

# Signs of hydrodynamic scaling in pseudorapidity distributions of p+p and heavy-ion collisions

GÁBOR KASZA, TAMÁS CSÖRGŐ, MÁTÉ CSANÁD

ZIMÁNYI SCHOOL'19, BUDAPEST

5TH OF DECEMBER, 2019

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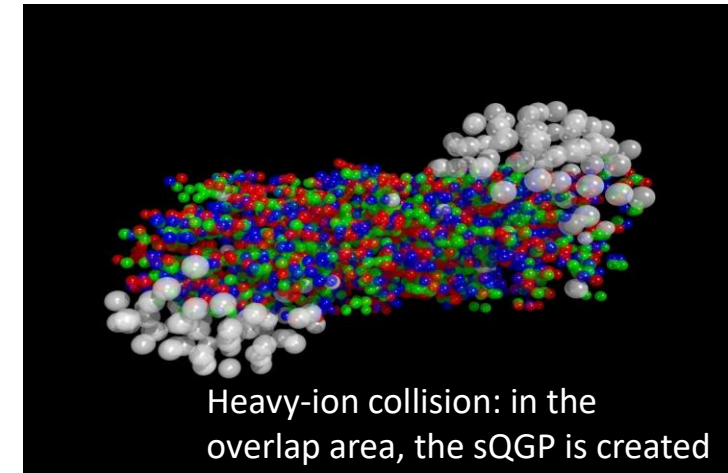
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# Various application of hydrodynamics

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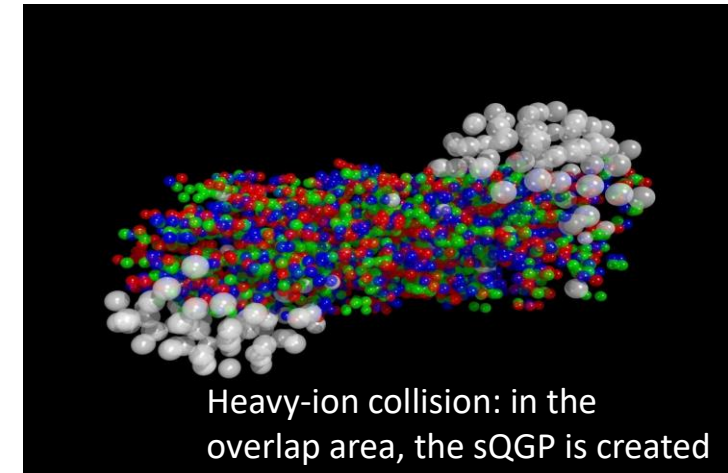
- Fluid dynamics: flow of liquid and gases
- Examples:
  - *Calculating forces and moments in aircrafts*
  - *Weather forecast*
  - *Describing nebulae*
  - *Inner structure of stars (magnetohydrodynamics)*
  - *Modelling fission weapon detonation*
  - *Describing the Quark Gluon Plazma (QGP)*
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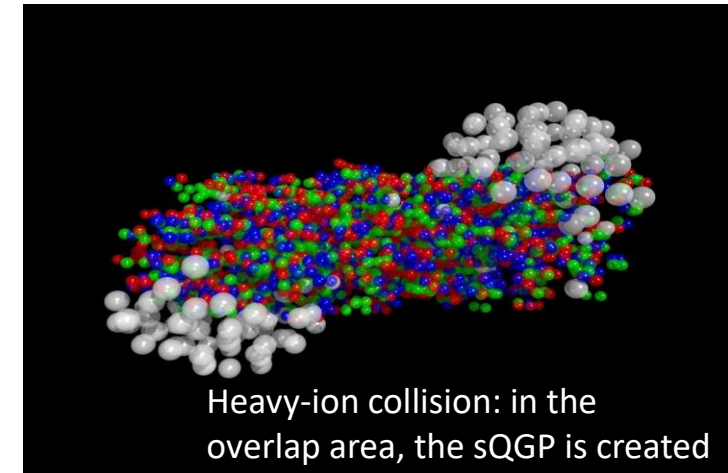
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- Different systems in many aspects: however, hydro works well in all cases
- Why is hydrodynamics so effective? *Hydrodynamics has no internal scale!*
- *Is the scaling behaviour violated on microscopic scales?*

# Our recent perfect fluid solution

- Rindler coordinates, velocity field:  $(\tau, \eta_x) = \left( \sqrt{t^2 - r_z^2}, \frac{1}{2} \ln \left[ \frac{t + r_z}{t - r_z} \right] \right), \quad u^\mu = (\cosh(\Omega), \sinh(\Omega))$
- 1+1 dimensional, parametric, almost self-similar solution:

**Csörgő T., Kasza G., Csanád M., Jiang Z.:**

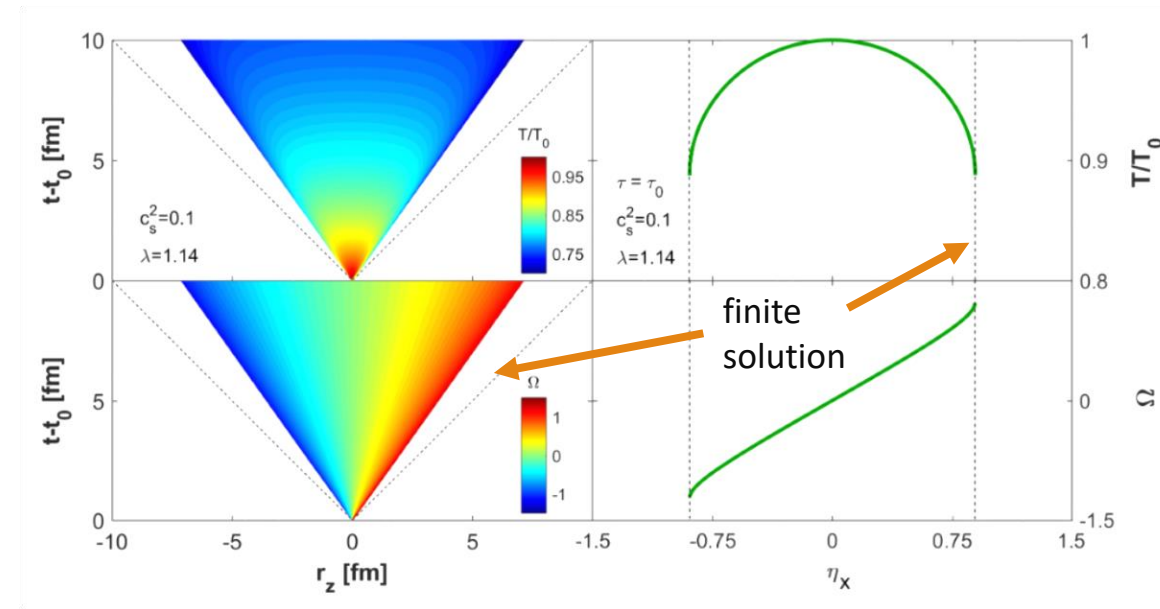
[arXiv:1805.01427](https://arxiv.org/abs/1805.01427), [arXiv:1806.06794](https://arxiv.org/abs/1806.06794)

$\lambda$ : acceleration parameter  
(Hwa-Bjorken:  $\lambda=1$ )

accelerating solution

realistic  $dN/d\eta$

$$\begin{aligned} \eta_x(H) &= \Omega(H) - H, \\ \Omega(H) &= \frac{\lambda}{\sqrt{\lambda-1}\sqrt{\kappa-\lambda}} \arctan \left( \sqrt{\frac{\kappa-\lambda}{\lambda-1}} \tanh(H) \right) \\ \sigma(\tau, H) &= \sigma_0 \left( \frac{\tau_0}{\tau} \right)^\lambda \mathcal{V}_\sigma(s) \left[ 1 + \frac{\kappa-1}{\lambda-1} \sinh^2(H) \right]^{-\frac{\lambda}{2}}, \\ T(\tau, H) &= T_0 \left( \frac{\tau_0}{\tau} \right)^\lambda \mathcal{T}(s) \left[ 1 + \frac{\kappa-1}{\lambda-1} \sinh^2(H) \right]^{-\frac{\lambda}{2\kappa}}, \\ \mathcal{T}(s) &= \frac{1}{\mathcal{V}_\sigma(s)}, \\ s(\tau, H) &= \left( \frac{\tau_0}{\tau} \right)^{\lambda-1} \sinh(H) \left[ 1 + \frac{\kappa-1}{\lambda-1} \sinh^2(H) \right]^{-\lambda/2} \end{aligned}$$



# Scaling behaviour in high-energy physics

- $dN/dy$  is obtained from the new solution (in self-similar approximation):

$$\frac{dN}{dy} \approx \left. \frac{dN}{dy} \right|_{y=0} \cosh^{-\frac{1}{2}\alpha(\kappa,\lambda)-1} \left( \frac{y}{\alpha(1,\lambda)} \right) \exp \left( -\frac{m}{T_{\text{eff}}} \left[ \cosh^{\alpha(\kappa,\lambda)} \left( \frac{y}{\alpha(1,\lambda)} \right) - 1 \right] \right)$$

**Physical parameters:**  
 $\lambda$ : acceleration parameter  
 $\kappa$ : inverse square of  $c_s$   
 $T_{\text{eff}}$ : effective temperature  
 $m$ : particle mass

- If  $|y| \ll 2+(\lambda-1)^{-1}$ , Gaussian rapidity-density:

$$\frac{dN}{dy} \approx \frac{\langle N \rangle}{(2\pi\Delta^2 y)^{1/2}} \exp \left( -\frac{y^2}{2\Delta^2 y} \right) \longrightarrow \frac{1}{\Delta^2 y} = (\lambda-1)^2 \left[ 1 + \left( 1 - \frac{1}{\kappa} \right) \left( \frac{1}{2} + \frac{m}{T_{\text{eff}}} \right) \right]$$

- Depends on the combination of the physical parameters through the width ( $\Delta y$ )
- $\lambda$ ,  $m$ ,  $T_{\text{eff}}$  and  $\kappa$  can be arbitrary, but their combination is not:  $\Delta y$  is determined by fits
- Physical differences are only apparent in the width of the distribution

Csörgő, Kasza, Csanád,  
 Jiang solution:

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*Beautiful example of scaling behaviour*

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# Scaling behaviour in high-energy physics

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- The pseudorapidity distribution is the product of  $dN/dy$  and the Jakobian:

$$\left( \eta_p(y), \frac{dN}{d\eta_p}(y) \right) = \left( \frac{1}{2} \log \left[ \frac{\bar{p}(y) + \bar{p}_z(y)}{\bar{p}(y) - \bar{p}_z(y)} \right], \frac{\bar{p}(y)}{\bar{E}(y)} \frac{dN}{dy} \right)$$

K. G. , Csörgő T.:  
[arXiv:1811.09990](https://arxiv.org/abs/1811.09990)  
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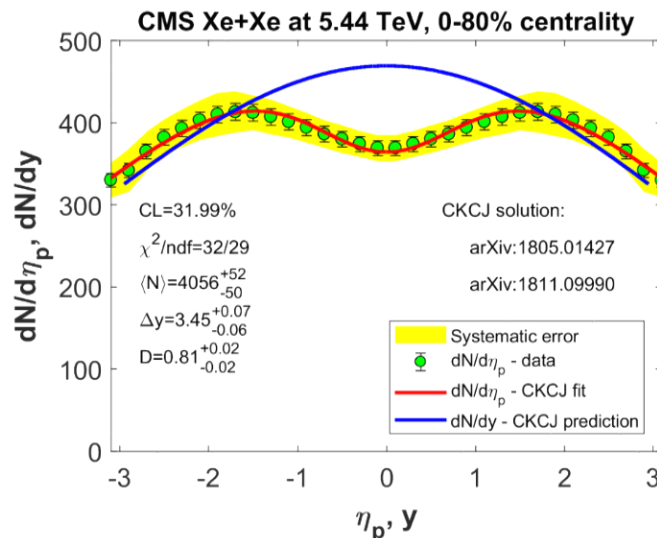
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- The rapidity density is depressed at midrapidity by  $\frac{1}{\sqrt{1+D^2}}$



$$D = \frac{m}{\bar{p}_T}$$

„Depression” or „Depth” parameter

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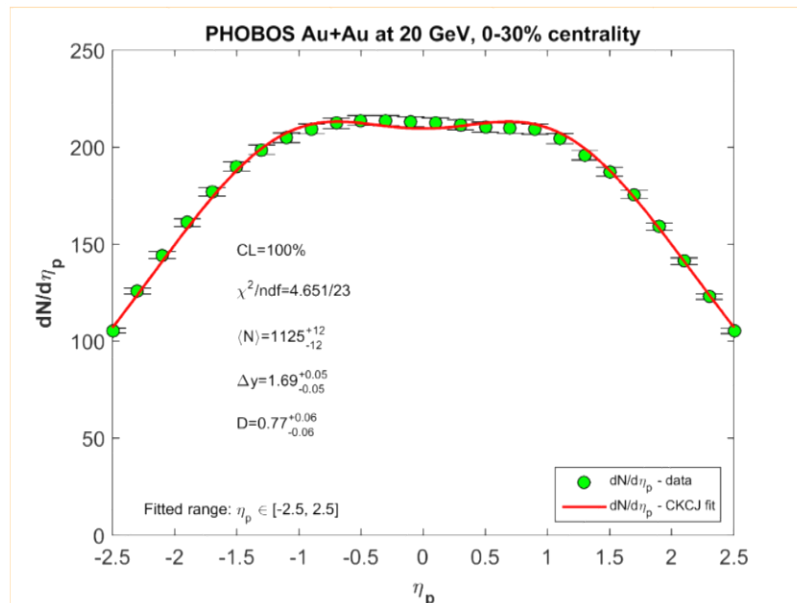
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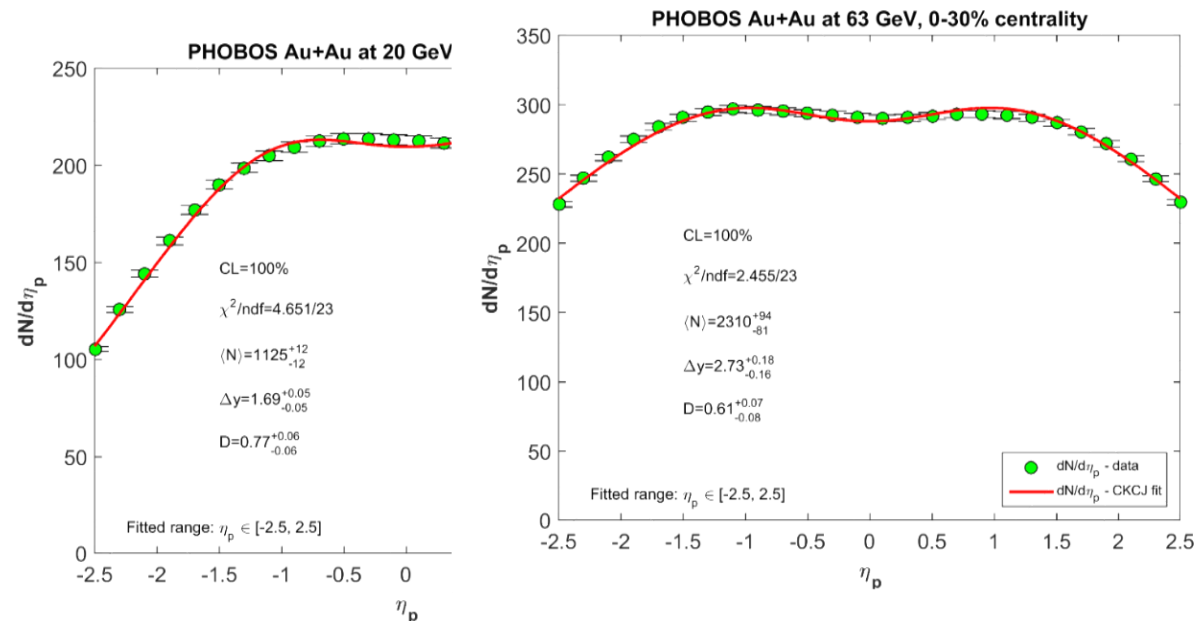
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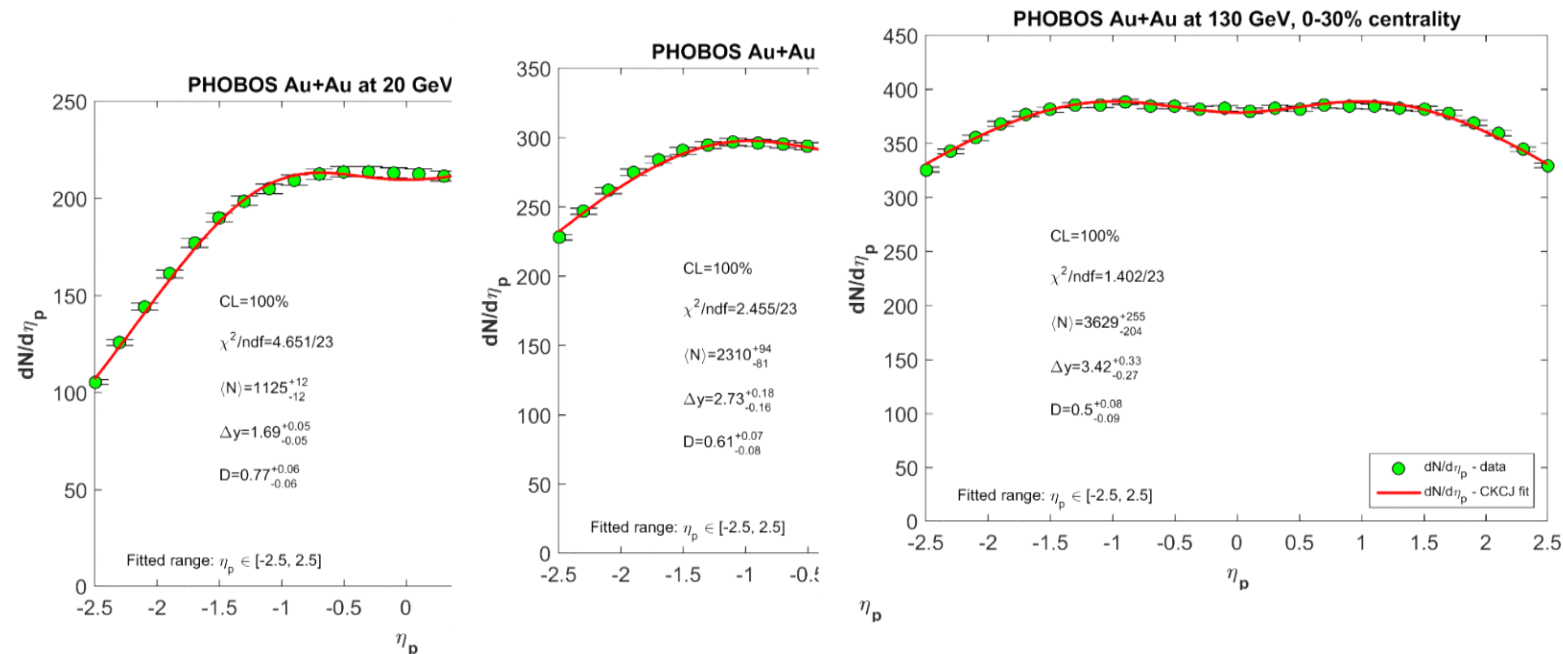
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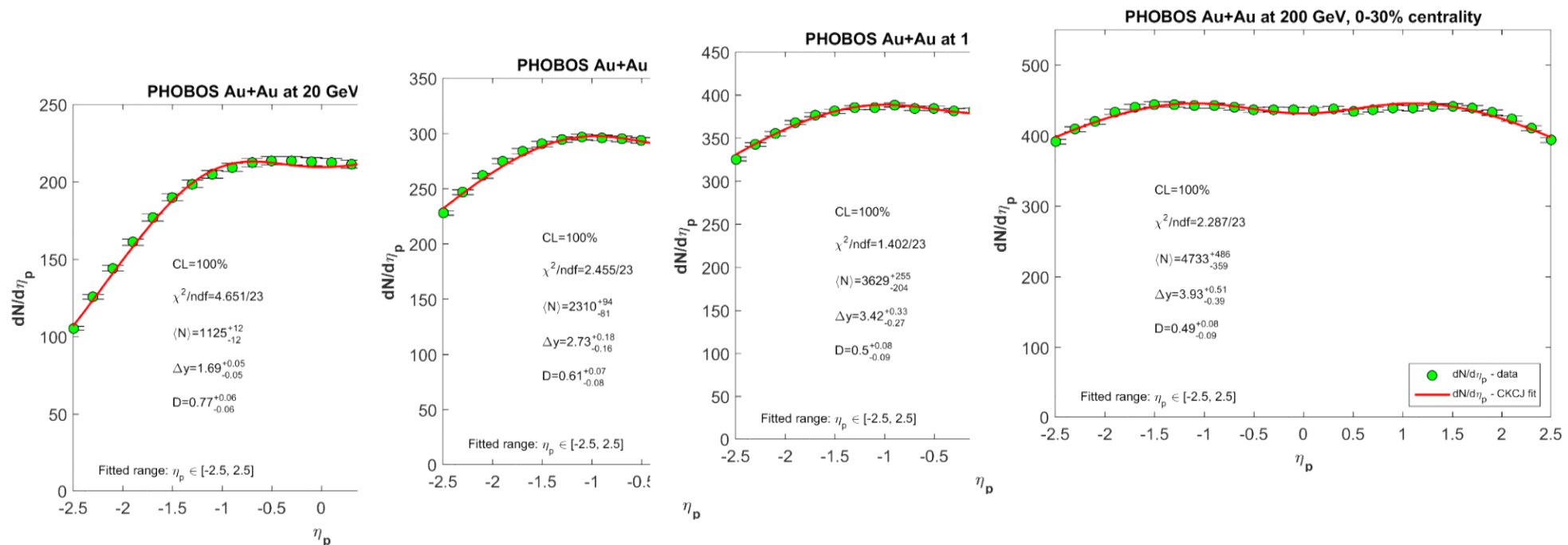
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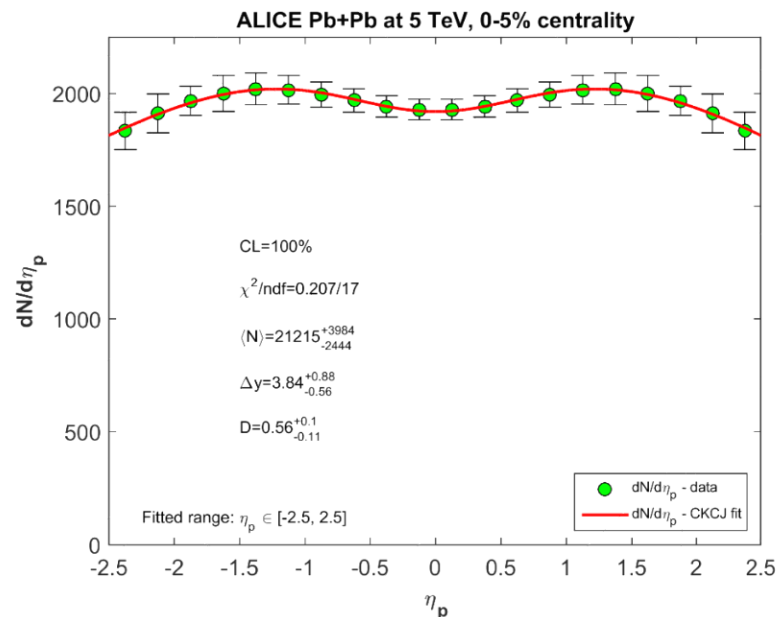
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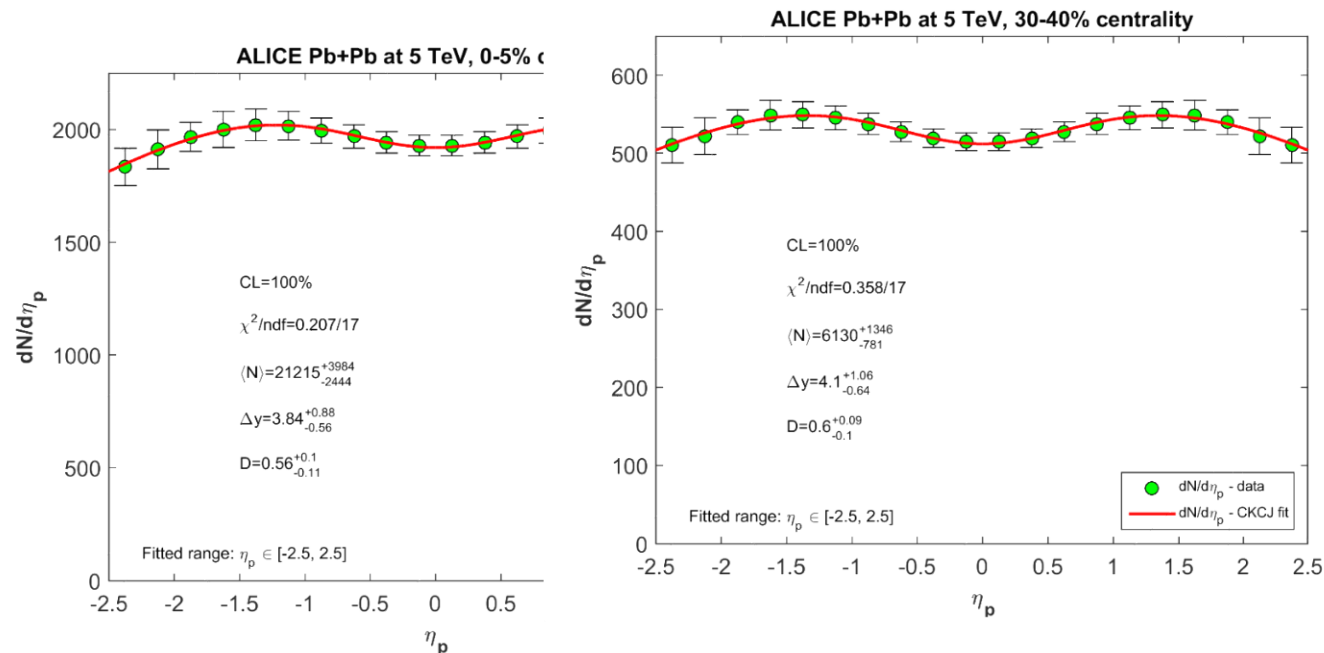
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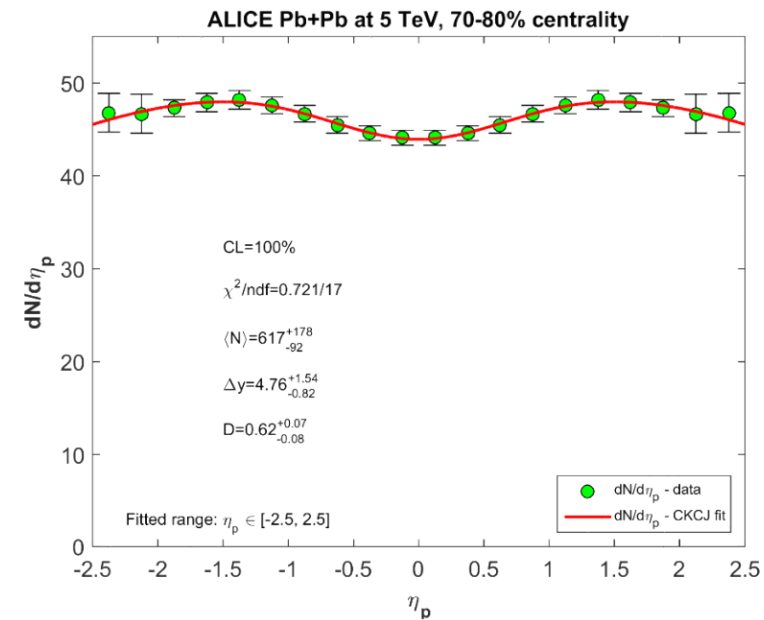
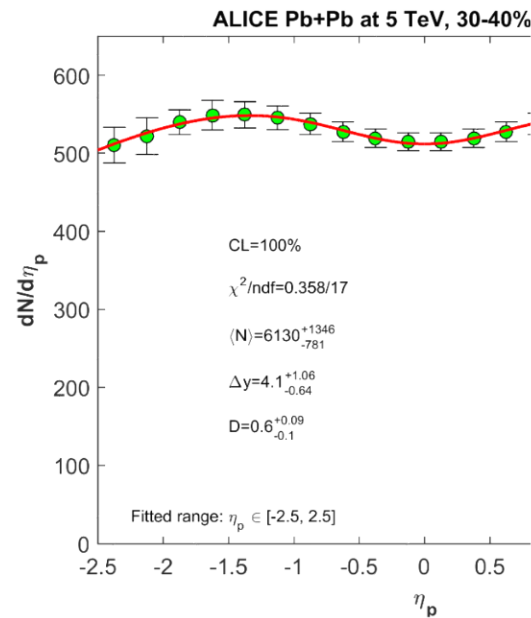
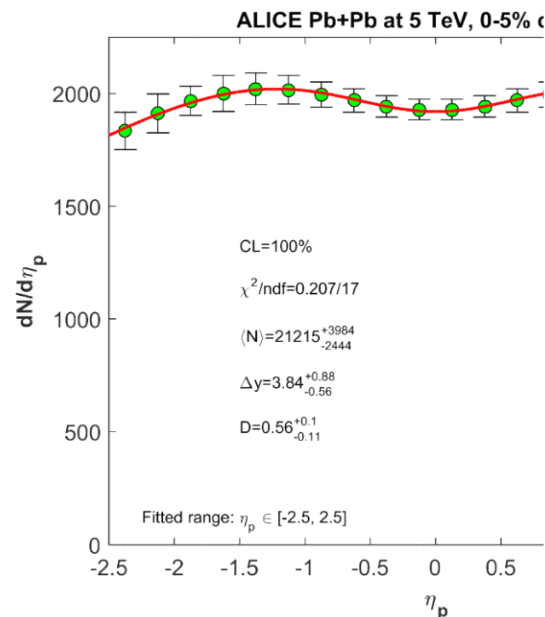
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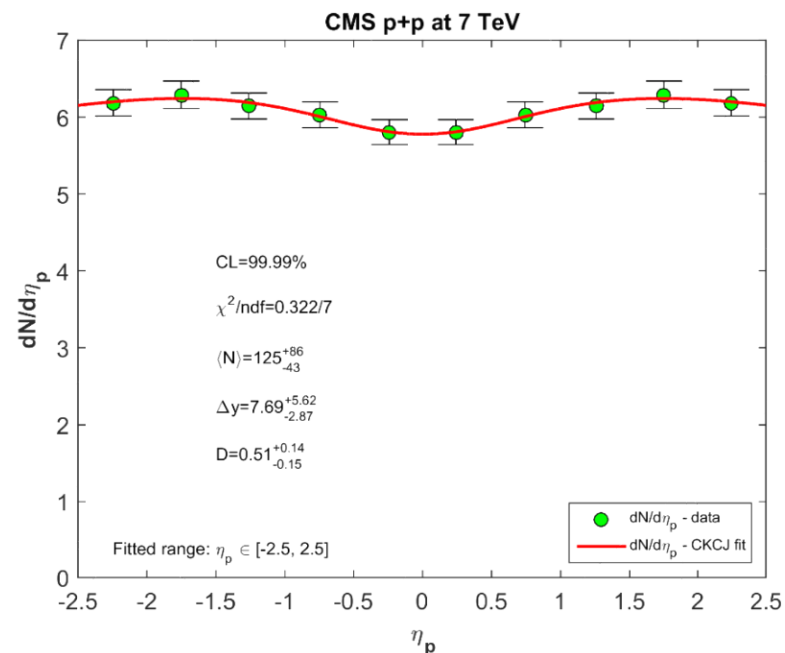
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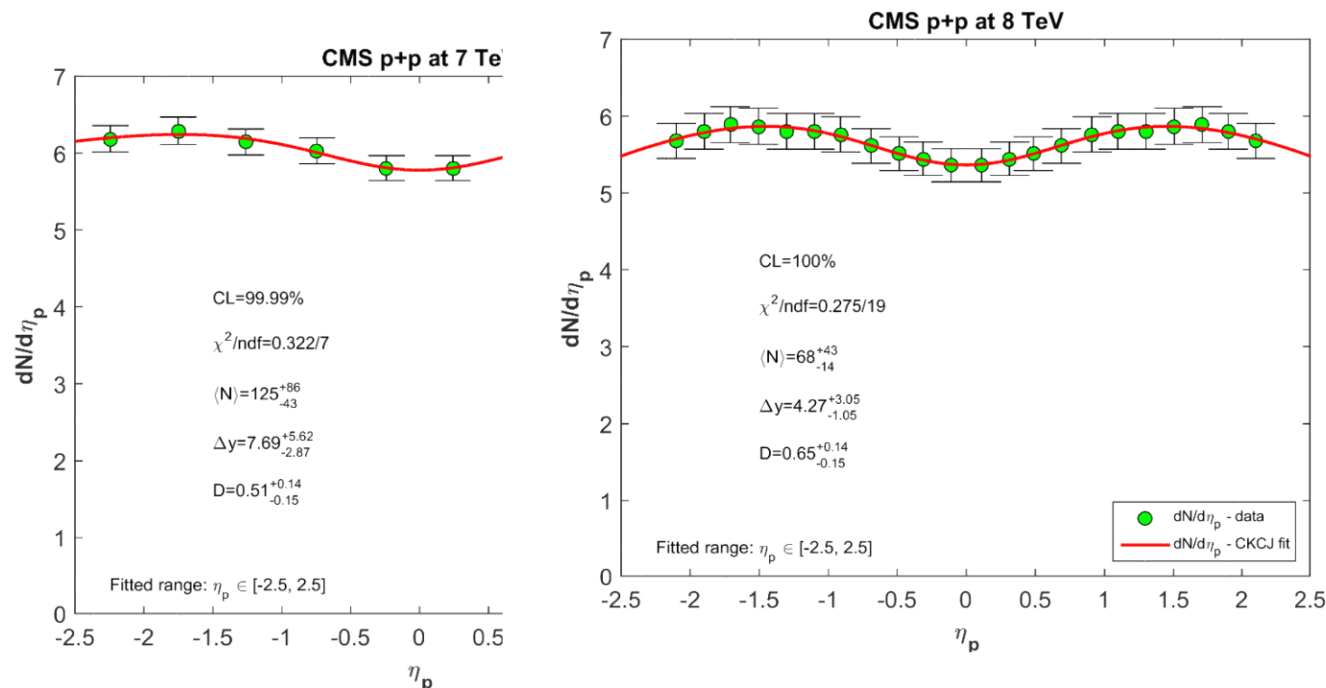
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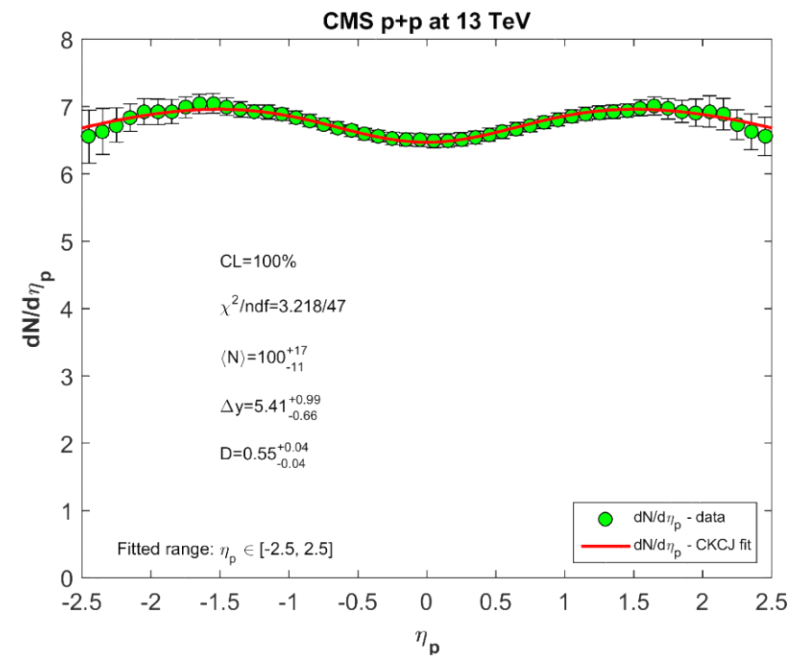
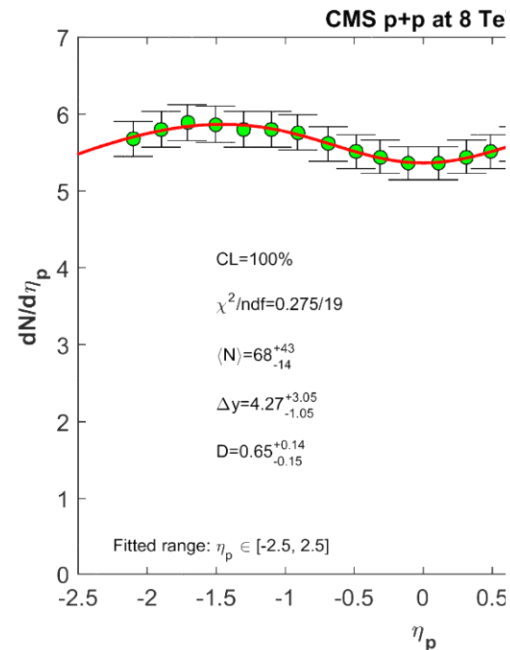
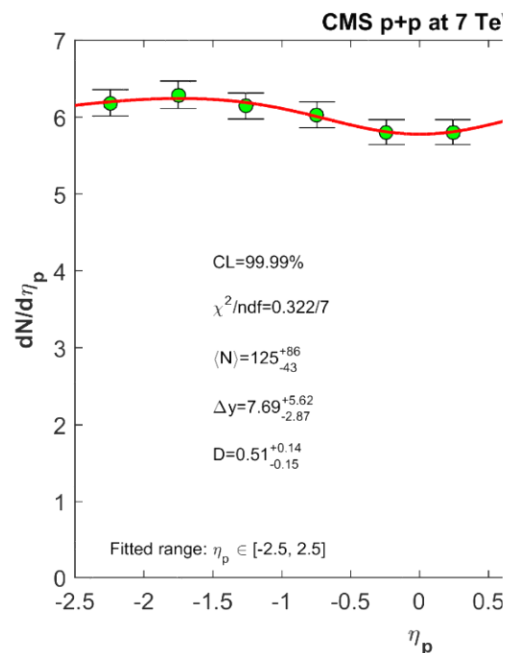
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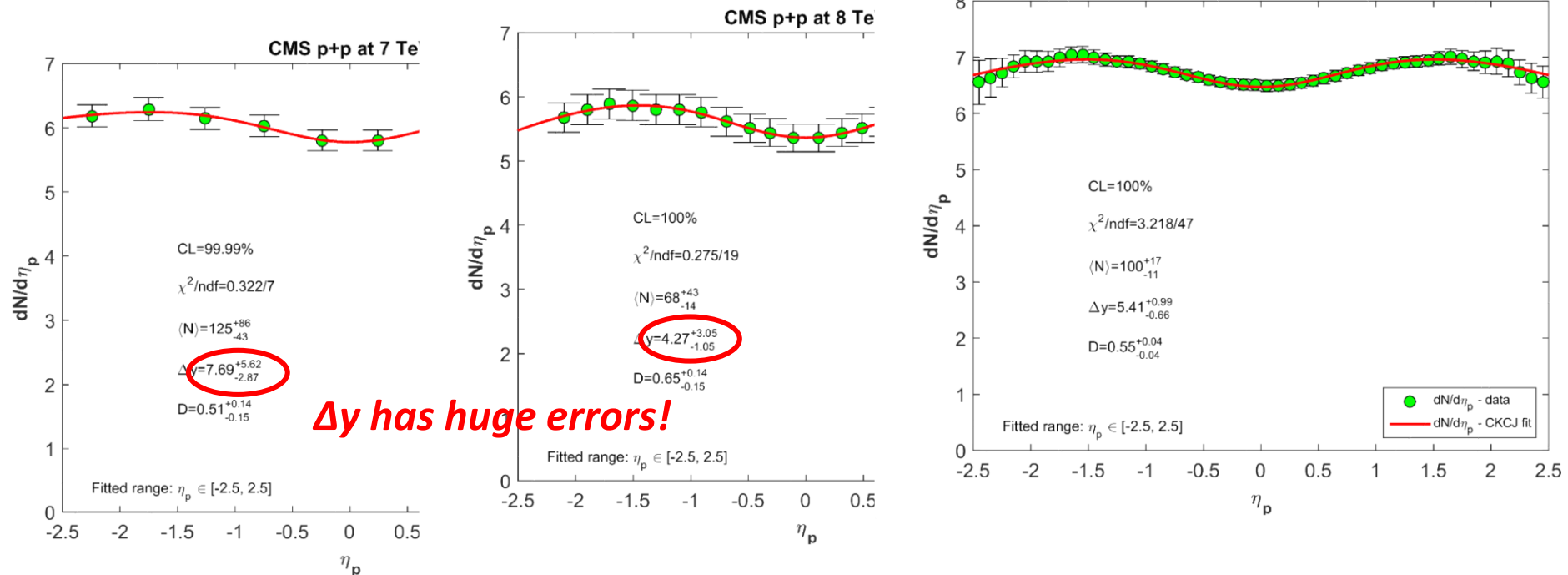
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- Scaling behaviour is evident: hydro works well independently on the system size ...

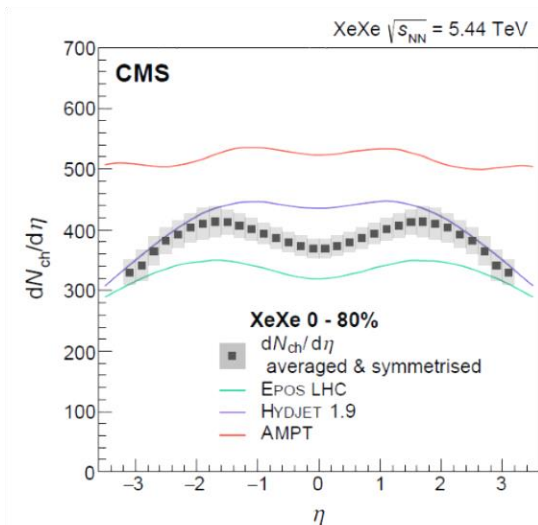
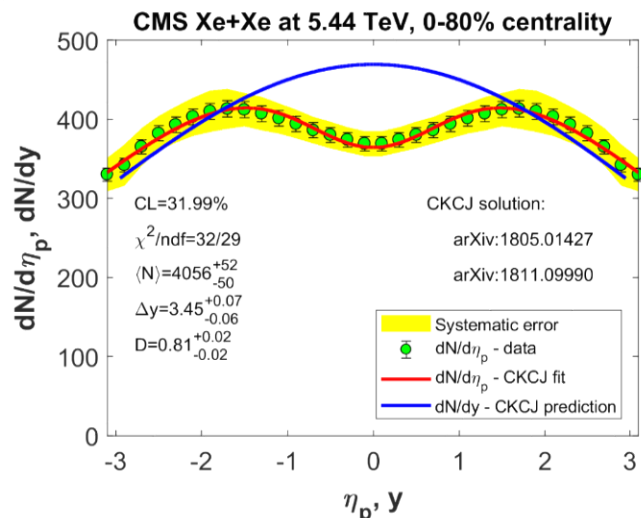
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*Our self-similar hydrodynamic calculations are succesful in such cases where other models fail.*

CMS collab.: [arXiv:1902.03603](https://arxiv.org/abs/1902.03603)

Werner, Liu, Pierog: [arXiv:hep-ph/0506232](https://arxiv.org/abs/hep-ph/0506232)

Pierog, Karpenko, et al.: [arXiv:1306.0121](https://arxiv.org/abs/1306.0121)

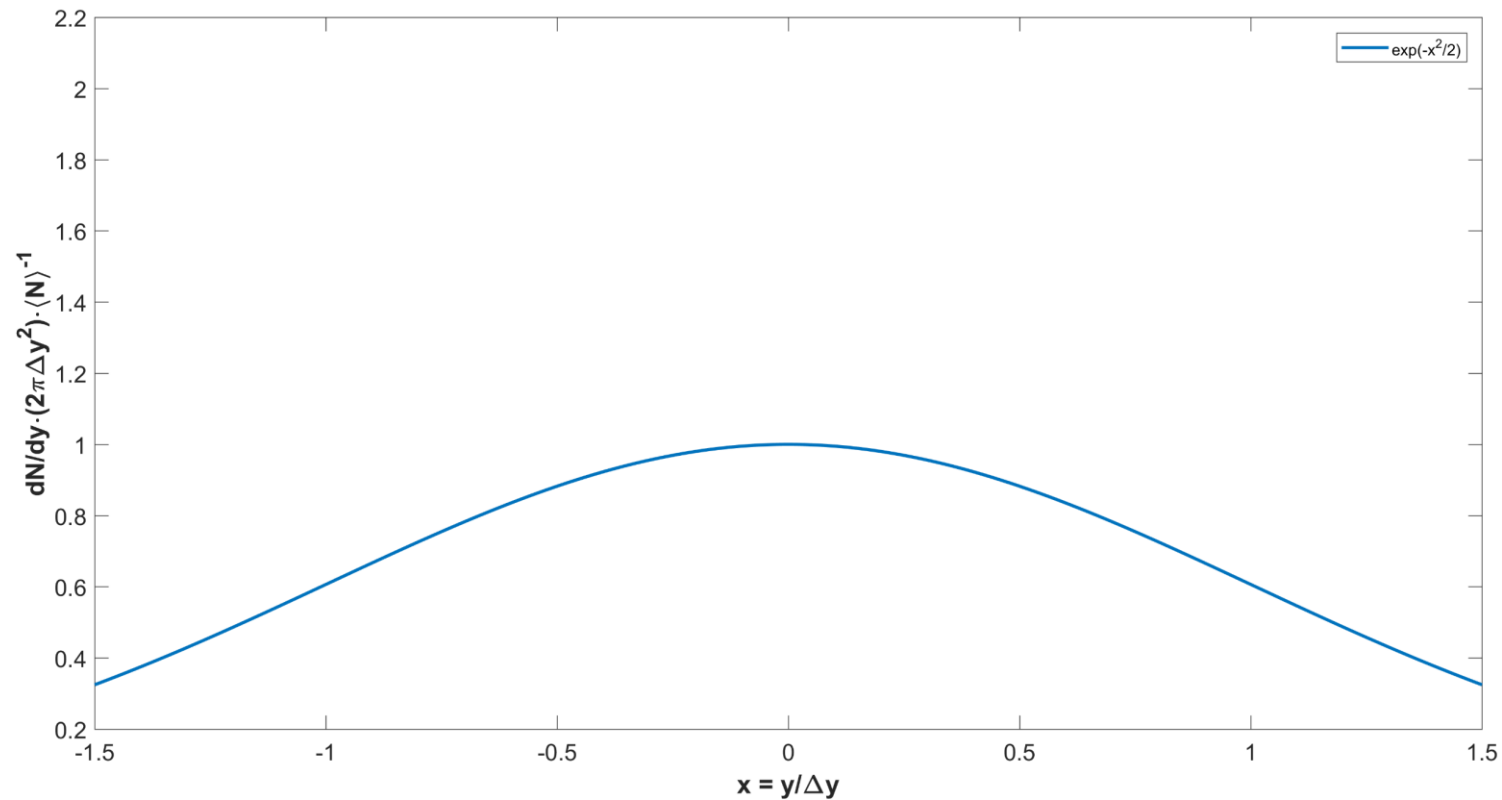
Lokhtin, Snigirev: [arXiv:hep-ph/0506189](https://arxiv.org/abs/hep-ph/0506189)

Lin, Ko, et al.: [arXiv:nucl-th/0411110](https://arxiv.org/abs/nucl-th/0411110)



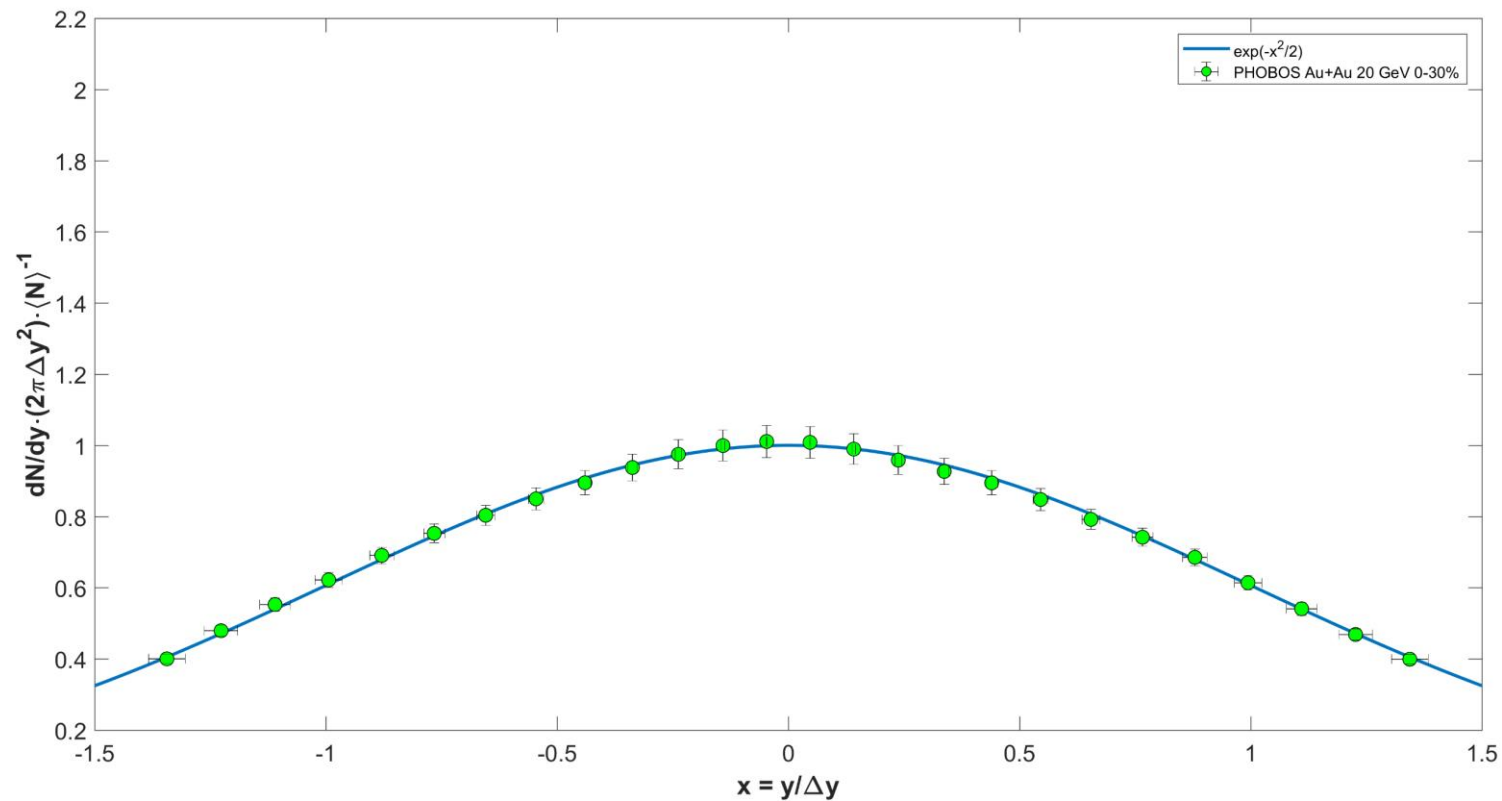
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- All the fitted dataset can be described by a normalized Gaussian:



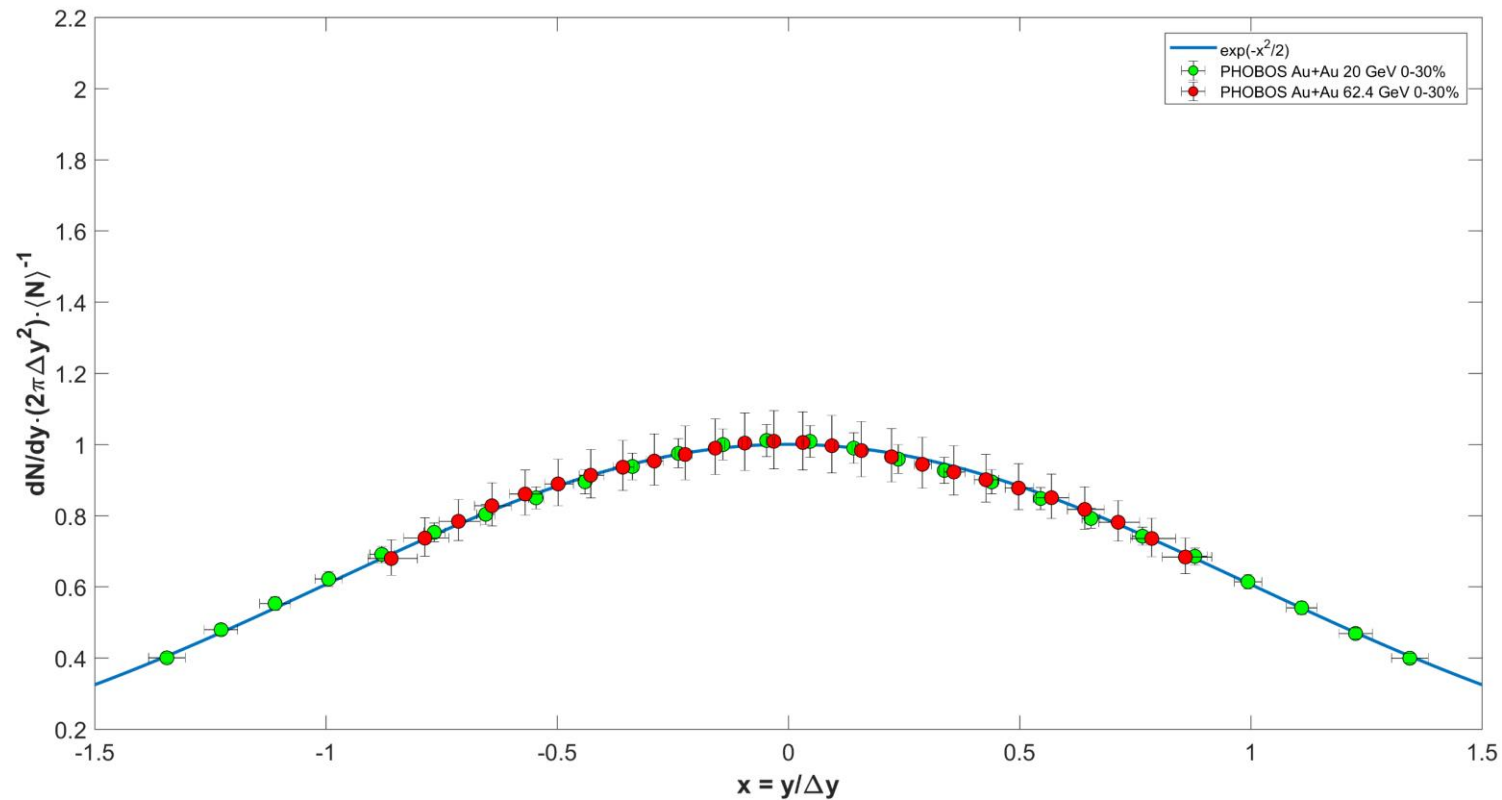
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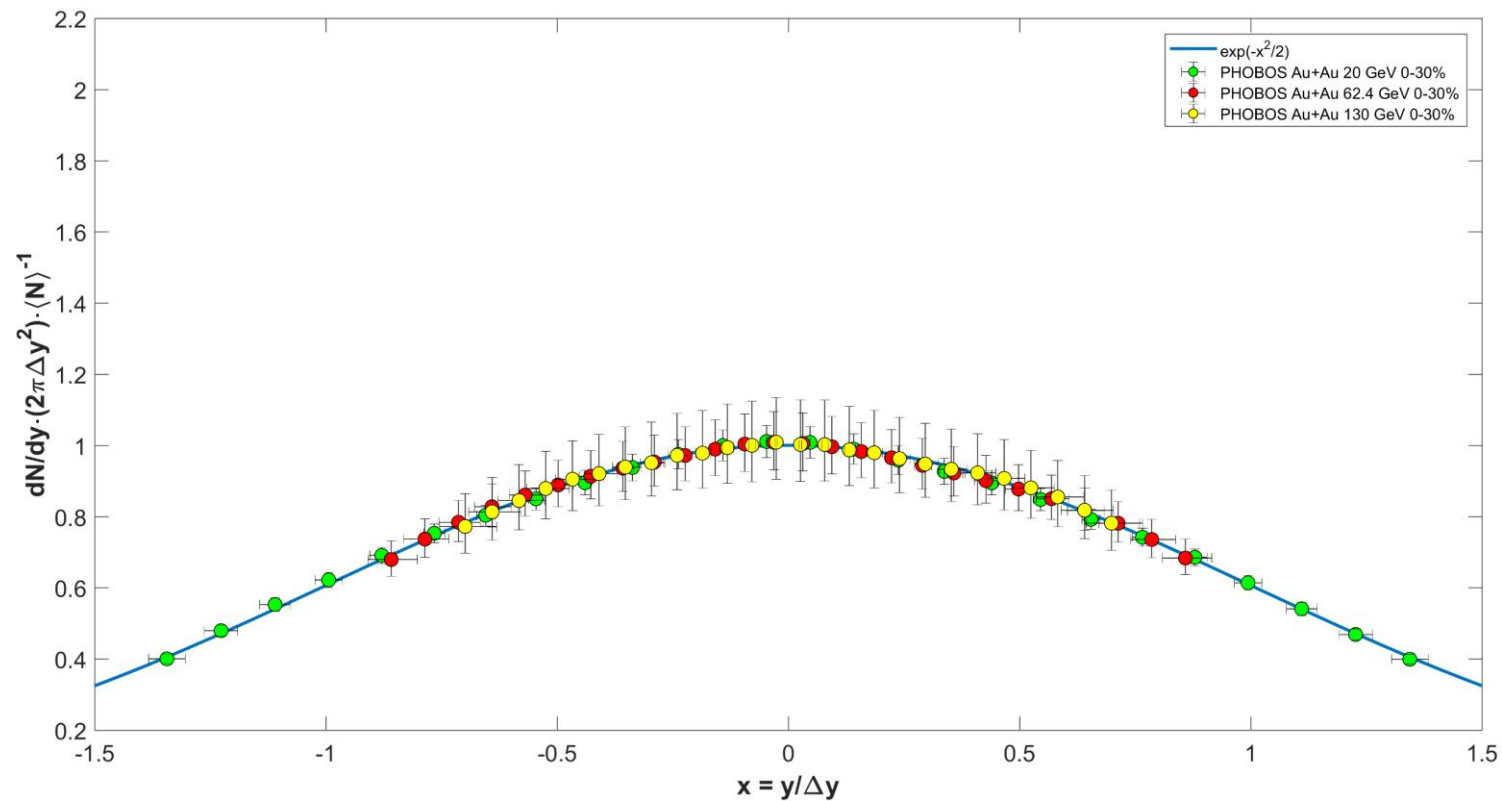
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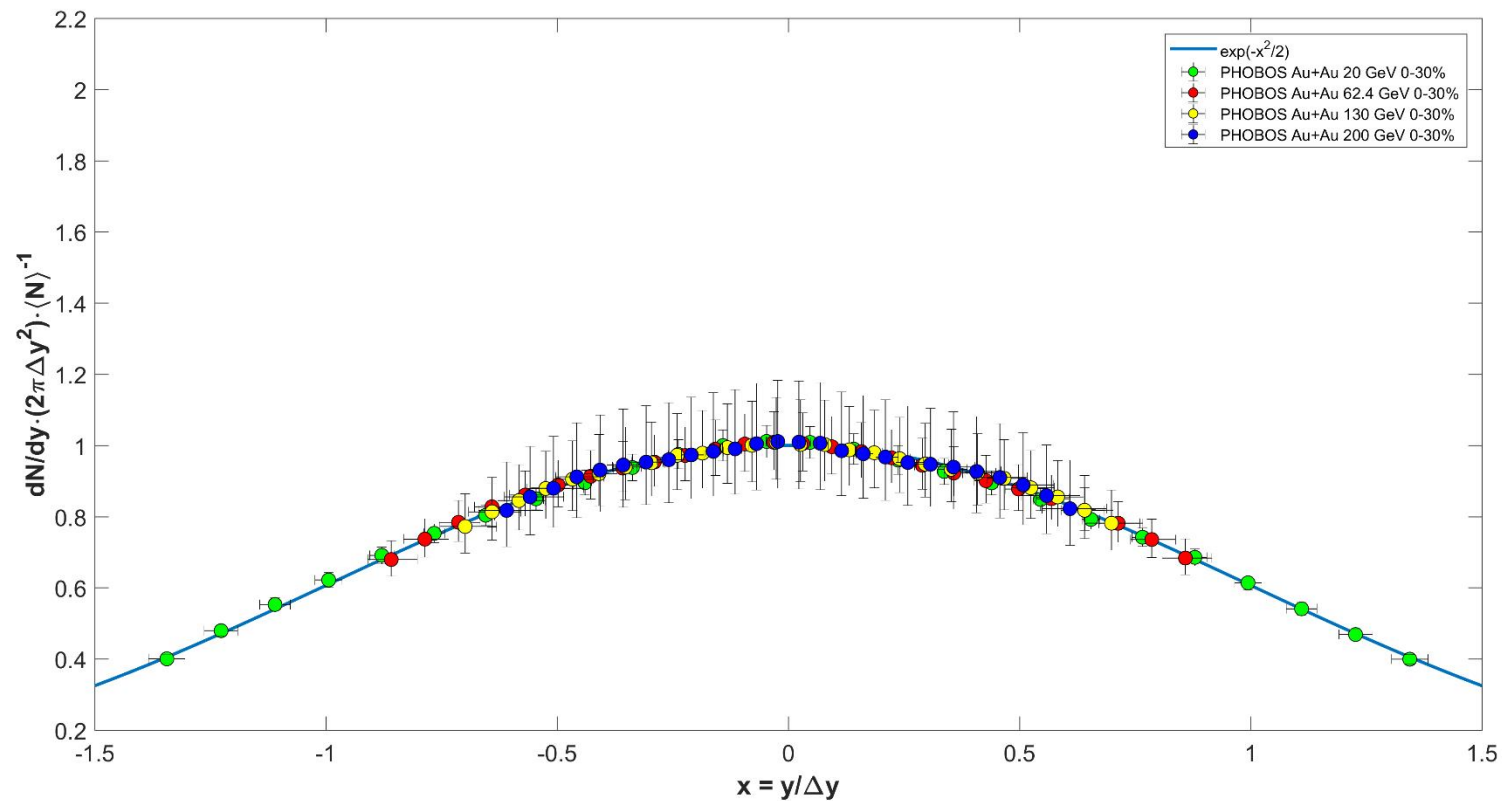
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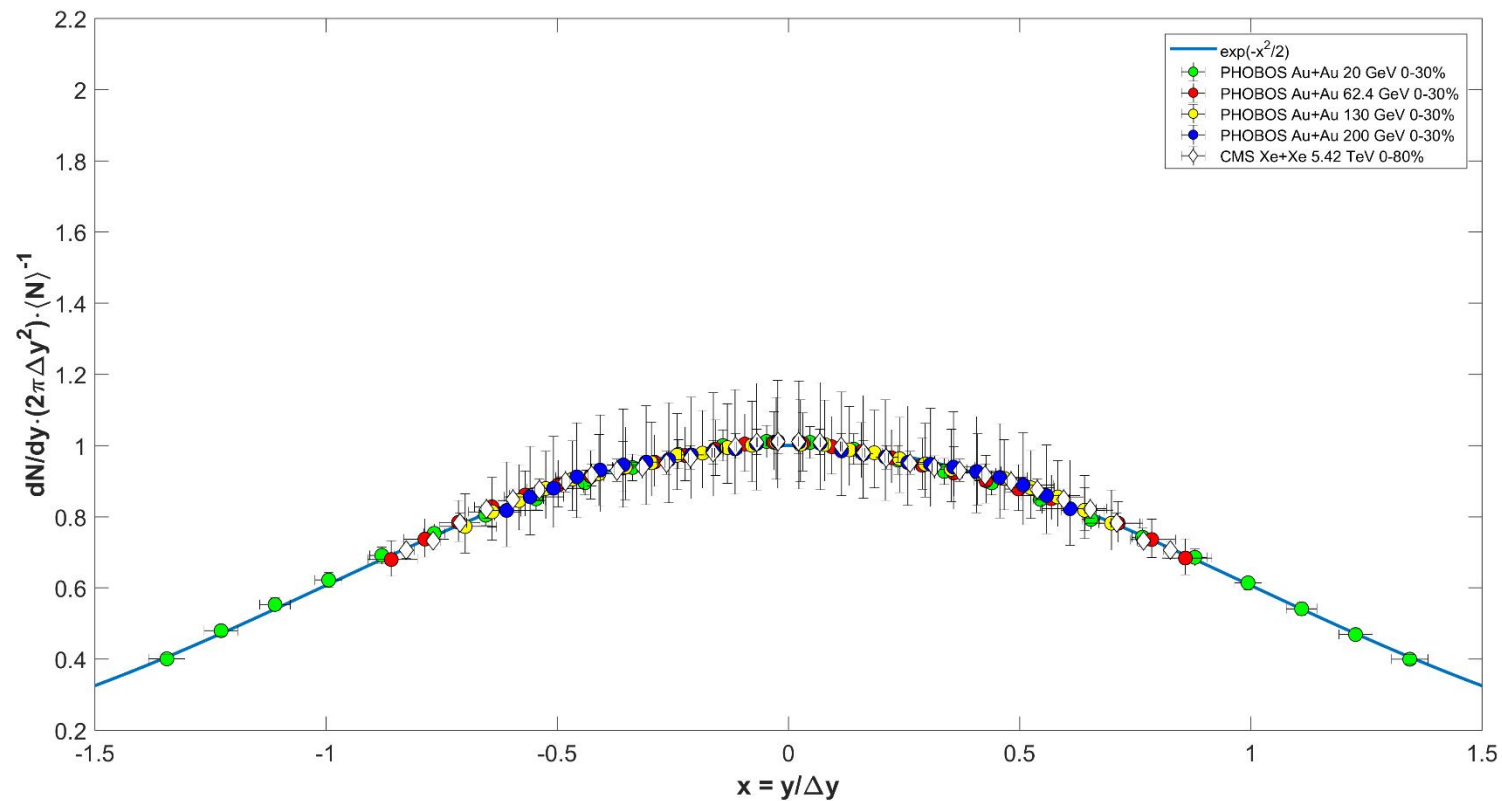
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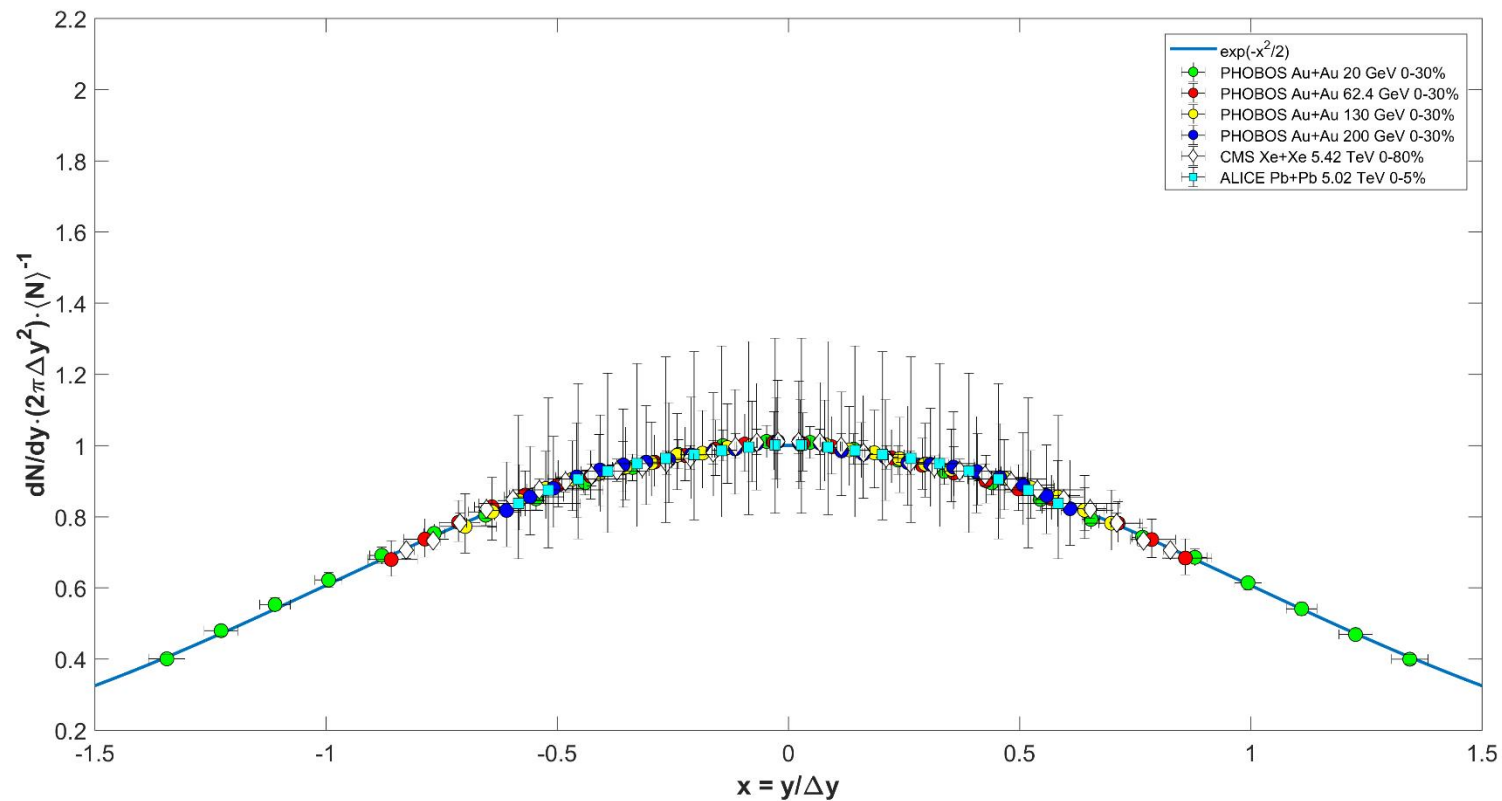
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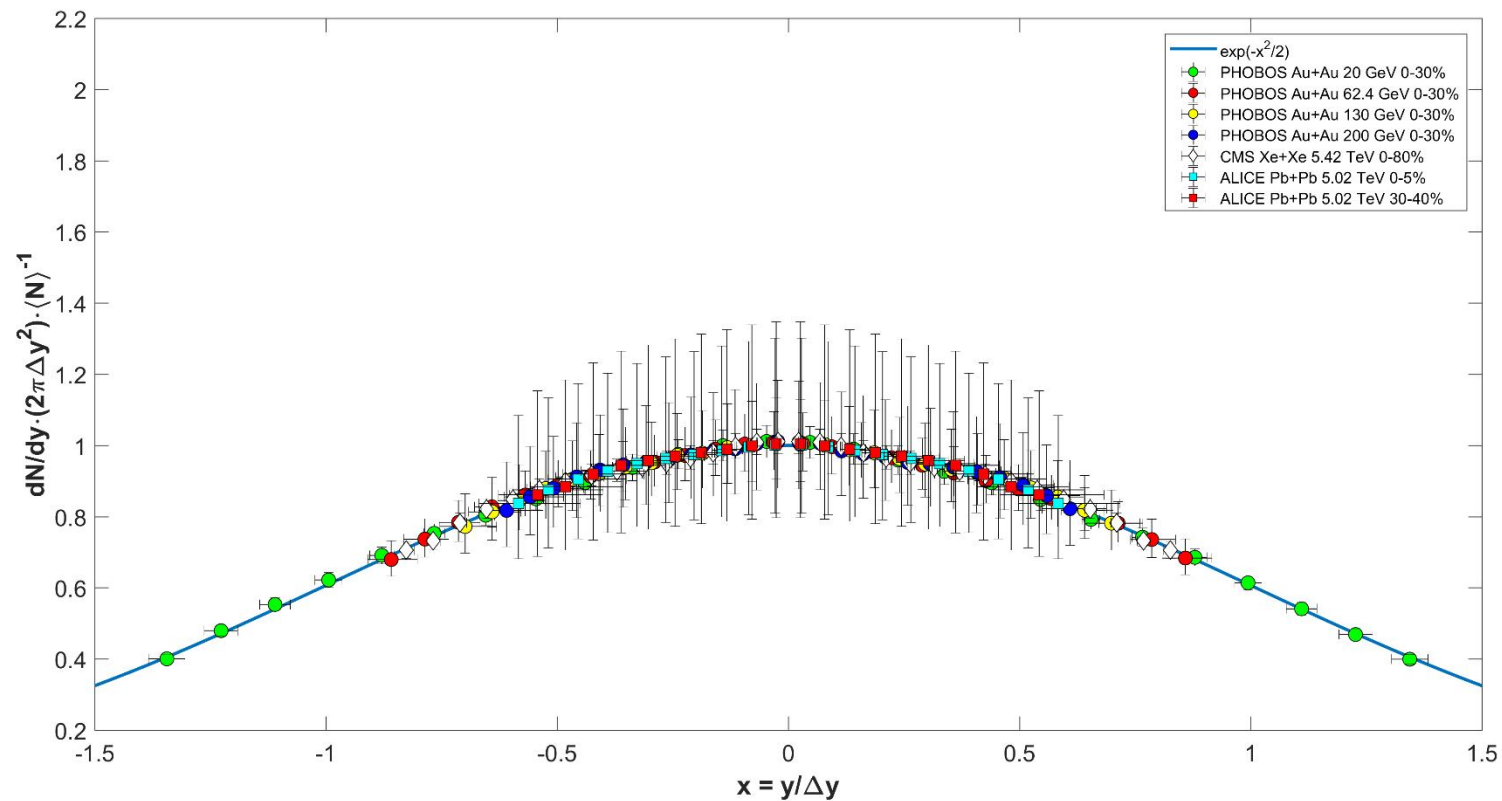
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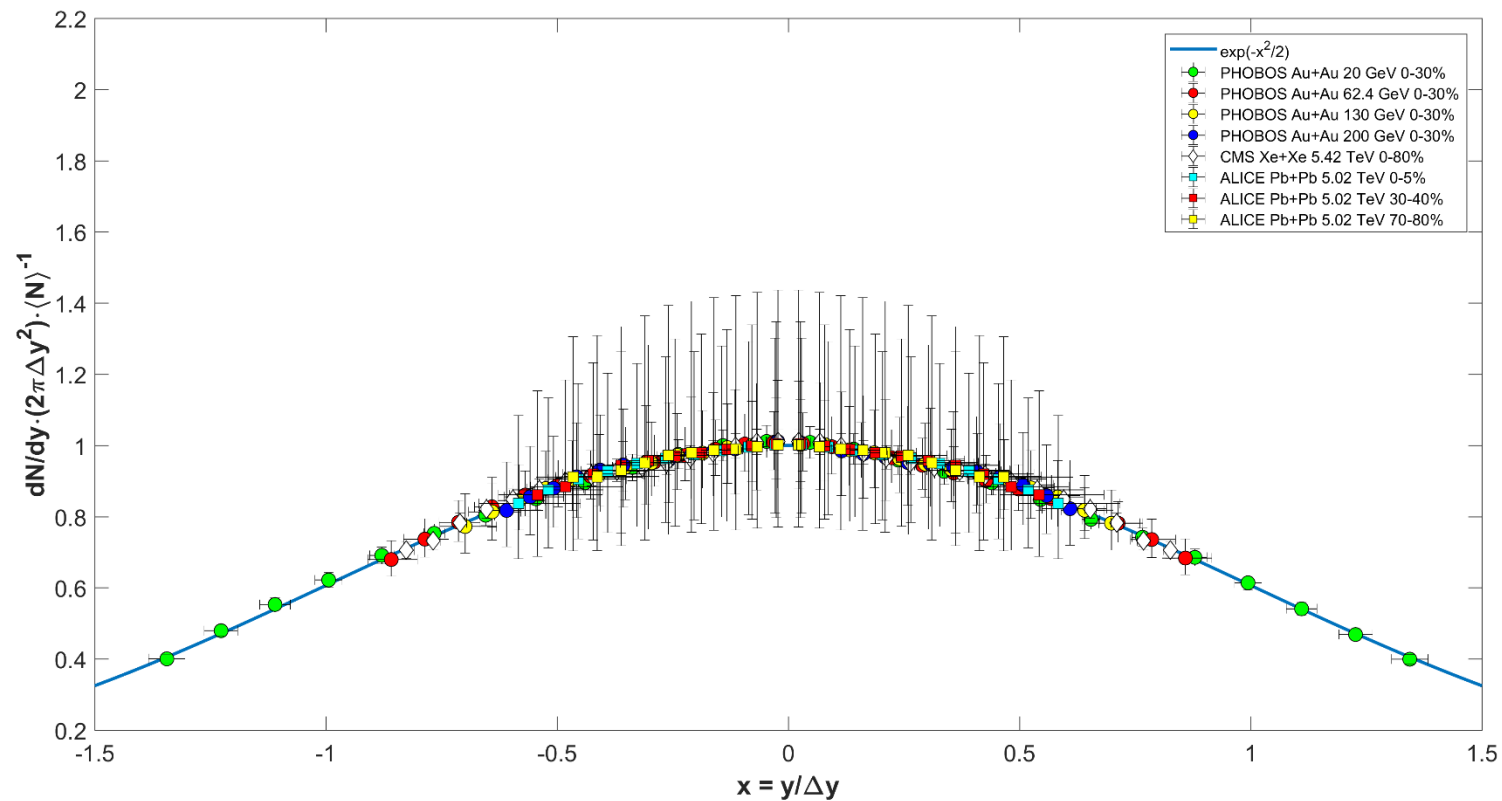
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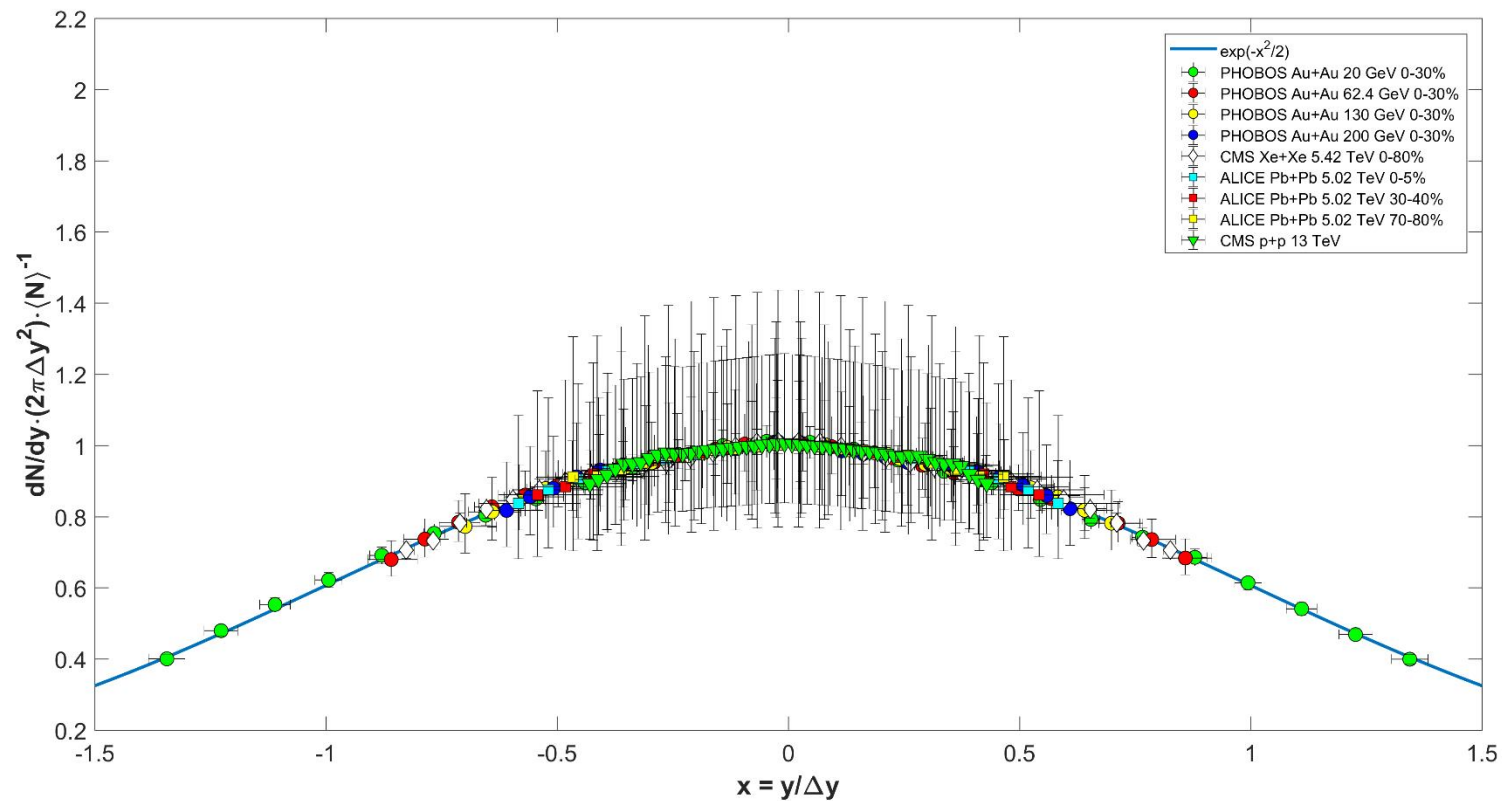
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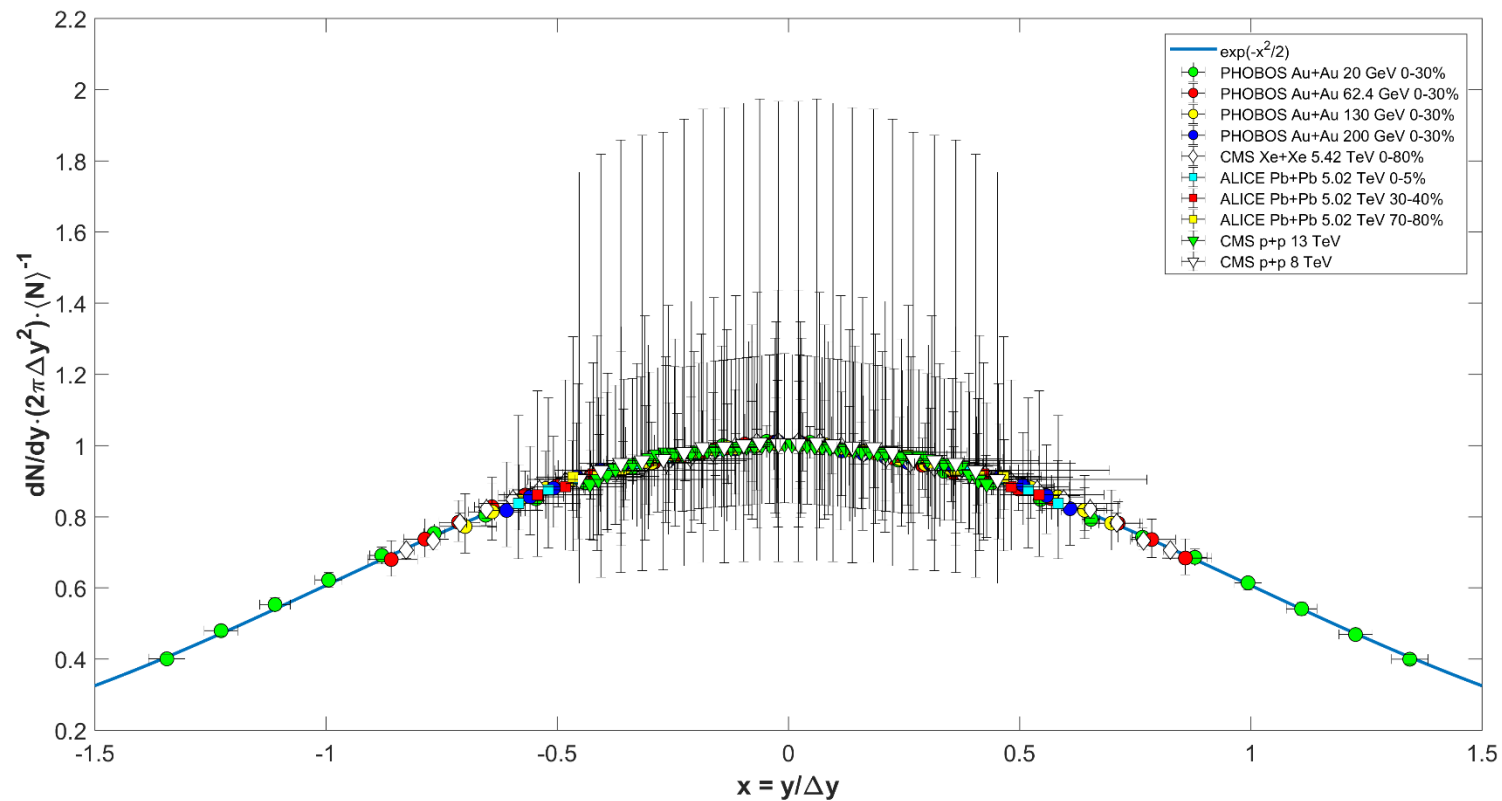
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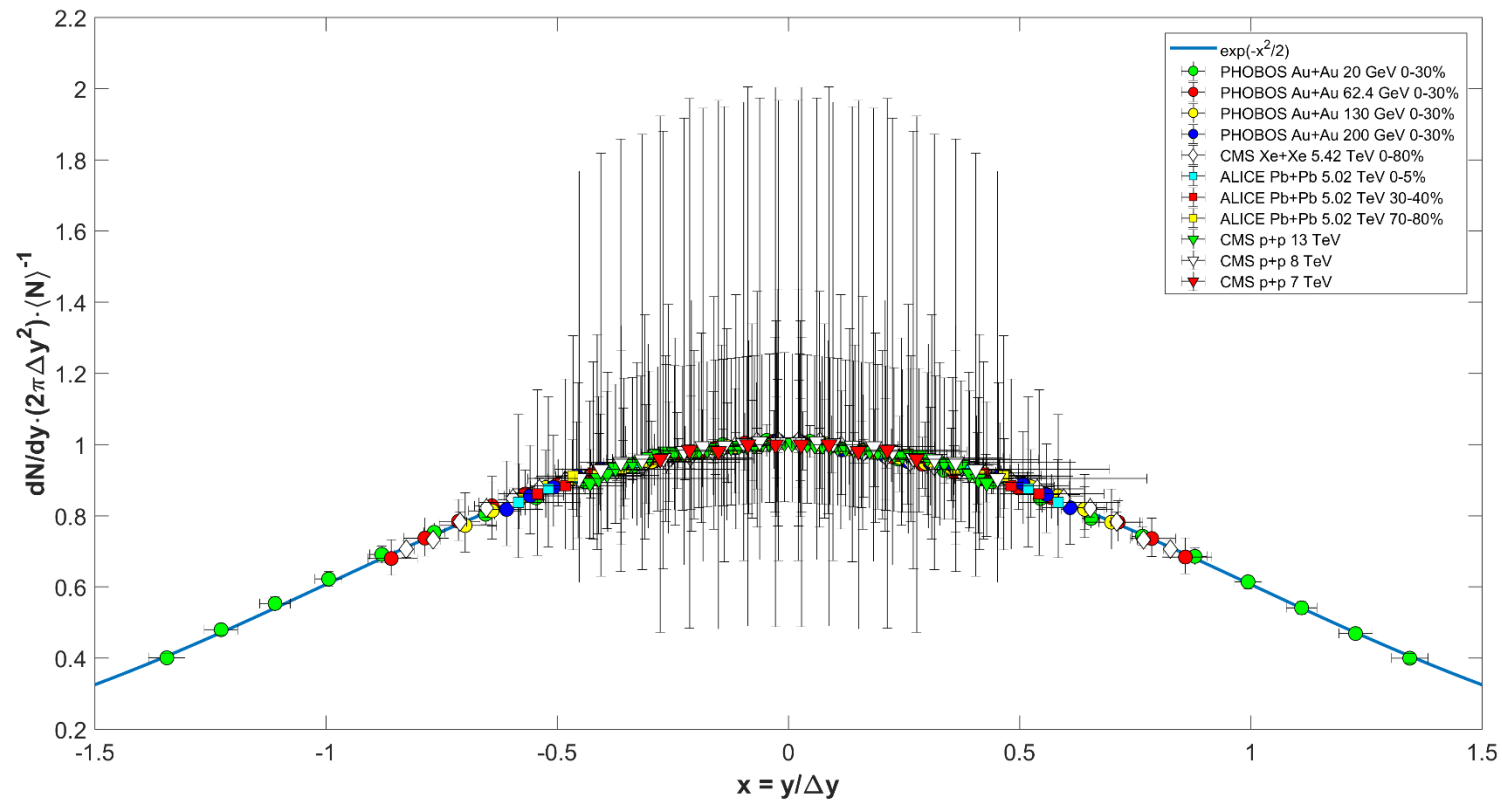
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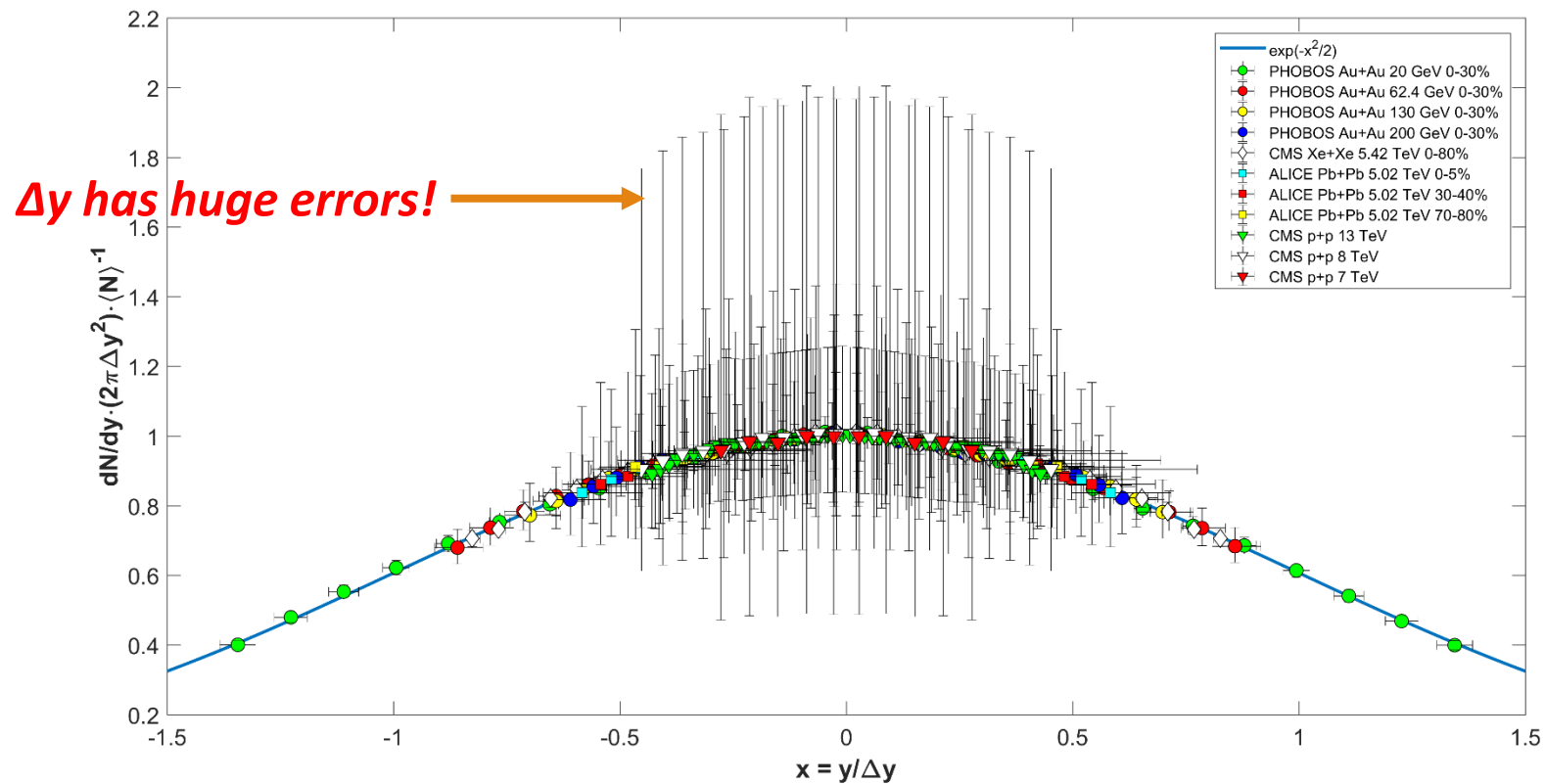
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# In conclusion...

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- p+p collisions can be described as collective systems
- Our fits indicate low  $c_s$  value ( $\approx 0.35$ )
- Low  $c_s$  value indicate the presence of fluid, so the presence of QGP
- p+p and A+A collisions: self-similar systems

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*We hope that these results help to confirm the legitimacy of hydro in p+p collisions!*



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*Thank you for your attention!*

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- p+p and A+A collisions: self-similar systems

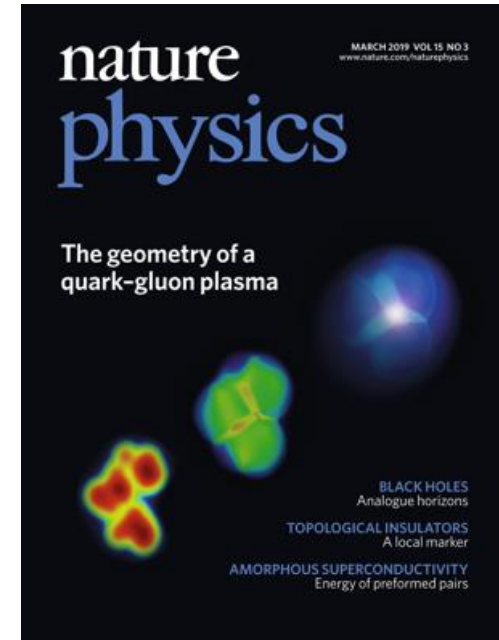
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- p+A, d+A and He+A collisions: accepted since 2019
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- However, describing H+H systems by hydro is not a recent idea

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ESTIMATION OF HYDRODYNAMICAL MODEL PARAMETERS FROM  
THE INVARIANT SPECTRUM AND THE BOSE-EINSTEIN CORRELATIONS OF  
 $\pi^-$  MESONS PRODUCED IN  $(\pi^+/K^+)p$  INTERACTIONS AT 250 GeV/c

EHS/NA22 Collaboration

N.M. Agababyan<sup>a</sup>, M.R. Atayan<sup>a</sup>, T. Csörgö<sup>b</sup>, E.A. De Wolf<sup>a,1</sup>, K. Dziunikowska<sup>b,2</sup>, A.M.F. Endler<sup>a</sup>,  
Z.Sh. Garutchava<sup>d</sup>, H.R. Gulikanyan<sup>e</sup>, R.Sh. Hakobyan<sup>e</sup>, J.K. Karamyan<sup>e</sup>, D. Kisielowski<sup>b,2</sup>,  
W. Kittel<sup>d</sup>, S.S. Mehrabyan<sup>e</sup>, Z.V. Metreveli<sup>f</sup>, K. Olkiewicz<sup>b,2</sup>, F.K. Rizatdinova<sup>e</sup>,  
E.K. Shabalina<sup>c</sup>, L.N. Smirnova<sup>c</sup>, M.D. Tabidze<sup>f</sup>, L.A. Tikhonova<sup>c</sup>, A.V. Tkabladze<sup>f</sup>,  
A.G. Tomaradze<sup>f</sup>, F. Verbeure<sup>g</sup>, S.A. Zotkin<sup>c</sup>

<sup>a</sup> Department of Physics, Universitaire Instelling Antwerpen, B-2610 Wilrijk, Belgium

<sup>b</sup> Institute of Physics and Nuclear Techniques of Academy of Mining and Metallurgy and Institute of Nuclear Physics, PL-30055 Krakow, Poland

<sup>c</sup> Nuclear Physics Institute, Moscow State University, RU-119899 Moscow, Russia

<sup>d</sup> High Energy Physics Institute Nijmegen (HEFIN), University of Nijmegen/NIKHEF, NL-6525

ED Nijmegen, The Netherlands

<sup>e</sup> Centro Brasileiro de Pesquisas Físicas, BR-22290 Rio de Janeiro, Brazil

<sup>f</sup> Institute for High Energy Physics of Tbilisi State University, GE-380086 Tbilisi, Georgia

<sup>g</sup> Institute of Physics, AM-375036 Yerevan, Armenia

<sup>h</sup> KFKI, Hungarian Academy of Sciences, H-1525 Budapest 114, Hungary

**Abstract:** The invariant spectra of  $\pi^-$  mesons produced in  $(\pi^+/K^+)p$  interactions at 250 GeV/c are analysed in the framework of the hydrodynamical model of three-dimensionally expanding cylindrically symmetric finite systems. A satisfactory description of experimental data is achieved. The data favour the pattern according to which the hadron matter undergoes predominantly longitudinal expansion and non-relativistic transverse expansion with mean transverse velocity  $\langle u_t \rangle = 0.20 \pm 0.07$ , and is characterized by a large temperature inhomogeneity in the transverse direction: the extracted freeze-out temperature at the center of the tube and at the transverse rms radius are  $140 \pm 3$  MeV and  $82 \pm 7$  MeV, respectively. The width of the (longitudinal) space-time rapidity distribution of the pion source is found to be  $\Delta\eta = 1.36 \pm 0.02$ . Combining this estimate with results of the Bose-Einstein correlation analysis in the same experiment, one extracts a mean freeze-out time of the source of  $\langle \tau_f \rangle = 1.4 \pm 0.1$  fm/c and its transverse geometrical rms radius,  $R_G(\text{rms}) = 1.2 \pm 0.2$  fm.