Traces of nuclear liquid-gas transition in analytic properties of hot QCD

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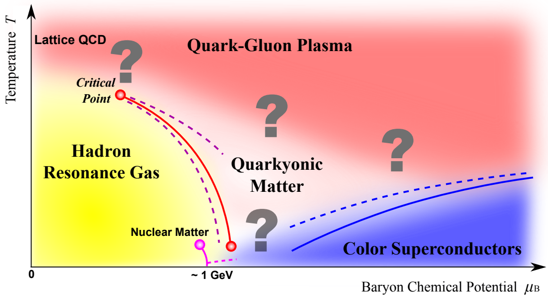
Based on O. Savchuk, V. Vovchenko, R. V. Poberezhnyuk, M. I. Gorenstein, and H. Stoecker, (2019), arXiv:1909.04461 [hep-ph]

Outline

1. Introduction
2. Branch points of the nuclear liquid-gas transition
   - Analytic expectations
   - Model dependence
   - Radius of convergence of QCD Taylor expansion
3. Conclusions

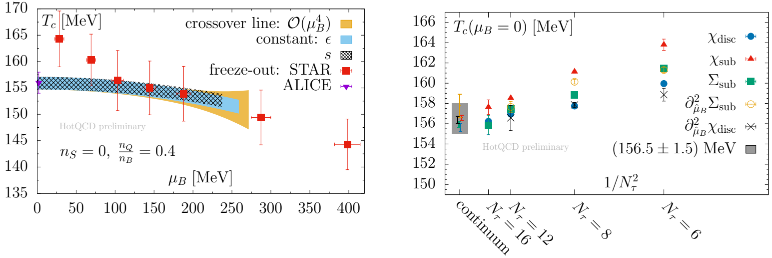
QCD phase diagram



K. Fukushima and T. Hatsuda, arXiv:1005.4814 [hep-ph]

- Crossover from hadron-resonance gas (HRG) to quark-gluon plasma (QGP) at high temperature and low density
- Hypothetical first order phase transition HRG-QGP at finite baryochemical potential with critical endpoint
- Liquid-gas phase transition with critical point at $T \approx 18\ MeV$, $\mu_B \approx 900\ MeV$

Lattice QCD



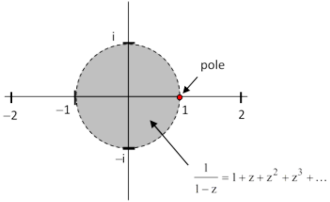
from P. Steinbrecher, arXiv:1807.05607 [hep-lat]

- Numerical study of QCD phase diagram performed via Monte Carlo sampling of configurations on the lattice.
- Due to the sign problem direct approach works only at $\mu_B = 0$.
- Results for nonzero baryochemical potential obtained either by analytic continuation from imaginary μ_B or by computing Taylor expansion coefficients.

Borsanyi et al. (Wuppertal-Budapest); Bazavov et al. (HotQCD); Philipsen, Endrodi, et al. (Frankfurt); . . .

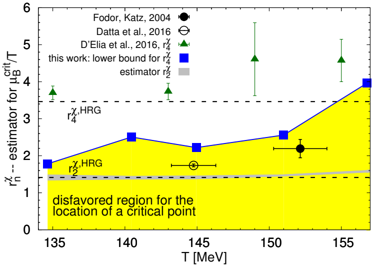
Taylor expansion

$$\frac{p(T,\mu_B)-p(T,0)}{T^4}=\sum_{n=1}^{\infty}\frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$



- Converges in a circle in complex plane, closest singularity on the boundary.
- One example is the critical point: singularity on the real μ_B-axis.
- The radius of convergence and the closest singularity can be estimated, commonly with:

$$r_n^{\chi}=\left|\frac{c_n}{c_{n+1}}\right|^{1/2},\quad c_n\equiv\chi_{2n}/(2n)!$$



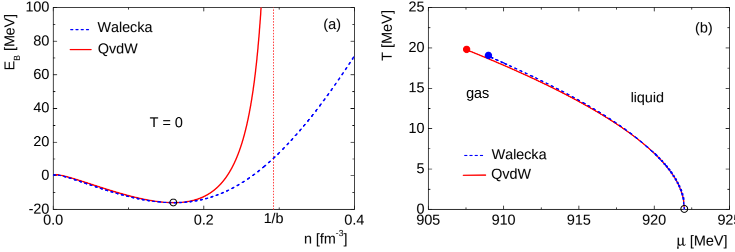
A. Bazavov et al., arXiv:1701.04325

Nuclear matter

Consists of nucleons: **protons and neutrons**. Its **ground state ($P = 0,\ T = 0$)** parameters estimated from properties of nuclei:

- Normal nuclear **density:** $\rho_0 = \mathbf{0.16\ fm^{-3}}$
- Binding energy **E/A = -16 MeV** from extrapolation of energy of finite nuclei

Evidence for nuclear liquid-gas transition found experimentally [ALADIN@GSI (1995)]



R. V. Poberezhnyuk, V. Vovchenko, D. V. Anchishkin, and M. I. Gorenstein, arXiv:1708.05605 [nucl-th]

Nuclear matter model parameters are commonly constrained to ground state properties. The phase diagram, e.g. the critical point location, are predicted.

How does the nuclear liquid-gas transition affect QCD analytic properties?

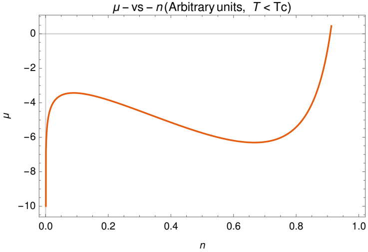
Branch points of a liquid-gas transition

van der Waals equation in the GCE:

$$p(T,n) = \frac{T\,n}{1-b\,n} - a\,n^2,$$

$$e^{\mu/T} = \frac{n}{n_{id}(T)\,(1-bn)}\,\exp\left[\frac{bn}{1-bn} - \frac{2an}{T}\right]$$

V. Vovchenko et al., arXiv:1501.03785



n is a **multi-valued** function of μ. This implies the existence of **branch points:**

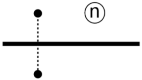
$$(\partial\mu/\partial n)_T = 0 \qquad \Rightarrow \qquad \frac{2an_{\rm br}}{T}\,(1-bn_{\rm br})^2 = 1$$

Branch points of a liquid-gas transition

$$\frac{2an_{\mathrm{br}}}{T}(1 - bn_{\mathrm{br}})^2 = 1$$

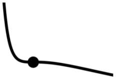
Solutions:

- $T > T_C$: two c.c. roots $n_{\mathrm{br1}} = (n_{\mathrm{br2}})^*$



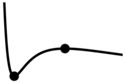
crossover singularities

- $T = T_C$: $n_{\mathrm{br1}} = n_{\mathrm{br2}} = n_c$



the critical point

- $T < T_C$: two real roots n_{sp1} and n_{sp2}



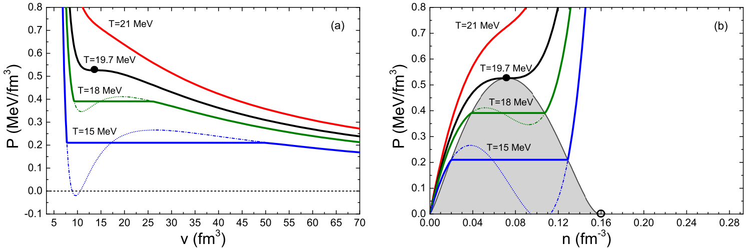
the spinodal points

see also V. Vovchenko, C. Greiner, V. Koch, and H. Stoecker, arXiv:1909.02276 [hep-ph]

Quantum van der Waals theory of nuclear matter

$$p(T,\mu) = p_{\rm id}(T,\mu^*) - a n^2$$
$$n(T,\mu) = (1 - b\,n)\, n_{\rm id}(T,\mu^*)$$
$$\mu^* = \mu - b\,p_{\rm id}(T,\mu^*) + 2\,a\,n$$

$$n_{\rm id}(T,\mu^*) = \frac{d}{2\,\pi^2}\int_0^\infty dk\,k^2 \times \left[\exp\left(\frac{\sqrt{m^2+k^2}-\mu^*}{T}\right) + 1\right]^{-1}$$



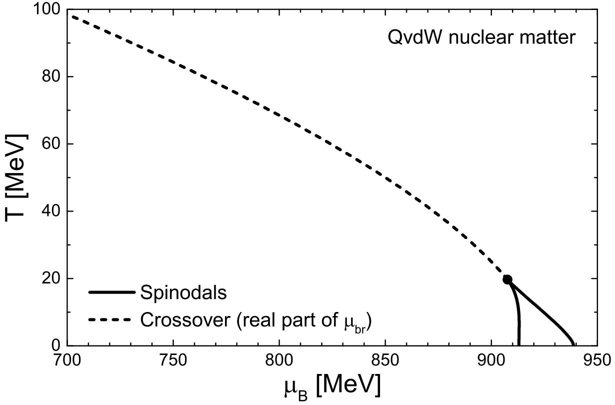
$a = 329$ MeV fm^3, $b = 3.42$ fm^3. **Critical point** at $T_c = 19.7$ MeV, $\mu_c = 908$ MeV.

V. Vovchenko, D. V. Anchishkin, and M. I. Gorenstein, arXiv:1504.01363 [nucl-th]

QvdW nuclear matter: branch points

Branch points equation for QvdW model:

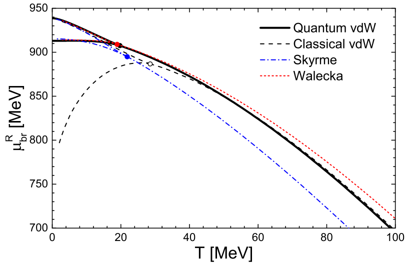
$$\frac{2\, a\, n_{\rm br}}{T}\,(1 - b n_{\rm br})^2\, \omega_{\rm id}(T, \mu^*_{\rm br}) = 1$$

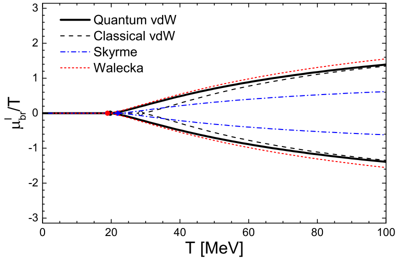


Nuclear matter branch points: Model dependence

Additional models:

- **Skyrme:** $\mu^* = \mu - U(n),\ U_{\rm sk}(n) = -\alpha\left(\frac{n}{n_0}\right) + \beta\left(\frac{n}{n_0}\right)^{\gamma}$ *mean-field*
- **Walecka:** $p(T,\mu) = p_{\rm id}(T,\mu^*;m^*) + \frac{(\mu-\mu^*)^2}{2c_v^2} - \frac{(m-m^*)^2}{2\,c_s^2}$ *RMF theory*





- Mild model dependence, behavior consistent with the classical vdW model
- Fermi statistics important at small temperatures, irrelevant at $T \gtrsim 80$ MeV

vdW-HRG model

At higher temperatures resonances cannot be neglected.
Treated using the **van der Waals hadron resonance gas (vdW-HRG) model:**

- Identical vdW interactions between all baryons
- Baryon-antibaryon, meson-meson, meson-baryon vdW terms neglected
- Baryon vdW parameters extracted from ground state of nuclear matter ($a = 329$ MeV fm^3, $b = 3.42$ fm^3)

Three independent subsystems: mesons + baryons + antibaryons

$$p(T,\mu) = p_M(T,\mu) + p_B(T,\mu) + p_{\bar{B}}(T,\mu)$$

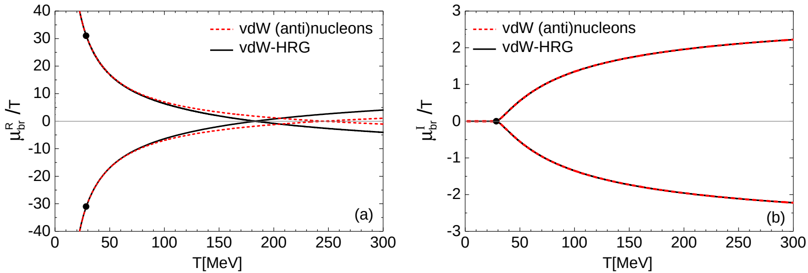
$$p_B(T,\mu) = \sum_{j\in B} p_j^{\mathrm{id}}(T,\mu_j^{B*}) - a\, n_B^2$$

$$p_M(T,\mu) = \sum_{j\in M} p_j^{\mathrm{id}}(T,\mu_j)$$

The main change: $n_{\mathrm{id}}(T,\mu_B^*) \rightarrow \sum_{j\in B} n_{\mathrm{id}}^j(T,\mu_B^*)$

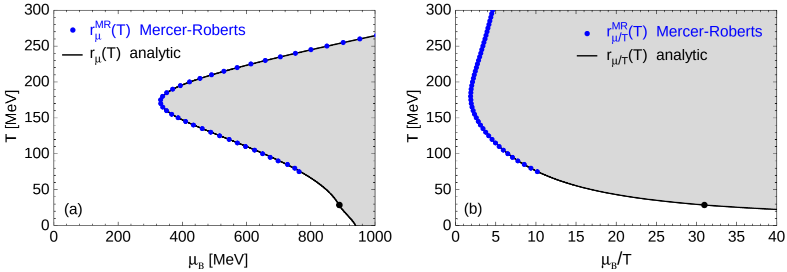
V. Vovchenko, M.I. Gorenstein, H. Stoecker, Phys. Rev. Lett. **118**, 182301 (2017)

vdW-HRG model



Inclusion of baryon resonances changes the real part of chemical potential branch point coordinates

Radius of convergence

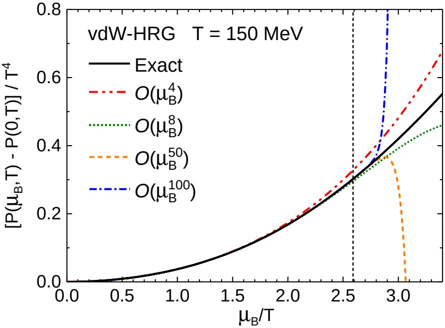


- Radius of convergence reaches values as small as $r_{\mu/T} \approx 2-3$ at $T = 140 - 170\ MeV$
- Ratio estimator of $r_{\mu/T}$ does not converge to a meaningful value, because singularity is located in the complex plane, rather than on the real axis.
- Mercer-Roberts estimator of $r_{\mu/T}$ converges to the correct value. About 8-10 Taylor coefficients required to reach a 10% accuracy.

see also V. Vovchenko, J. Steinheimer, O. Philipsen, and H. Stoecker, arXiv:1711.01261 [hep-ph]

Testing the Taylor expansion

$$\frac{p(T,\mu_B)-p(T,0)}{T^4}=\sum_{n=1}^{\infty}\frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$



- Taylor expansion diverges for $\mu_B/T > r_{\mu/T}$ but many coefficients needed to see this.
- For $\mu_B/T > r_{\mu/T}$ the Taylor expansion can at best be viewed as an asymptotic series.

Conclusions

- Nuclear liquid-gas transition implies the existence of branch points in QCD thermodynamic potential.
- At $T > T_c \simeq 20$ MeV the branch points are located in the complex μ_B plane. Model dependence is mild.
- Radius of convergence of QCD Taylor expansion may be as small as $r_{\mu/T} \approx 2-3$ at the $T = 140 - 170$ *MeV due to the nuclear matter critical point alone*.
- Important to be able to distinguish signals of a hypothetical chiral critical point from nuclear matter critical point.

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Thank you for attention!