sk

collaborators:
Ivan Melo, Lukáš Lafférs, Marcus Bleicher

Zimányi School
Budapest, Hungary

5.12.2019
\[ \langle N \rangle = \sum_i N_i P_i = \frac{\sum_i N_i w_i}{\sum_i w_i} = \frac{\sum_i N_i \exp \left( -\frac{E_i - \mu N_i}{T} \right)}{\sum_i \exp \left( -\frac{E_i - \mu N_i}{T} \right)} = \frac{\partial Z}{\partial \mu_T} = \frac{\partial \ln Z}{\partial \mu_T} \]
\[ \langle N \rangle = \sum_i N_i P_i = \frac{\sum_i N_i w_i}{\sum_i w_i} = \frac{\sum_i N_i \exp \left( -\frac{E_i - \mu N_i}{T} \right)}{\sum_i \exp \left( -\frac{E_i - \mu N_i}{T} \right)} = \frac{\partial Z}{\partial \mu} \frac{T}{Z} = \frac{\partial \ln Z}{\partial \mu} \]

Relativistic system:

- creation and annihilation of particle-antiparticle pairs
- study charges which are \textbf{conserved in microscopic interactions}
- fluctuations by exchange with the heatbath
\[ \langle N \rangle = \sum_i N_i P_i = \frac{\sum_i N_i w_i}{\sum_i w_i} = \frac{\sum_i N_i \exp \left( -\frac{E_i - \mu N_i}{T} \right)}{\sum_i \exp \left( -\frac{E_i - \mu N_i}{T} \right)} = \frac{\partial Z}{\partial \mu} T Z = \frac{\partial \ln Z}{\partial \mu} T \]

Relativistic system:
- creation and annihilation of particle-antiparticle pairs
- study charges which are conserved in microscopic interactions
- fluctuations by exchange with the heatbath

mean baryon number

\[ \langle B \rangle = \frac{\partial \ln Z}{\partial \mu_{\text{B}} T} \]
Higher moments of the (net) baryon number distribution obtained via derivatives of $\ln Z$:

\[
\frac{\partial^2 \ln Z}{\partial \left( \frac{\mu}{T} \right)^2} = \langle N^2 \rangle - \langle N \rangle^2 = \mu_2 = \kappa_2 = \sigma^2 = VT^3 \chi_2
\]

\[
\frac{\partial^3 \ln Z}{\partial \left( \frac{\mu}{T} \right)^3} = \langle N^3 \rangle - 3\langle N^2 \rangle \langle N \rangle + 2\langle N \rangle^3 = \mu_3 = \kappa_3 = VT^3 \chi_3
\]

\[
\frac{\partial^4 \ln Z}{\partial \left( \frac{\mu}{T} \right)^4} = \langle N^4 \rangle - 4\langle N^3 \rangle \langle N \rangle - 3\langle N^2 \rangle^2 + 12\langle N^2 \rangle \langle N \rangle^2 - 6\langle N \rangle^4
\]

\[
= \mu_4 - 3\mu_2^2 = \kappa_4 = VT^3 \chi_4
\]

Here:

- $\mu_i$ : central moments
- $\kappa_i$ : central cumulants
- $\chi_i$ : susceptibilities
Other coefficients that characterise statistical distribution

Skewness:

\[ S = \frac{\kappa_3}{\kappa_2^{3/2}} = \frac{\mu_3}{\sigma^3} \]

Kurtosis:

\[ \kappa = \frac{\kappa_4}{\kappa_2^2} = \frac{\mu_4}{\mu_2^2} - 3 \]

Volume-independent ratios

\[ S\sigma = \frac{\kappa_3}{\kappa_2} = \frac{\mu_3}{\sigma^2} = \frac{\chi_3}{\chi_2} \]

\[ \kappa\sigma^2 = \frac{\kappa_4}{\kappa_2} = \frac{\mu_4}{\sigma^2} - 3\sigma^2 = \frac{\chi_4}{\chi_2} \]
Why is this interesting?
Why is this interesting?

Because we look for the state of matter where $\ln Z$ changes dramatically (phase transition, crossover).
This should be visible via its derivatives.
Why is this interesting?

Because we look for the state of matter where \( \ln Z \) changes dramatically (phase transition, crossover). This should be visible via its derivatives.

Example: \( r_{42}^{B,0} = \chi_4^B / \chi_2^B = \kappa \sigma^2 \) at \( \mu_B = 0 \)

Why is this even more interesting?
Why is this even more interesting?

Because it could reveal the position of the critical point!
Why is this even more interesting?

Because it could reveal the position of the critical point!
Example: susceptibilities in the Ising model (same universality class)

![Diagram showing phase transitions and susceptibilities](image)

Why is this totally exciting?!
Why is this totally exciting?!

Because STAR collaboration measured data which no theoretical model can reproduce!
Why is this totally exciting?!

Because STAR collaboration measured data which no theoretical model can reproduce!

Net proton number fluctuations.


Huge increase of $\kappa \sigma^2 = \chi_4/\chi_2$ at $\sqrt{s_{NN}} = 7.7$ GeV.

No theoretical understanding, but look at A. Bzdak et al.
Measure the net proton number fluctuations

- baryon number susceptibilities $\chi_i^B$ calculated on the lattice
- enhancement of susceptibilities near the critical point
- susceptibilities are measurable as cumulants of baryon number distribution
- $B$-number not measurable, since no neutrons are measured
- Conflict!
  - susceptibilities are calculated in grand-canonical ensemble
  - cumulants are measured in real collisions which conserve $B$, have limited acceptance, and measure (almost) only protons
- many papers devoted to these subjects (!!!)
- 100% detector efficiency assumed here
- Original features here:
  - rapidity distribution of wounded vs. produced (anti)baryons
  - isospin memory in wounded nucleons
Our approach: Monte Carlo simulation

- Baryon number is conserved
- Only protons and neutrons (and their antiparticles) in the simulations
- Only a (fluctuating) part of incoming nucleons participate
- Isospin of individual wounded nucleons is kept
- Wounded nucleons have double-Gaussian rapidity distribution
  - Protons from this source fluctuate due to:
    - Fluctuations of number of wounded nucleons
    - Random number of protons out of wounded nucleons, track isospin
    - Limited acceptance out of the whole rapidity distribution
- Additionally produced $B\bar{B}$-pairs flat in rapidity
  - Net) protons from this source fluctuate due to:
    - Poissonian fluctuations of $B\bar{B}$ pairs with mean proportional to $N_{\text{wound}}$
    - Random number of protons and antiprotons ($p = 1/2$)
    - Limited acceptance out of the whole rapidity distribution

$\Rightarrow$ Composition wounded/produced protons depends on energy, centrality, and rapidity window
Rapidity distribution of wounded nucleons

\[
\frac{dN_{w}}{dy}(y) = \frac{N_{w}}{2\sqrt{2\pi}\sigma_{y}^{2}} \left\{ \exp \left( -\frac{(y - y_{m})^{2}}{2\sigma_{y}^{2}} \right) + \exp \left( -\frac{(y + y_{m})^{2}}{2\sigma_{y}^{2}} \right) \right\}
\]

Parameter settings:

- \( \sigma_{y} = 0.8 \)
- obtain \( y_{m} \) from

\[
N_{p-\bar{p}} = \frac{Z}{A} \int_{-y_{b}}^{y_{b}} \frac{dN_{w}}{dy} \, dy
\]

where

\( N_{p-\bar{p}} \) in \( |y| < y_{b} = 0.25 \)

is taken from STAR:

- PRC79 (2009) 034909,
- PRC96 (2017) 044904

Illustration for: \( y_{m} = 1, \ dy = 0.8 \)
Rapidity distribution of produced $N\bar{N}$ pairs

$$\frac{dN_{B\bar{B}}}{dy} = N_{B\bar{B}} \frac{C}{1 + \exp\left(\frac{y - y_m}{a}\right)}$$

Parameter settings:
- $C = \left(2a \ln \left(e^{y_m/a} + 1\right)\right)^{-1}$
- $a = \sigma_y/10$
- obtain $N_{B\bar{B}}$ from

$$N_{\bar{p}} = \frac{1}{2} \int_{-y_b}^{y_b} \frac{dN_{B\bar{B}}}{dy} \, dy$$

where
- $N_{\bar{p}}$ in $|y| < y_b = 0.25$

is taken from STAR:
- PRC79 (2009) 034909,
- PRC96 (2017) 044904

Illustration for: $y_m = 1$, $a = 0.08$
Other model features

Isospin determination

- Wounded nucleons remember their isospin. This feature can be turned off and on.
- Wounded proton number thus follows hypergeometric distribution.
- A produced nucleon becomes proton with probability 1/2.

Glauber Monte Carlo

- we use GLISSANDO 2
  [M. Rybczyński et al., Comp. Phys. Commun. 185 (2014) 1759]
- centrality is determined based on deposited energy measure (analogically to experiment)
Exercise: Baryon number conservation

Moments of baryon number distribution around midrapidity.

\[ N_w = 338, \quad N_{\bar{B}B} = 16.94, \quad y_m = 1.019, \quad (\sqrt{s_{NN}} = 19.6 \text{ GeV}), \quad 5 \times 10^7 \text{ events} \]
Net proton number: dependence on rapidity window width

Moments of net proton number distribution around midrapidity.

\[ N_w = 338, \quad N_{B\bar{B}} = 16.94, \quad y_m = 1.019, \quad (\sqrt{s_{NN}} = 19.6 \text{ GeV}), \quad 2 \times 10^7 \text{ events} \]
Dependence on $\Delta y$: fixed $N_w$ vs. Glauber MC

Moments of $p - \bar{p}$ distribution around $y = 0$

$N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, ($\sqrt{s_{NN}} = 19.6$ GeV), $2 \times 10^7$ events

Glauber MC: $1.2 \times 10^6$ events

[see also: Braun-Munzinger, Rustamov, Stachel]
Dependence on $\Delta y$: fixed $N_w$ vs. Glauber MC

Moments of $p - \bar{p}$ distribution around $y = 0$: zoom into detector coverage

$N_w = 338, N_{B\bar{B}} = 16.94, y_m = 1.019, (\sqrt{s_{NN}} = 19.6 \text{ GeV}), 2 \times 10^7$ events

Glauber MC: $1.2 \times 10^6$ events

[see also: Braun-Munzinger, Rustamov, Stachel]
Net proton number: dependence on rapidity

Moments of $p - \bar{p}$ distribution for $\Delta y = 0.5$

$N_w = 338, \ N_{B\bar{B}} = 16.94, \ y_m = 1.019, \ (\sqrt{s_{NN}} = 19.6 \text{ GeV}), \ 2 \times 10^7 \text{ events}$

Glauber MC: $1.2 \times 10^6 \text{ events}$

Net proton number: dependence on rapidity

Moments of $p - \bar{p}$ distribution for $\Delta y = 0.5$: zoom into detector coverage

$N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, ($\sqrt{s_{NN}} = 19.6$ GeV), $2 \times 10^7$ events

Glauber MC: $1.2 \times 10^6$ events

Dependence on rapidity for different collision energies

Fixed $N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, $2 \times 10^7$ events,

Glauber MC, $1.2 \times 10^6$ events
Net proton number: dependence on centrality

$\sqrt{s_{NN}} = 19.6$ GeV: $y_m = 1.019$, $N_{B\bar{B}}/N_w = 0.050$
Statistics: $2 \times 10^7$ for fixed $N_w$, $\sim 5 \times 10^5$ for Glauber MC

$S\sigma$ and $\kappa\sigma^2$ are lowered towards more central events of wounded protons nucleons remember their isospin.
Net proton number: dependence on collision energy

rapidity bin $\Delta y = 0.5$ around $y = 0$
Statistics: $2 \times 10^7$ events for fixed $N_w$, $1.2 \times 10^6$ events for Glauber MC

The importance of produced $B\bar{B}$ pairs grows with increasing energy.
Conclusions

- Net baryon number fluctuations are sensitive to the statistical properties of the matter in the phase diagram.
- Only (net) proton number in limited detector acceptance is measurable—this involves other effects on which fluctuations depend.
- Exciting data on $\chi_4/\chi_2$ at $\sqrt{s_{NN}} = 7.7$ GeV.

A “minimal” model for proton number fluctuations:

- rapidity dependent composition through two components: wounded nucleons and produced $B\bar{B}$ pairs
- possible “isospin memory” of wounded nucleons
- Glauber MC (GLISSANDO 2)

Findings:

- rapidity dependence of $\kappa \sigma^2$ with $\sqrt{s_{NN}}$-dependent minimum
- isospin effect: decrease of $S\sigma$ and $\kappa \sigma^2$ with higher centrality
- baryon number conservation: decrease of $S\sigma$ and $\kappa \sigma^2$ with lower energies