General relativistic Mass twin stars and their astrophysical aspects

Noshad Khosravi Largani
David Blaschke
David Edwin Alvarez Castillo
Alexander Ayriyan
Hovik Grigorian
Fridolin Weber

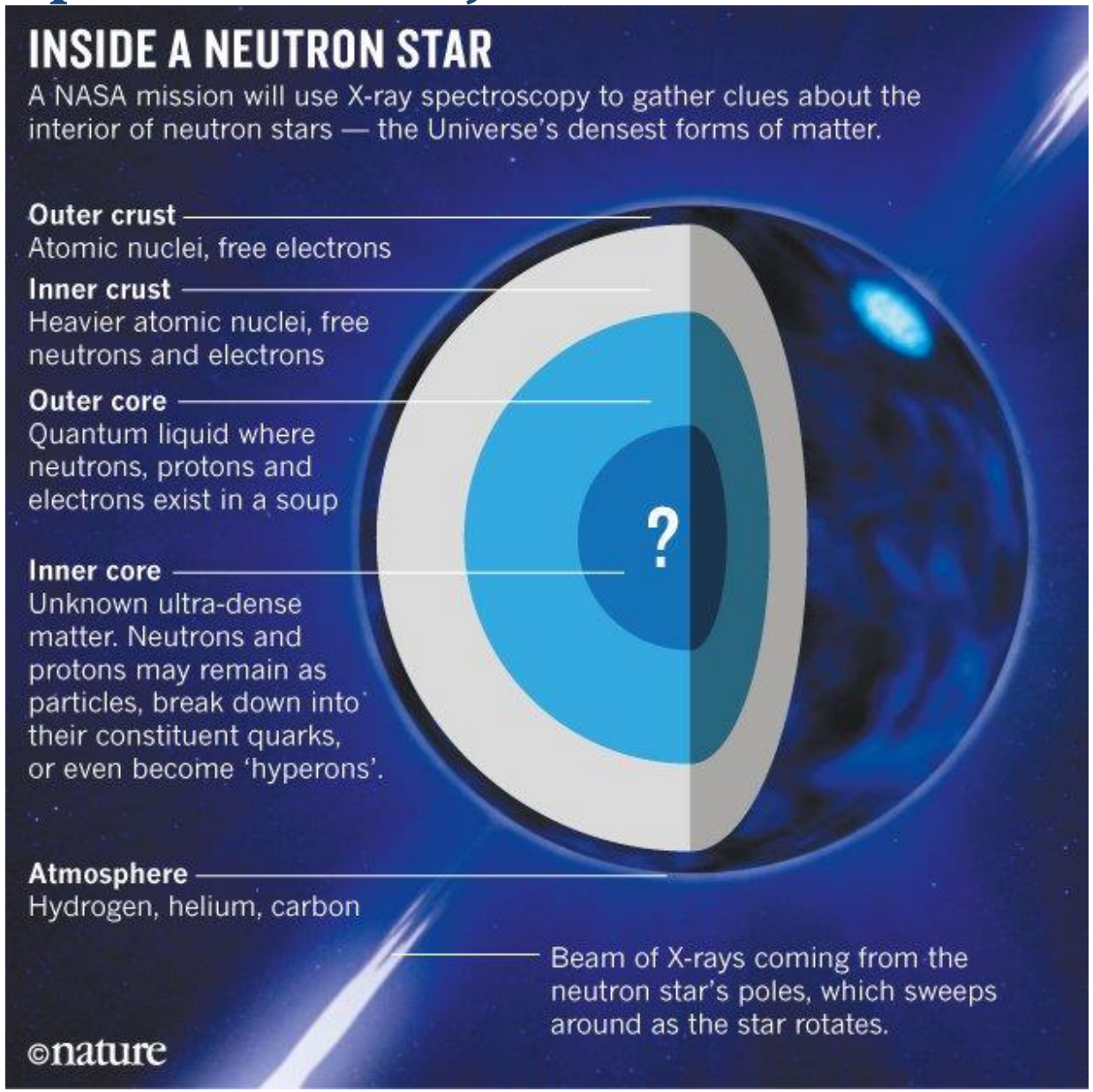
Based on: arXiv:1906.02522 [astro-ph.HE]

Uniwersytet Wrocławski

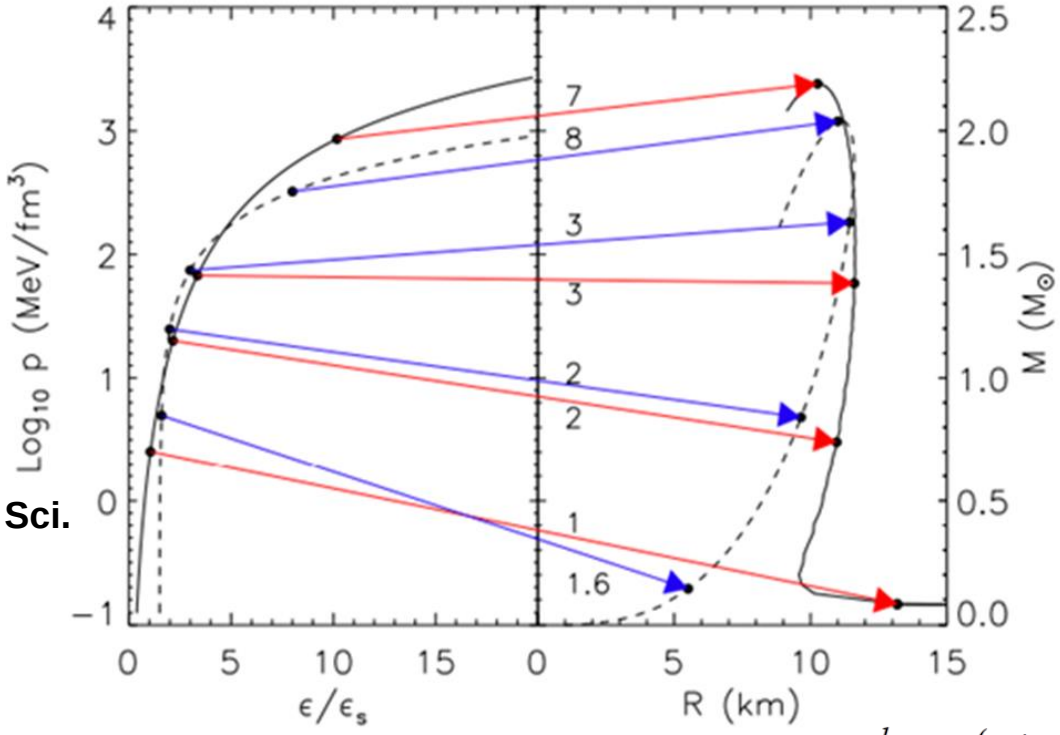
contents

- A brief introduction to the neutron star equation of state and its location within the QCD phase diagram.
- Multi polytrope Equation of state
- The compact star mass twins hypothesis.
- Effects of the Mixed phase (pasta phases)
- Rotation of compact Stars
- Maximum mass constraints.

Superdense objects – what is inside?



Compact Star Sequences
(M-R ⇔ EoS)



Lattimer,
Annu. Rev. Nucl. Part. Sci.
62, 485 (2012)
arXiv: 1305.3510

- TOV Equations

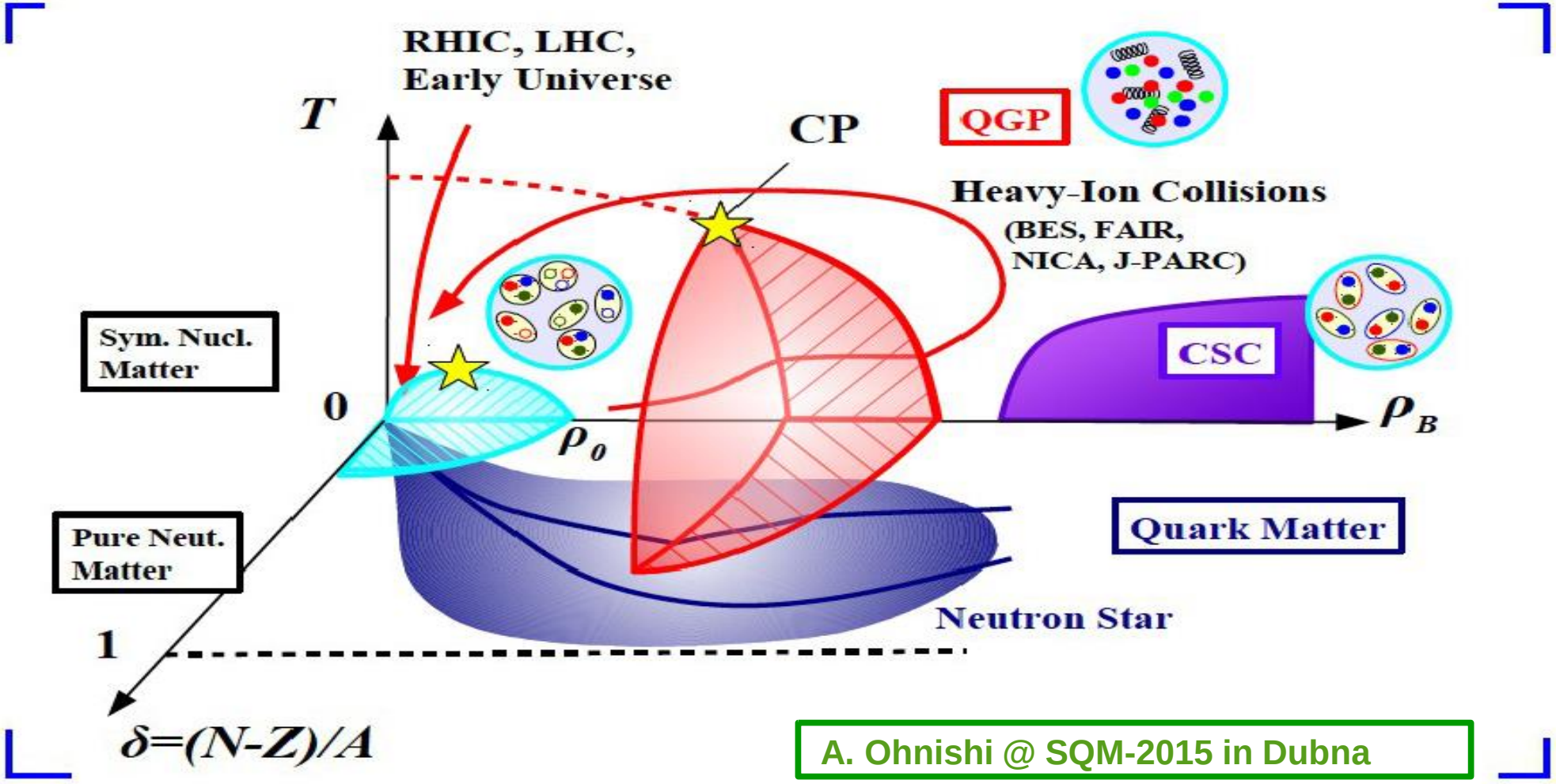
$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

- Equation of State (EoS)

$$p(\varepsilon)$$

Support a CEP in QCD phase diagram with Astrophysics?



A. Ohnishi @ SQM-2015 in Dubna

Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics)
=> Critical endpoint exists!

Compact Star Mass Twins and the AHP scheme

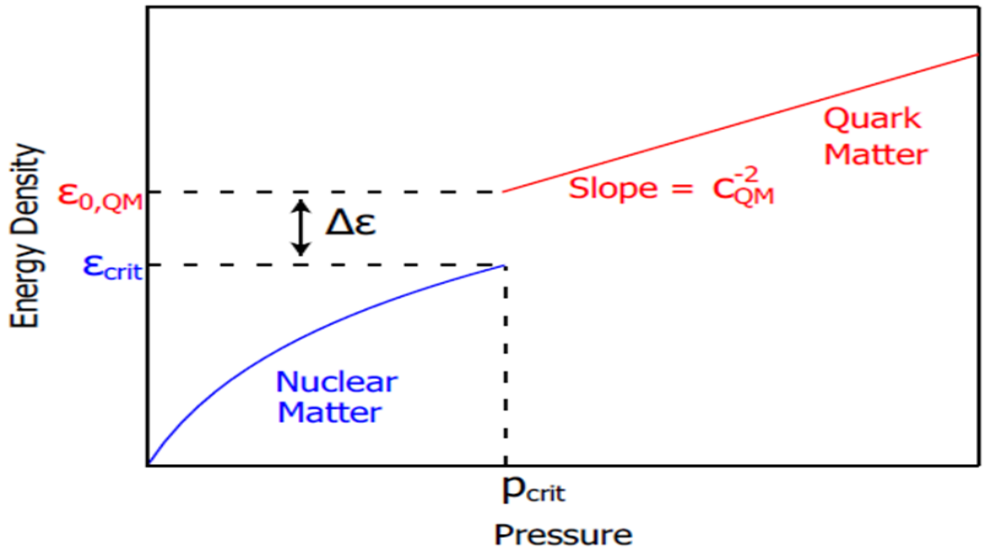
- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the "latent heat" (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → "**third family of CS**".

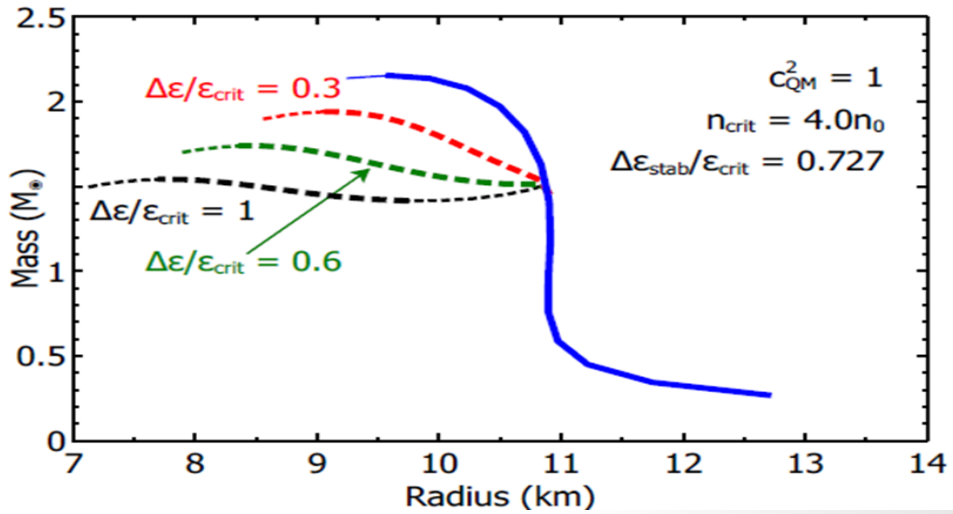
- Measuring two **disconnected populations** of compact stars in the M-R diagram would represent **the detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

Seidov criterion:

$$\frac{\Delta\varepsilon}{\varepsilon_{crit}} \geq \frac{1}{2} + \frac{3}{2}\frac{P_{crit}}{\varepsilon_{crit}}$$

Alford, Han, Prakash,
Phys. Rev. D 88, 083013 (2013)
arxiv:1302.4732



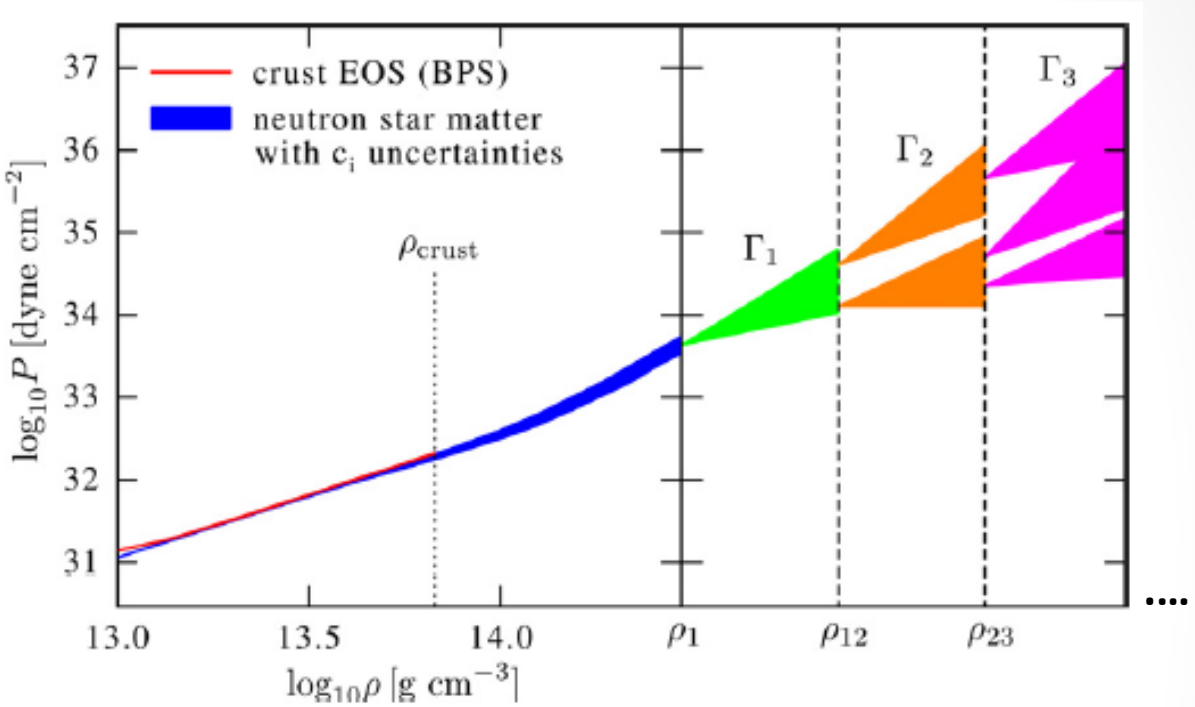


Piecewise polytrope EoS

Hebeler et al., ApJ 773, 11 (2013)

$$P_i(n) = \kappa_i n^{\Gamma_i}$$

$$i = 1 \ : \ n_1 \le n \le n_{12}$$
$$i = 2 \ : \ n_{12} \le n \le n_{23}$$
$$i = 3 \ : \ n \ge n_{23} \ ,$$



....

Here, 1st order PT in region 2:

$$\Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\rm crit}$$

$$P(n) = n^2 \frac{d(\varepsilon(n)/n)}{dn},$$

$$\varepsilon(n)/n = \int dn \, \frac{P(n)}{n^2} = \int dn \, \kappa n^{\Gamma-2} = \frac{\kappa \, n^{\Gamma-1}}{\Gamma - 1} + C,$$

$$\mu(n) = \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa \, \Gamma}{\Gamma - 1} n^{\Gamma-1} + m_0,$$

Seidov criterion for instability: $\dfrac{\Delta\varepsilon}{\varepsilon_{\rm crit}} \ge \dfrac{1}{2} + \dfrac{3}{3}\dfrac{P_{\rm crit}}{\varepsilon_{\rm crit}}$

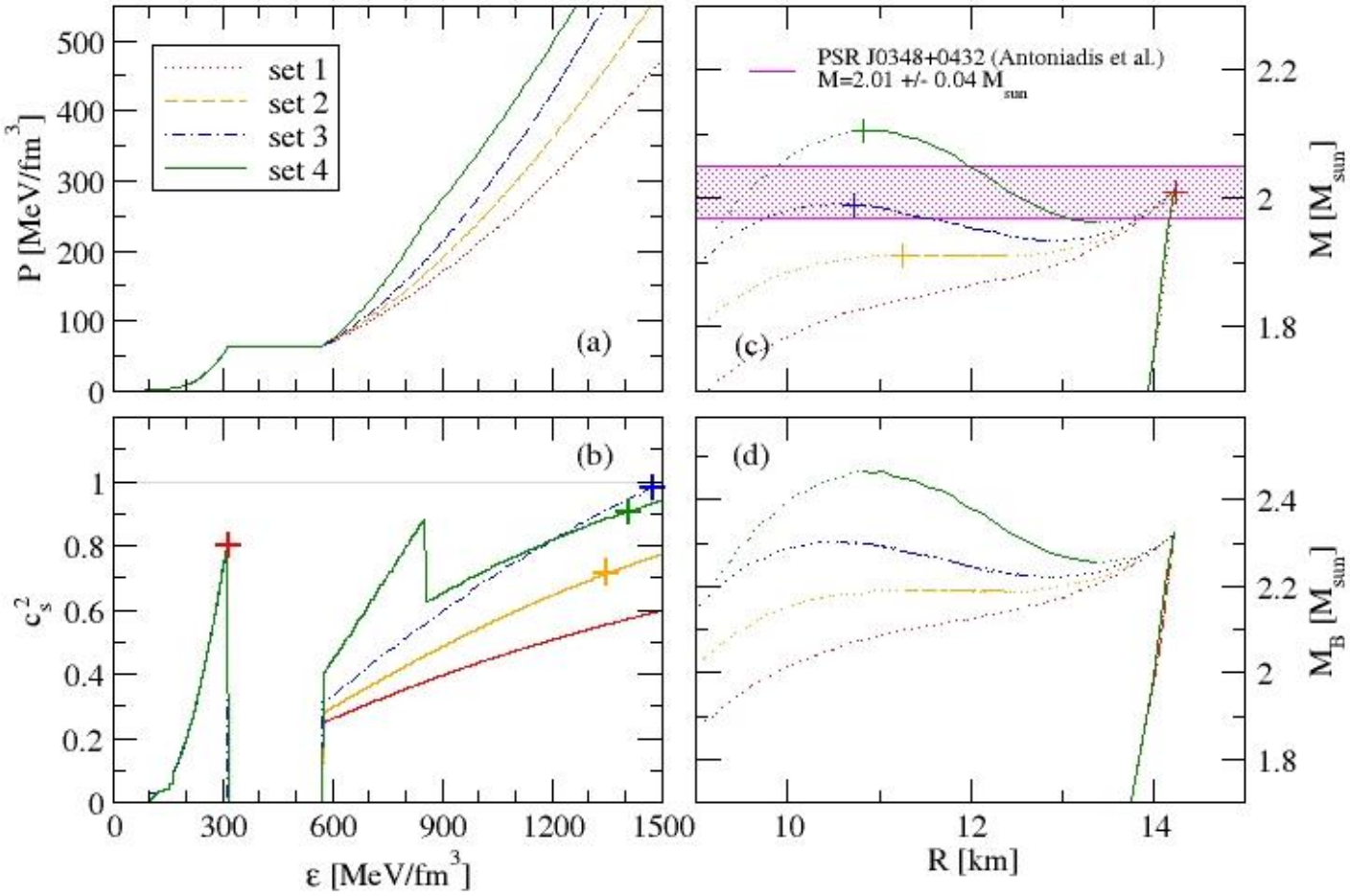
$$n(\mu) = \left[(\mu - m_0)\frac{\Gamma - 1}{\kappa\Gamma}\right]^{1/(\Gamma-1)}$$

$$P(\mu) = \kappa \left[(\mu - m_0)\frac{\Gamma - 1}{\kappa\Gamma}\right]^{\Gamma/(\Gamma-1)}$$

Maxwell construction:

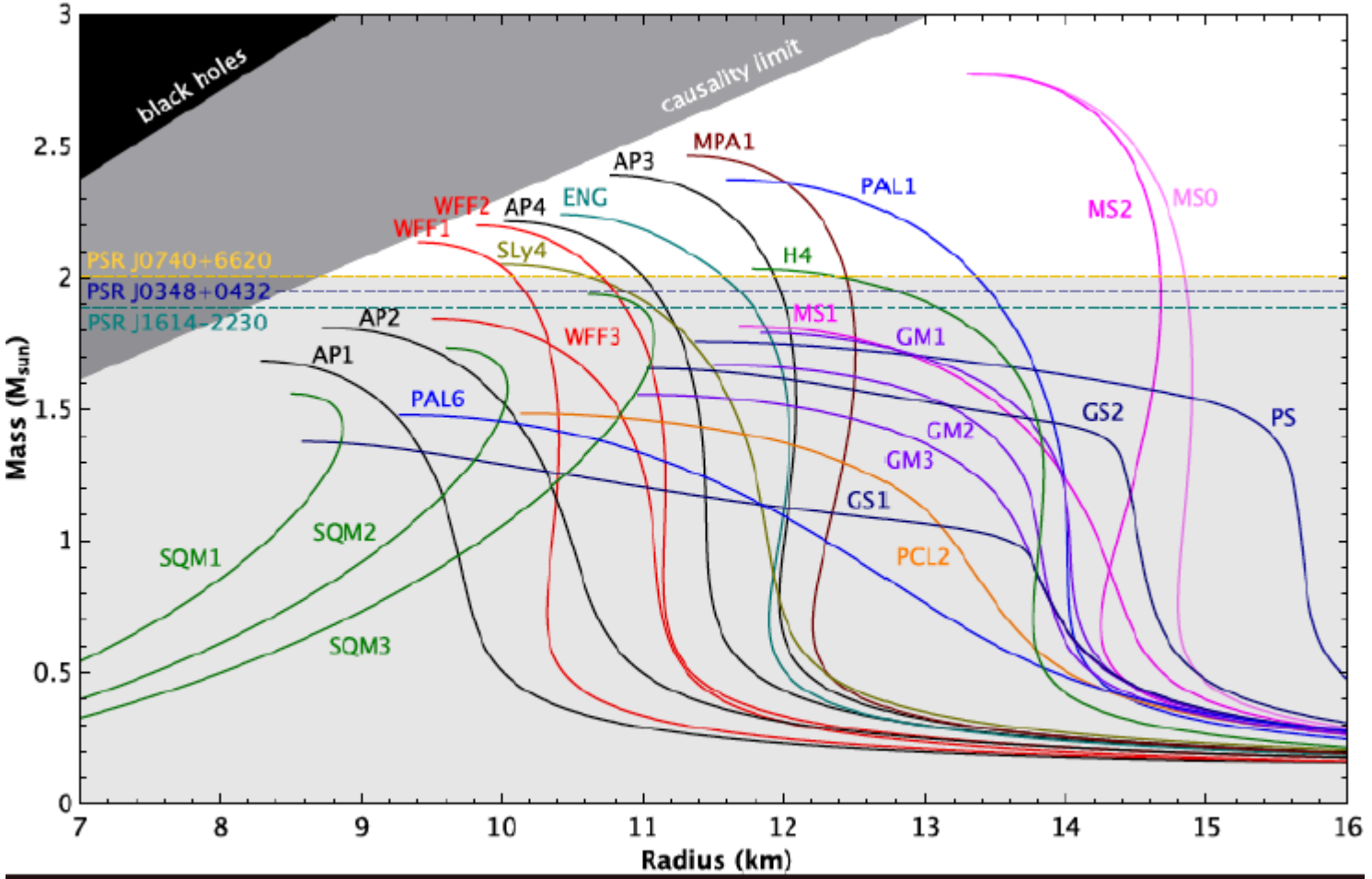
$$P_1(\mu_{\rm crit}) = P_3(\mu_{\rm crit}) = P_{\rm crit}$$
$$\mu_{\rm crit} = \mu_1(n_{12}) = \mu_3(n_{23})$$

Compact Star Twins



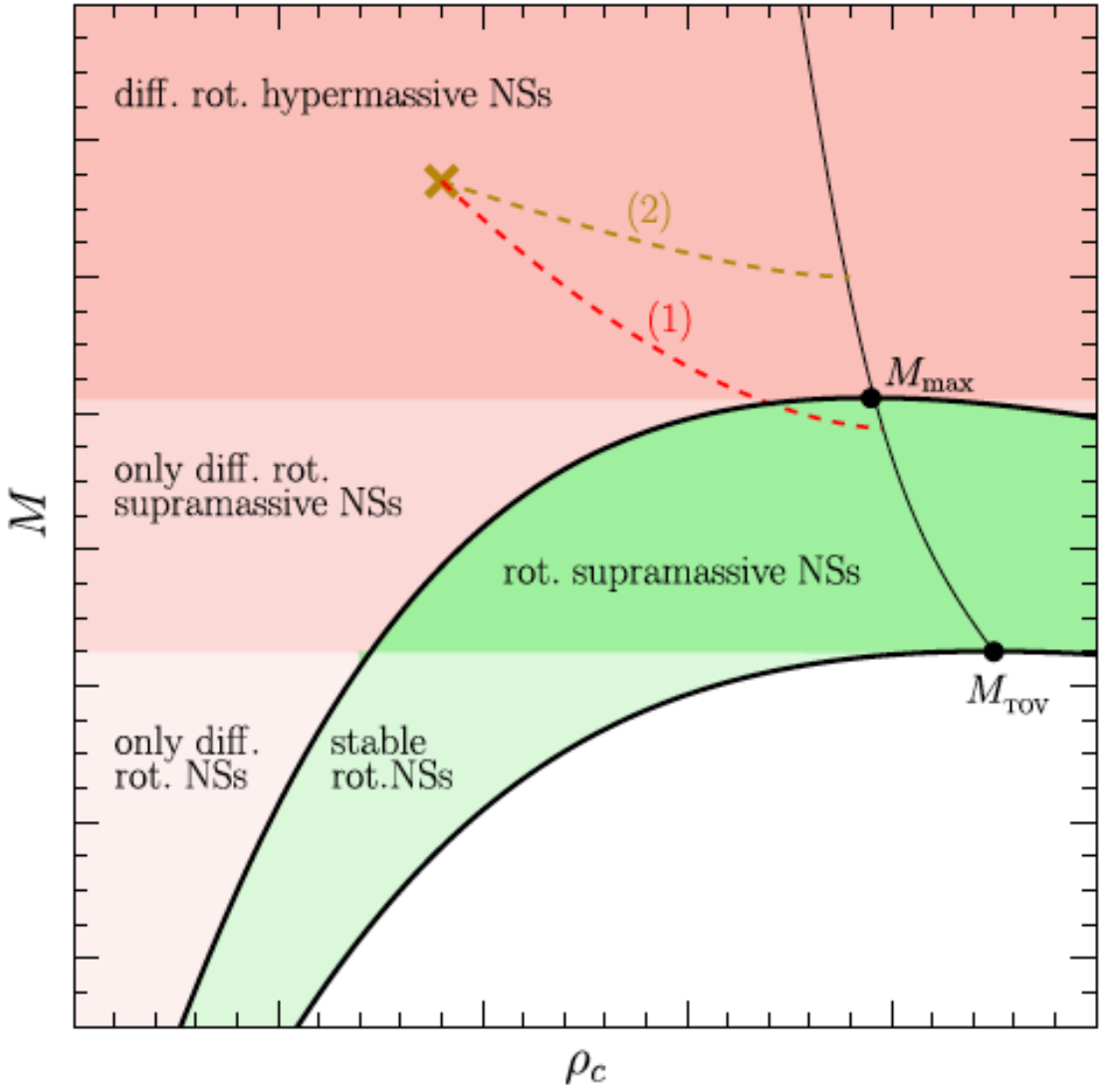
Alvarez-Castillo, Blaschke (2017)
High mass twins from multi-polytrope equations of state
arXiv: 1703.02681v2, Phys. Rev. C 96, 045809 (2017)

Massive Neutron stars:
Is there a concrete upper limit for the mass?



www3.mpifr-bonn.mpg.de/staff/pfreire/NS_masses.html

Upper limit on the Maximum Mass of static compact stars?



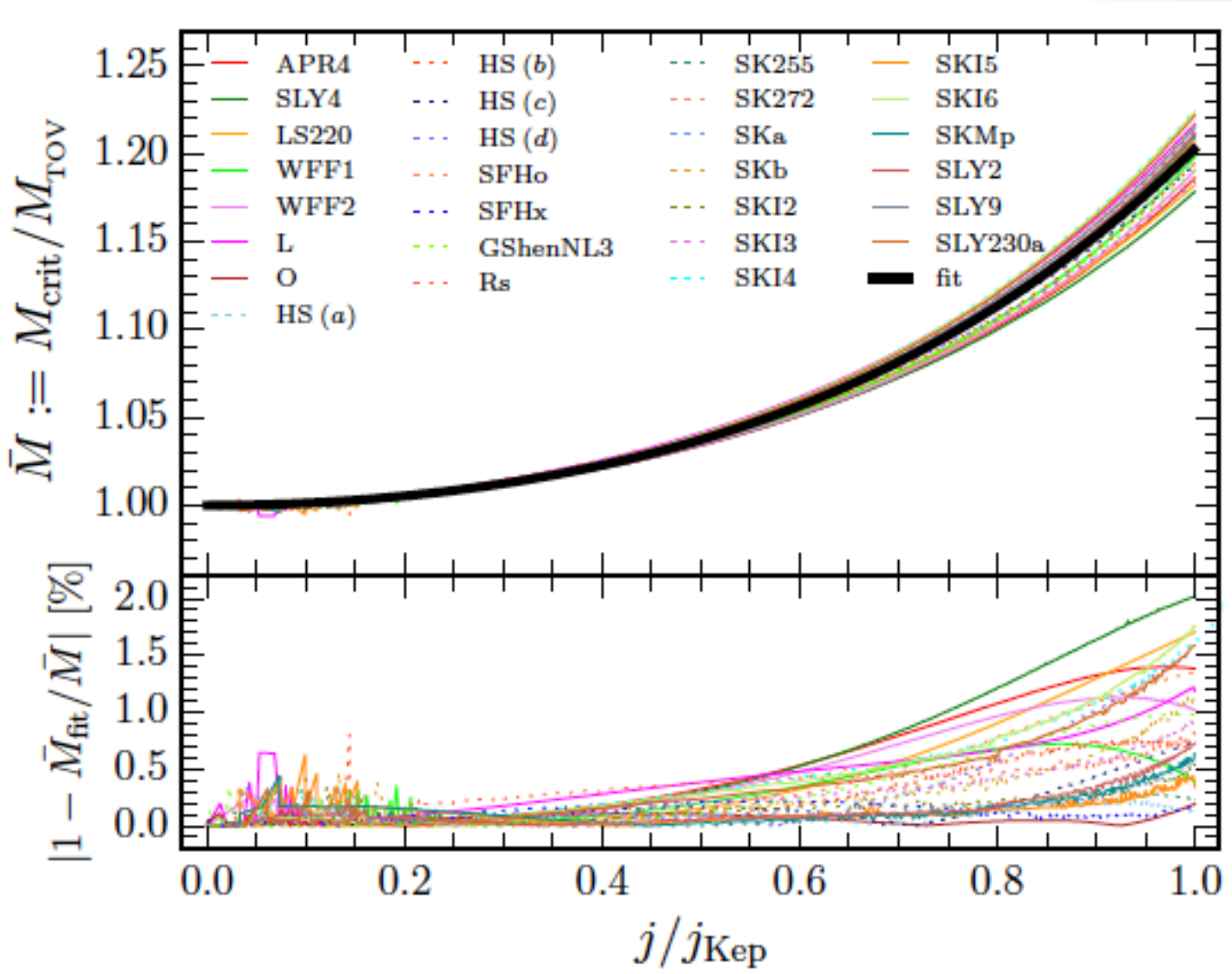
Rezzolla et al., ApJ Lett 852 (2018) L25

Universal relation for maximum mass increase upon rigid rotation

$$\frac{M_{\rm crit}}{M_{\rm TOV}} = 1 + a_2 \left(\frac{j}{j_{\rm Kep}}\right)^2 + a_4 \left(\frac{j}{j_{\rm Kep}}\right)^4$$

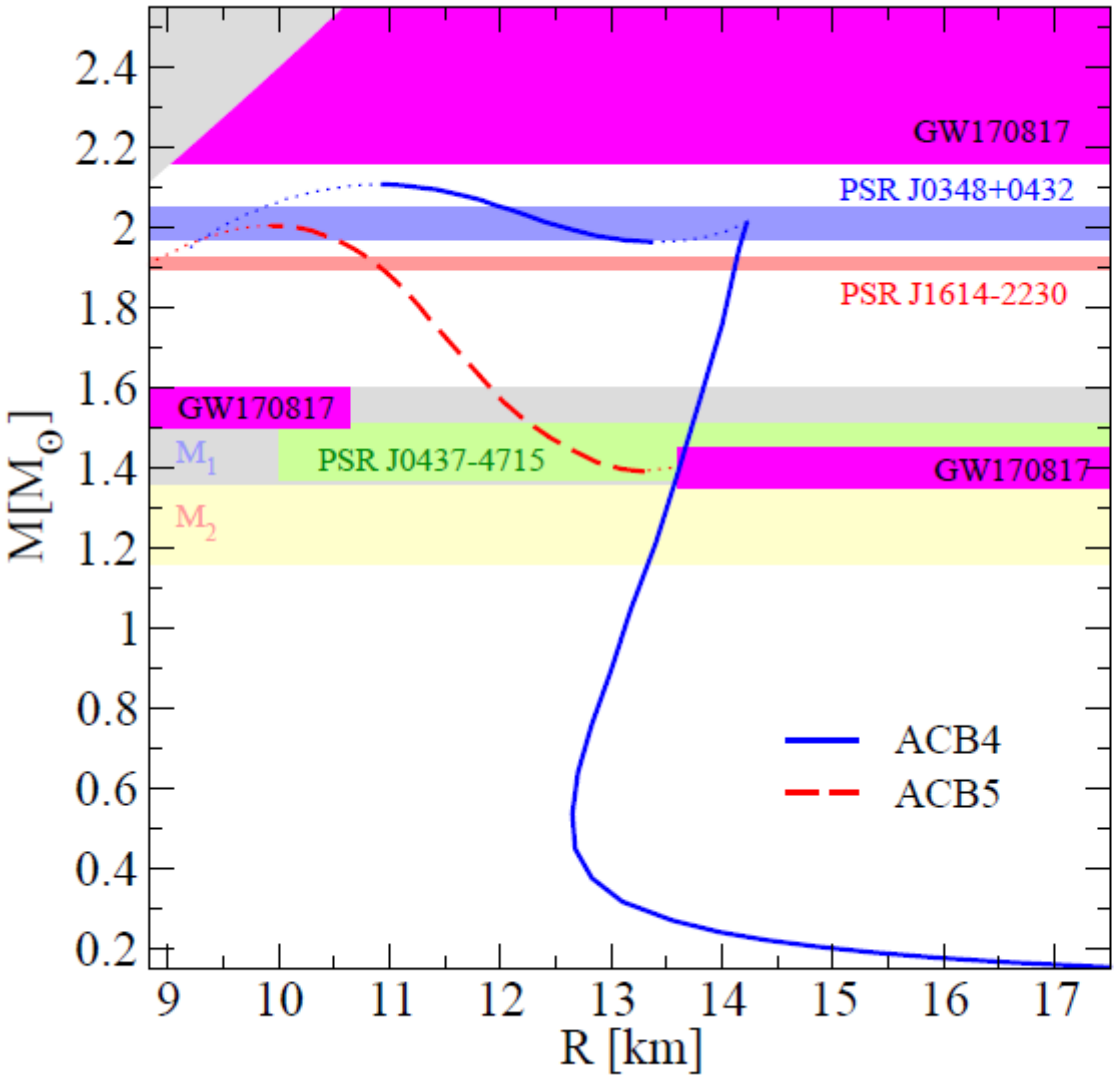
$$M_{\rm max} := M_{\rm crit}(j = j_{\rm Kep}) = (1 + a_2 + a_4)\, M_{\rm TOV}$$
$$\simeq (1.203 \pm 0.022)\, M_{\rm TOV}$$

"universal" increase of maximum mass by 20% due to rigid rotation at maximum (critical) angular momentum



Breu & Rezzolla et al., MNRAS (2016)

ACB4 and ACB5 M-R: observations and constraints



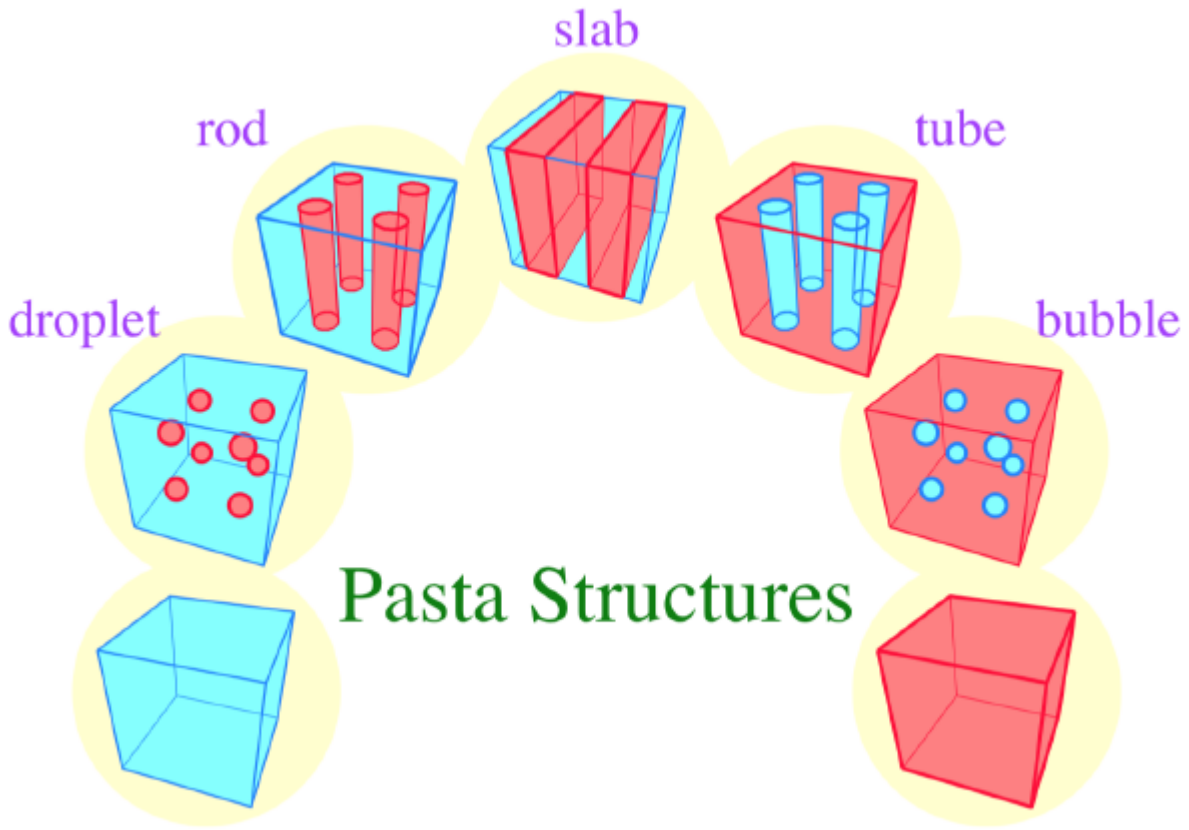
Blaschke et al., arXiv:1906.02522 [astro-ph.HE](2019)

Mixed phases

Coulomb interaction

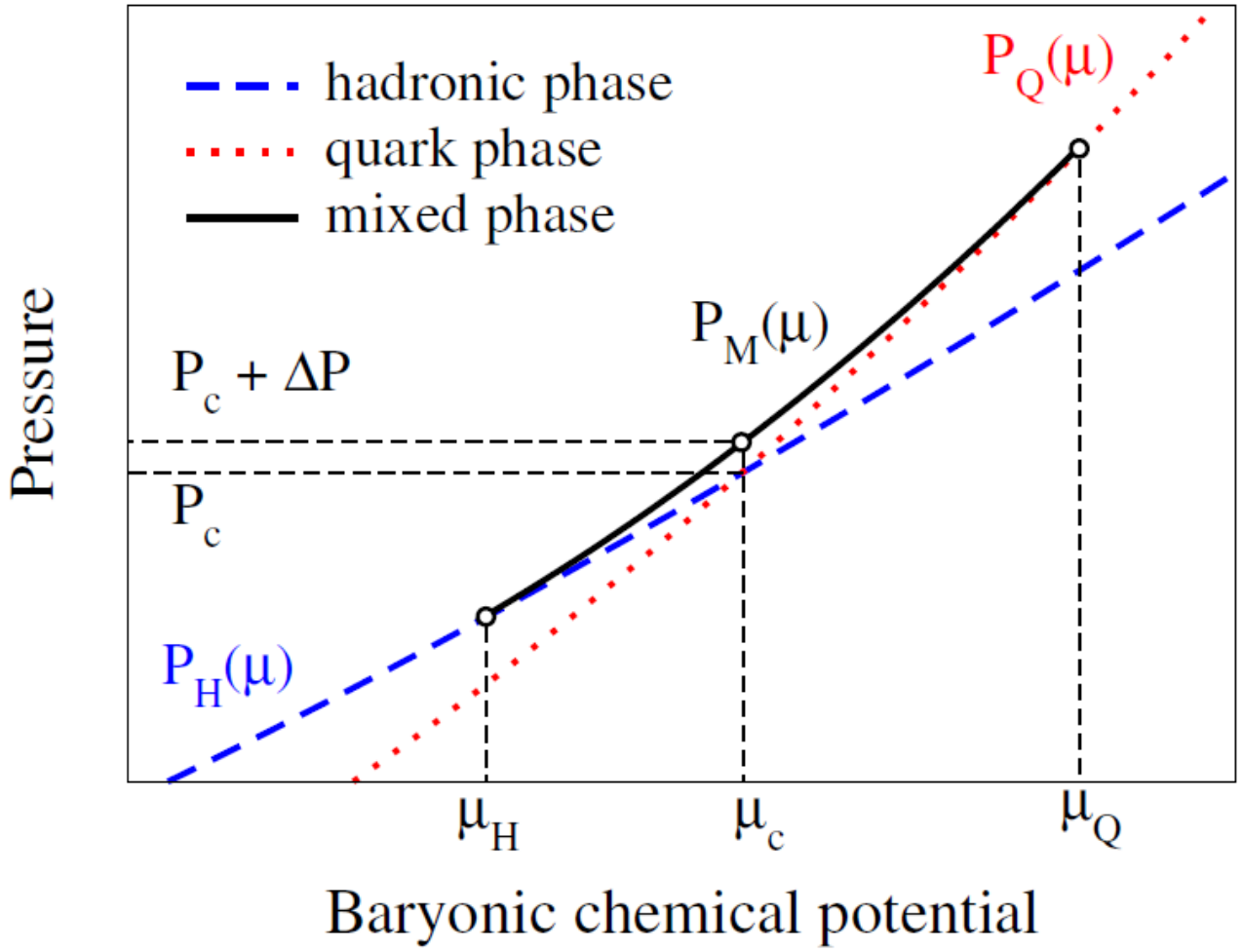
Vs

Surface tension



Yasutake, Maruyama, Tatsumi, Phys. Rev. D80 (2009) 123009

The mixed phase



Schematic representation of the interpolation function PM(μ), it has to go though three points: PH(μ H), Pc + P and PQ(μ Q).

The interpolation method

$$P_M(\mu) = \sum_{q=1}^{N} \alpha_q (\mu - \mu_c)^q + (1 + \Delta_P) P_c$$

where Δ_P is a free parameter representing additional pressure of the mixed phase at μ_c.

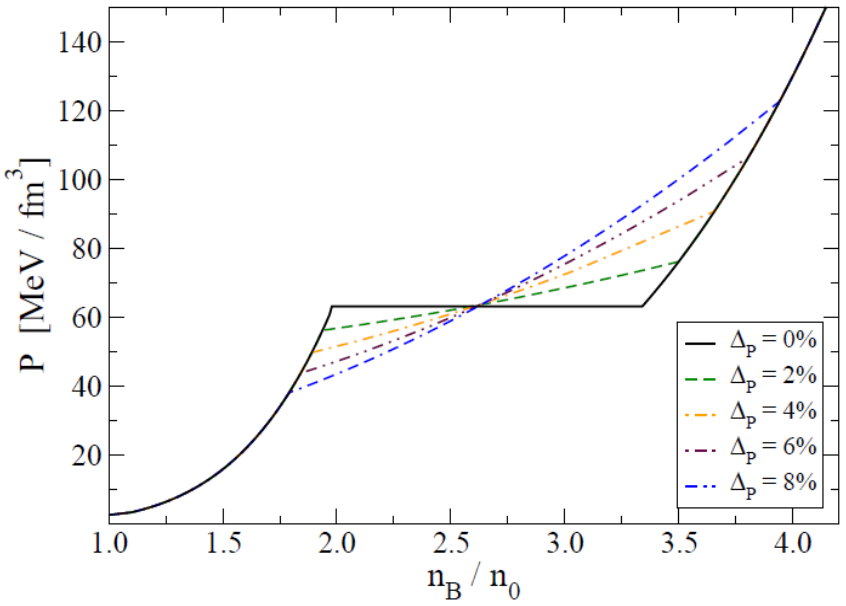
$$P_H(\mu_H) = P_M(\mu_H) \qquad P_Q(\mu_Q) = P_M(\mu_Q)$$

$$\frac{\partial^q}{\partial \mu^q} P_H(\mu_H) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_H) \qquad \frac{\partial^q}{\partial \mu^q} P_Q(\mu_Q) = \frac{\partial^q}{\partial \mu^q} P_M(\mu_Q)$$

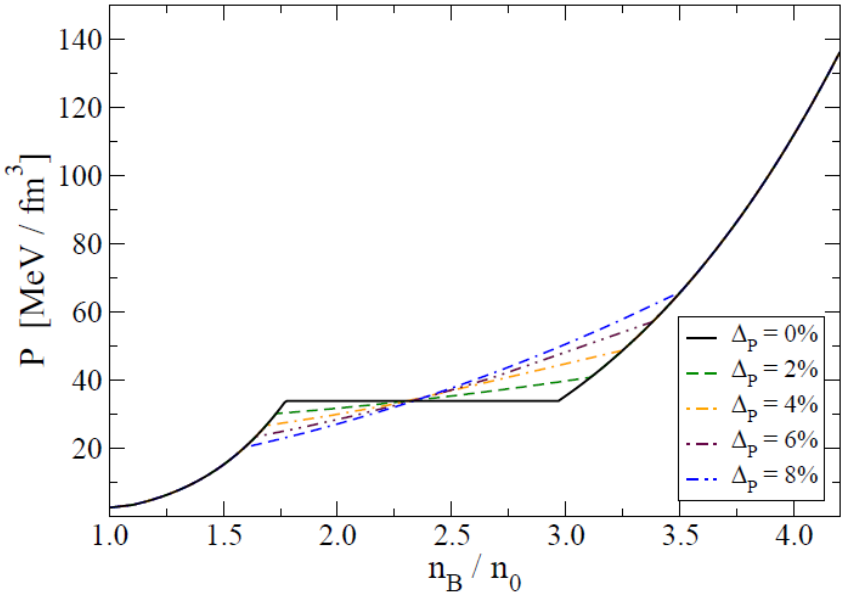
where $q = 1, 2, \ldots, k$. All $N + 2$ parameters (μ_H, μ_Q and α_q, for $q = 1, \ldots, N$) can be found by solving the above system of equations, leaving one parameter (ΔP) as a free one.

Ayriyan and Grigorian, *EPJ Web Conf.* **173**, 03003 (2018)
Abgaryan, Alvarez-Castillo, Ayriyan et al. *Universe* **4(9)**, 94 (2018)

Mixed phase effects (Pasta phases)

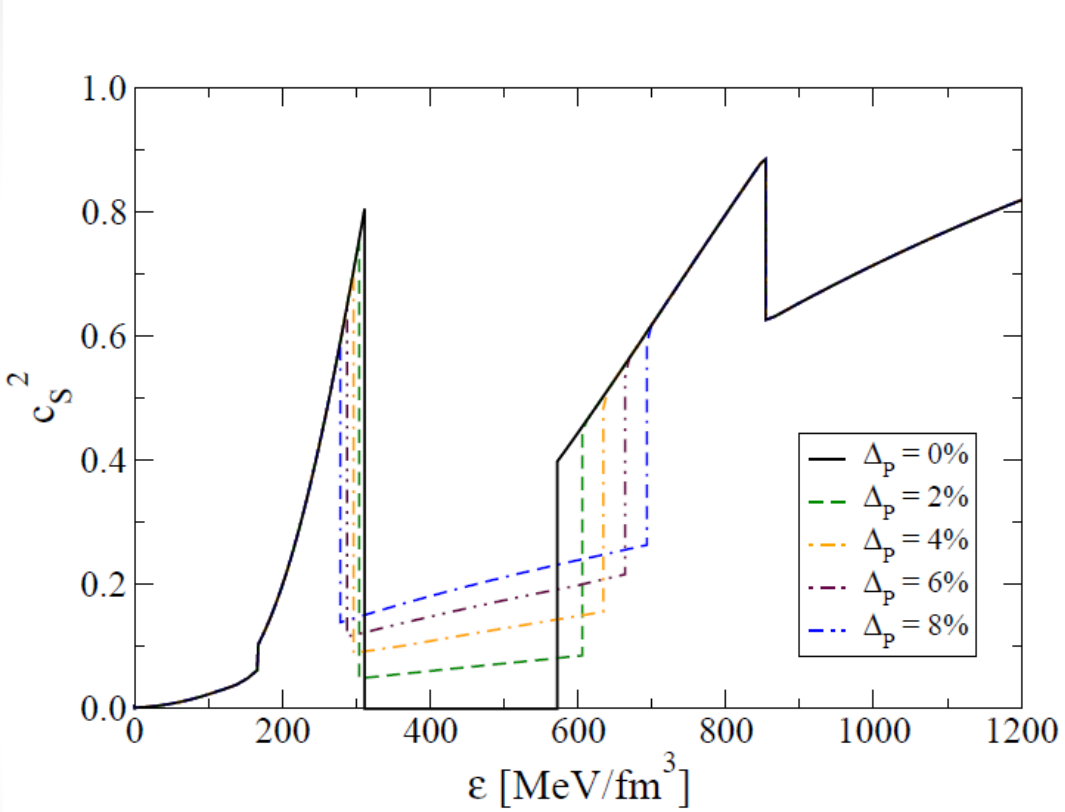


High mass twin stars
(ACB4)

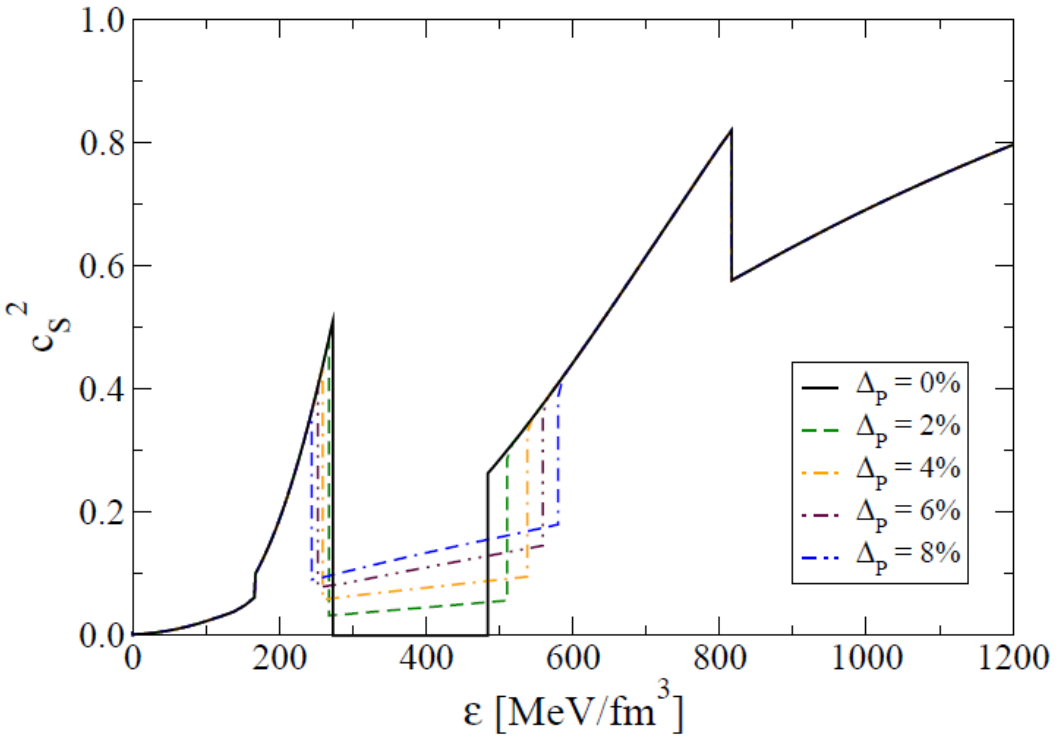


Low mass twin stars
(ACB5)

Speed of sound and Causality

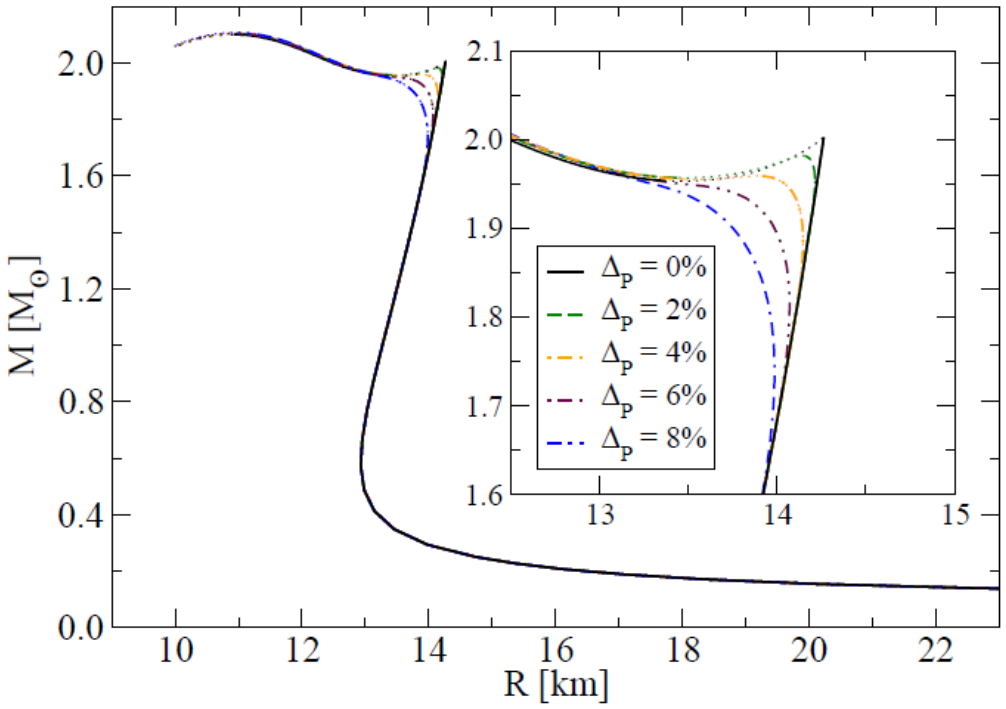


High mass twin stars
(ACB4)

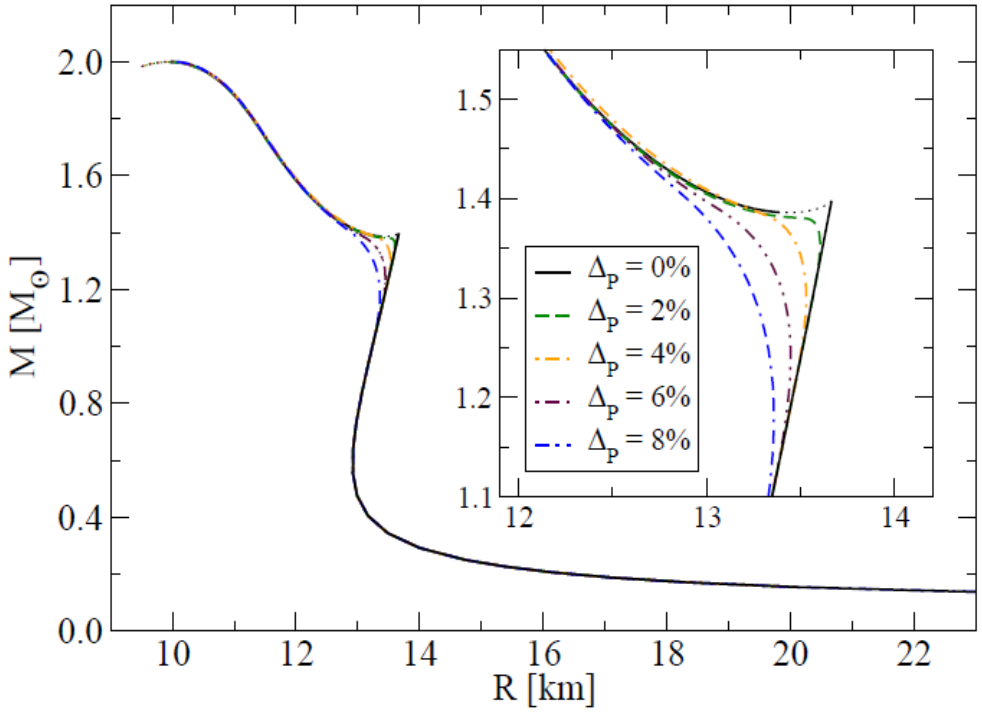


Low mass twin stars
(ACB5)

Mass-Radius relations



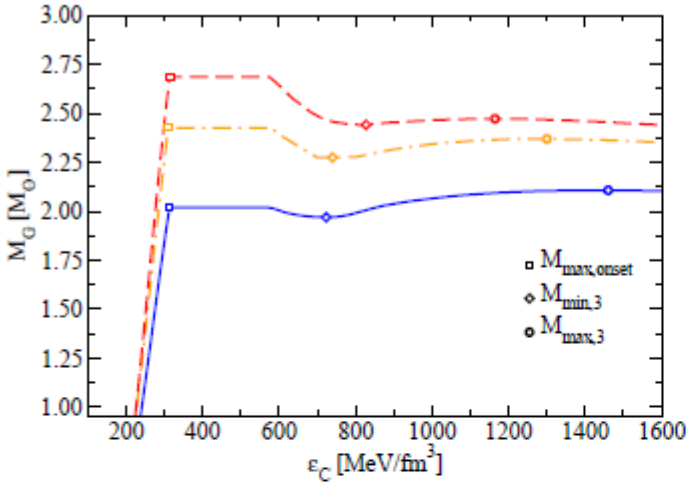
ACB4

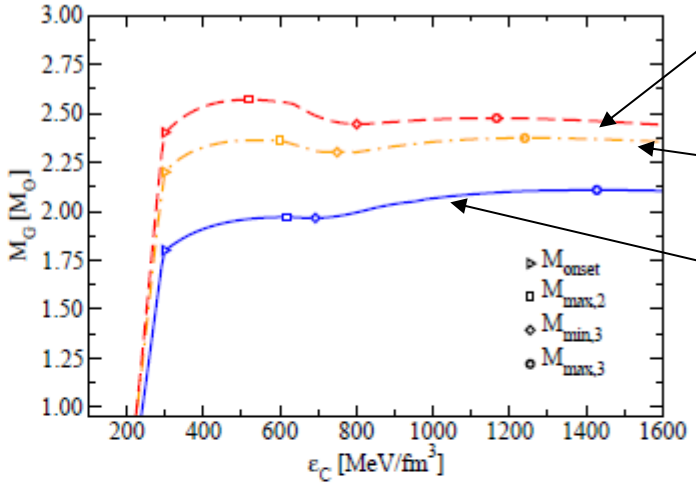


ACB5

Effect of Rotation and mixed phase

ACB4



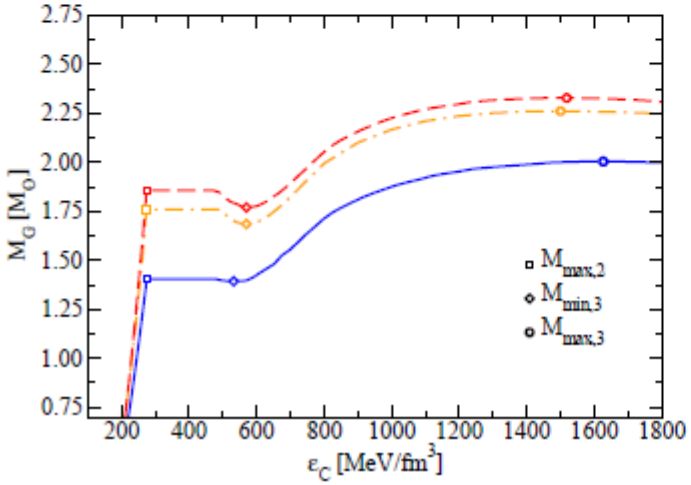


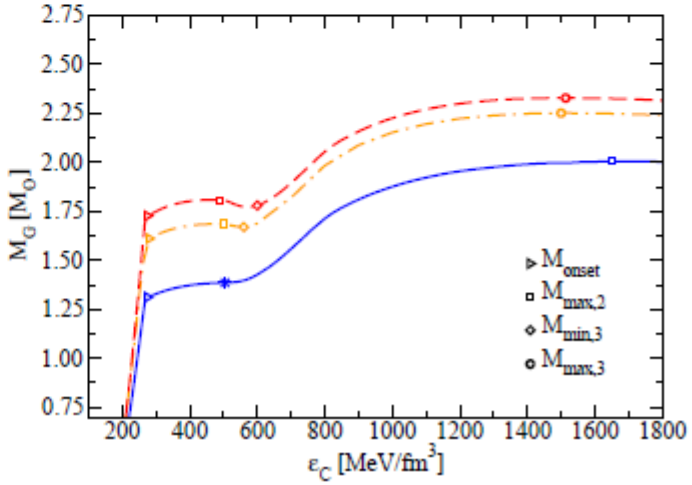
Full GR

Perturbative $_2$ Approach ~Ω

Static = TOV

ACB5





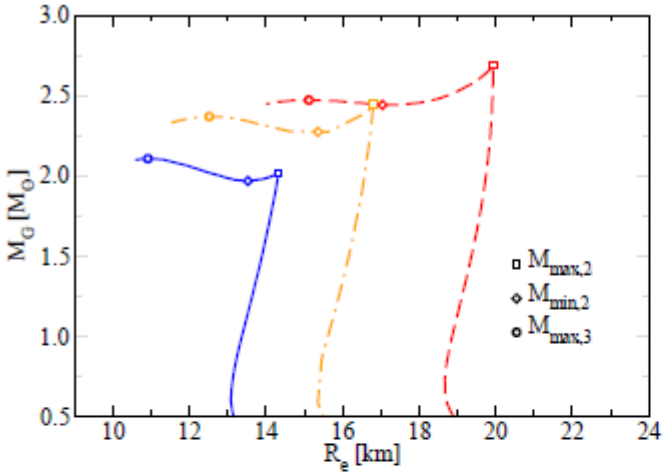
Maxwell construction

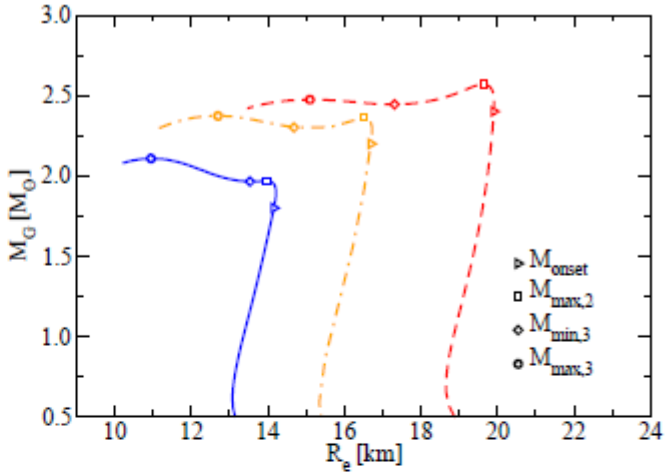
Critical value of Mixed-phase parameter

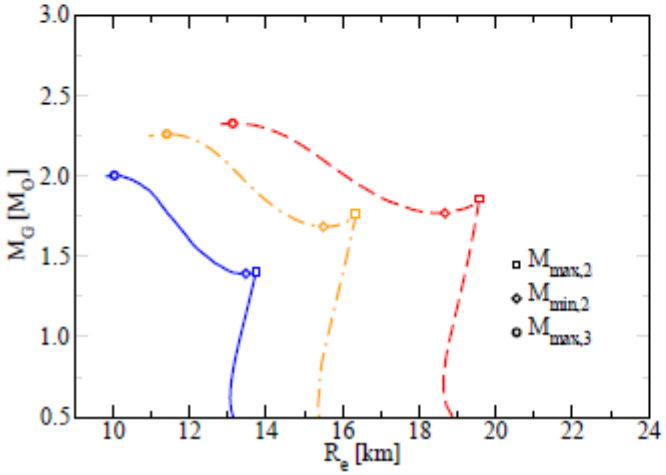
Effect of Rotation and mixed phase

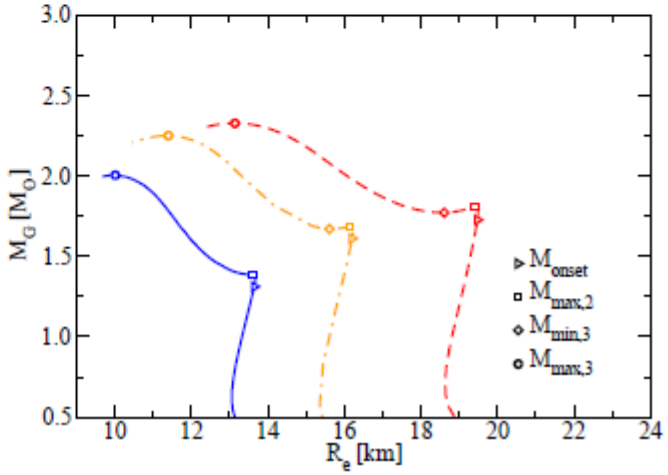
ACB4

ACB5





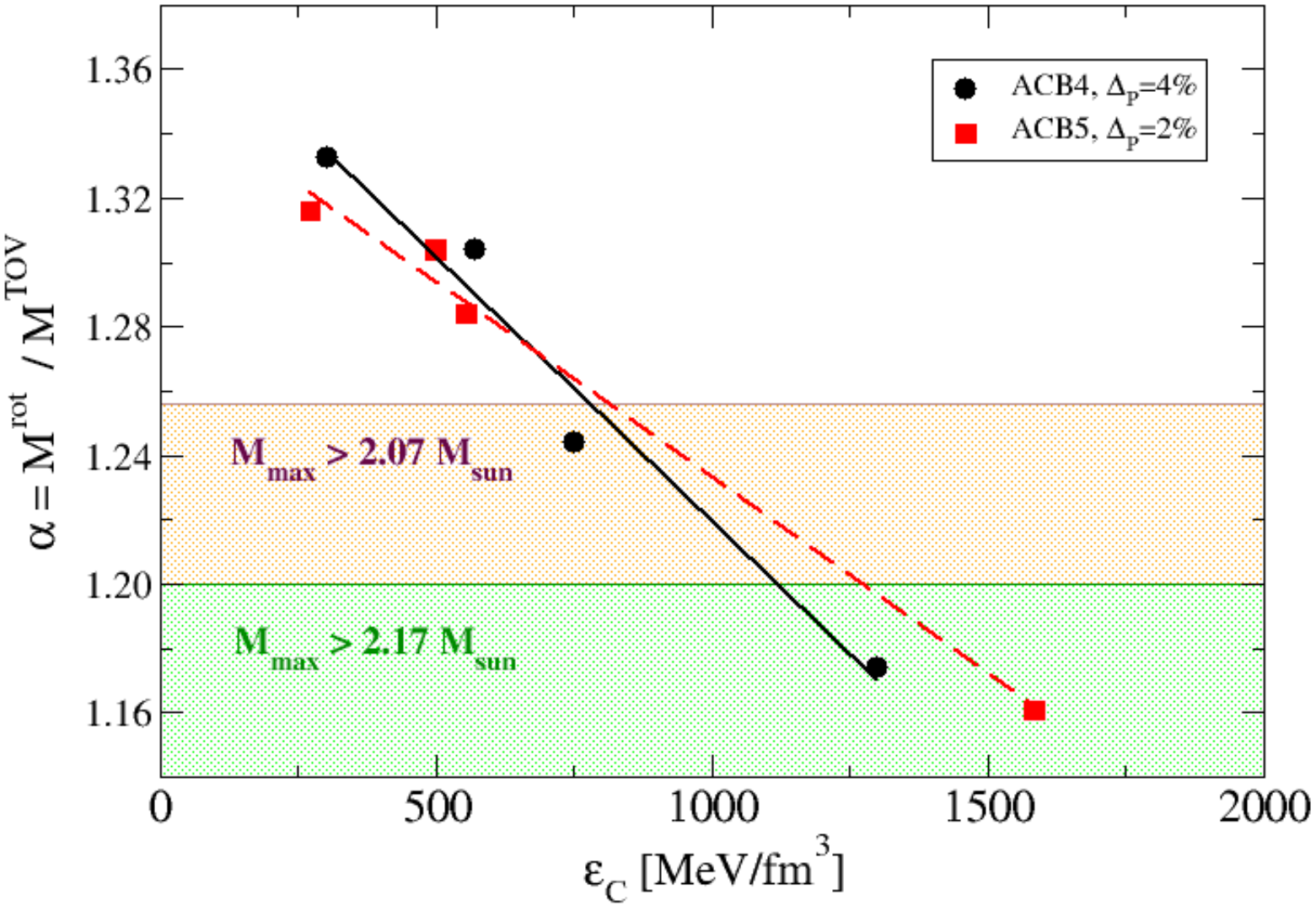




Maxwell construction

Critical value of Mixed-phase parameter

Universal relation for rotating compact stars



Conclusions

1)
The existence of mass twins invariably signals the existence of a strong phase transition in compact star matter at supersaturation densities.
“Strong transition”: energy density changes by about the value of the critical energy density in a sufficiently small region of pressures around the critical one of the Maxwell construction.
This can also support the existence of a critical endpoint (CEP) of first order phase transitions in the QCD phase diagram.

2)
The mixed phase construction mimics the pasta phase in accordance with a full pasta calculation. The result is a “broadening” of the phase transition over a certain pressure region, similar to the Gibbs construction in matter with more than one conserved charge and global charge conservation [Glendenning (1992)]. The construction makes the approach more realistic and Has advantages for numerical treatment of hybrid stars in general relativity.

3)
The conjecture of an upper limit on the maximum mass of nonrotating compact stars derived from GW170817 is revisited. We find a criterion for the minimal central energy density in the maximum mass configuration that would correspond to the core of GW170817. The equation of state at high densities must be effectively soft, either as a relatively soft hadronic one or a hybrid one with a strong phase transition. The NICER radius measurement could be decisive.

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TACK
ÞAKKA ÞÉR