Proton number fluctuations due to mundane effects

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5.12.2019

Statistical physics: Fluctuations of a conserved charge 1

$$\langle N \rangle = \sum_i N_i P_i = \frac{\sum_i N_i w_i}{\sum_i w_i} = \frac{\sum_i N_i \exp\left(-\frac{E_i - \mu N_i}{T}\right)}{\sum_i \exp\left(-\frac{E_i - \mu N_i}{T}\right)} = \frac{\frac{\partial Z}{\partial \frac{\mu}{T}}}{Z} = \frac{\partial \ln Z}{\partial \frac{\mu}{T}}$$

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Relativistic system:

- creation and annihilation of particle-antiparticle pairs
- study charges which are **conserved in microscopic interactions**
- fluctuations by exchange with the heatbath

Statistical physics: Fluctuations of a conserved charge 1

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mean baryon number

$$\langle B \rangle = \frac{\partial \ln Z}{\partial \frac{\mu_B}{T}}$$

Statistical physics: Fluctuations of a conserved charge 2

Higher moments of the (net) baryon number distribution obtained via derivatives of $\ln Z$:

$$
\begin{aligned}
\frac{\partial^2 \ln Z}{\partial \left(\frac{\mu}{T}\right)^2} &= \langle N^2\rangle - \langle N\rangle^2 = \mu_2 = \kappa_2 = \sigma^2 = VT^3\chi_2 \\
\frac{\partial^3 \ln Z}{\partial \left(\frac{\mu}{T}\right)^3} &= \langle N^3\rangle - 3\langle N^2\rangle\langle N\rangle + 2\langle N\rangle^3 = \mu_3 = \kappa_3 = VT^3\chi_3 \\
\frac{\partial^4 \ln Z}{\partial \left(\frac{\mu}{T}\right)^4} &= \langle N^4\rangle - 4\langle N^3\rangle\langle N\rangle - 3\langle N^2\rangle^2 + 12\langle N^2\rangle\langle N\rangle^2 - 6\langle N\rangle^4 \\
&= \mu_4 - 3\mu_2^2 = \kappa_4 = VT^3\chi_4
\end{aligned}
$$

Here:

μ_i : central moments

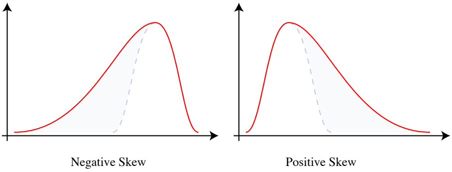
κ_i : central cumulants

χ_i : susceptibilities

Other coefficients that characterise statistical distribution

Skewness:

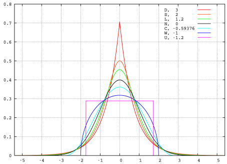
$$S = \frac{\kappa_3}{\kappa_2^{3/2}} = \frac{\mu_3}{\sigma^3}$$



[Rodolfo Hermans on Wikipedia, and Wikipedia]

Kurtosis:

$$\kappa = \frac{\kappa_4}{\kappa_2^2} = \frac{\mu_4}{\mu_2^2} - 3$$



Volume-independent ratios

$$S\sigma = \frac{\kappa_3}{\kappa_2} = \frac{\mu_3}{\sigma^2} = \frac{\chi_3}{\chi_2}$$

$$\kappa\sigma^2 = \frac{\kappa_4}{\kappa_2} = \frac{\mu_4}{\sigma^2} - 3\sigma^2 = \frac{\chi_4}{\chi_2}$$

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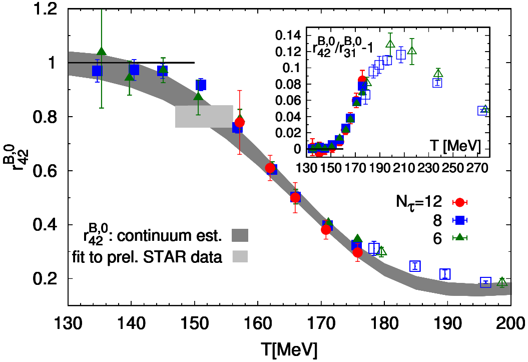
Example: $r_{42}^{B,0} = \chi_4^B/\chi_2^B = \kappa\sigma^2$ at $\mu_B = 0$



[A. Bazavov *et al.*, Phys. Rev. D **96** (2017) 074510]

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Because it could reveal the position of the critical point!

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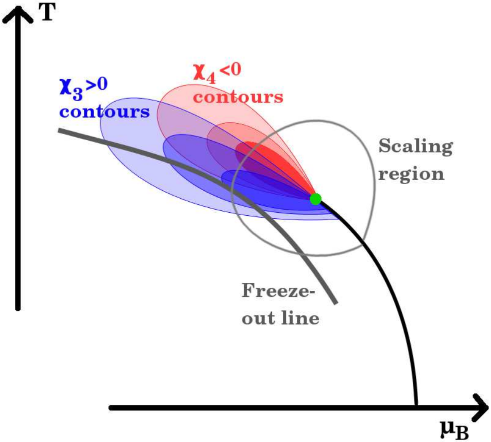
Example: susceptibilities in the Ising model (same universality class)



[J.W. Chen et al.: Phys. Rev. D 95 (2017) 014038]

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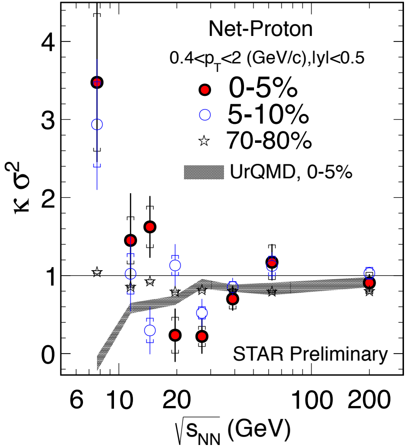

Net proton number fluctuations.

[STAR, PRL 112 (2014) 032302, CPOD2014, QM2015]

Huge increase of $\kappa\sigma^2 = \chi_4/\chi_2$ at $\sqrt{s_{NN}} = 7.7$ GeV.

No theoretical understanding, but look at A. Bzdak et al.

Measure the net proton number fluctuations

- baryon number susceptibilities χ_i^B calculated on the lattice
- enhancement of susceptibilities near the critical point
- susceptibilities are measurable as cumulants of baryon number distribution
- B-number not measurable, since no neutrons are measured
- Conflict!
    - susceptibilities are calculated in grand-canonical ensemble
    - cumulants are measured in real collisions which conserve B, have limited acceptance, and measure (almost) only protons
- many papers devoted to these subjects (!!!)
- 100% detector efficiency assumed here
- Original features here:
    - rapidity distribution of wounded vs. produced (anti)baryons
    - isospin memory in wounded nucleons

Our approach: Monte Carlo simulation

- baryon number is conserved
- only protons and neutrons (and their antiparticles) in the simulations
- only a (fluctuating) part of incoming nucleons participate
- isospin of individual wounded nucleons is kept
- wounded nucleons have double-Gaussian rapidity distribution
  protons from this source fluctuate due to:
  - fluctuations of number of wounded nucleons
  - random number of protons out of wounded nucleons, track isospin
  - limited acceptance out of the whole rapidity distribution
- additionally produced $B\bar{B}$-pairs flat in rapidity
  (net) protons from this source fluctuate due to:
  - Poissonian fluctuations of $B\bar{B}$ pairs with mean proportional to N_{wound}
  - random number of protons and antiprotons ($p = 1/2$)
  - limited acceptance out of the whole rapidity distribution

$\Rightarrow$ **composition wounded/produced protons depends on energy, centrality, and rapidity window**

Rapidity distribution of wounded nucleons

$$\frac{dN_w}{dy}(y) = \frac{N_w}{2\sqrt{2\pi\sigma_y^2}} \left\{ \exp\left(-\frac{(y-y_m)^2}{2\sigma_y^2} \right) + \exp\left(-\frac{(y+y_m)^2}{2\sigma_y^2} \right) \right\}$$

Parameter settings:

- $\sigma_y = 0.8$
- obtain y_m from

$$N_{p-\bar{p}} = \frac{Z}{A} \int_{-y_b}^{y_b} \frac{dN_w}{dy}\, dy$$

where
$N_{p-\bar{p}}$ in $|y| < y_b = 0.25$
is taken from STAR:
PRC**79** (2009) 034909,
PRC**96** (2017) 044904

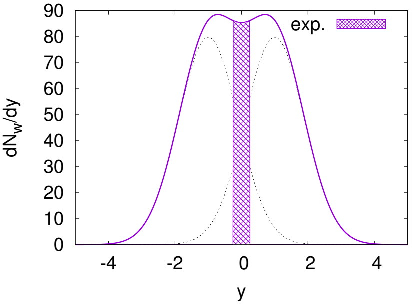

Illustration for: $y_m = 1$, $dy = 0.8$

Rapidity distribution of produced $N\bar{N}$ pairs

$$\frac{dN_{B\bar{B}}}{dy} = N_{B\bar{B}} \frac{C}{1 + \exp\left(\frac{|y|-y_m}{a}\right)}$$

Parameter settings:

- $C = \left(2a \ln\left(e^{y_m/a} + 1\right)\right)^{-1}$
- $a = \sigma_y/10$
- obtain $N_{B\bar{B}}$ from

$$N_{\bar{p}} = \frac{1}{2}\int_{-y_b}^{y_b} \frac{dN_{B\bar{B}}}{dy}\, dy$$

where
$N_{\bar{p}}$ in $|y| < y_b = 0.25$
is taken from STAR:
PRC**79** (2009) 034909,
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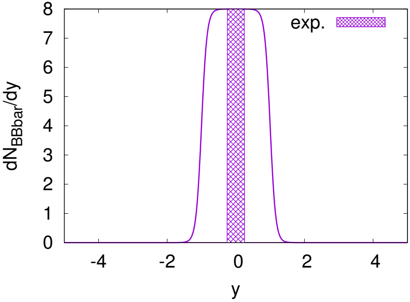


Illustration for: $y_m = 1$, $a = 0.08$

Other model faetures

Isospin determination

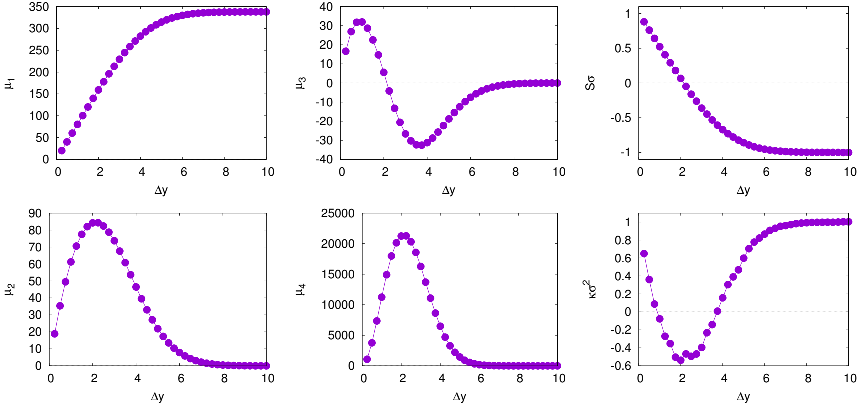
- Wounded nucleons remember their isospin. This feature can be turned off and on.
- Wounded proton number thus follows hypergeometric distribution.
- A produced nucleon becomes proton with probability $1/2$.

Glauber Monte Carlo

- we use GLISSANDO 2
  [M. Rybczyński *et al.*, Comp. Phys. Commun. **185** (2014) 1759]
- centrality is determined based on deposited energy measure (analogically to experiment)

Exercise: Baryon number conservation

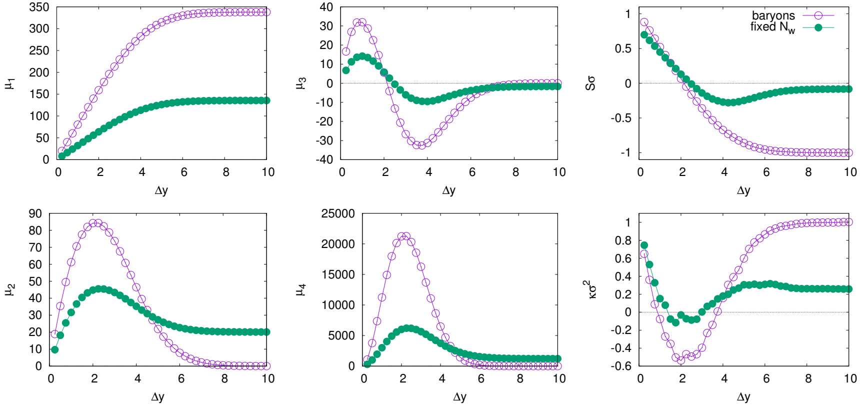
Moments of baryon number distribution around midrapidity.



$N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, ($\sqrt{s_{NN}} = 19.6$ GeV), 5×10^7 events

Net proton number: dependence on rapidity window width

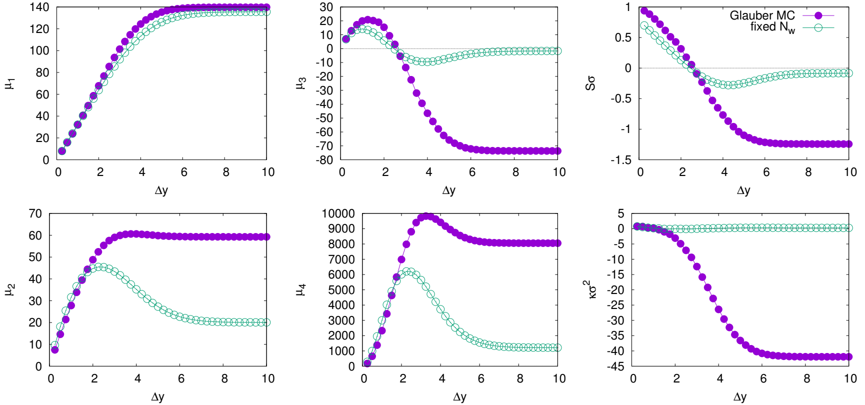
Moments of net proton number distribution around midrapidity.



$N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, ($\sqrt{s_{NN}} = 19.6$ GeV), 2×10^7 events

Dependence on Δy: fixed N_w vs. Glauber MC

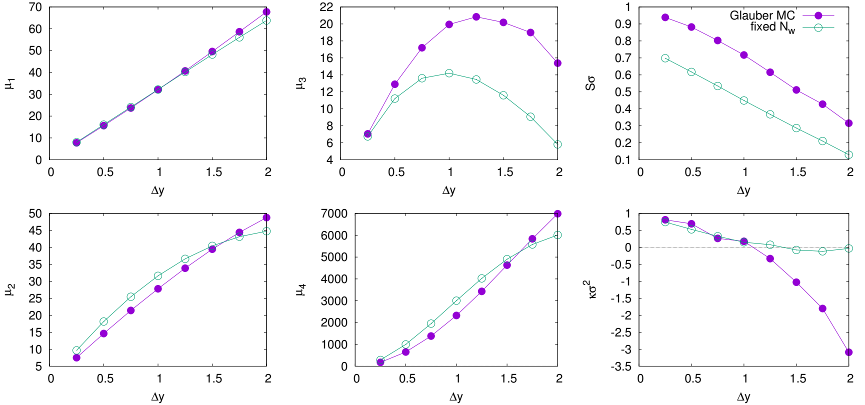
Moments of $p - \bar{p}$ distribution around $y = 0$



$N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, ($\sqrt{s_{NN}} = 19.6$ GeV), 2×10^7 events
Glauber MC: 1.2×10^6 events
[see also: Braun-Munzinger, Rustamov, Stachel]

Dependence on Δy: fixed N_w vs. Glauber MC

Moments of $p - \bar{p}$ distribution around $y = 0$: zoom into detector coverage



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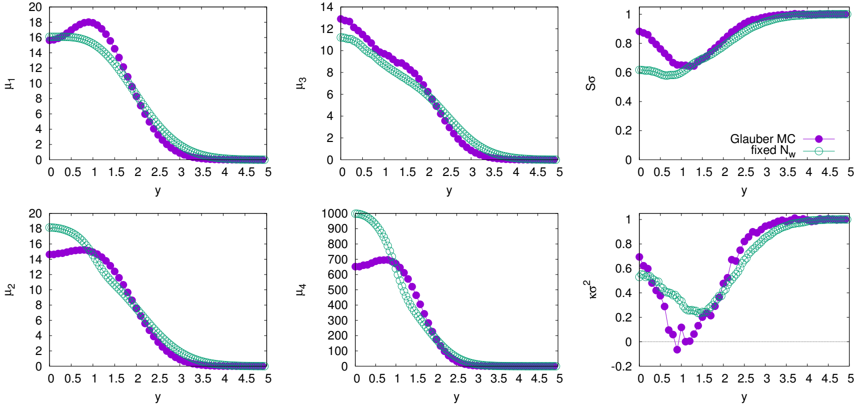
Net proton number: dependence on rapidity

Moments of $p - \bar{p}$ distribution for $\Delta y = 0.5$



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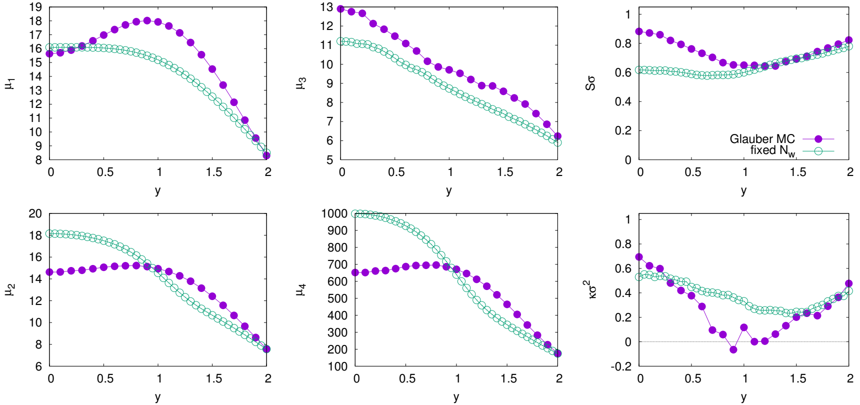
cf: [J. Brewer, S. Mukharjee, K. Rajagopal, Y. Yin, Phys. Rev. C 98 (2018) 061901]

Net proton number: dependence on rapidity

Moments of $p - \bar{p}$ distribution for $\Delta y = 0.5$: zoom into detector coverage



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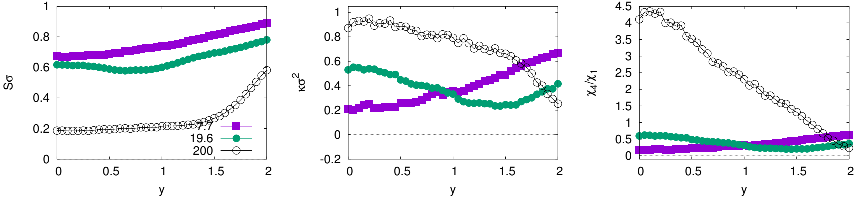
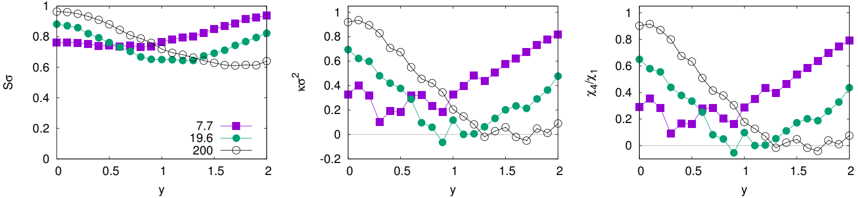
Glauber MC: 1.2×10^6 events
cf: [J. Brewer, S. Mukharjee, K. Rajagopal, Y. Yin, Phys. Rev. C 98 (2018) 061901]

Dependence on rapidity for different collision energies

Fixed $N_w = 338$, $N_{B\bar{B}} = 16.94$, $y_m = 1.019$, 2×10^7 events,

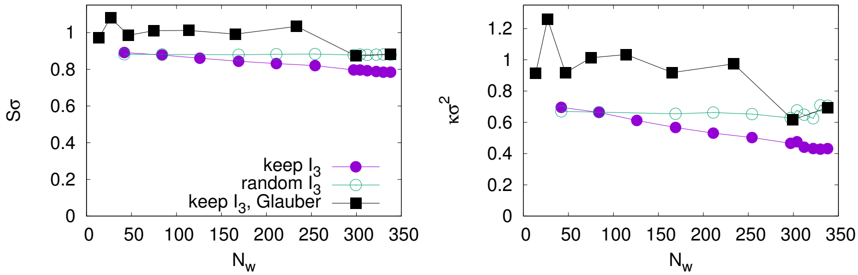


Glauber MC, 1.2×10^6 events



Net proton number: dependence on centrality

$\sqrt{s_{NN}} = 19.6$ GeV: $y_m = 1.019$, $N_{B\bar{B}}/N_w = 0.050$
Statistics: 2×10^7 for fixed N_w, $\sim 5 \times 10^5$ for Glauber MC

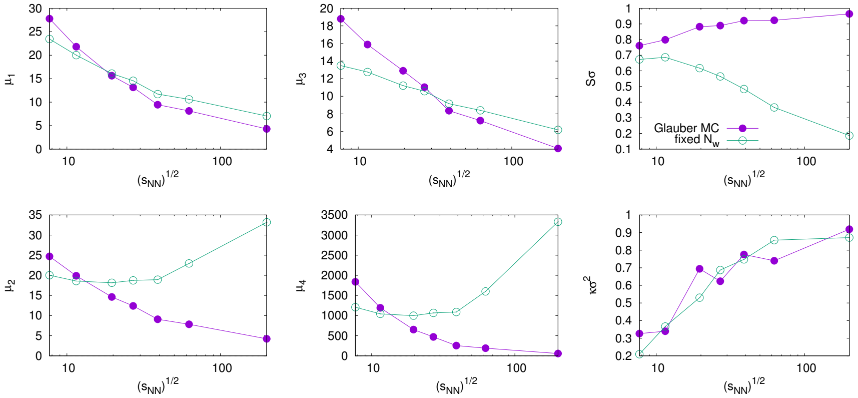


$S\sigma$ and $\kappa\sigma^2$ are lowered towards more central events of wounded protons nucleons remember their isospin.

Net proton number: dependence on collision energy

rapidity bin $\Delta y = 0.5$ around $y = 0$

Statistics: 2×10^7 events for fixed N_w, 1.2×10^6 events for Glauber MC



The importance of produced $B\bar{B}$ pairs grows with increasing energy.

Conclusions

- Net baryon number fluctuations are sensitive to the statistical properties of the matter in the phase diagram.
- Only (net) proton number in limited detector acceptance is measurable—this involves other effects on which fluctuations depend.
- Exciting data on χ_4/χ_2 at $\sqrt{s_{NN}} = 7.7$ GeV.

A "minimal" model for proton number fluctuations:

- rapidity dependent composition through two components: wounded nucleons and produced $B\bar{B}$ pairs
- possible "isospin memory" of wounded nucleons
- Glauber MC (GLISSANDO 2)

Findings:

- rapidity dependence of $\kappa\sigma^2$ with $\sqrt{s_{NN}}$-dependent minimum
- isospin effect: decrease of $S\sigma$ and $\kappa\sigma^2$ with higher centrality
- baryon number conservation: decrease of $S\sigma$ and $\kappa\sigma^2$ with lower energies