Axial anomaly and hadronic properties in a nuclear medium

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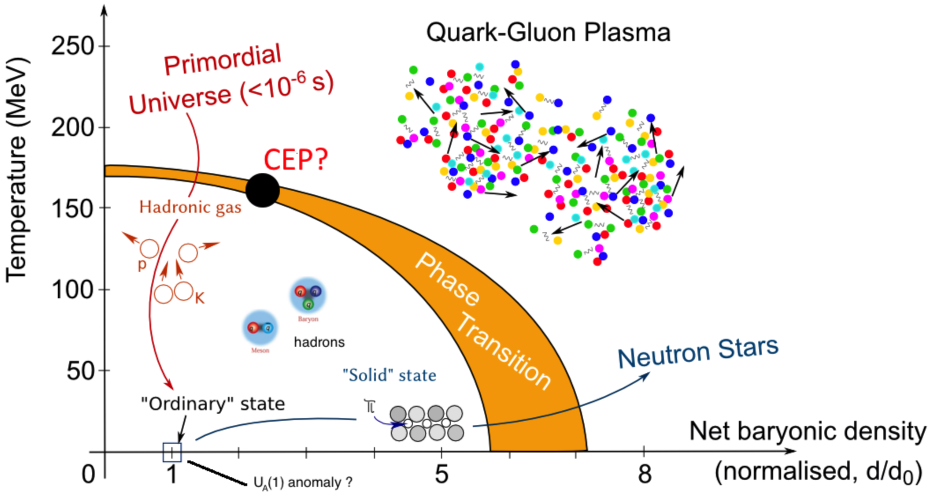
Új Nemzeti Kiválóság Program

GF & A. Hosaka, Phys. Rev. D **95**, 116011 (2017)
GF & A. Hosaka, Phys. Rev. D **98**, 036009 (2018)

Motivation



Quark-Gluon Plasma
Primordial Universe (<10^-6 s)
CEP?
Hadronic gas
p
K
Baryon
Meson
hadrons
Phase Transition
Neutron Stars
"Solid" state
"Ordinary" state
Temperature (MeV)
250
200
150
100
50
0
1
5
8
Net baryonic density (normalised, d/d0)
UA(1) anomaly ?

Motivation

AXIAL ANOMALY OF QCD:

- $U_A(1)$ anomaly: anomalous breaking of the $U_A(1)$ subgroup of $U_L(N_f) \times U_R(N_f)$ chiral symmetry
  $\longrightarrow$ vacuum-to-vacuum topological fluctuations (instantons)

$$\partial_\mu j_A^{\mu a} = -\frac{g^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,[T^a F_{\mu\nu} F_{\rho\sigma}]$$

- $U_A(1)$ breaking interactions depend on instanton density
  $\longrightarrow$ suppressed at high T[1] (valid beyond T_c)
  $\longrightarrow$ is the anomaly present at the phase transition?
- Very little is known at finite baryochemical potential (μ_B)[2]
  $\longrightarrow$ sign problem in lattice simulations
  $\longrightarrow$ effective models have not been extensively explored

[1] R. D. Pisarski, and L. G. Yaffe, Phys. Lett. B**97**, 110 (1980).
[2] T. Schaefer, Phys. Rev. D**57**, 3950 (1998).

Motivation

η' - NUCLEON BOUND STATE:

- Effective models at finite T and/or density:
  $\longrightarrow$ mean field calculations (NJL[3], linear sigma models[4]) predict a $\sim$150 MeV drop in $m_{\eta'}$ at finite μ_B
- Effective description of the mass drop:
  $\longrightarrow$ attractive potential in medium $\Rightarrow$ $\eta' N$ bound state
  $\longrightarrow$ Analogous to $\Lambda(1405) \sim \bar{K}N$ bound state

[3] P. Costa, M. C. Ruivo & Yu. L. Kalinovsky, Phys. Lett. B 560, 171 (2003).
[4] S. Sakai & D. Jido, Phys. Rev. C**88**, 064906 (2013).

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- Problem with mean field calculations: they treat model parameters as environment independent constants
  $\longrightarrow$ „$A \cdot v$" type of terms decrease (A-constant, v-decreases)
  $\longrightarrow$ evolution of the „A" anomaly at finite T and μ_B?

- What is the role of fluctuations?

[3] P. Costa, M. C. Ruivo & Yu. L. Kalinovsky, Phys. Lett. B 560, 171 (2003).
[4] S. Sakai & D. Jido, Phys. Rev. C**88**, 064906 (2013).

Motivation

- Fluctuation effects in a quantum system is encoded in the effective action
- Partition function and effective action in field theory:
  [$\mathcal{S}$: classical action, ϕ: dynamical variable, $\bar{\phi}$: mean field, J: source field]

$$Z[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi)}, \quad \Gamma[\bar{\phi}] = -\log Z[J] - \int J\bar{\phi}$$

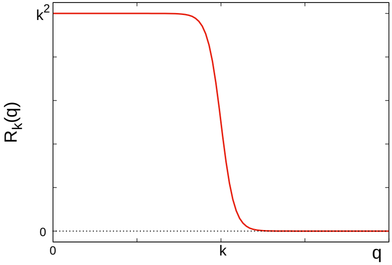
- Γ contains the truncated 1PI *n-point functions*
- How to calculate the effective action? $\Rightarrow$ perturbation theory!
  $\longrightarrow$ find a <u>small parameter</u> in $\mathcal{S}$ and Taylor expand
  $\longrightarrow$ fails in QCD & eff. models are not weakly coupled either
- Non-perturbative methods are necessary:
  Functional Renormalization Group (FRG)[5]

[5] C. Wetterich, Phys. Lett. B**301**, 90 (1993)

- FRG generalizes the idea of the Wilsonian RG: fluctuations are taken into account at the level of the quantum effective action
- Introduce a flow parameter k and include fluctuations for which $q \gtrsim k$

$$Z_k[J] = \int \mathcal{D}\phi e^{-(\mathcal{S}[\phi]+\int J\phi)} \times e^{-\frac{1}{2}\int \phi R_k \phi}$$

$\longrightarrow$ regulator: mom. dep. mass term suppressing low modes



Functional Renormalization Group

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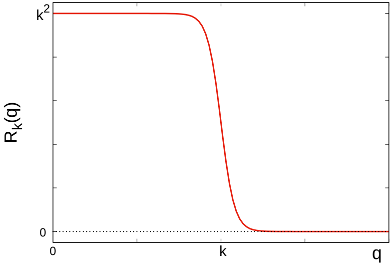
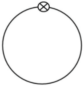


- Scale dependent effective potential and its flow equation:

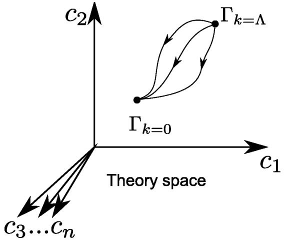
$$\Gamma_k[\bar\phi] = -\log Z_k[J] - \int J\bar\phi - \frac{1}{2}\int \bar\phi R_k \bar\phi$$

$$\partial_k \Gamma_k = \frac{1}{2}\int_{q,p}^{(T)} \mathrm{Tr}\left[\partial_k R_k(q,p)(\Gamma_k^{(2)} + R_k)^{-1}(p,q)\right] = \frac{1}{2}$$



Functional Renormalization Group

- The scale dependent effective action (Γ_k) is an average action
  $\longrightarrow$ fluctuations with wavelenghts $\lambda \sim k^{-1}$ are integrated out
  $\longrightarrow$ $k \to \infty$: no fluctuations $\Rightarrow$ $\Gamma_{k\to\infty}[\bar{\phi}] = \mathcal{S}[\bar{\phi}]$
  $\longrightarrow$ $k = 0$: all fluctuations $\Rightarrow$ $\Gamma_{k=0}[\bar{\phi}] = \Gamma[\bar{\phi}]$
- The scale-dependent effective action interpolates between classical- and quantum effective actions
- The trajectory depends on R_k but the endpoint does not
- Choice of R_k $\leftrightarrow$ choice of scheme



Chiral effective nucleon-meson theory at finite μ_B

3 FLAVOR CHIRAL NUCLEON-MESON MODEL:

- Effective model of chiral symmetry breaking: order par. M
  [excitations of M: π, K, η, η' and a_0, κ, f_0, σ]

$$
\begin{aligned}
\mathcal{L}_M &= \mathrm{Tr}\,[\partial_i M^\dagger \partial_i M] - \mathrm{Tr}\,[H(M^\dagger + M)] \\
&+ V_{ch}(M) + A \cdot (\det M^\dagger + \det M) \\
\mathcal{L}_{\omega+N} &= \frac{1}{4}(\partial_i \omega_j - \partial_j \omega_i)^2 + \frac{1}{2} m_\omega \omega_i^2 + \bar{N}(\partial\!\!\!/ - \mu_B \gamma_0) N, \\
\mathcal{L}_{\mathrm{Yuk}} &= \bar{N}(g_Y \tilde{M}_5 - i g_\omega \omega\!\!\!/) N
\end{aligned}
$$

$\longrightarrow$ nucleon mass: entirely from Yukawa coupling

- Fluctuation effects are calculated in the mesonic potentials:

$$
V_k = V_{ch,k}(M) + A_k(M) \cdot (\det M^\dagger + \det M)
$$

$\longrightarrow$ solve a set of functional differential equations on a grid

Chiral effective nucleon-meson theory at finite μ_B

- Baryon Silver Blaze property:
  $\longrightarrow$ no change in the effective action for $T = 0$ if
  $\mu_B < m_N - B \equiv \mu_{B,c}$

[6] M. Drews and W. Weise, Prog. Part. Nucl. Phys. 93, 69 (2017).

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- At $\mu_B = \mu_{B,c}$:[6]
  $\longrightarrow$ 1st order phase transition from nuclear gas to liquid
  $\longrightarrow$ nuclear density jumps from zero to $n_0 \approx 0.17\,\mathrm{fm}^{-3}$
  $\longrightarrow$ non-strange chiral condensate jumps from f_π to $v_{ns,nucl}$
  (Landau mass $M_L \approx 0.8 m_N \Rightarrow v_{ns,nucl} \approx 69.5\,\mathrm{MeV}$)

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- The first order transition is related to the condensation of the timelike component of the ω vector particle

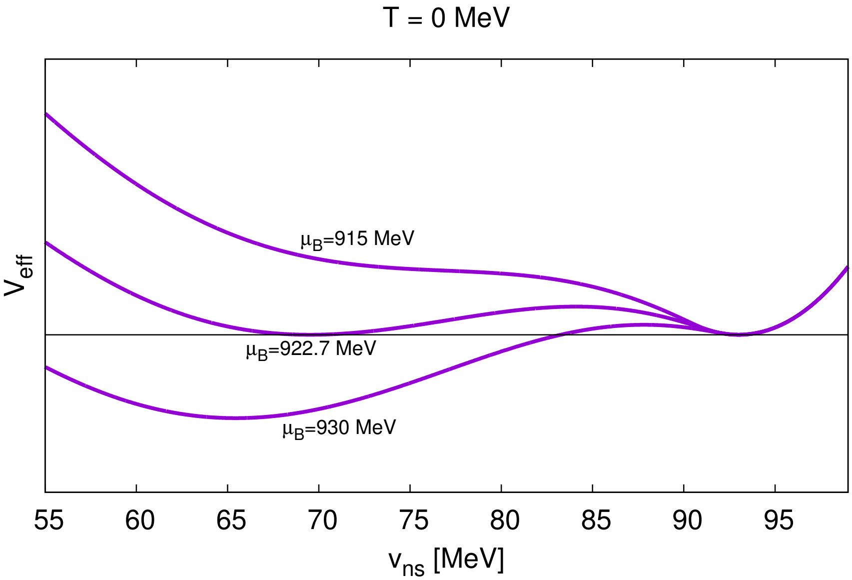
- ω couples to v_{ns} that couples to v_s
  $\longrightarrow$ jump in all order parameters

[6] M. Drews and W. Weise, Prog. Part. Nucl. Phys. 93, 69 (2017).

PARAMETRIZATION:

- The model consists of the following parameters:
  $\rightarrow$ $V(M)$: m^2, g_1, g_2, b_i $(i = 1..4)$ [b_i are non-renormalizable interactions!]
  $\rightarrow$ explicit breaking, anomaly: h_0, h_8, A
  $\rightarrow$ $\omega + N$: $g_\omega^2/m_\omega^2, g_Y$
- 12 parameters in total. Input:
  $\rightarrow$ masses in the vacuum: m_π, m_K, m_η, $m_{\eta'}$, m_{a_0}, m_N
  $\rightarrow$ normal nuclear density: n_0
  $\rightarrow$ critical chemical potential: $\mu_{B,c}$
  $\rightarrow$ nucleon mass drop in the medium: Δm_N
  $\rightarrow$ 2 PCAC relations (decay constants f_π, f_K)
  $\rightarrow$ temperature of the critical endpoint T_{CEP}
  [Compression modulus: prediction! $K = \frac{9n_0}{\partial n_0/\partial \mu_B} \approx 287\,\mathrm{MeV}$]

Numerical results



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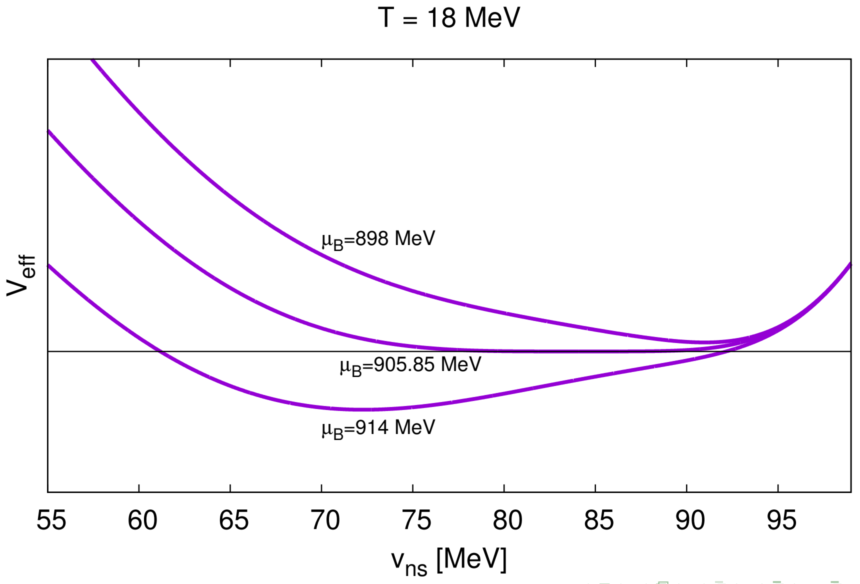

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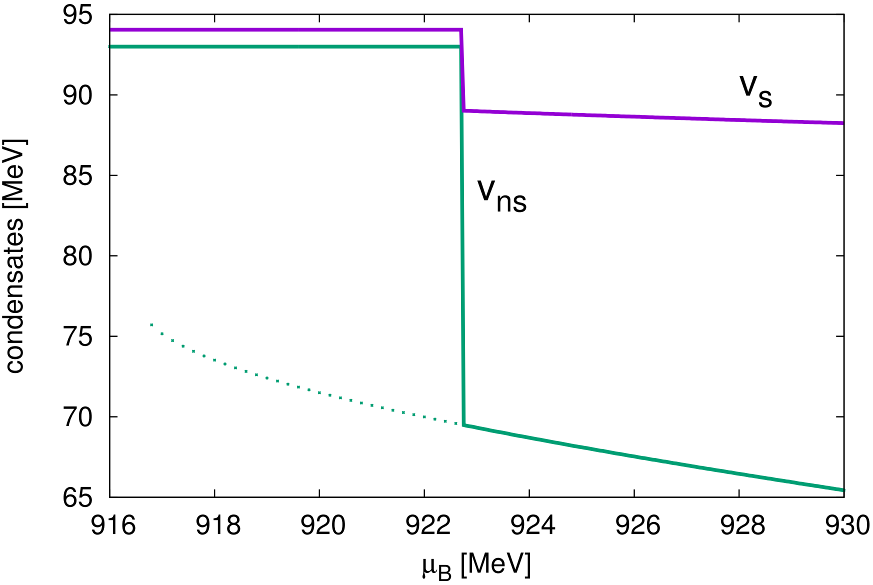

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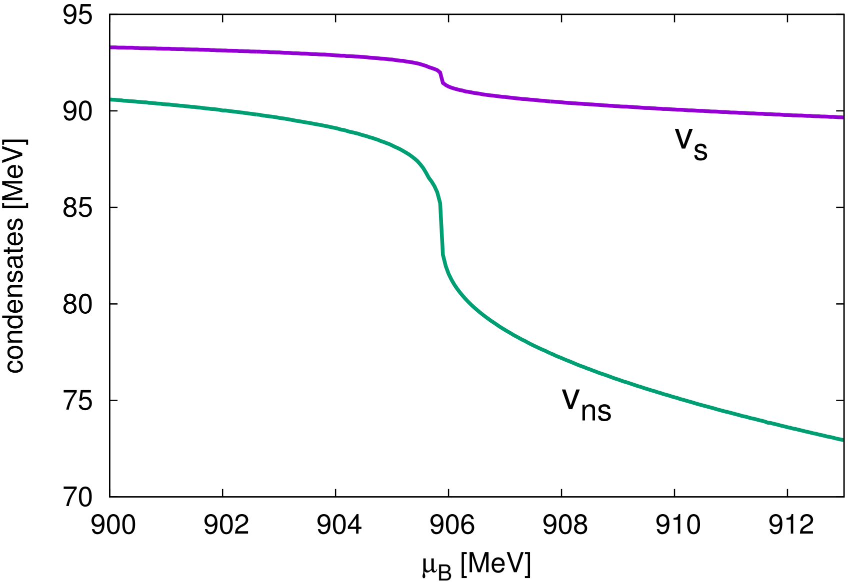

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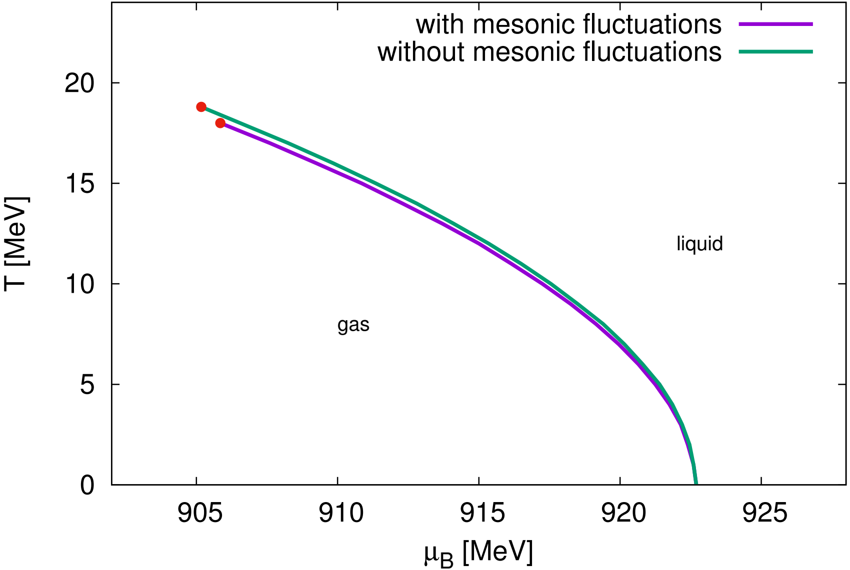

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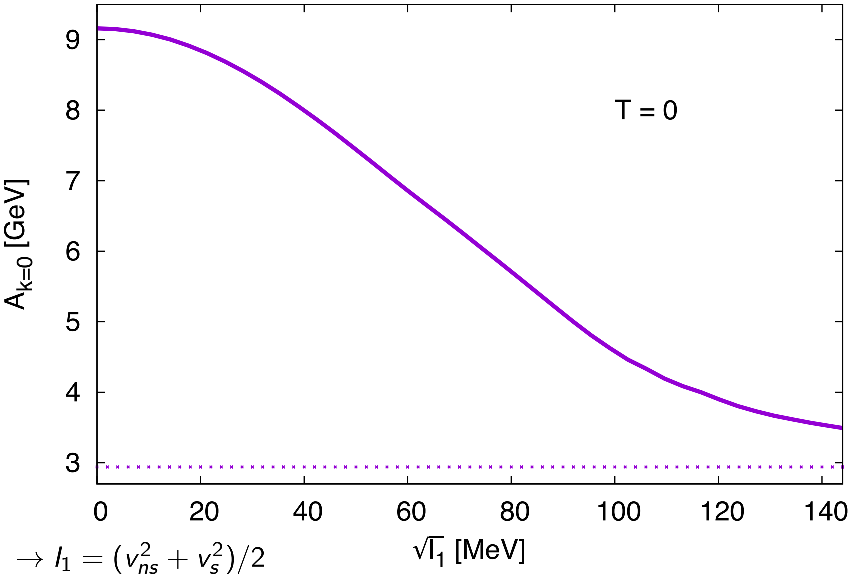

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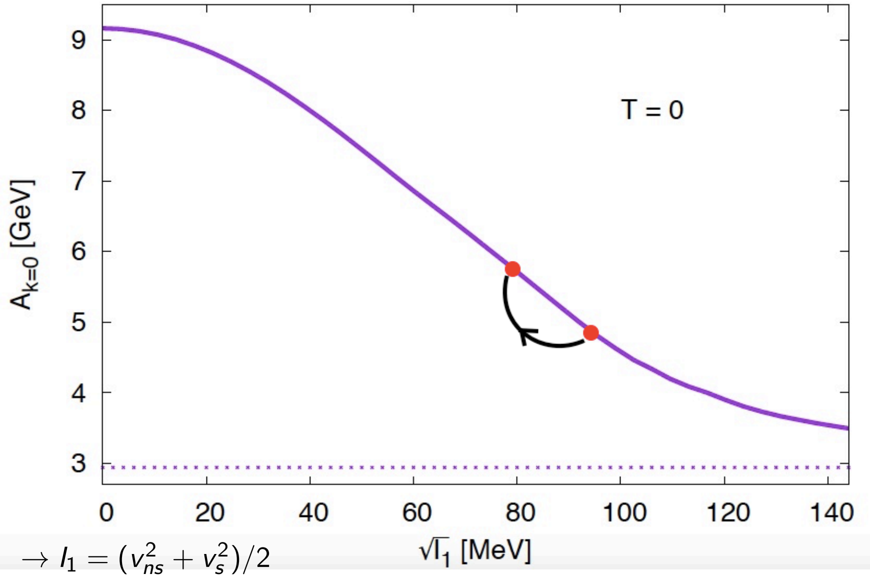

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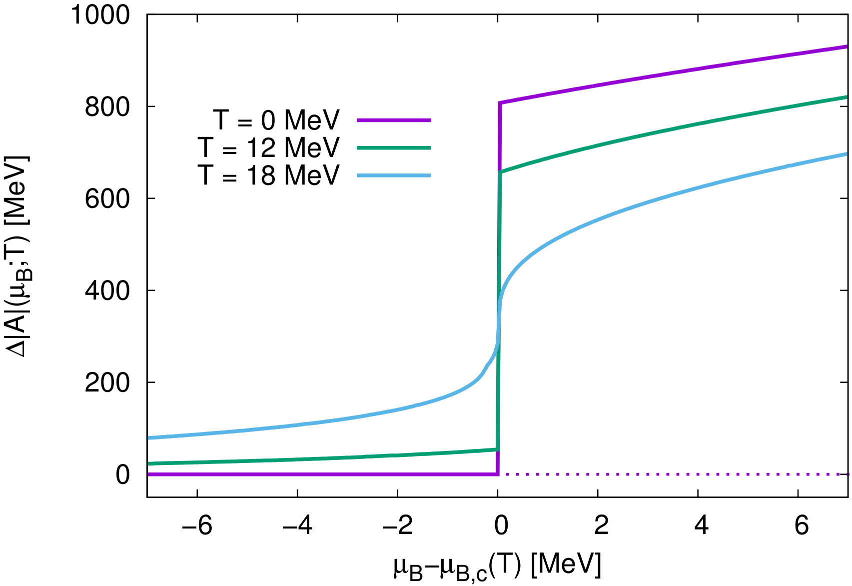
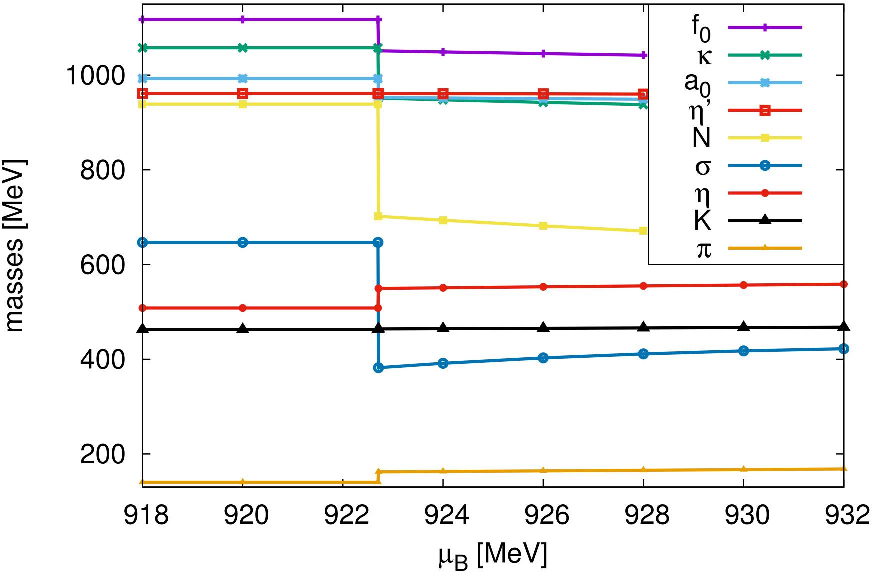


1000
800
600
400
200
0
Δ|A|(μB;T) [MeV]
T = 0 MeV
T = 12 MeV
T = 18 MeV
−6
−4
−2
0
2
4
6
μB−μB,c(T) [MeV]

Numerical results



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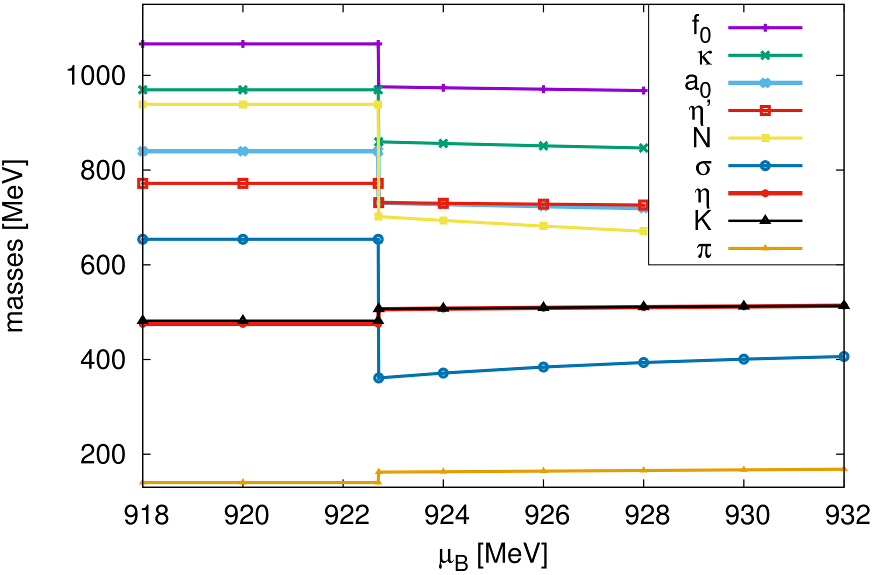

Numerical results

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- Current effect: mesonic quantum fluctuations, <u>not</u> instanton contributions
  $\longrightarrow$ backreaction of the anomaly on itself
  $\longrightarrow$ mean field theory is questionable
- Even the bare anomaly coefficient A can depend explicitly on T and μ_B!
  $\longrightarrow$ competition between instantons and mesonic loop effects
  $\longrightarrow$ extension: assume a form of $A = A(T, \mu_B)$

Summary

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$\longrightarrow$ (partial) restoration of chiral symmetry seem to increase the anomaly ($\Delta|A| \gtrsim 15\%$ relative difference)

$\longrightarrow$ nuclear transition: $\sim 20\%$ drop in (n.s.) chiral cond.

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  $\longrightarrow$ η' mass is smooth at the transition point
  $\Rightarrow \eta' N$ bound state?
- Important:

  $\longrightarrow$ no instanton effects have been included!

  $\longrightarrow$ environment dependence of the bare anomaly coefficient could be relevant!