Traces of nuclear liquid-gas transition in analytic properties of hot QCD

Oleh Savchuk

Physics Department, Taras Shevchenko National University of Kyiv, Ukraine

December 5, 2019

Outline

1. Introduction
2. Branch points of the nuclear liquid-gas transition
   - Analytic expectations
   - Model dependence
   - Radius of convergence of QCD Taylor expansion
3. Conclusions
Crossover from hadron-resonance gas (HRG) to quark-gluon plasma (QGP) at high temperature and low density

Hypothetical first order phase transition HRG-QGP at finite baryochemical potential with critical endpoint

Liquid-gas phase transition with critical point at $T \approx 18 \text{ MeV}$, $\mu_B \approx 900 \text{ MeV}$
Numerical study of QCD phase diagram performed via Monte Carlo sampling of configurations on the lattice.

Due to the sign problem direct approach works only at $\mu_B = 0$.

Results for nonzero baryochemical potential obtained either by analytic continuation from imaginary $\mu_B$ or by computing Taylor expansion coefficients.

Borsanyi et al. (Wuppertal-Budapest); Bazavov et al. (HotQCD); Philipsen, Endrodi, et al. (Frankfurt); ...
Taylor expansion

\[ p(T, \mu_B) - p(T, 0) = \frac{T^4}{\infty} \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!} \left( \frac{\mu_B}{T} \right)^{2n} \]

- Converges in a circle in complex plane, closest singularity on the boundary.
- One example is the critical point: singularity on the real \( \mu_B \)-axis.
- The radius of convergence and the closest singularity can be estimated, commonly with:

\[ r_n^\chi = \left| \frac{c_n}{c_{n+1}} \right|^{1/2}, \quad c_n \equiv \chi_{2n}/(2n)! \]

A. Bazavov et al., arXiv:1701.04325
Nuclear matter

Consists of nucleons: **protons and neutrons**. Its **ground state** \((P = 0, \ T = 0)\) parameters estimated from properties of nuclei:

- Normal nuclear **density**: \(\rho_0 = 0.16 \text{ fm}^{-3}\)
- Binding energy \(E/A = -16 \text{ MeV}\) from extrapolation of energy of finite nuclei

Evidence for nuclear liquid-gas transition found experimentally [ALADIN@GSI (1995)]

![](chart1.png)


Nuclear matter model parameters are commonly constrained to ground state properties. The phase diagram, e.g. the critical point location, are predicted.

**How does the nuclear liquid-gas transition affect QCD analytic properties?**
van der Waals equation in the GCE:

\[ p(T, n) = \frac{T n}{1 - bn} - a n^2, \]

\[ e^{\mu/T} = \frac{n}{n_{id}(T)(1 - bn)} \exp \left[ \frac{bn}{1 - bn} - \frac{2an}{T} \right] \]

V. Vovchenko et al., arXiv:1501.03785

\( n \) is a multi-valued function of \( \mu \). This implies the existence of branch points:

\( (\partial \mu/\partial n)_T = 0 \implies \frac{2an_{br}}{T} (1 - bn_{br})^2 = 1 \)
Branch points of a liquid-gas transition

\[
\frac{2an_{br}}{T} \left(1 - bn_{br}\right)^2 = 1
\]

**Solutions:**

- \( T > T_C \): two c.c. roots \( n_{br1} = (n_{br2})^* \)
- \( T = T_C \): \( n_{br1} = n_{br2} = n_c \)
- \( T < T_C \): two real roots \( n_{sp1} \) and \( n_{sp2} \)

see also V. Vovchenko, C. Greiner, V. Koch, and H. Stoecker, arXiv:1909.02276 [hep-ph]
Quantum van der Waals theory of nuclear matter

\[ p(T, \mu) = p_{id}(T, \mu^*) - a n^2 \]
\[ n(T, \mu) = (1 - b n) n_{id}(T, \mu^*) \]
\[ \mu^* = \mu - b p_{id}(T, \mu^*) + 2an \]

\[ n_{id}(T, \mu^*) = \frac{d}{2 \pi^2} \int_0^\infty dk k^2 \times \left[ \exp \left( \frac{\sqrt{m^2 + k^2 - \mu^*}}{T} \right) + 1 \right]^{-1} \]

\[ a = 329 \text{ MeV fm}^3, \quad b = 3.42 \text{ fm}^3. \textbf{Critical point at } T_c = 19.7 \text{ MeV, } \mu_c = 908 \text{ MeV.} \]

QvdW nuclear matter: branch points

Branch points equation for QvdW model:

\[ \frac{2 a n_{br}}{T} (1 - b n_{br})^2 \omega_{id}(T, \mu^*_br) = 1 \]
Nuclear matter branch points: Model dependence

Additional models:

- **Skyrme**: \( \mu^* = \mu - U(n) \), \( U_{sk}(n) = -\alpha \left( \frac{n}{n_0} \right) + \beta \left( \frac{n}{n_0} \right)^\gamma \) **mean-field**

- **Walecka**: \( p(T, \mu) = p_{id}(T, \mu^*; m^*) + \frac{(\mu - \mu^*)^2}{2c_v^2} - \frac{(m - m^*)^2}{2c_s^2} \) **RMF theory**

- Mild model dependence, behavior consistent with the classical vdW model

- Fermi statistics important at small temperatures, irrelevant at \( T \lesssim 80 \text{ MeV} \)
vdW-HRG model

At higher temperatures resonances cannot be neglected. Treated using the van der Waals hadron resonance gas (vdW-HRG) model:

- Identical vdW interactions between all baryons
- Baryon-antibaryon, meson-meson, meson-baryon vdW terms neglected
- Baryon vdW parameters extracted from ground state of nuclear matter ($a = 329$ MeV fm$^3$, $b = 3.42$ fm$^3$)

Three independent subsystems: mesons + baryons + antibaryons

$$\begin{align*}
p(T, \mu) &= p_M(T, \mu) + p_B(T, \mu) + p_{\bar{B}}(T, \mu) \\
p_B(T, \mu) &= \sum_{j \in B} p_{id}^j(T, \mu_j^B) - a n_{B}^2 \\
p_M(T, \mu) &= \sum_{j \in M} p_{id}^j(T, \mu_j)
\end{align*}$$

The main change: $n_{id}(T, \mu_B^*) \rightarrow \sum_{j \in B} n_{id}^j(T, \mu_{B}^*)$

Inclusion of baryon resonances changes the real part of chemical potential branch point coordinates.
Radius of convergence reaches values as small as $r_{\mu/T} \approx 2 - 3$ at $T = 140 - 170$ MeV.

Ratio estimator of $r_{\mu/T}$ does not converge to a meaningful value, because singularity is located in the complex plane, rather than on the real axis.

Mercer-Roberts estimator of $r_{\mu/T}$ converges to the correct value. About 8-10 Taylor coefficients required to reach a 10% accuracy.

Testing the Taylor expansion

\[
p(T, \mu_B) - p(T, 0) = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!} \left( \frac{\mu_B}{T} \right)^{2n}
\]

- Taylor expansion diverges for \( \mu_B / T > r_{\mu/T} \) but many coefficients needed to see this.
- For \( \mu_B / T > r_{\mu/T} \) the Taylor expansion can at best be viewed as an asymptotic series.
Nuclear liquid-gas transition implies the existence of branch points in QCD thermodynamic potential.

At $T > T_c \simeq 20$ MeV the branch points are located in the complex $\mu B$ plane. Model dependence is mild.

Radius of convergence of QCD Taylor expansion may be as small as $r_{\mu/T} \approx 2 - 3$ at the $T = 140 - 170$ MeV due to the nuclear matter critical point alone.

Important to be able to distinguish signals of a hypothetical chiral critical point from nuclear matter critical point.
Conclusions

- Nuclear liquid-gas transition implies the existence of branch points in QCD thermodynamic potential.

- At $T > T_c \simeq 20 \text{ MeV}$ the branch points are located in the complex $\mu_B$ plane. Model dependence is mild.

- Radius of convergence of QCD Taylor expansion may be as small as $r_{\mu/T} \approx 2 - 3$ at the $T = 140 - 170 \text{ MeV}$ due to the nuclear matter critical point alone.

- Important to be able to distinguish signals of a hypothetical chiral critical point from nuclear matter critical point.

Thank you for attention!