PROTON HOLOGRAPHY

SCALING PROPERTIES OF ELASTIC SCATTERING

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Motivation: Odderon

Proton Holography:
phase reconstruction
H(x) scaling at ISR
H(x) scaling at TeV

Conclusion
Addendum

INTRODUCTION: HOLOGRAPHY

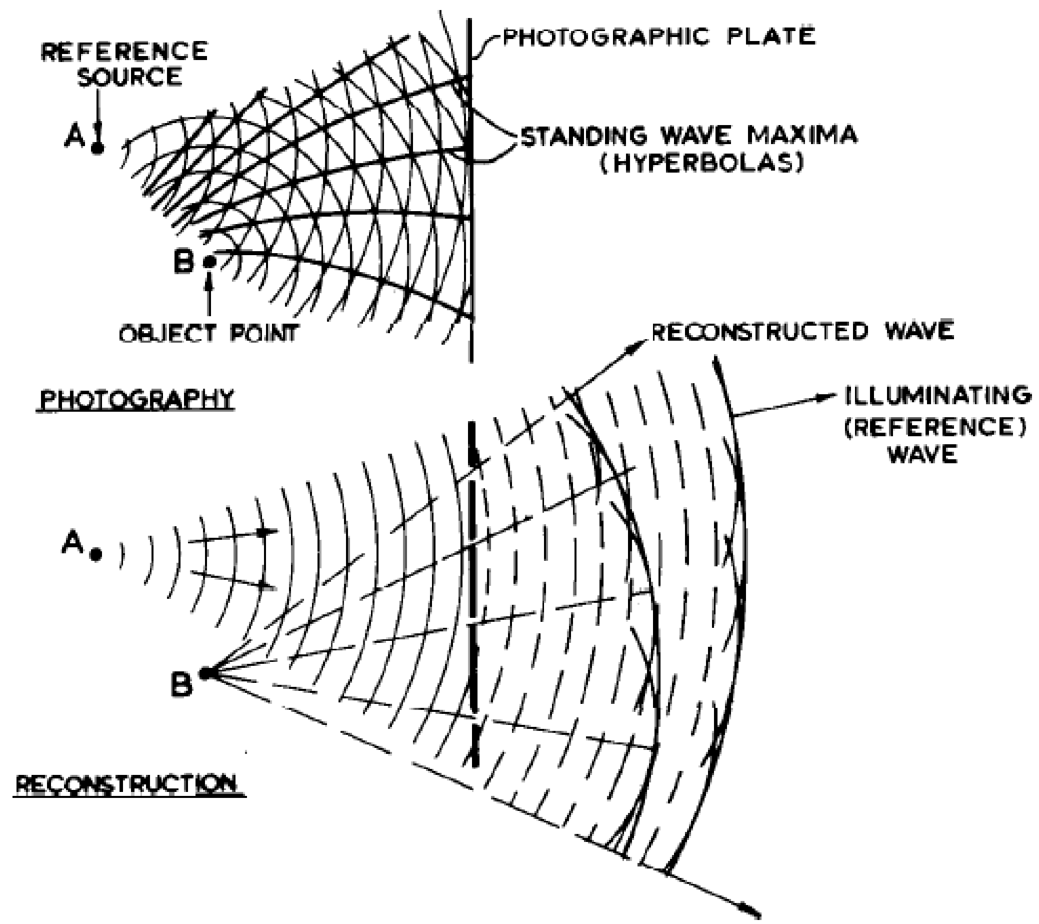


Fig. 2.
The Basic Idea of Holography, 1947.

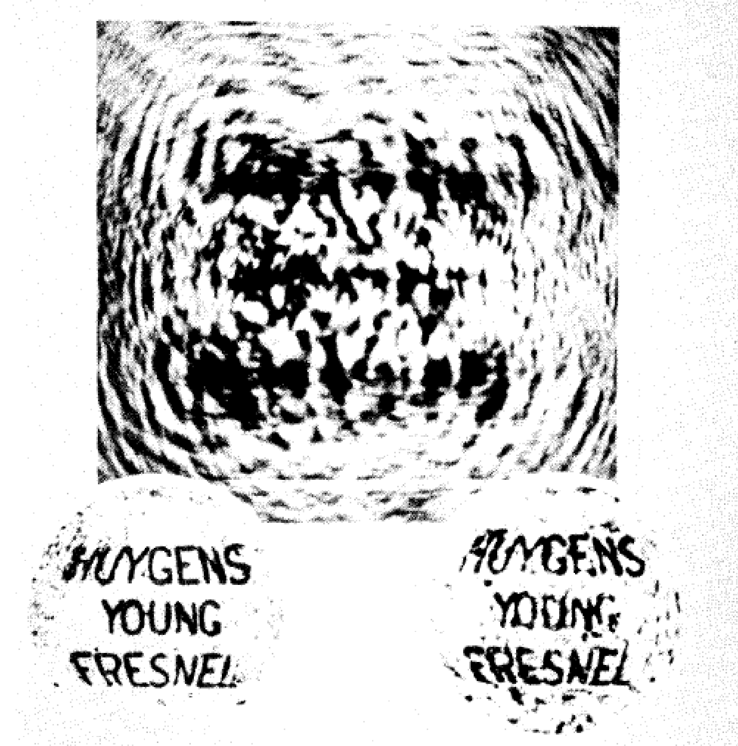


Fig 4.
First Holographic Reconstruction, 1948

Basic idea of holography (1947): amplitude level reconstruction.
First hologram (1948) from D. Gabor's Nobel lecture (1967).
https://www.nobelprize.org/uploads/2018/06/gabor-lecture.pdf

Formalism: elastic scattering

$$\sigma_{el}(s) = \int_0^\infty d|t| \frac{d\sigma(s)}{dt}$$

$$\boxed{\frac{d\sigma(s)}{dt} = \frac{1}{4\pi} |T_{el}(s,\Delta)|^2 , \qquad \Delta = \sqrt{|t|} .}$$

$$B(s,t) = \frac{d}{dt} \ln \frac{d\sigma(s)}{dt}$$

$$B(s) \equiv B_0(s) \; = \; \lim_{t\to 0} B(s,t),$$

$$\sigma_{\mathrm{tot}}(s) \equiv 2\,\mathrm{Im}\,T_{el}(\Delta = 0, s)$$

$$\rho(s,t) \equiv \frac{\mathrm{Re}\,T_{el}(s,\Delta)}{\mathrm{Im}\,T_{el}(s,\Delta)}$$

$$\rho(s) \equiv \rho_0(s) \; = \; \lim_{t\to 0} \rho(s,t)$$

Basic problem: dσ/dt measures an amplitude, *modulus squared*.
How to achieve amplitude level reconstruction? Phase info lost…

MODEL INDEPENDENT LEVY EXPANSION

$$\frac{d\sigma}{dt} = A\, w(z|\alpha) \left| 1 + \sum_{j=1}^{\infty} c_j l_j(z|\alpha) \right|^2 ,$$

$$w(z|\alpha) = \exp(-z^\alpha) ,$$

non-exponential behavior (NEB) in a single parameter

$$z = |t| R^2 \ge 0 ,$$

α

idea: complete set of orthonormal functions, put NEB to the weight

$$c_j = a_j + i b_j ,$$

$$l_j(z\,|\,\alpha) = D_j^{-\frac{1}{2}} D_{j+1}^{-\frac{1}{2}} L_j(z\,|\,\alpha),$$

$$D_0(\alpha) = 1,$$

$$D_1(\alpha) = \mu_{0,\alpha},$$

$$D_2(\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} \end{pmatrix},$$

$$D_3(\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ \mu_{2,\alpha} & \mu_{3,\alpha} & \mu_{4,\alpha} \end{pmatrix},$$

$$\int_0^\infty dz \exp(-z^\alpha) l_n(z\,|\,\alpha) l_m(z\,|\,\alpha) = \delta_{n,m}$$

$$\mu_{n,\alpha} = \int_0^\infty dz\; z^n \exp(-z^\alpha) = \frac{1}{\alpha} \Gamma\left(\frac{n+1}{\alpha}\right)$$

T. Csörgő, R. Pasechnik, A. Ster, arxiv.org:1807.02897

$$L_0(z\,|\,\alpha) = 1,$$

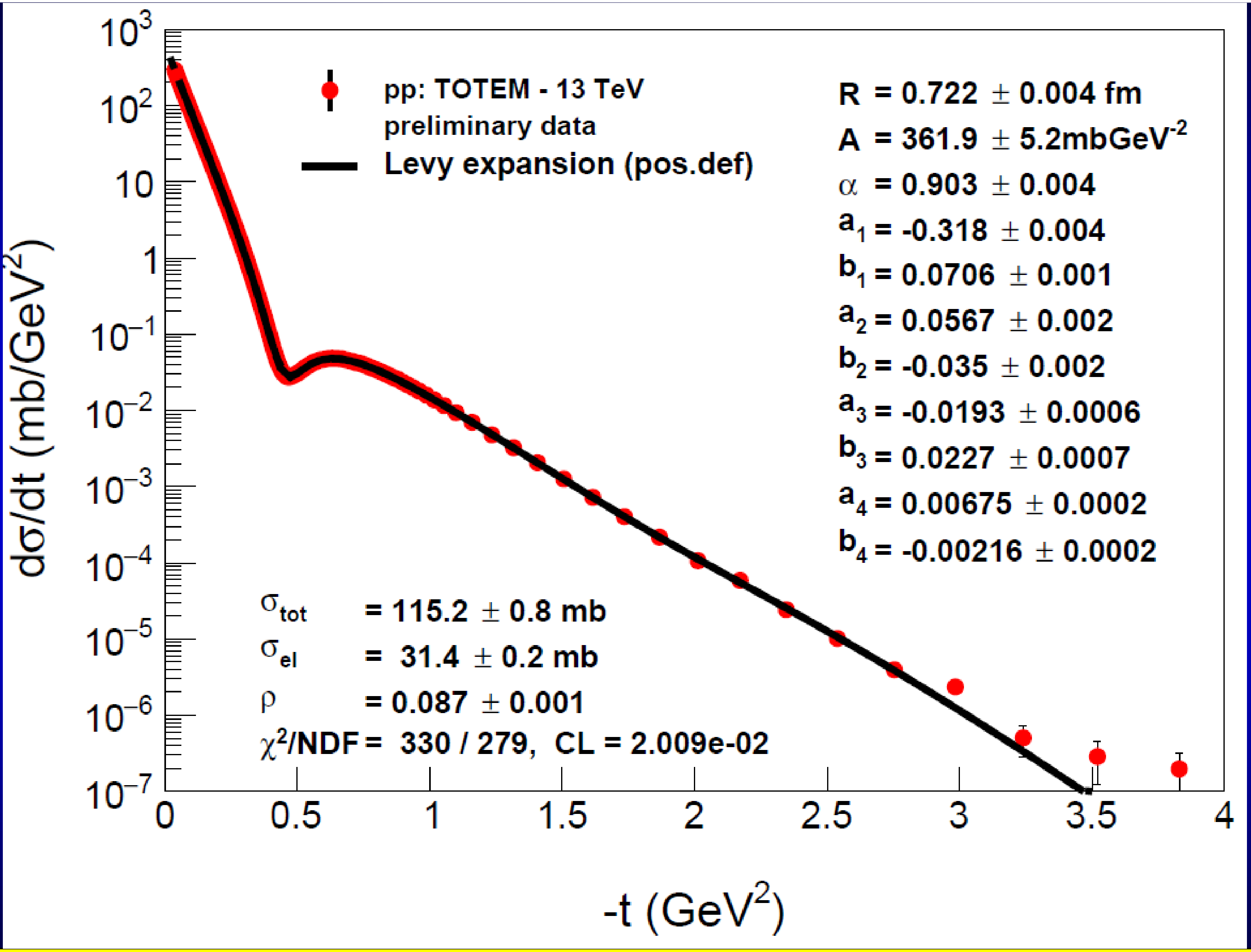
$$L_1(z\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & z \end{pmatrix},$$

$$L_2(z\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & z & z^2 \end{pmatrix},$$

$$L_3(z\,|\,\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} & \mu_{4,\alpha} \\ \mu_{2,\alpha} & \mu_{3,\alpha} & \mu_{4,\alpha} & \mu_{5,\alpha} \\ 1 & z & z^2 & z^3 \end{pmatrix},$$

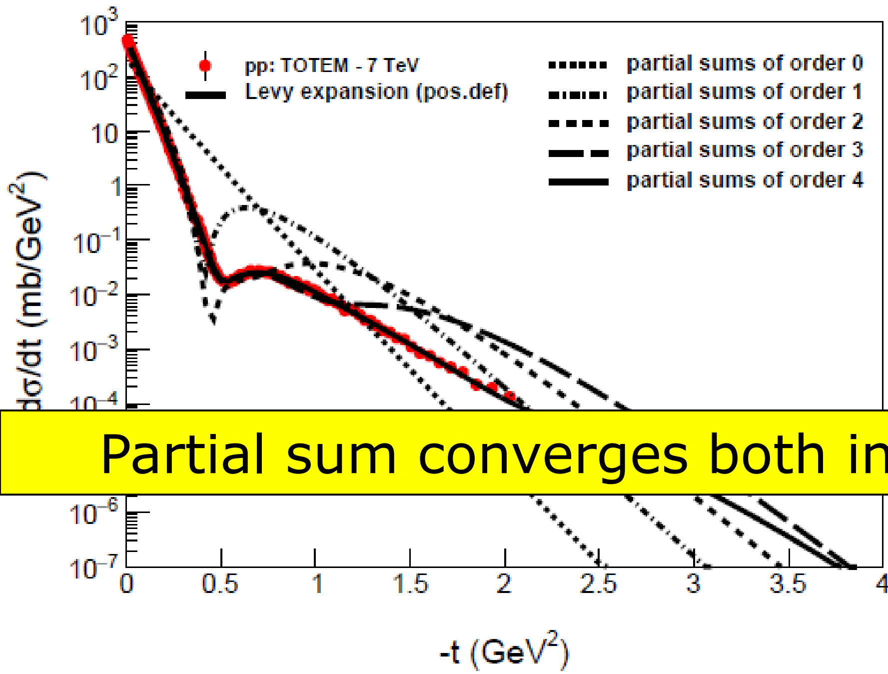
Levy series ~ Taylor series: orthonormal wrt $w(z) = \exp(-z^\alpha)$. T. Cs., R. Pasechnik, A. Ster, arXiv:1807.02897, arXiv:1811.08913, arXiv:1902.00109, arXiv:1903.08235

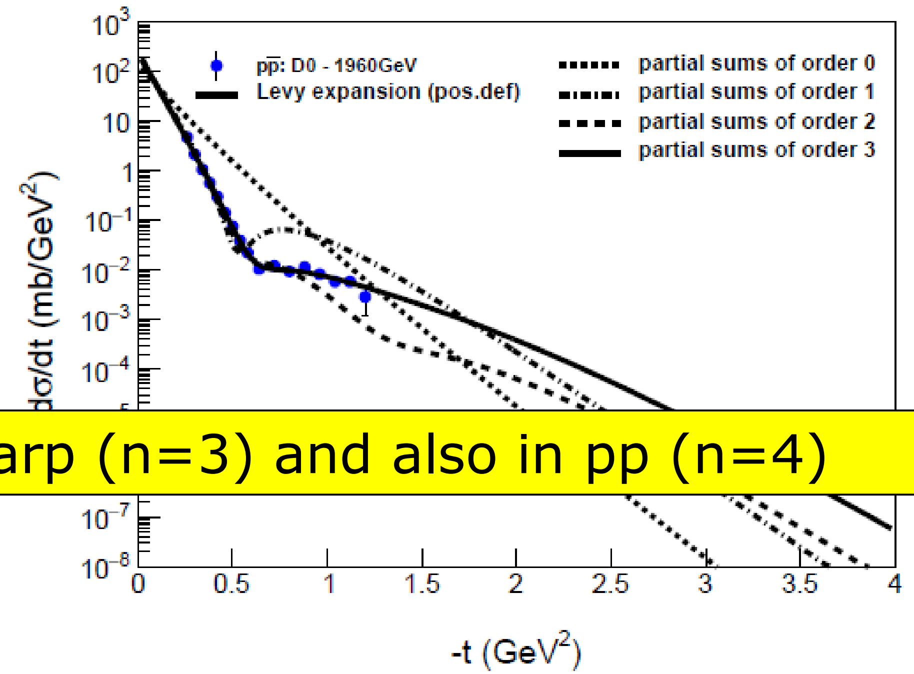
ABILITIES: CONVERGES TO pp dσ/dt @ 13 TeV



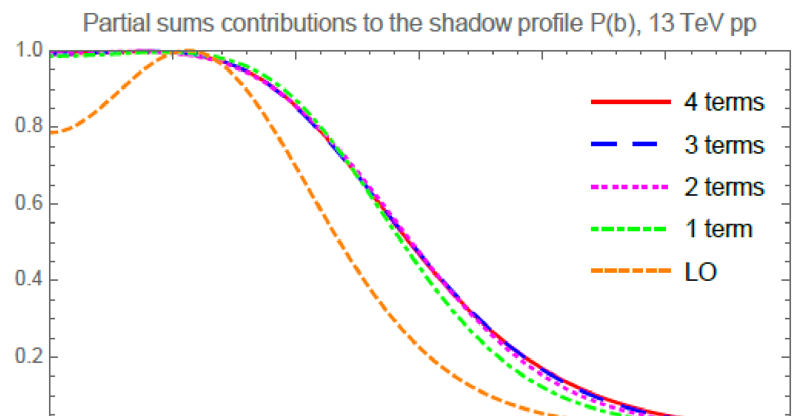
Levy series ~ Taylor series: orthonormal wrt $w(z) = \exp(-z^{\alpha})$. T. Cs., R. Pasechnik, A. Ster, arXiv:1807.02897, arXiv:1811.08913, arXiv:1902.00109, arXiv:1903.08235

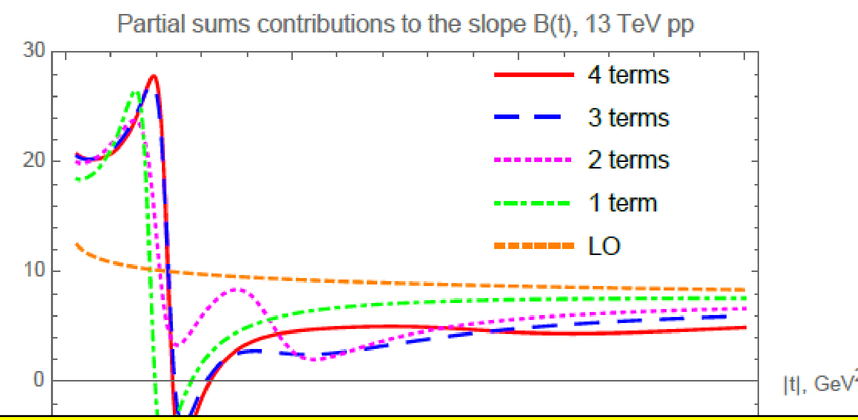
CONVERGENCE PROPERTIES OF LEVY SERIES





Partial sum converges both in pbarp (n=3) and also in pp (n=4)

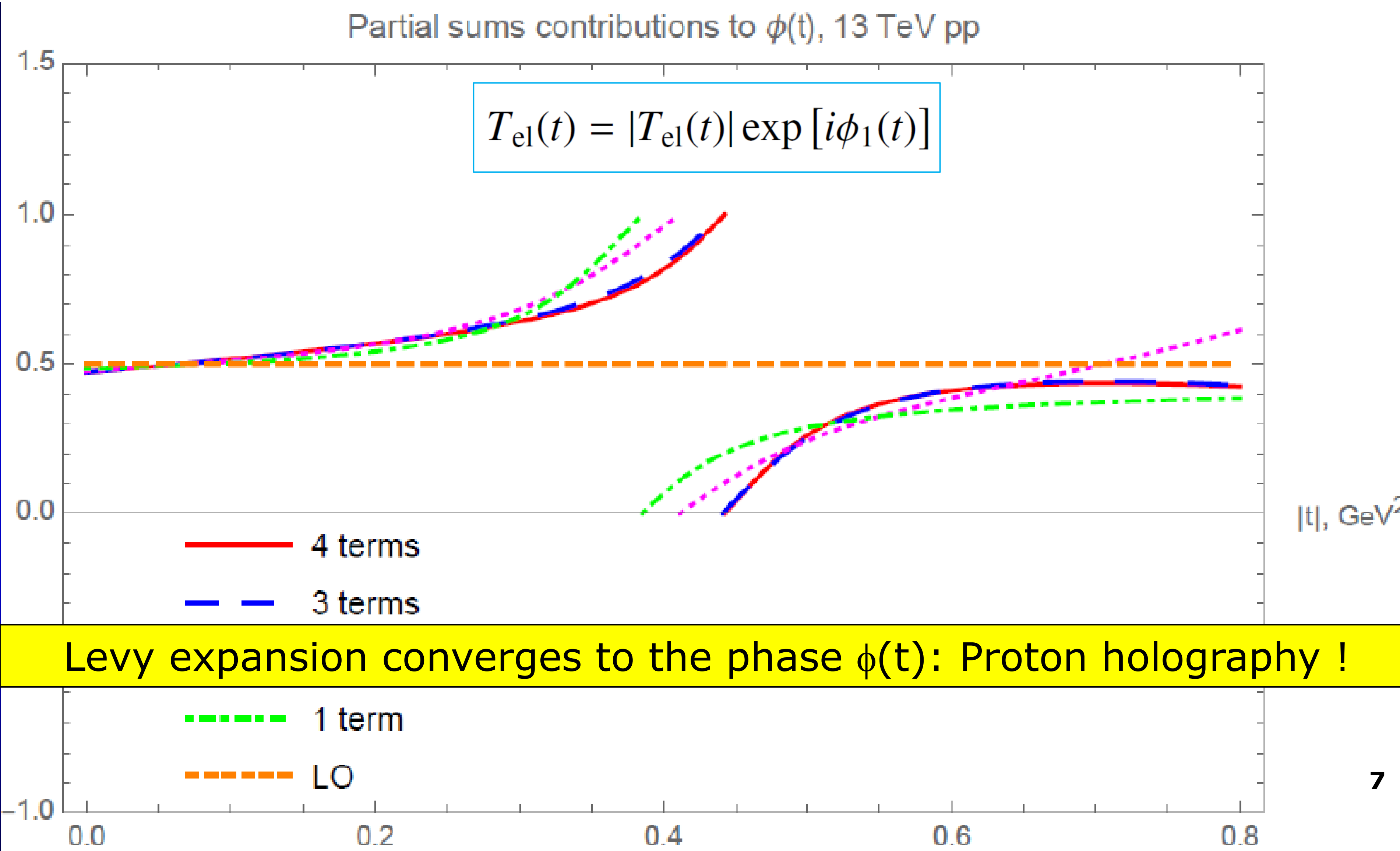




Partial sum converges to profile function P(b) and slope B(t)

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235

CONVERGENCE OF PHASE RECONSTRUCTION



Levy expansion converges to the phase $\phi(t)$: Proton holography !

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235

Scaling in the region of diffractive dip

$$\begin{aligned} \varepsilon &= \frac{t_d - t_0}{t_d} \ll 1 \\ R_d &= \Re T_{el}(t = t_d) \\ I_d &= \Im T_{el}(t = t_d) \\ R'_d &= \frac{d}{dt}\Re T_{el}(t = t_d) \\ I'_d &= \frac{d}{dt}\Im T_{el}(t = t_d) \end{aligned}$$

$$\begin{aligned} t_0 &= t_d(1-\varepsilon) \\ I_d &= \varepsilon R_d \\ R'_d &= -\varepsilon I'_d \\ R(t) &= \Re T_{el}(t) \simeq R_d - \varepsilon I'_d(t - t_d) \\ I(t) &= \Im T_{el}(t) \simeq \varepsilon R_d + I'_d(t - t_d) \end{aligned}$$

$$\rho(t) \equiv \frac{\mathrm{Re}\, T_{el}(\Delta)}{\mathrm{Im}\, T_{el}(\Delta)} = - \left. \frac{\sum_{i=1}^{\infty} b_i l_i(z|\alpha)}{1 + \sum_{i=1}^{\infty} a_i l_i(z|\alpha)} \right|_{z=tR^2}$$

$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{1}{4\pi}(1+\varepsilon^2)\left[R_d^2 + I'_d(t - t_d)^2\right] \\ \varepsilon &= \frac{1}{\rho(t = t_d)} = \frac{I_d}{R_d} \end{aligned}$$

Linear Taylor series around the dip position $t_d = t(\mathrm{dip})$
Information on the ratio of imaginary to real part of amplitude
$\rho(t_d) = 1/\varepsilon$ connects to $\rho_0 = \rho(t=0)$. CNI measures ρ_0 .
CNI: Coulomb-Nuclear Interference.

Odderon search: a possible strategy

Our research strategy in this paper is to try to scale out the s-dependence of the differential cross-section by scaling out its dependencies on $\sigma_{tot}(s)$, $\sigma_{el}(s)$, $B(s)$ and $\rho(s)$. The residual scaling functions will be compared for proton-proton and proton-antiproton elastic scattering to see if any difference remains. Such residual difference is a clear as a signal for Odderon-exchange, if the differential cross-sections were measured at exactly the same energies. However, currently such data are lacking. So we may expect that after scaling out the trivial s-dependences, only small scaling violating terms remain that depends on s, which can be estimated by the scaling violations of differential cross-sections measured at various nearby energies. If we see larger differences between the scaling functions of proton-proton and proton-antiproton collisions as compared to the s-dependent scaling violating term, that will be an indication for the Odderon effect.

Odderon: L. Lukaszuk, B. Nicolescu, Lett. Nuovo Cim. 8, 405 (1973)

Known trivial s-dependences in
$\sigma_{tot}(s)$, $\sigma_{el}(s)$, $B(s)$, $\rho(s)$

Try to scale this out
Data collapsing (scaling)

Look for scaling violations

Looking for Odderon effects

$$
\begin{aligned}
T_{el}^{pp}(s,t) &= T_{el}^{+}(s,t) + T_{el}^{-}(s,t),\\
T_{el}^{p\bar{p}}(s,t) &= T_{el}^{+}(s,t) - T_{el}^{-}(s,t),\\
T_{el}^{+}(s,t) &= T_{el}^{P}(s,t) + T_{el}^{f}(s,t),\\
T_{el}^{-}(s,t) &= T_{el}^{O}(s,t) + T_{el}^{\omega}(s,t).
\end{aligned}
$$

$$
\begin{aligned}
T_{el}^{P}(s,t) &= \frac{1}{2}\left(T_{el}^{pp}(s,t) + T_{el}^{p\bar{p}}(s,t)\right) \quad \text{for } \sqrt{s} \geq 1 \text{ TeV},\\
T_{el}^{O}(s,t) &= \frac{1}{2}\left(T_{el}^{pp}(s,t) - T_{el}^{p\bar{p}}(s,t)\right) \quad \text{for } \sqrt{s} \geq 1 \text{ TeV}.
\end{aligned}
$$

Three simple consequences:

$$T_{el}^{O}(s,t) = 0 \implies \frac{d\sigma^{pp}}{dt} = \frac{d\sigma^{p\bar{p}}}{dt} \quad \text{for } \sqrt{s} \geq 1 \text{ TeV}$$

$$\frac{d\sigma^{pp}}{dt} = \frac{d\sigma^{p\bar{p}}}{dt} \quad \text{for } \sqrt{s} \geq 1 \text{ TeV} \;\not\Longrightarrow\; T_{el}^{O}(s,t) = 0.$$

$$\boxed{\frac{d\sigma^{pp}}{dt} \neq \frac{d\sigma^{p\bar{p}}}{dt} \quad \text{for } \sqrt{s} \geq 1 \text{ TeV} \implies T_{el}^{O}(s,t) \neq 0}$$

Odderon differential cross-section from pp and ppbar collisions, Reggeized Philips-Barger:
A. Ster, L. Jenkovszky, T. Cs, arxiv:1501.03860

Scaling in the diffractive cone region

$$\frac{d\sigma}{dt} = A(s)\,\exp\left[B(s)t\right],$$

$$A(s) = B(s)\,\sigma_{\rm el}(s) = \frac{1+\rho_0^2(s)}{16\,\pi}\,\sigma_{\rm tot}^2(s),$$

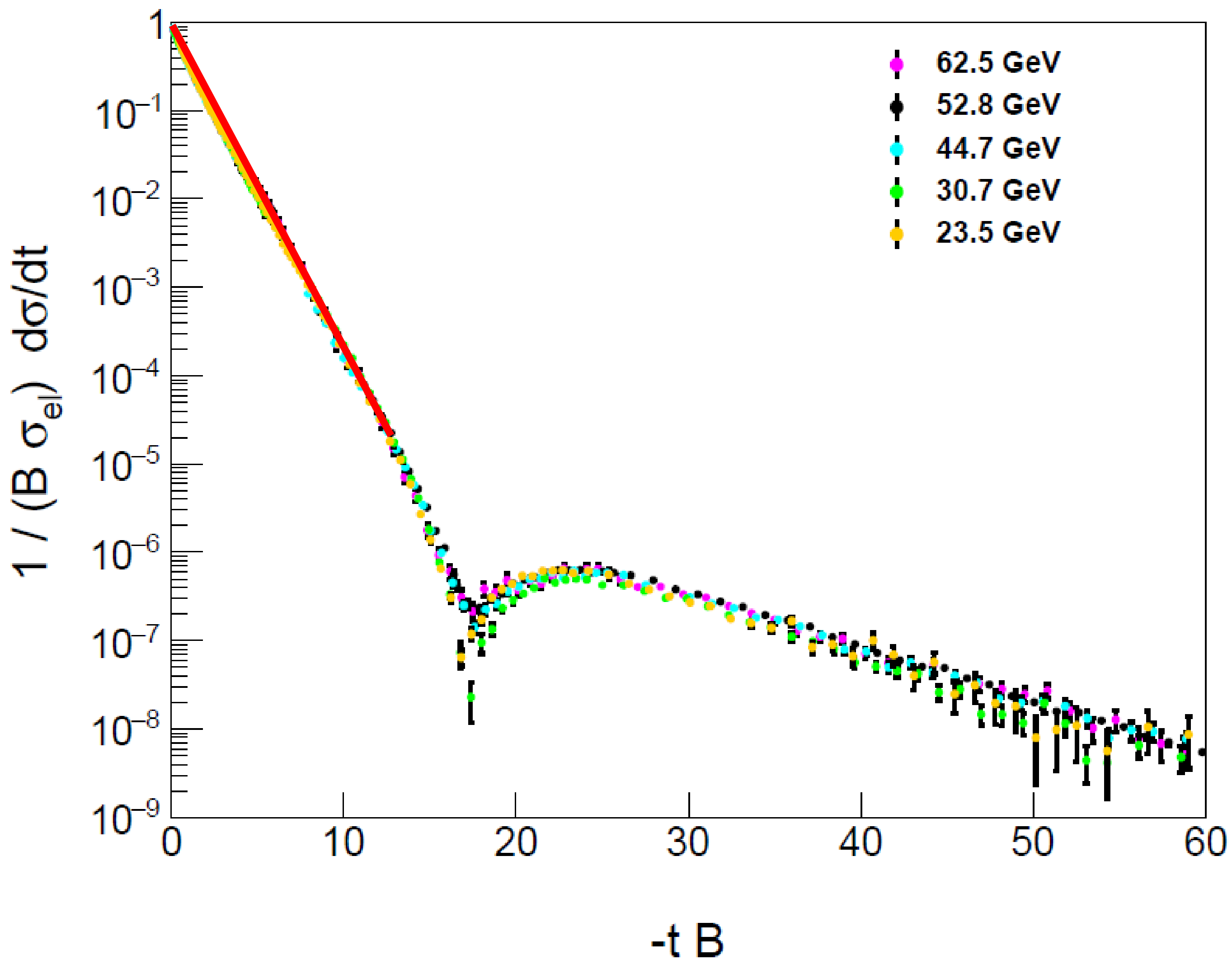
$$B(s) = \frac{1+\rho_0^2(s)}{16\,\pi}\,\frac{\sigma_{\rm tot}^2(s)}{\sigma_{\rm el}(s)}.$$

$$\frac{1}{B(s)\sigma_{el}(s)}\frac{d\sigma}{dt} = \exp\left[-tB(s)\right] \qquad \text{versus} \quad x = -tB(s).$$

$$H(x) = \frac{1}{B(s)\sigma_{el}(s)}\frac{d\sigma}{dt}, \qquad x = -tB(s).$$

Advantages:
H(x) = exp(-x) in the cone
Measurable both for pp and p-antip

Test of the H(x) scaling on ISR data



Energy range: 23.5 – 62.5 GeV (nearly factor of 3)
H(x) works in the cone, shape ~ exp(-x)
H(x) scaling works also in the dip and bump region

H(x) scaling in greater x region

$$t_{el}(s, \mathrm{b}) = (i + \rho_0)\, r(s) E(\tilde{\mathrm{x}}).$$

$$\begin{aligned}
\mathrm{Re}\, \exp\left[-\Omega(s, b)\right] &= 1 - r(s) E(\tilde{\mathrm{x}}), \\
\mathrm{Im}\, \exp\left[-\Omega(s, b)\right] &= \rho_0\, r(s) E(\tilde{\mathrm{x}}), \\
\tilde{\mathrm{x}} &= \mathrm{b}/R(s), \\
R(s) &= \sqrt{B(s)},
\end{aligned}$$

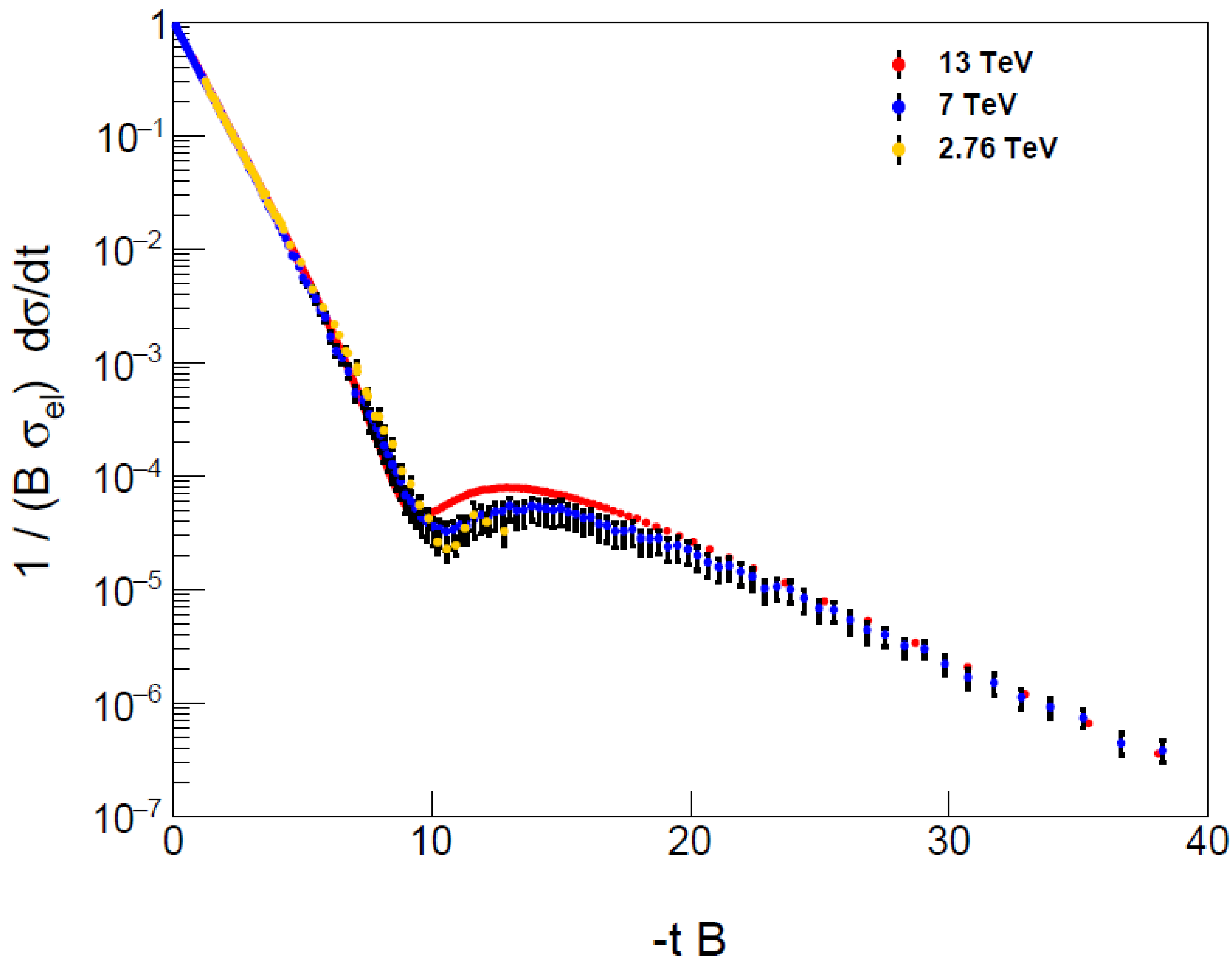
$$\frac{d\sigma}{dt} = \frac{1}{4\pi} |T_{el}(\Delta)|^2 = \frac{1+\rho_0^2}{4\pi} r^2(s) R^2(s) |\tilde{E}(R(s)\Delta)|^2$$

$$A = \left.\frac{d\sigma}{dt}\right|_{t=0} = \frac{1+\rho_0^2}{4\pi} r^2(s) R^2(s) |\tilde{E}(0)|^2,$$

$$\frac{1}{A}\frac{d\sigma}{dt} = \frac{|\tilde{E}(\sqrt{x})|^2}{|\tilde{E}(x=0)|^2} = H(x),$$

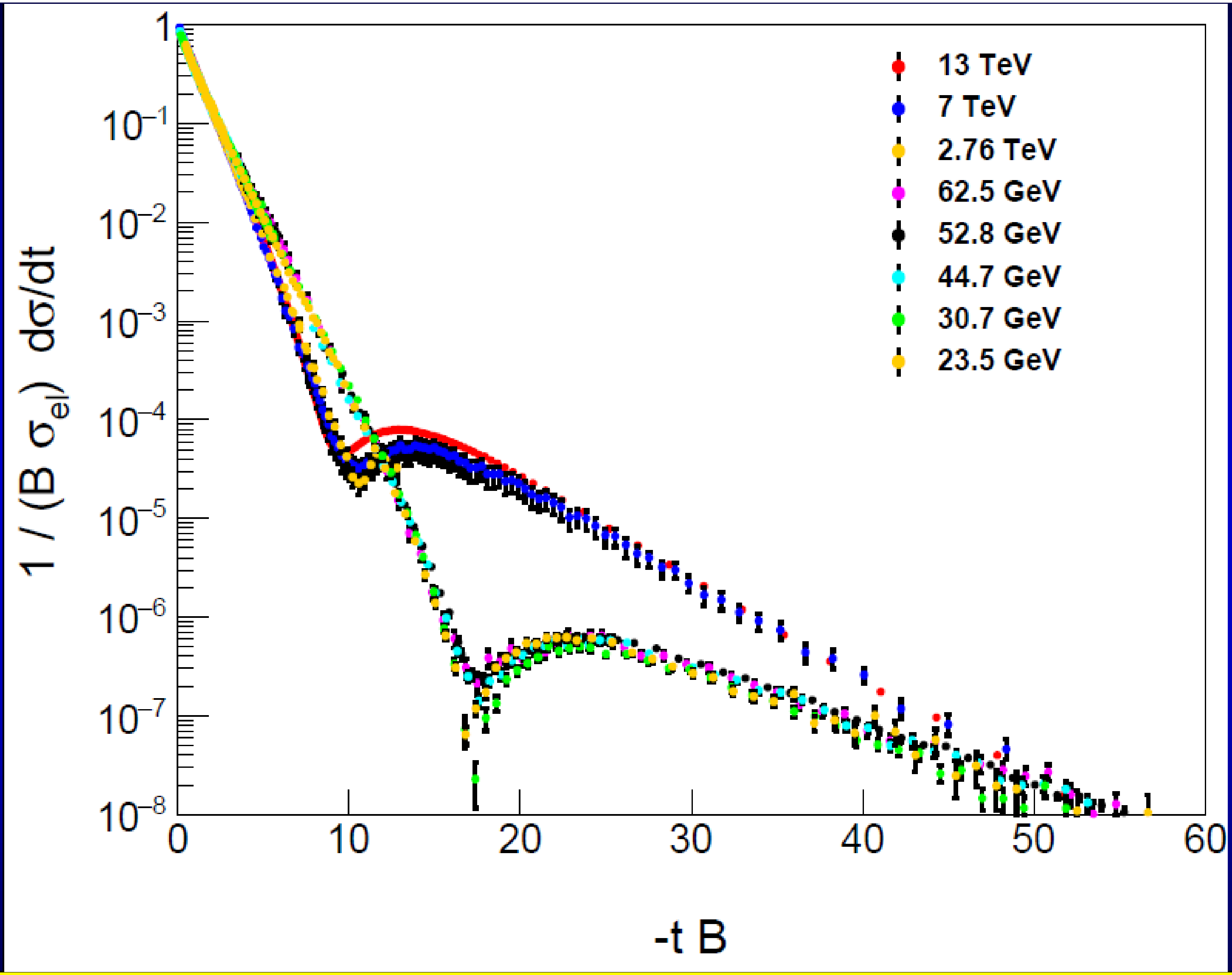
Advantages:
H(x) ≠ exp(-x) arbitrary positive def. in the dip-bump region
Measurable both for pp and p-antip. Normalized as H(0) = 1.

Test of H(x) scaling on LHC pp data



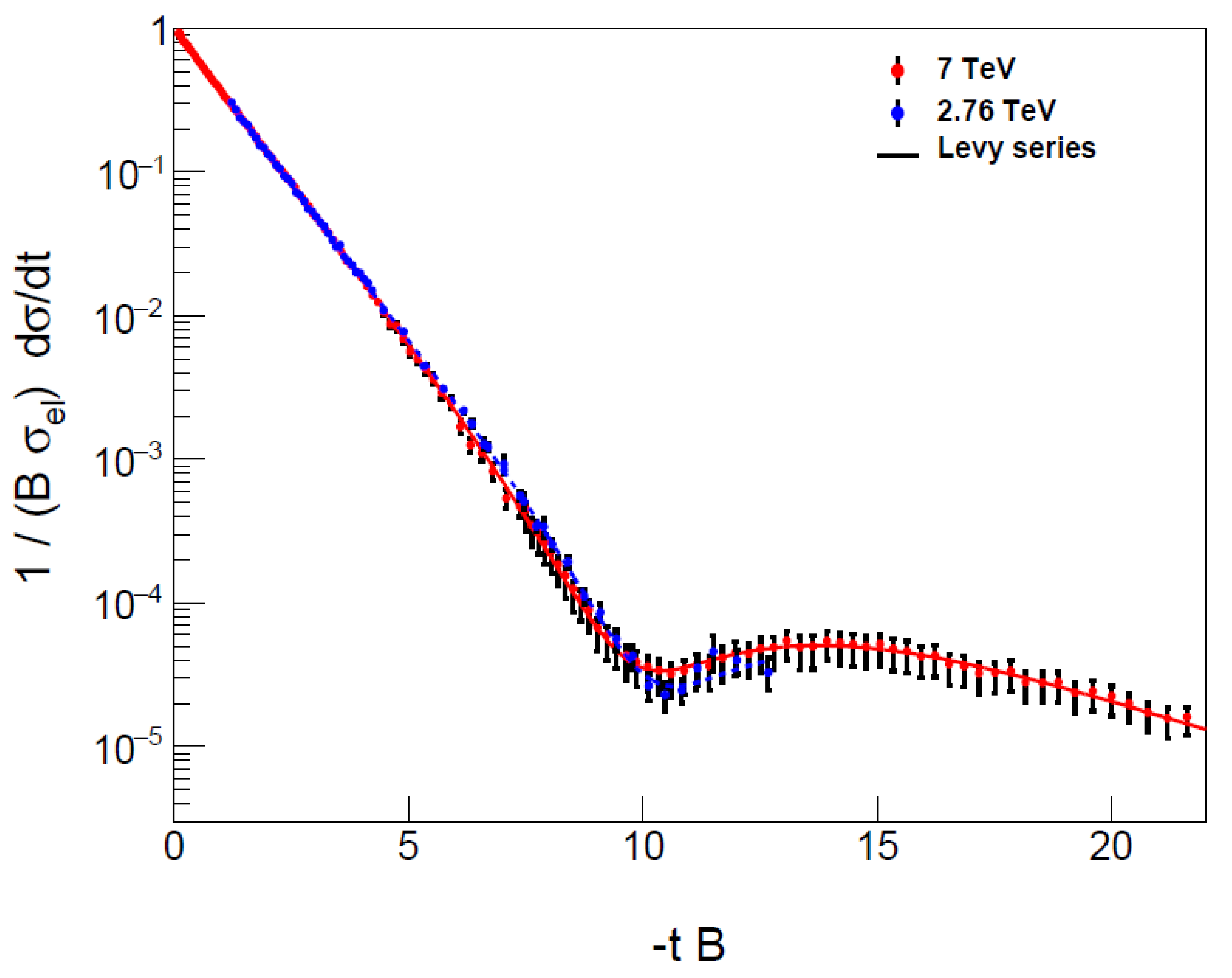
Energy range: 2.76 – 13 TeV (nearly factor of 4)
H(x) works within errors: 2.76 – 7 TeV, scaling violation @13 TeV

H(x) scaling vs LHC + ISR pp data



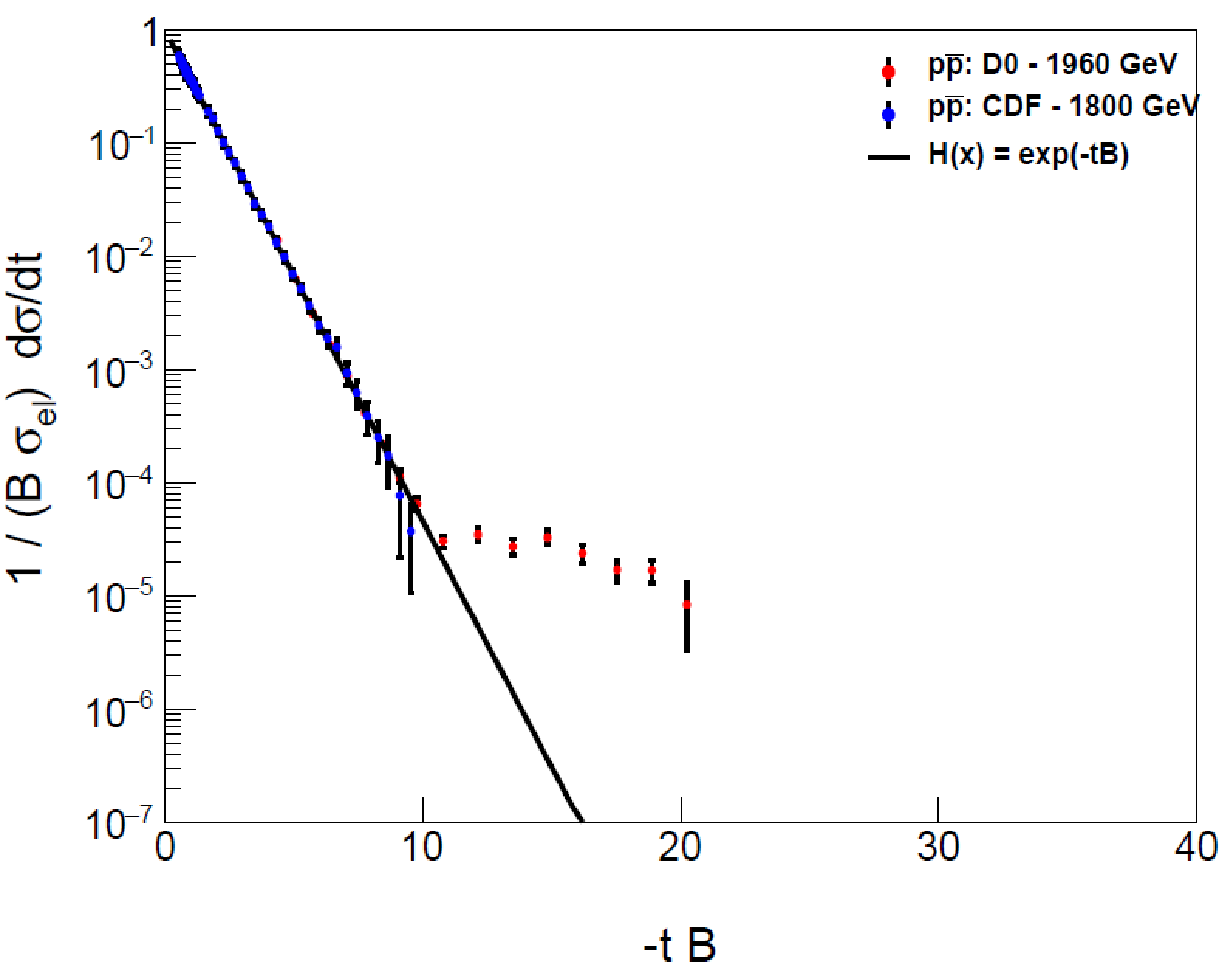
Energy range: 23.5 GeV – 13 TeV (nearly factor of 100)
scaling violating terms are large

H(x) scaling, 2.76 vs 7 TeV pp



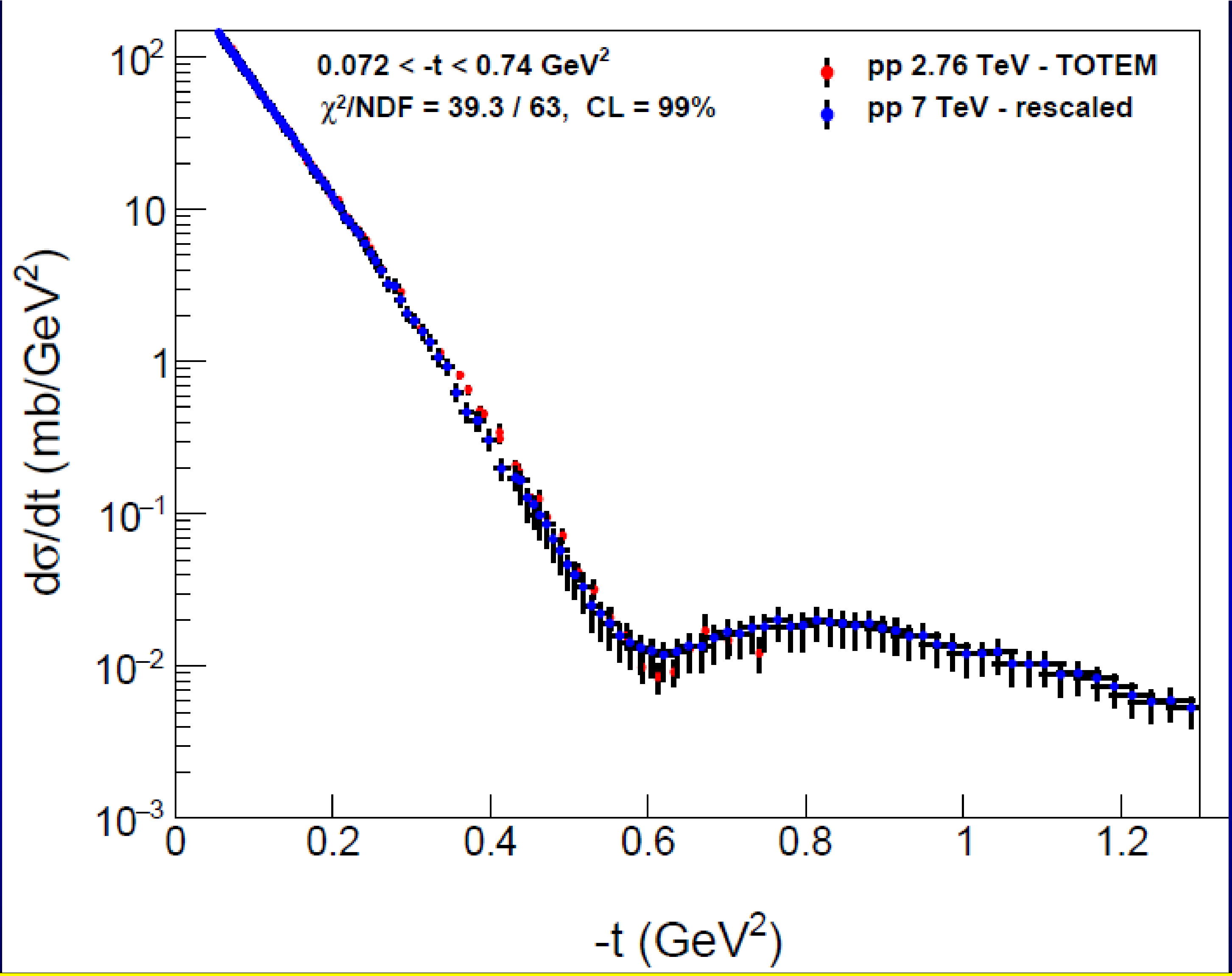
H(x) scaling works for pp in the energy range 2.76 – 7 TeV (factor of 2.5). Levy fits to guide the eye only.

H(x) scaling, 1.96 vs 1.8 TeV pbarp



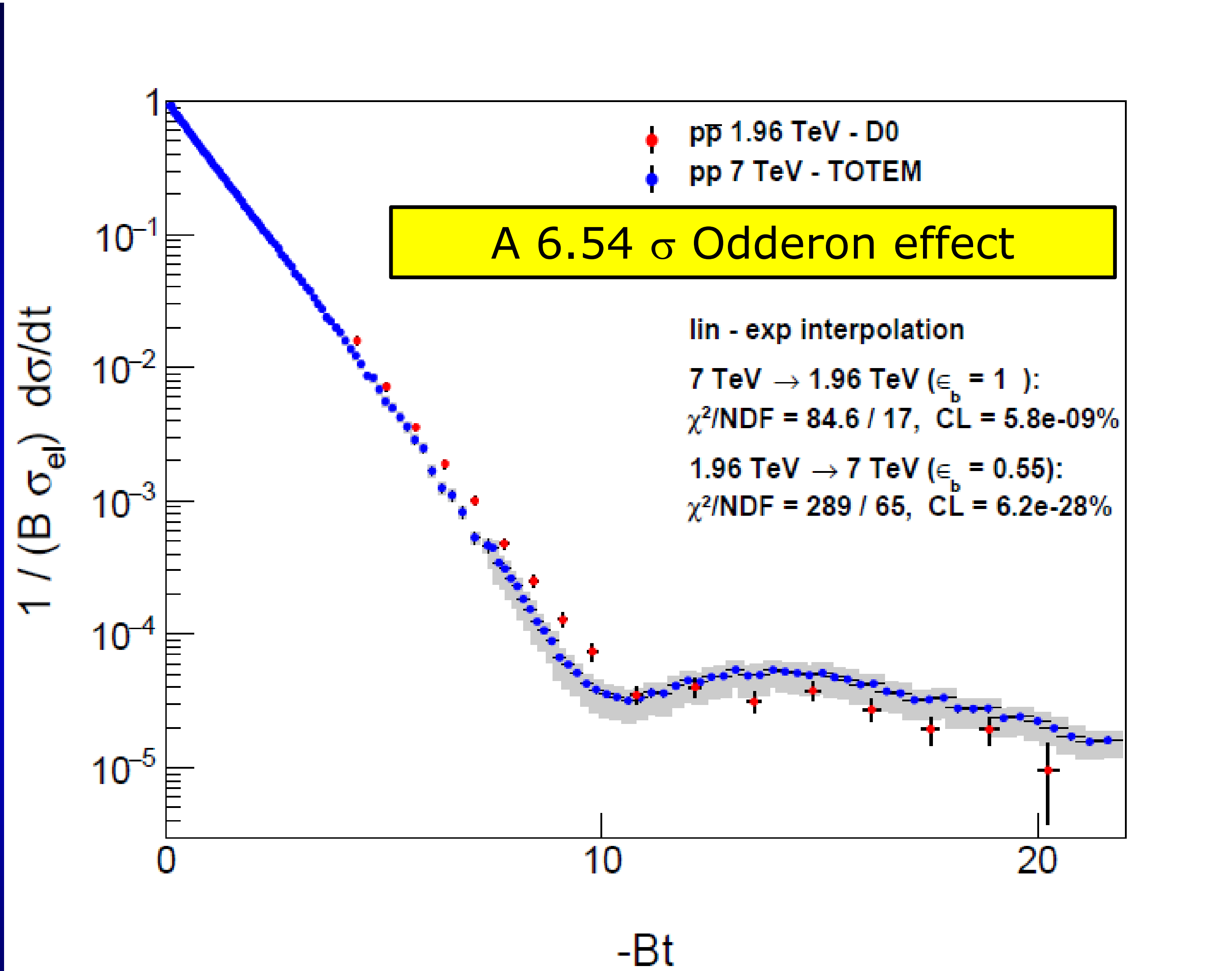
H(x) scaling works for pbarp in the energy range 1.8 – 1.96 TeV.
H(x) = exp(-x) for x ≤ 10.

ADDENDUM: SIGNIFICANCE



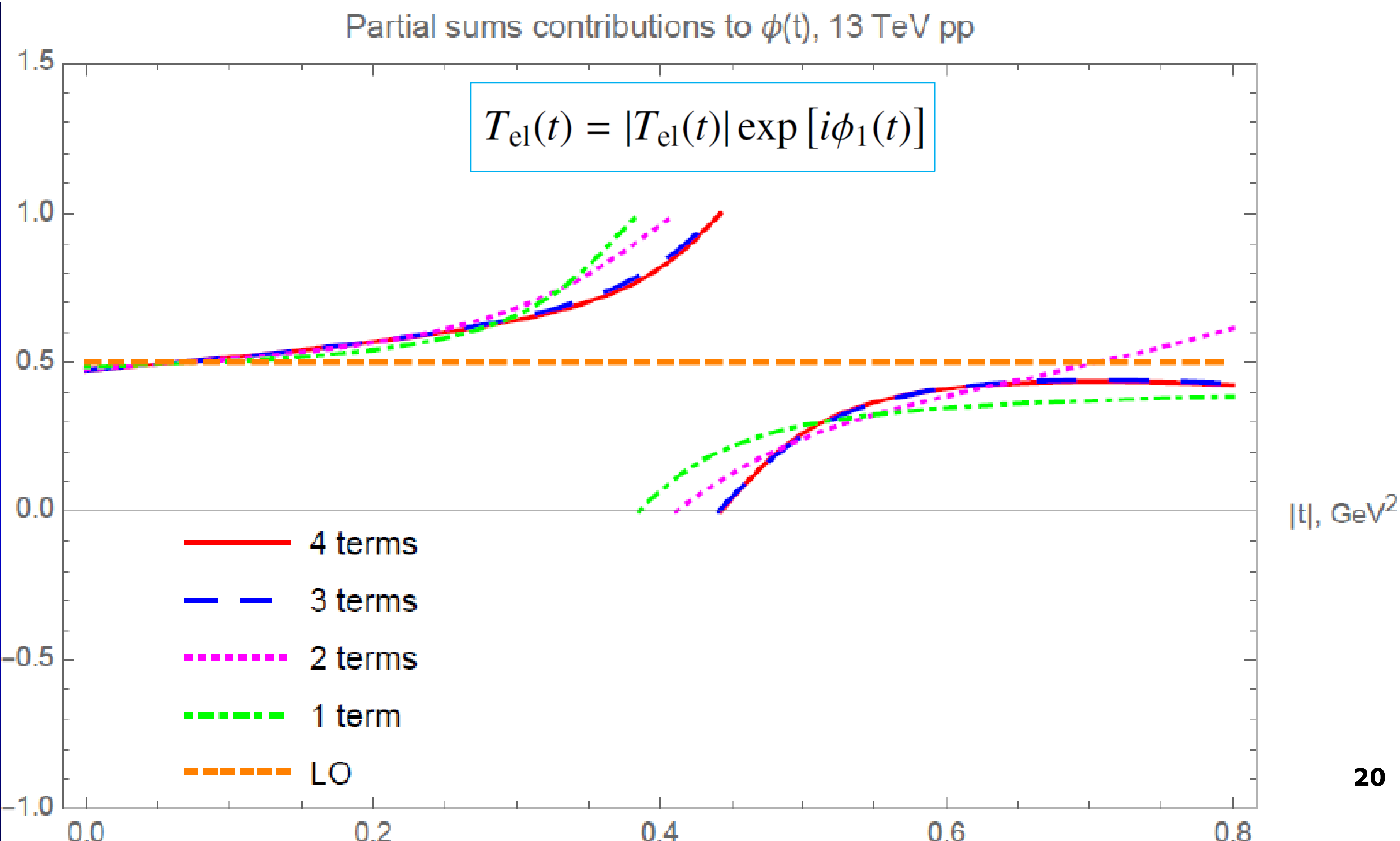
Scaled pp differential cross-section from 7 to 2.76 TeV:
CL = 99%, agreement with data at 2.76 TeV, rescaling works

A SIGNIFICANT ODDERON SIGNAL



Scaled pp differential cross-section from 7 to 1.96 TeV:
CL ≤ 5.8×10^{-9} %: disagreement with pbarp data at 1.96 TeV.
Significance ≥ 6.54 σ : a **significant** Odderon effect. Details: A Ster.

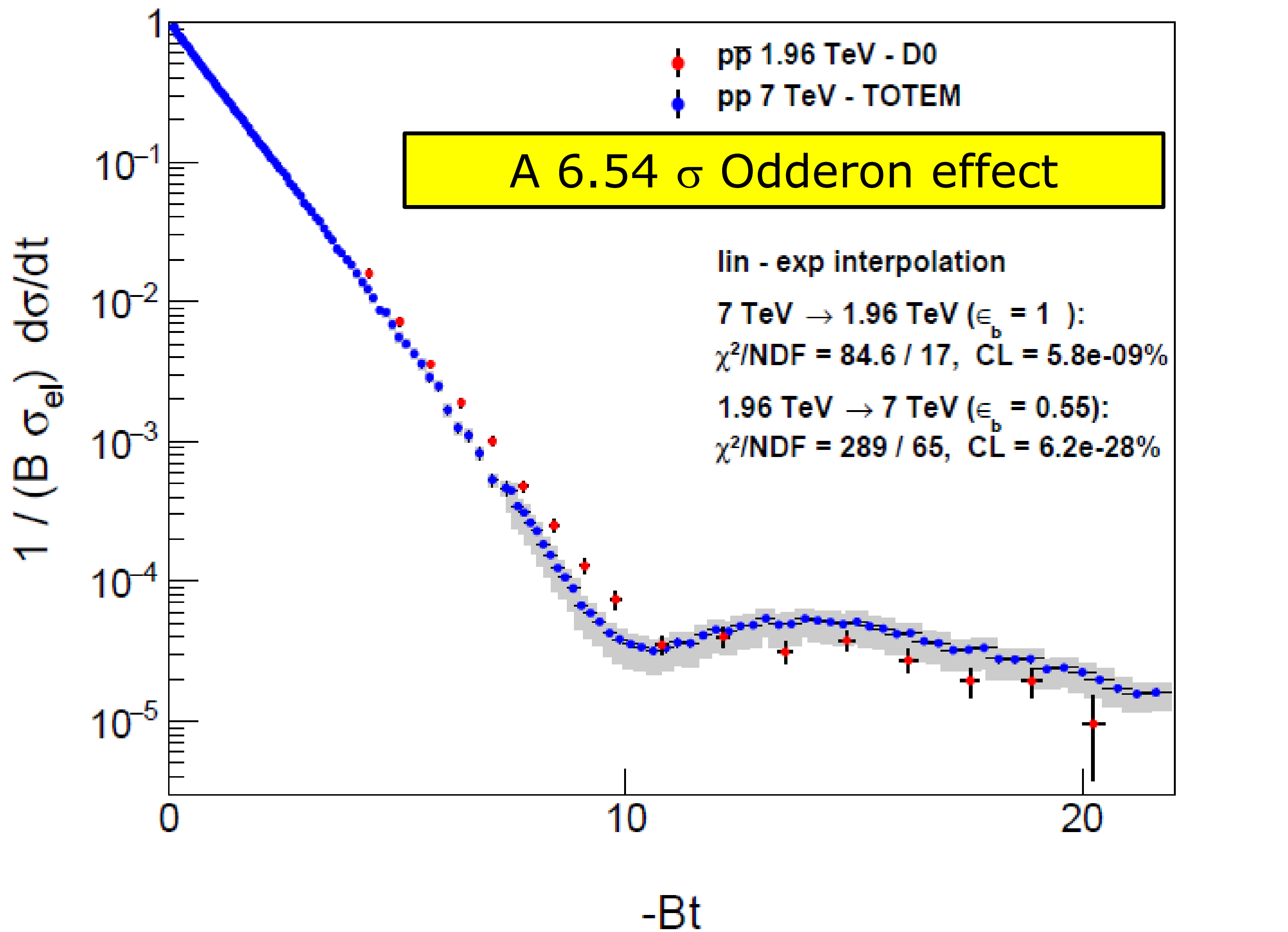
SUMMARY 1: PROTON HOLOGRAPHY



Levy expansion converges to the phase $\phi(t)$: Proton holography !

T. Cs., R. Pasechnik, A. Ster, arXiv:1903.08235

SUMMARY 2: SIGNIFICANT ODDERON



Significance ≥ 6.54 σ : a **significant** Odderon effect.
For more details: see the talk of A Ster.