How, how much, and when are $U_A(1)$ and chiral symmetry restored: T-dependence of axions, η and η'

Talk presented at

ZIMÁNYI SCHOOL'19, Winter Workshop on Heavy Ion Physics

Budapest, Hungary, 2. – 6. of December 2019.

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The issue of the (effective) restoration of the $U_A(1)$ symmetry

• In QCD, $U_A(1)$ and $SU_A(3)$ chiral symmetry are explicitly broken by current quark masses: only slightly by m_u and m_d & not too badly by s-quark mass m_s $\longrightarrow$ **chiral limit(s)** make sense [with 3 (or 2) $m_q \to 0$].

• **But approximate chiral $SU_A(3)$ symmetry = absent due to DChSB, signaled by $\langle\bar{q}q\rangle$ condensates and by the octet of very light (almost) Goldstone bosons: $\pi^{0,\pm}, K^{0,\pm}, \bar{K}^0, \eta$.**
... But as lattice now agrees, chiral symmetry should be restored as a crossover (for $\mu \sim 0$) around $T_{\rm Ch} \sim 155$ MeV: $\langle\bar{q}q\rangle(T) \to 0$.

• η' **very massive**, as even in chiral limit, $m_q \to 0$, $U_A(1)$ is broken explicitly on the quantum level by nonabelian ("gluon") axial anomaly:

$$\partial_\alpha \bar{\psi}(x)\gamma^\alpha\gamma_5 \frac{\lambda^0}{2}\psi(x) \propto F^a(x)\cdot\widetilde{F}^a(x) \equiv F^a_{\mu\nu}(x)\,\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}F^a_{\rho\sigma}(x) \neq 0\,,$$

which holds at any E and T $\Rightarrow$? Does $U_A(1)$ remains unrestored ?!?!

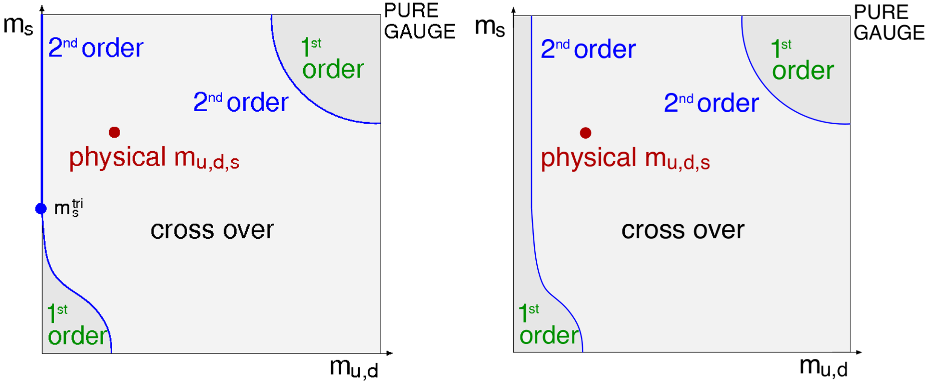
NO, since DChSB and $U_A(1)$ anomaly are tied through quark bilinears such as $\langle\bar{q}q\rangle$ and QCD topological susceptibility χ $\Rightarrow$ Expect an effective restoration signaled by vanishing or diminishing of $U_A(1)$-violating quantities (e.g., large $M_{\eta'}$, difference π-$a_0(980)$, ...) **over the chiral symmetry crossover** ... **BUT** ...

... still debatable what happens with $U_A(1)$ symmetry restoration!

- Presently, no consensus within lattice community whether $U_A(1)$ is badly broken or effectively restored at the chiral crossover critical temper. $T = T_{\rm Ch}$ [Sharma for HotQCD collaboration, e-Print: arXiv:1801.08500]
- Already older works found sizable $U_A(1)$ breaking above $T_{\rm Ch}$ [Bernard+al, PRL78 (1997)598, Chanrasekharan+al, PRL82(1999)2463, Ohno+al, PoS LATTICE 2011(2011)210 arXiv:1111.1939] ... and, this is confirmed by some recent works: notably by HotQCD collab. [Bazavov+al,PRD86(2012)9094503] and by Karsch & collaborators [Buchoff+al,PRD89(2014) 054514, Sharma+al, NPA956(2016)793, Dick+al,PRD91(2015)095504] as high as $T = 1.5\ T_{\rm Ch}$.
- BUT, some recent works claim that $U_A(1)$ breaking above $T_{\rm Ch}$ is overestimated in the continuum limit (blaming lattice artifacts near ChLim). Some then conclude that $U_A(1)$ anomaly is consistent with zero above $T_{\rm Ch}$, including also Graz group Rohrhofer+al, Phys.Rev.D96(2017)094501 arXiv 1707.01881, but most vocal were researchers around JLQCD collaboration [Cossu+al, PRD93(2016)034507 arXiv:1510.07395, PRD87&88 (2013)114514&019901 ....
These disappearances of $U_A(1)$ anomaly seem to be associated with the chiral limit - see, *e.g.*, Tomiya+al, PRD96(2017)034509.
- Then our model approach to η-η' may show that these two kinds of results can be reconciled, since it is consistent with both - depending whether one uses "massless" $\langle \bar{q}q \rangle_0$ or "massive" $q\bar{q}$ condensates: Horvatić, Kekez & D.K., Phys.Rev. D99 (2019) 014007, and spinoff for axions in Universe 5 (2019) 208.

What happens with $U_A(1)$ symmetry restoration matters a lot - see Columbia plot!

Left: $U_A(1)$ broken by anomaly, right: $U_A(1)$ restored (C.Fischer arXiv1810.12938)



General renorm-group arguments (Pisarski:1983ms) ⇒ QCD with 3 degenerate light flavors has a $1^{\rm st}$order phase transition in chiral limit, whereas in QCD with (2+1) flavors (*i.e.*, *s*-quark significantly more massive), a $2^{\rm nd}$order chiral-limit transition is also possible and even more likely (*e.g.*, Ejiri:2009ac,Ding:2019fzc). A $2^{\rm nd}$ order chiral-limit transition is exhibited by most DSE models – *e.g.*, clearly through the characteristic drop of their "massless" condensates $\langle\bar{q}q\rangle_0$.

Quantum-level breaking of $U_A(1)$ causes anomalously high $\eta' \approx \eta_0$ mass

QCD chiral behavior (reproduced by, e.g., DS approach) of the non-anomalous parts of masses of light $q\bar{q}'$ pseudoscalars: $M^2_{q\bar{q}'} = \mathrm{const}\,(m_q + m_{q'})$.

$\Rightarrow$ non-anomalous parts of the masses cancel in Witten-Veneziano rel. (WVR):

$$M_{\eta'}{}^2 + M_\eta{}^2 - 2\,M_K{}^2 = \frac{2N_f}{f_\pi^2}\,\chi_{\mathrm{YM}} = \text{anomalous mass}^2 \equiv M_{U_A(1)}{}^2 \approx \Delta M_{\eta_0}{}^2\,,$$

$$\chi \equiv \int d^4x\,\langle 0|Q(x)Q(0)|0\rangle = m_{\mathrm{a}}^2\,f_{\mathrm{a}}^2\,, \qquad Q(x) = \frac{g^2}{32\,\pi^2}F^a_{\mu\nu}(x)\widetilde{F}^a_{\mu\nu}(x)$$

QCD topological susceptibility χ = a direct measure of $U_A(1)$ breaking $\Rightarrow$ (partial) $U_A(1)$ restoration is indicated by vanishing or reduction of χ and related quantities, like $M_{U_A(1)} \approx \Delta M_{\eta_0} \approx \Delta M_{\eta'}$.

- $Q(x)$ = topological charge density operator
- In WVR, χ is pure-gauge, YM one, $\chi_{\mathrm{YM}} \leftrightarrow \chi_{quench}$, obtained long ago by lattice - harder for χ of light-flavorQCD, but can use DiVecchia-Veneziano

relation: $$\chi = \frac{-\langle\bar{q}q\rangle_0}{\sum\limits_{q=u,d,s}\frac{1}{m_q}} + \mathcal{C}_m(\text{unknown corrections, higher}\,\mathcal{O}\,\text{in small}\,m_q)$$

$\Rightarrow$ 1st & simplest example of $U_A(1)$ breaking given by chiral symmetry breaking

The 2nd example of tied breaking of $U_A(1)$ and chiral symmetries:

Leutwyler-Smilga relation (LS), also connecting YM and full QCD quantities (like WVR), "making" $\chi_{\rm YM}$ out of much smaller χ:

$$At \quad T = 0 \qquad \chi_{\rm YM} = \frac{\chi}{1 + \frac{\chi}{\langle \bar{q}q \rangle_0} \sum\limits_{q=u,d,s} \frac{1}{m_q}} \equiv \widetilde{\chi} \to \widetilde{\chi}(T)$$

$$\text{where for the light quarks} \qquad \chi = \frac{-1}{\sum\limits_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0}} + \mathcal{C}_m$$

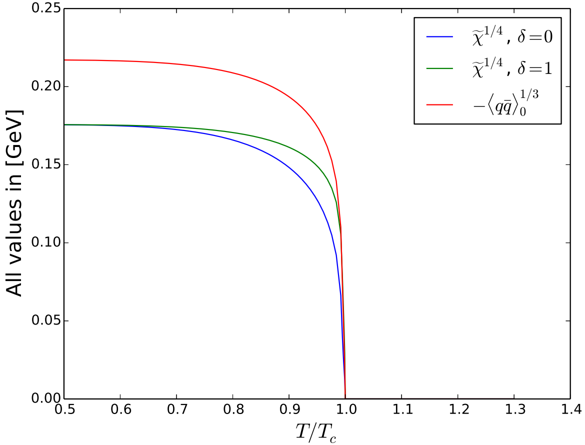
.

- $\mathcal{C}_m$ = small corrections of higher orders in small m_q.
  However, neglecting it, *i.e.*, $\mathcal{C}_m = 0$, would imply $\chi_{\rm YM} = \infty$.
  Conversely, $\chi_{\rm YM} = \infty$ in LS returns the leading term of χ.
  For axions, $\chi_{\rm YM}$ is not needed $\Rightarrow$ the leading term of χ will suffice.
- LS relation fixes the value of the correction at $T = 0$:

$$\frac{1}{\mathcal{C}_m} = \sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} - \chi_{\rm YM}(0) \left(\sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} \right)^2 .$$

- The conjecture on $\widetilde{\chi}(T)$ supported by Shore's generalization of WV relation.

Chiral-limit condensate $\langle\bar{q}q\rangle_0(T)$ and resulting $\widetilde{\chi}(T)$



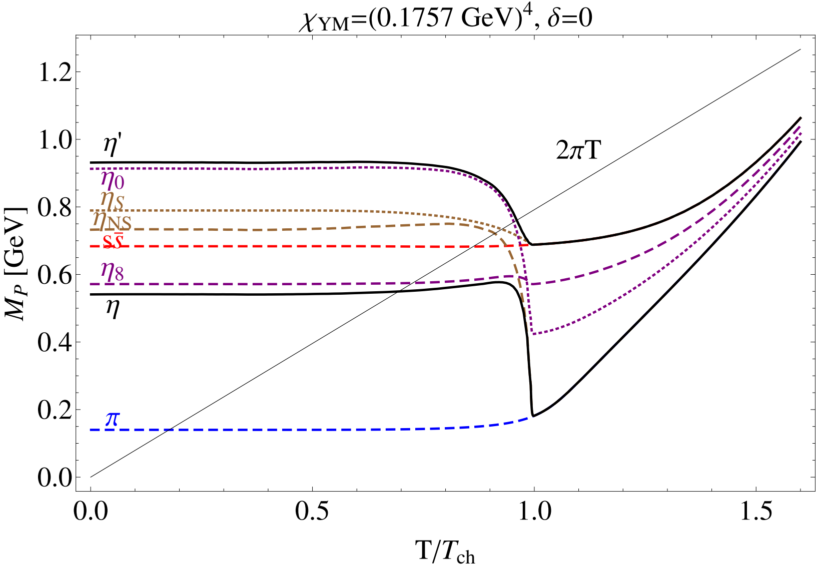
At density $= 0$, **2nd order** chiral transition forces at $T = T_c \equiv T_{\mathrm{Ch}}$ the abrupt transition to the NS-S asymptotic regime of vanishing $U_A(1)$ anomaly influence: $M_{\eta'}(T) \to M_{s\bar{s}}(T)$, and $M_\eta(T) \to M_{\mathrm{NS}}(T) \to M_\pi(T)$, and $\phi(T) \to 0$.

Acceptable or even good for η', but η would be in conflict with experiment.

Prediction good for η', but for η not supported by any experiment

[Benić, Horvatić, Kekez and Klabučar, Phys. Rev. D **84** (2011) 016006.]:

Anomalous contribution from WVR:



Shore's generalized WV = 3rd example of tying $U_A(1)$ and $SU_A(3)$

$$(f^0_{\eta'})^2 M^2_{\eta'} + (f^0_{\eta})^2 M^2_{\eta} = \frac{1}{3}\Big(f^2_\pi M^2_\pi + 2 f^2_K M^2_K\Big) + 2N_f\, A \qquad (1)$$

$$f^0_{\eta'} f^8_{\eta'} M^2_{\eta'} + f^0_{\eta} f^8_{\eta} M^2_{\eta} = \frac{2\sqrt{2}}{3}\Big(f^2_\pi M^2_\pi - f^2_K M^2_K\Big) \qquad (2)$$

$$(f^8_{\eta'})^2 M^2_{\eta'} + (f^8_{\eta})^2 M^2_{\eta} = -\frac{1}{3}\Big(f^2_\pi M^2_\pi - 4 f^2_K M^2_K\Big) \qquad (3)$$

The role of $\chi_{\rm YM}$ taken over by the full QCD topological charge parameter A ,

$$A = \frac{\chi}{1 + \chi\big(\frac{1}{\langle \bar{u}u\rangle\, m_u} + \frac{1}{\langle \bar{d}d\rangle\, m_d} + \frac{1}{\langle \bar{s}s\rangle\, m_s}\big)} \qquad (4)$$

A behaves with T as a full QCD quantity, **but**, at $T = 0$, $A = \chi_{\rm YM} + \mathcal{O}(\frac{1}{N_c})$
Again, $A = \infty$ returns the leading term of

$$\chi = \frac{-1}{\frac{1}{m_u\,\langle \bar{u}u\rangle} + \frac{1}{m_d\,\langle \bar{d}d\rangle} + \frac{1}{m_s\,\langle \bar{s}s\rangle}} + \mathcal{C}'_m \qquad (5)$$

Massive-quark condensates employed $\Rightarrow$ crossover around $T \sim T_{\rm Ch}$

(Large N_c limit & approximating 3 condensates by $\langle \bar{q}q\rangle_0$, returns the LS relation.)

QCD topological susceptibility $\sqrt{\chi(T)}$ gives axion mass $\times f_a$

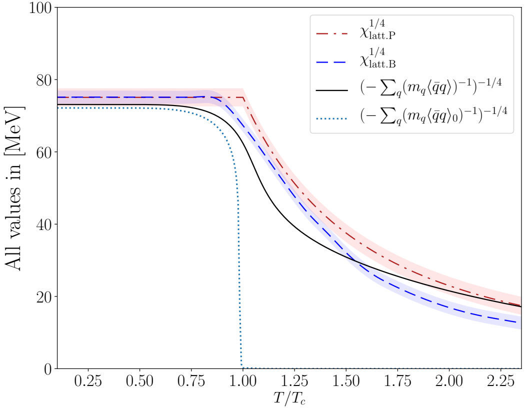
- For all temperatures: $m_a^2(T)\, f_a^2 = \chi(T) =$ **full** QCD topological susceptibility

- At $T = 0$, $$m_a^2\, f_a^2 = \frac{m_u\, m_d}{(m_u + m_d)^2}\, f_\pi^2\, M_\pi^2 \to \frac{\text{isospin}}{\text{limit}} \to (78.9\,\text{MeV})^4$$

- This agrees well with results, including $\chi(T)$, from "our" DS-BSE chirally well-behaved model (separable: simplified, but phenomenologically successful)

- Agrees well with $\chi(T)$ from lattice studies of axion mass: Petreczky & al. PLB (2016) and Borsany & al. Nature (2016)

- $\chi(T)$ from our usual DS-BSE model: successful at $T = 0$, no additional fitting for $T > 0$: condensates $\langle \bar{q}q \rangle(T)$ of **massive** $q = u, d, s$ essential to yield good **crossover** T-dependence of $\chi(T)$ for good T-dependence of η and η' masses.



Briefly on axions as solutions for Strong CP problem

• QCD has the Strong CP problem: no experimental evidence of any CP-symmetry violation in strong interactions, in spite of its θ-term:

$$\mathcal{L}_{\rm QCD} = \mathcal{L}^{\rm QCD}_{\rm CPsymmetric} + \bar{\theta}\, \frac{g^2}{32\pi^2}\, F^b_{\mu\nu} \widetilde{F}^{b\mu\nu} \qquad (\widetilde{F}^{b\mu\nu} \equiv \tfrac{1}{2}\, \epsilon^{\mu\nu\rho\sigma} F^b_{\rho\sigma})$$

• The θ-term is **a total divergence, but it cannot be discarded. It contributes anyway** (unlike in QED) due to **nontrivial topological structures in QCD** – *e.g.*, **instantons** (probably yielding, *e.g.*, anomalously large $M_{\eta'}$ $\Rightarrow$ important for solving the $U_A(1)$ problem).

• **The experimental bound is mysteriosly small:** $|\bar{\theta}| < 10^{-10}$. **Why?!?**

$\bar{\theta} = \theta_{\rm QCD} + Arg\, Det \hat{M}_q \Rightarrow$ setting $\bar{\theta} = 0$ just "by hand" is **fine-tuning**.

How to get $\bar{\theta} \approx 0$?!?

• Nowadays preferred solution: a new particle beyond SM: **axion** a

• **Axions are very interesting also for cosmology as candidates for dark matter.**

Axions as quasi-Goldstone bosons

• Peccei & Quinn introduced a new axial global symmetry $U(1)_{PQ}$ which is broken spontaneously at some scale $f_{\rm a}$
($f_{\rm a}$ = free parameter of axion theories, determines absolute value of the axion mass $m_{\rm a}$, but cancels from combinations such as $m_{\rm a}(T)/m_{\rm a}(0)$.)

• the pseudoscalar axion field ${\rm a}(x)$ is the (would-be massless) Goldstone boson of this spontaneous breaking. Then,

$$\mathcal{L}^{\rm QCD+}_{\rm axion} = \mathcal{L}^{\rm QCD}_{\rm CPsymmetric} + \left(\bar{\theta} + \frac{\rm a}{f_{\rm a}}\right) \frac{g^2}{32\pi^2} F^b_{\mu\nu} \widetilde{F}^{b\mu\nu} + \frac{1}{2}\partial_\mu {\rm a}\, \partial^\mu {\rm a} + \mathcal{L}^{{\rm a}\,\psi}_{\rm int}$$

• But, the $U(1)_{PQ}$ symmetry is also broken explicitly by the gluon axial anomaly through axion's coupling with gluons $\Rightarrow m_{\rm a} \neq 0$.
• Gluons generate an effective axion potential, which leads to the axion expectation value $\langle {\rm a} \rangle$ such that $(\bar{\theta} + \langle {\rm a} \rangle / f_{\rm a}) = 0$, minimizing the potential
$\Rightarrow$ **strong CP problem solved**, irrespective of the initial $\bar{\theta}$.

("Misalignment production" is relaxation from any value in the early Universe towards the effective potential minimum at $\bar{\theta} = -\langle {\rm a} \rangle / f_{\rm a}$. The resulting axion oscillation energy is a "cold dark matter" candidate.)

Evaluation of $q\bar{q}$ condensates from propagators

Solving the gap SD equation $\Rightarrow$ dressed propagators $S_q(p)$

The usual expression for the condensate of the flavor q for $T > 0$ becomes

$$\langle \bar{q}q \rangle = -N_c \sum\!\!\!\!\!\!\!\int_p \mathrm{Tr}\,[S_q(p)] \equiv -N_c\, T \sum_{n_q \in \mathbb{Z}} \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\mathrm{Tr}\,[S_q(p)]$$

Tr = trace in Dirac space. The combined integral-sum symbol says: when $T > 0$, the 4-momentum integration $\longrightarrow$ 3-momentum integr. & sum over Matsubaras $\omega_q = (2n_q + 1)\pi T$, $n_q \in \mathbb{Z}$.

• Well known: $q\bar{q}$ condensates are finite only for massless quarks, $m_q = 0$. "Massive" condensates must be subtracted of their divergences.

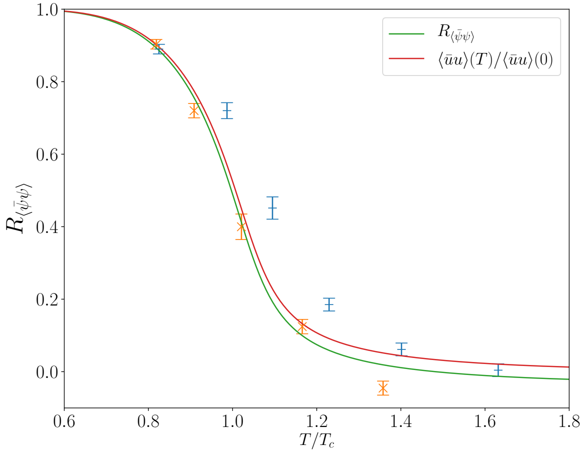
• The arbitrariness of sensible procedures is in practice slight, *i.e.,* only small differences between the results of various sensible subtractions.

• First consider the (normalized) subtraction proposed on lattice by Burger+al (2011), here applied to our condensates of light (u- and d-) quarks:

$$R_{\langle\bar{\psi}\psi\rangle}(T) = R_{\langle\bar{u}u\rangle}(T) = \frac{\langle\bar{u}\,u\rangle(T) - \langle\bar{u}\,u\rangle(0) + \langle\bar{q}\,q\rangle_0(0)}{\langle\bar{q}\,q\rangle_0(0)}\,.$$

Comparison of subtracted & normalized lattice- and DS-condensates

Relative T-dependence of the subtracted (and normalized) condensate $R_{\langle\bar{\psi}\psi\rangle}$. The lattice data points are from Fig. 6 of Kotov, Lombardo & Trunin, PLB794 (2019), scaled for the critical temperatures T_χ from their Table 2, which are different for the "crosses" (lattice data for $m_\pi \approx 370$ MeV) and "bars" (lattice data for $m_\pi \approx 210$ MeV).

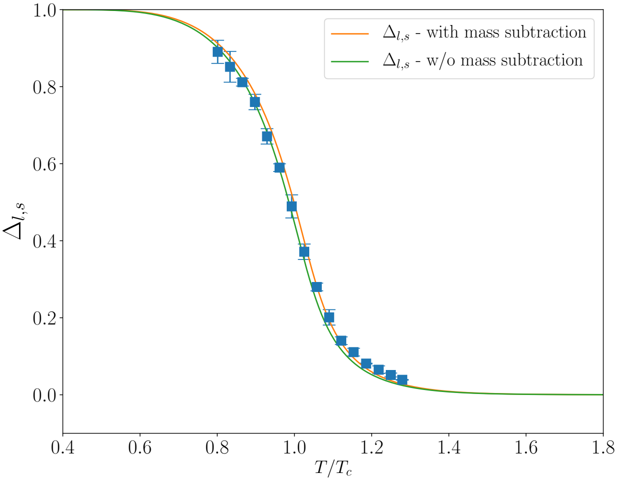


The lower, green curve results from only the $R_{\langle\bar{\psi}\psi\rangle}$-subtraction of our u-quark condensate. The upper, red curve is $R_{\langle\bar{u}u\rangle}(T)$ when our u-quark condensate is regularized in the usual way, by subtracting the current quark mass parameter m_u from the numerator of the dressed quark propagator.

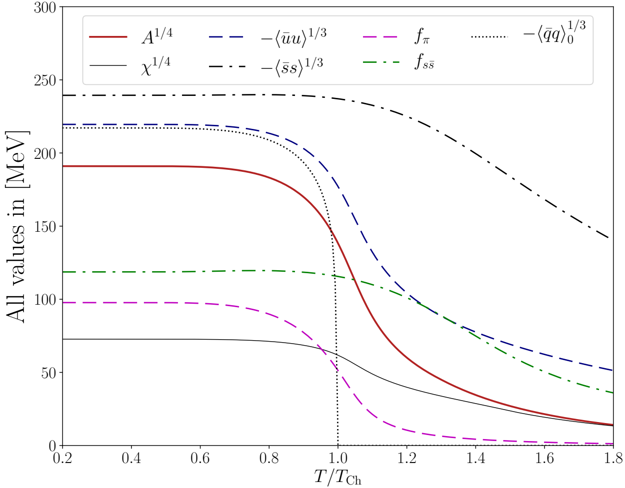
Compare $\Delta_{l,s}(T)$ regularization of the lattice and DS condensates

$$\Delta_{l,s}(T) = \frac{\langle \bar{l}\, l(T)\rangle - \frac{m_l}{m_s}\langle \bar{s}\, s(T)\rangle}{\langle \bar{l}\, l(0)\rangle - \frac{m_l}{m_s}\langle \bar{s}\, s(0)\rangle} \qquad (l = u \text{ or } d \text{ in isosymmetric limit})$$

This is the most usual (normalized) subtraction on the lattice. Very good agreement with Isserstedt+al 2019 & the lattice (Borsanyi+al 2010). Red and green curve, respectively, again result from our model DS condensate $\langle \bar{u}\, u(T)\rangle$ with and without subtraction of m_u from the quark propagator numerator.



T-dependence of $\langle \bar{q}q \rangle$ & decay const's f_P with χ & A



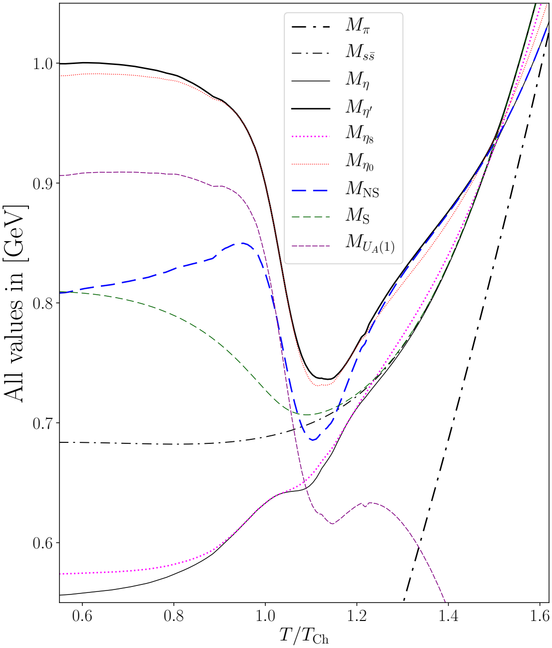
FKS scheme on Shore $\Rightarrow$ how f_P influence elements of the η-η' mass matrix:

$$X = \frac{f_\pi}{f_{s\bar{s}}}, \qquad M^2_{\rm NS} = M^2_\pi + \frac{4A}{f^2_\pi}, \qquad M^2_{\rm NSS} = \frac{2\sqrt{2}A}{f_\pi f_{s\bar{s}}}, \qquad M^2_{\rm S} = M^2_{s\bar{s}} + \frac{2A}{f^2_{s\bar{s}}}$$

Zoomed η-η' complex

- $M_{\eta'}(T)$ is not changed much as condensates are changed from chiral to massive: $M_{\eta'}(T)$ falls again around T_{Ch} by 300 to 200 MeV, corresponding to melting of $\sim \frac{1}{3} M_{U_A(1)}$.
  - But η does not exhibit any mass drop at all, now. It stays predominantly η_8 till anticrossing at $\sim 1.5\ T_{\mathrm{Ch}}$.

Similarly $\eta' \sim \eta_0$ long after T_{Ch}, and only after this anti-X with η, η' tends to a pure $s\bar{s}$.



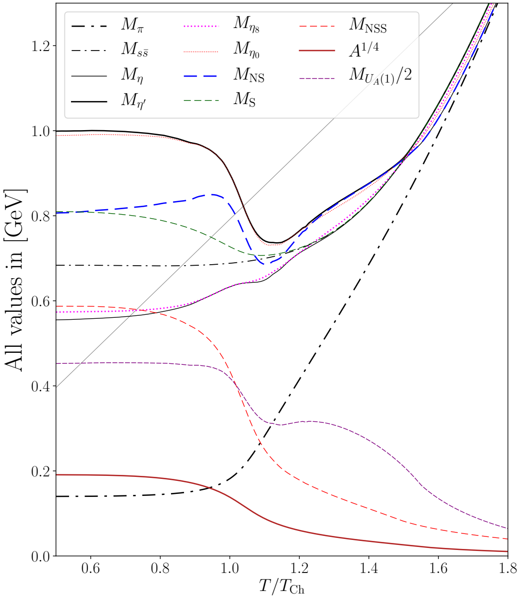
T-dependence of $M_P(T)$ up to $T = 1.8\,T_{\rm Ch}$ [Horvatić& al. Phys.Rev. D99 (2019) 014007]

- $\mathcal{C}(T) \neq$ *const*, adjusted to enable reaching arbitrary high T's, results otherwise very similar to previous case with $\mathcal{C}(T) = \mathcal{C}(0)$.
- Other limitations of rank-2 separable model make it hard to find solutions beyond $\sim 1.8\,T_c$.

But it is enough to exhibit cleanly the asymptotic regime beyond anticrossing at $1.5\,T_{\rm Ch}$.

Along with A, influence on anomalous masses is given by $M_{\rm NS\,S}$ and $(\frac{1}{2})M_{U_A(1)}$.

Utopistic in practice? - but in principle, accurate experimental knowledge of $M_{\eta'}(T)$ would tell us about $A(T)$ and thus about $\chi(T) \propto m_{\rm a}(T)^2$.

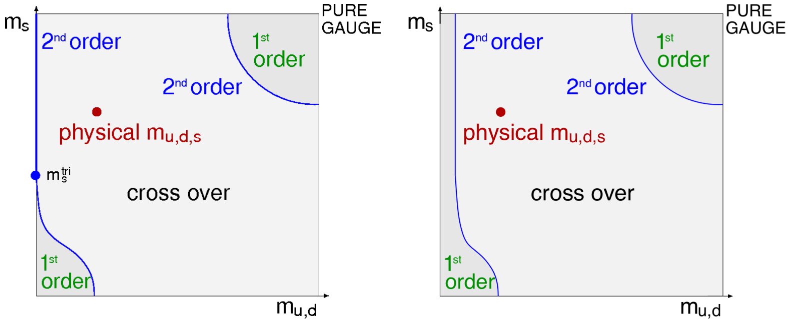


Summary of model results

- Our approach ties the $U_A(1)$ SB to the DChSB so closely, expressing χ & A through $m_q\langle\bar{q}q\rangle$, $q = u, d, s$, that the restoration of the chiral symmetry must lead to the restoration of the $U_A(1)$ symmetry at least partially, and surely on the level of the η' & η masses.
- **Full $U_A(1)$ restoration occurs together with the chiral one at $T = T_{Ch}$ only for the chiral-limit condensate $\langle q\bar{q}\rangle_0$ exhibiting sharp phase transition. However, such an abrupt restoration is in the real world excluded by the behavior of η.**
- Condensates with **explicit** ChSB exhibit crossover, *i.e.*, fall with T much more slowly than $\langle q\bar{q}\rangle_0$. Our "massive" condensates yield $\chi(T)$ in reasonable agreement with $\chi(T)$ from lattice studies of the T-dependence of axion mass.

  Now, η does not exhibit any mass drop at all, while the **significant drop of the η' mass signals only a partial restoration of $U_A(1)$ symmetry, consuming only about $\frac{1}{3}M_{U_A(1)}$ in the vicinity of $T = T_{Ch}$.**

Conclusion: our results consistent with "Left C-plot", with broken $U_A(1)$



- DS $\langle \bar{q}q \rangle$ consistent with chiral limit enabling 2^{nd} order transition, *i.e.*, $m_u = 0 = m_d$ only for the 2 lightest flavors, but $m_s > m_s^{\rm tri}$.
- For realistic explicit chiral breaking, the crossover creates an intermediate region between the chiral restoration at $T = T_{Ch}$ and the η-η' anticrossing at $T = 1.5\, T_{Ch}$ which marks the effective $U_A(1)$ restoration. The anomalous contributions then become so weak, that the η-η' complex enters the NS-S asymptotic regime: $\phi(T) \to 0$ and $M_{\eta'}(T) \to M_{\rm S}(T) \to M_{s\bar{s}}(T)$ and $M_{\eta}(T) \to M_{\rm NS}(T) \to M_{\pi}(T)$.