

Hybrid star constructions with effective field theories

Péter Kovács

Senior Research Fellow
Wigner RCP

Collaborator:
János Takátsy

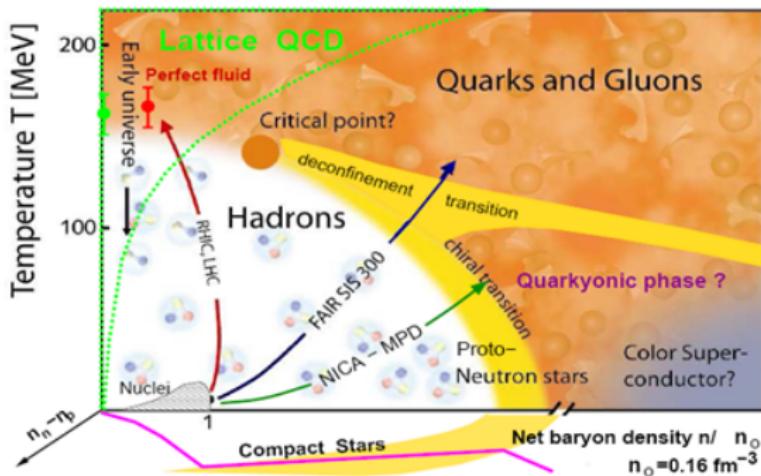


Zimányi School'19, 2-6 December 2019

Supported by the ÚNKP-19-4 New National Excellence Program of the
Ministry for Innovation and Technology.



Envisaged phase diagram of QCD



Compact stars living at $T \approx 0 \text{ MeV}$.

Observation of compact stars can help to understand strongly interacting matter in medium and investigation of strongly interacting matter in medium can be used to describe compact stars

Structure of compact stars

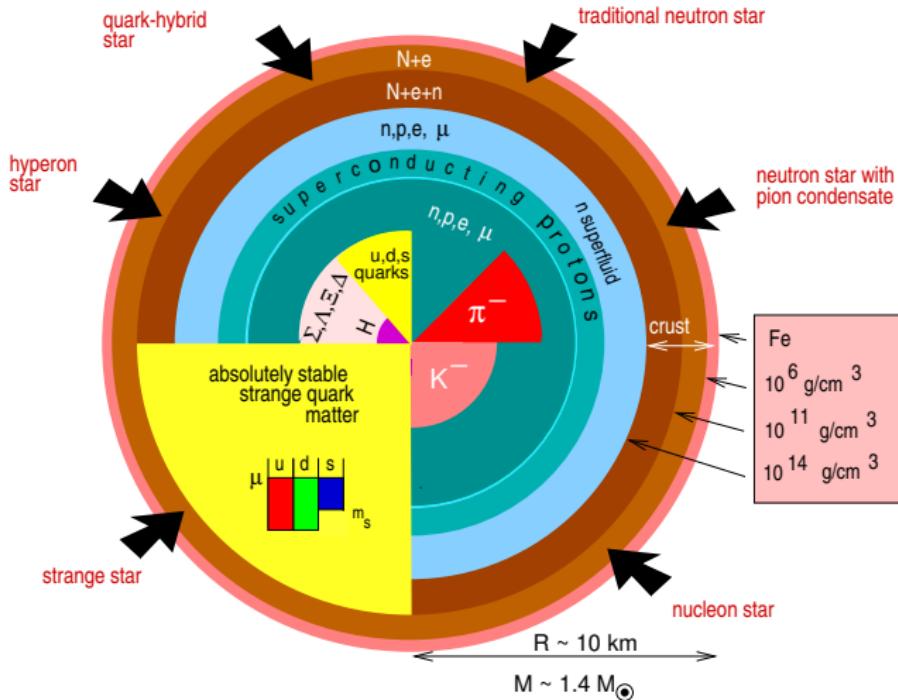
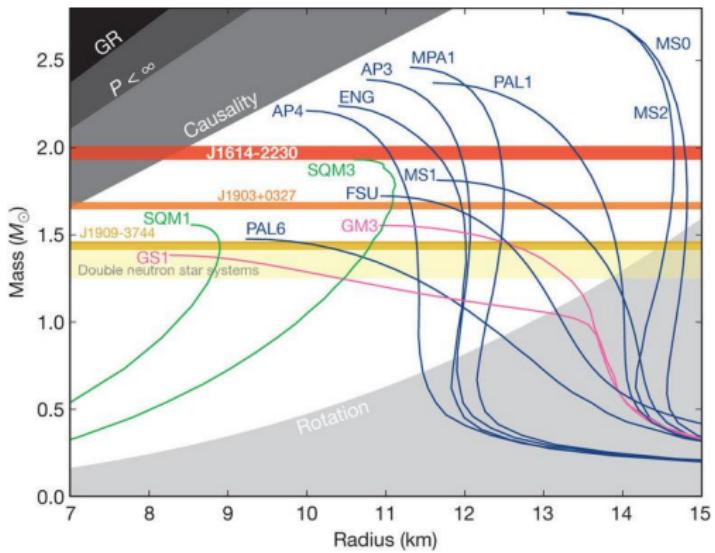


Fig. from F. Weber, J. Phys. G **27**, 465 (2001)

Various $M - R$ curves for different compact star EoS's

- ▶ QCD directly unsolvable at finite density
- ▶ One can use effective models in the zero temperature finite density region
- ▶ Neutron star observations restrict such models [1,2]



$M - R$ relations from various models [1]

[1] P. Demorest et al. (2010), *Nature* **467**, 1081

[2] J. Antoniadis et al. (2013), *Science* **340**, 6131

Neutron star models (two flavours)

EoS Name	Reference	$\rho \leq \rho_{ND}$	$\rho_{ND} \leq \rho < \rho_0$	$\rho_0 \leq \rho$	Method	Comments
BPS	Baym et al. (1971)	B.En.+El.+ Lattice	X	X	Empirical binding energies	Commonly used for low-density regime
HP (HP94)	Haensel & Pichon (1994)	B.En.+El.+ Lattice	X	X	Empirical binding energies	Includes electron screening
NV	Negele & Vauth. (1974)	X	Hartree- Fock	X	Variational	Used for intermediate densities
FPS	Lorenz (1991)	CLDM	CLDM	$np+e\mu$	Density functional	Unified EoS with FP Skyrme potential
Sly	Douchin & Haensel (2001)	CLDM	CLDM	$np+e\mu$	Density functional	Unified EoS with Skyrme Lyon potential
WFF (WFF1)	Wiringa et al. (1988)	X	X	$np+e\mu$	Variational	A14+UVII
APR (AP4)	Akmal & Pand. (1997)	X	X	$np+e\mu$	Variational	A18+UIX*+ δv
MPA (MPA1)	Müther et al. (1987)	X	X		Rel. Brueckner HF	
ENG	Engvik et al. (1996)	X	X		Rel. Brueckner HF	
PAL	Prakash et al. (1988)	X	X	$np+e(\mu?)$	Schematic potential	Parameterizing E and S (symmetry energy)
GM	Glend. & Moszk. (1991)	X	X	$npH+e\mu$	Field theoretical	Same as the model in Glendenning's book
H4	Lackey et al. (2006)	X	X	$npH+e\mu$	Field theoretical	Stiffest EoS compatible with nuclear constraints
MS (MS1)	Müller & Serot (1996)	X	X	$np(+e\mu?)$	Field theoretical	Nonlinear mesonic potentials
SQM	Farhi & Jaffe (1984)	uds+e			Bag model	

These models are used for the entire neutron star.

BPS, NV and APR are commonly used together

Hybrid star models

Hybrid stars: Compact stars with quark matter in the core.

Different approaches in the literature:

- ▶ BPS or BPS + NV at very low ρ_B
- ▶ Some nuclear model at low ρ_B (2 or 3 flavour): Walecka model, Parity doublet model, Relativistic Mean-Field (RMF) models
- ▶ Quark matter at high ρ_B (2 or 3 flavour): Nambu-Jona-Lasinio (NJL) model, Linear sigma model (LSM)

How to combine models at low density with models at high density?

→ Various approaches exist: Quark-Meson-Nucleon model (QMN) with statistical confinement; Hadron-quark crossover with P-interpolation; Energy minimization method; Coexisting phases method; Gibbs construction; Maxwell construction

QMN with statistical confinement

based on: *S. Benić et al., Phys. Rev. D, 91, 125034 (2015);
M. Marczenko et al., Phys. Rev. D, 98, 103021 (2018)*

Features of the model:

- ▶ Two flavour parity doublet model with mirror assignment (*N(938)*, *N(1500)*, $\pi(138)$, $f_0(500)$ (or σ), $\omega(782)$, $\rho(770)$)
- ▶ Linear sigma model (*u, d* constituent quarks, $\pi(138)$, $f_0(500)$), quarks are not coupled to vectors
- ▶ Tree-level mesons, one-loop fermions (mean-field approximation)

Grand canonical potential:

$$\Omega = \sum_{x \in (\textcolor{red}{p}_{\pm}, \textcolor{red}{n}_{\pm}, \textcolor{red}{u}, \textcolor{red}{d})} \Omega_x + V_\sigma + V_\omega + \textcolor{blue}{V}_b + V_\rho$$

$$\Omega_x = \gamma_x \int \frac{\mathbf{d}^3 p}{(2\pi)^3} T [\ln(1 - \textcolor{blue}{n}_x) + \ln(1 - \bar{n}_x)].$$

Statistical confinement with auxiliary field

Concatenation at the level of the grand potential. Nucleons have to be suppressed at high ρ_B , while quarks at low ρ_B

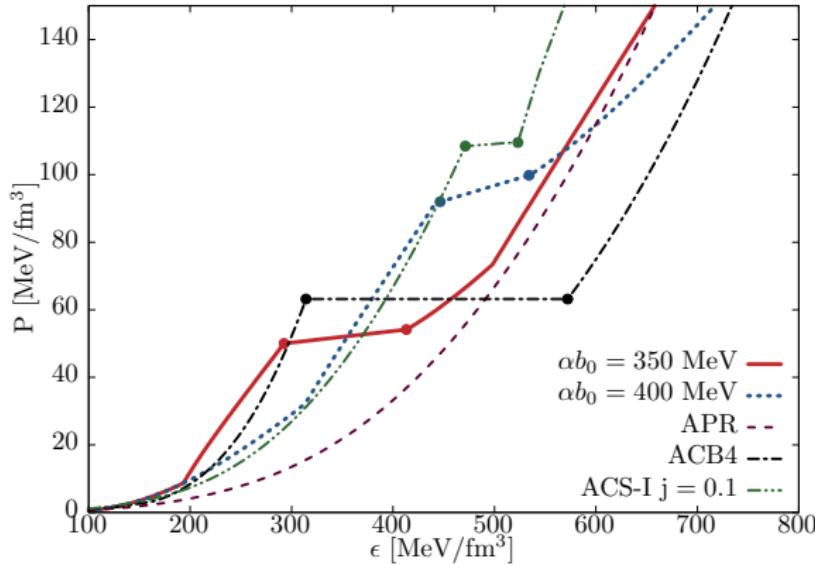
⇒ Modified Fermi-Dirac distributions:

$$\begin{aligned} n_{\pm} &= \theta(\alpha^2 b^2 - \mathbf{p}^2) f_{\pm} \\ \bar{n}_{\pm} &= \theta(\alpha^2 b^2 - \mathbf{p}^2) \bar{f}_{\pm} \\ n_q &= \theta(\mathbf{p}^2 - b^2) f_q \\ \bar{n}_q &= \theta(\mathbf{p}^2 - b^2) \bar{f}_q \end{aligned}$$

b is a T and μ_B dependent bag field with $\langle b \rangle = b_0$

b might be associated with chromoelectric part of the gluon sector

EoSs for the QMN model



Chiral phase transition is shown, deconfinement starts at very large ρ_B ; Other EoSs are shown for comparison

Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter \rightarrow TOV eqs.:

$$\frac{dp}{dr} = -\frac{[p(r) + \varepsilon(r)] [M(r) + 4\pi r^3 p(r)]}{r[r - 2M(r)]} \quad (1)$$

with

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- ▶ For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- ▶ Varying ε_c a series of compact stars is obtained (with given M and R)
- ▶ Once the maximal mass is reached, the stable series of compact stars ends

$M - R$ curves for the QMN model

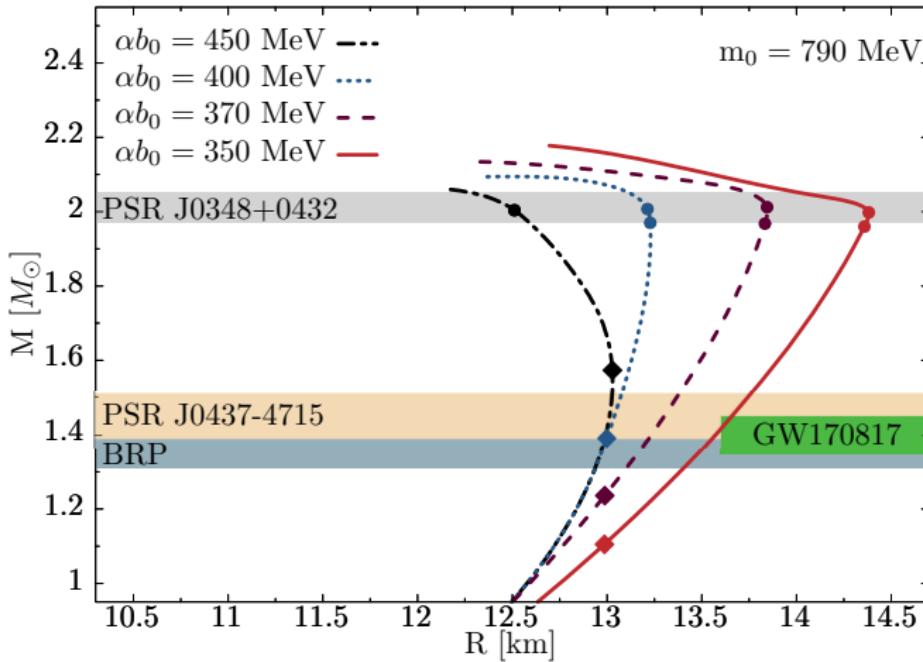
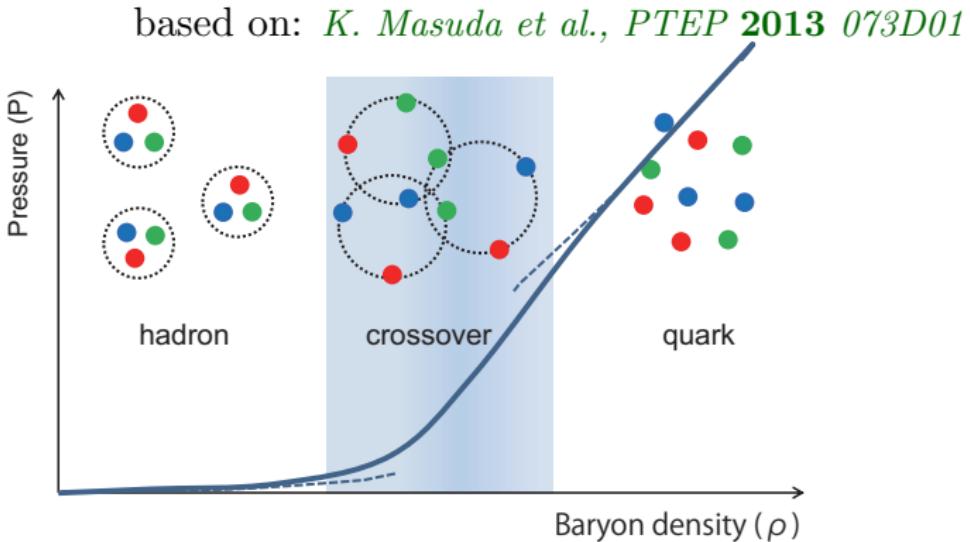


Fig. from *M. Marczenko et al., Phys. Rev. D, 98, 103021 (2018)*

Schematic picture of pressure (H-Q crossover)



In the crossover region hadrons starts to overlap
→ both low and high ρ_B models loose their validity.
Gibbs condition (extrapolation from the dashed lines) can be misleading.

Hadron-quark crossover with P -interpolation

Features of the model:

- ▶ H-EOS: Two and three flavour hadronic EoSs with Y -mixing: TNI2, TNI3, TNI2u, TNI3u, AV18+TBF, SCL3 $\Lambda\Sigma$
- ▶ Q-EOS: NJL-model with u, d, s quarks and vector interaction
- ▶ mean-field approximation

P -interpolation ($\rho = \rho_B$):

$$P(\rho) = P_H(\rho)f_-(\rho) + P_Q(\rho)f_+(\rho), \quad (2)$$

$$f_{\pm}(\rho) = \frac{1}{2} \left(1 \pm \tanh \left(\frac{\rho - \bar{\rho}}{\Gamma} \right) \right) \quad (3)$$

$$\varepsilon(\rho) = \varepsilon_H(\rho)f_-(\rho) + \varepsilon_Q(\rho)f_+(\rho) + \Delta\varepsilon \quad (4)$$

$$\Delta\varepsilon = \rho \int_{\bar{\rho}}^{\rho} (\varepsilon_H(\rho') - \varepsilon_Q(\rho')) \frac{g(\rho')}{\rho'} d\rho' \quad (5)$$

$P(\rho)$ for hadronic matter for diff. models

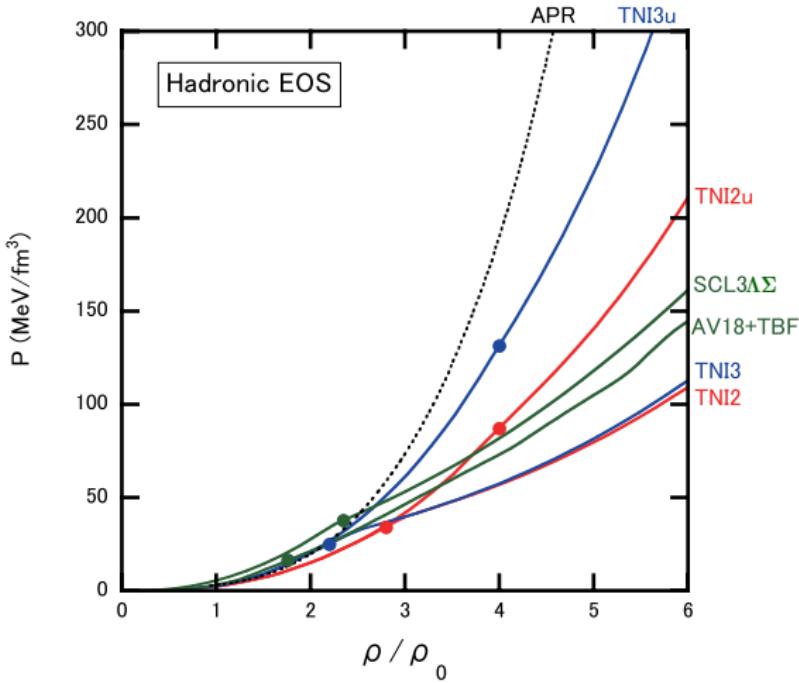


Fig. from *K. Masuda et al., PTEP 2013 073D01*

$P(\rho)$ for pure quark matter for diff. g_V s

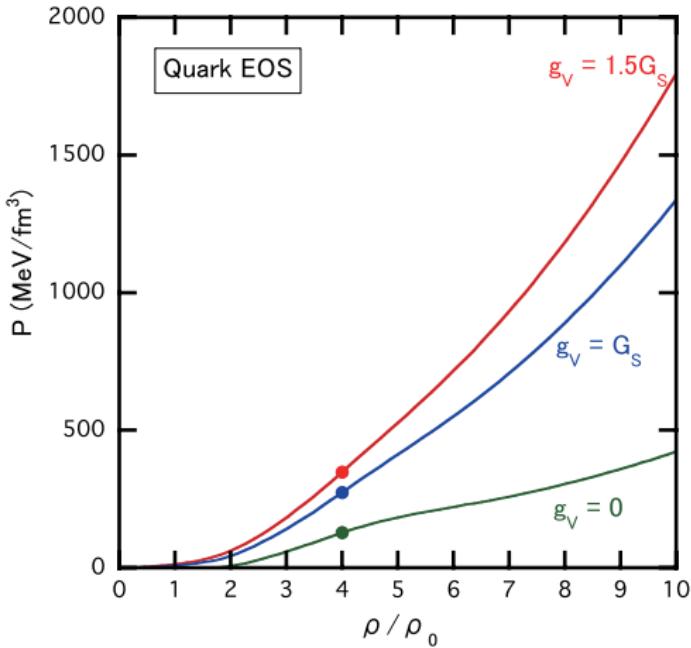


Fig. from *K. Masuda et al., PTEP 2013 073D01*

Interpolated pressure

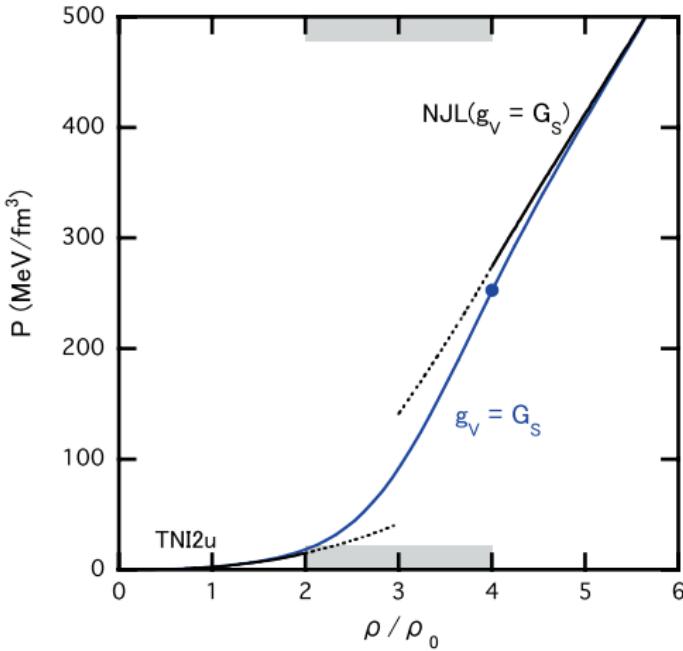
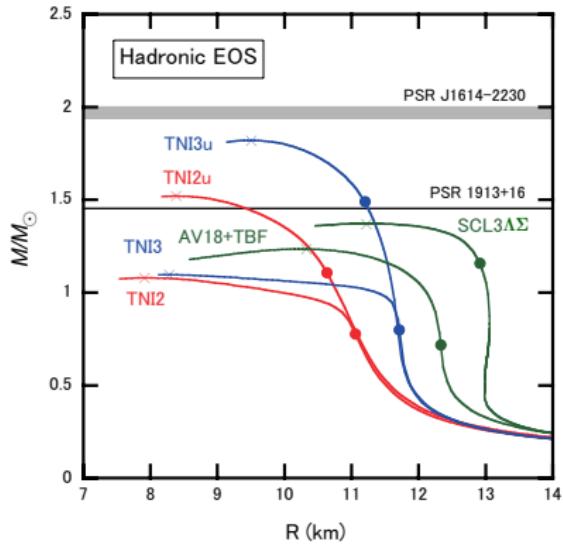
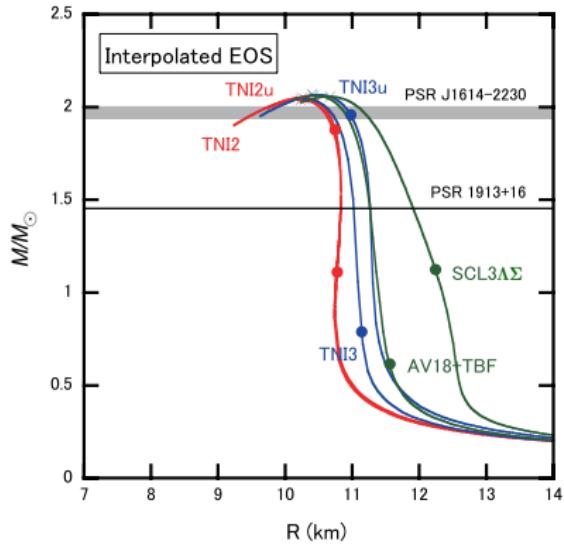


Fig. from [K. Masuda et al., PTEP 2013 073D01](#)

$M - R$ curves for hadron-quark crossover model



(a)



(b)

Hyperons make the EoS softer $\rightarrow 2M_\odot$ limit not reached

Energy minimization (EM) method

based on: *X. H. Wu et al., PRC **99**, 065802 (2019)*

Features of the model:

- ▶ Hadronic matter: Relativistic mean-field (RMF) model (2 flavours, nucleons interacting through σ , ω and ρ mesons, mesons treated at tree-level, additional $\omega - \rho$ interaction)
- ▶ Quark matter: NJL-model with u, d, s quarks and vector interaction
- ▶ mean-field approximation

Total energy of the mixed phase:

$$\varepsilon_{\text{MP}} = u\varepsilon_{\text{QP}} + (1-u)\varepsilon_{\text{HP}} + \varepsilon_{\text{surf}} + \varepsilon_{\text{Coul}} \quad (6)$$

$$u = V_{\text{QP}} / (V_{\text{QP}} + V_{\text{HP}}) \quad (7)$$

minimization w.r.t. the densities ($n_\rho, n_n, n_u, n_d, n_s, n_e, n_\mu$) and u gives equilibrium conditions (under global charge neutrality and baryon number conservation)

Special cases of the EM method

- ▶ Coexisting phases method: $\varepsilon_{\text{surf}}$ and $\varepsilon_{\text{Coul}}$ are treated perturbatively (minimization or Gibbs condition without surf. and Coul. terms) $\Rightarrow P_{HP} = P_{QP}$, and
$$\varepsilon_{\text{MP}} = u\varepsilon_{\text{QP}} + (1 - u)\varepsilon_{\text{HP}} + \varepsilon_{\text{surf}} + \varepsilon_{\text{Coul}}$$
- ▶ Gibbs construction: $\varepsilon_{\text{surf}}$ and $\varepsilon_{\text{Coul}}$ are neglected, $\sigma \approx 0$, global charge neutrality, hadronic and quark phases can be charged separately, $\Rightarrow P_{HP} = P_{QP}$, and
$$\varepsilon_{\text{MP}} = u\varepsilon_{\text{QP}} + (1 - u)\varepsilon_{\text{HP}}$$
- ▶ Maxwell construction: $\varepsilon_{\text{surf}}$ and $\varepsilon_{\text{Coul}}$ are neglected, $\sigma >> 0$, local charge neutrality, both hadronic and quark phases charge neutral $\Rightarrow P_{HP} = P_{QP}$, and
$$\varepsilon_{\text{MP}} = u\varepsilon_{\text{QP}} + (1 - u)\varepsilon_{\text{HP}}$$

Pressure with the EM method

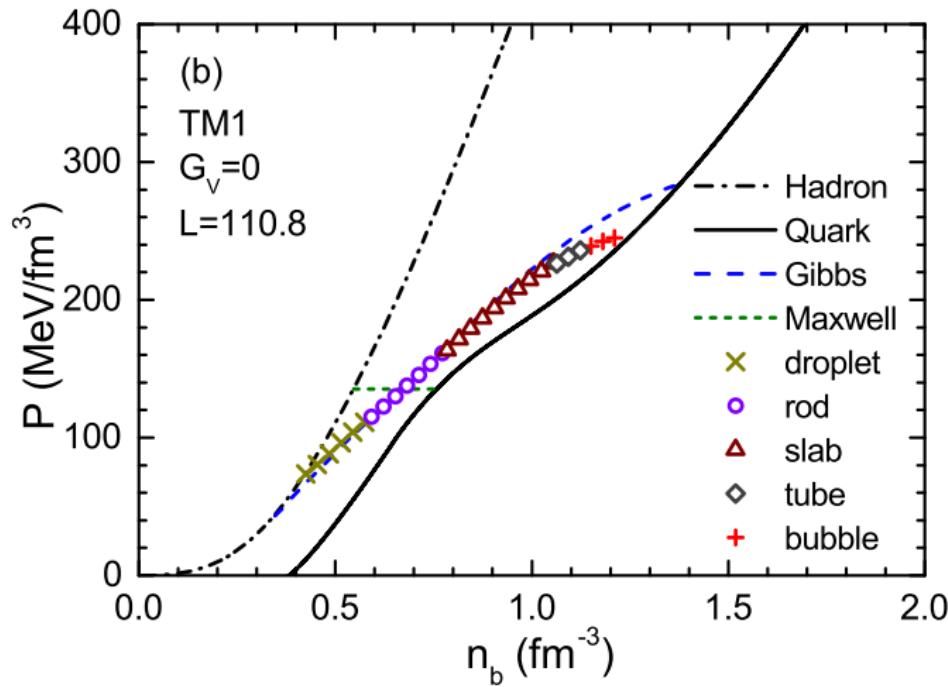
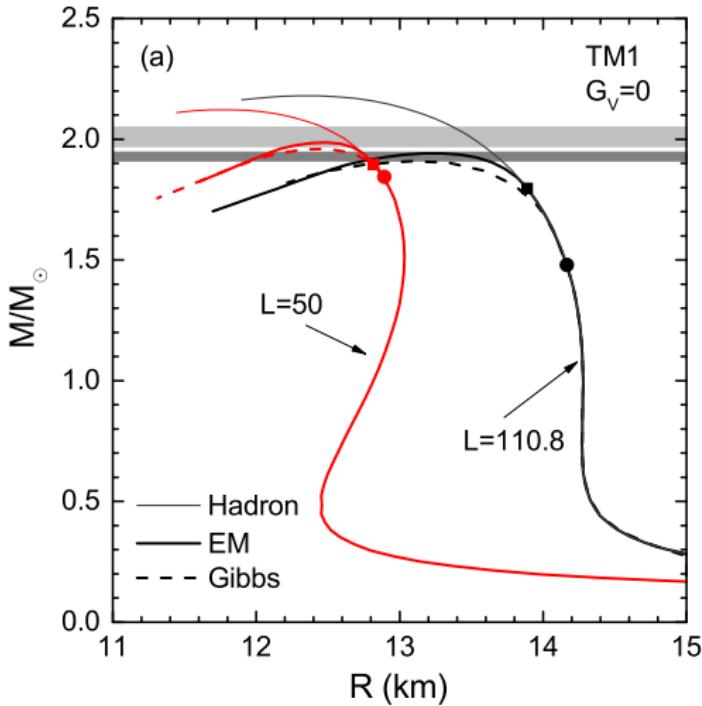


Fig. from X. H. Wu et al., PRC 99, 065802 (2019)

$M - R$ curves with the EM method



$M - R$ curves with the EM method

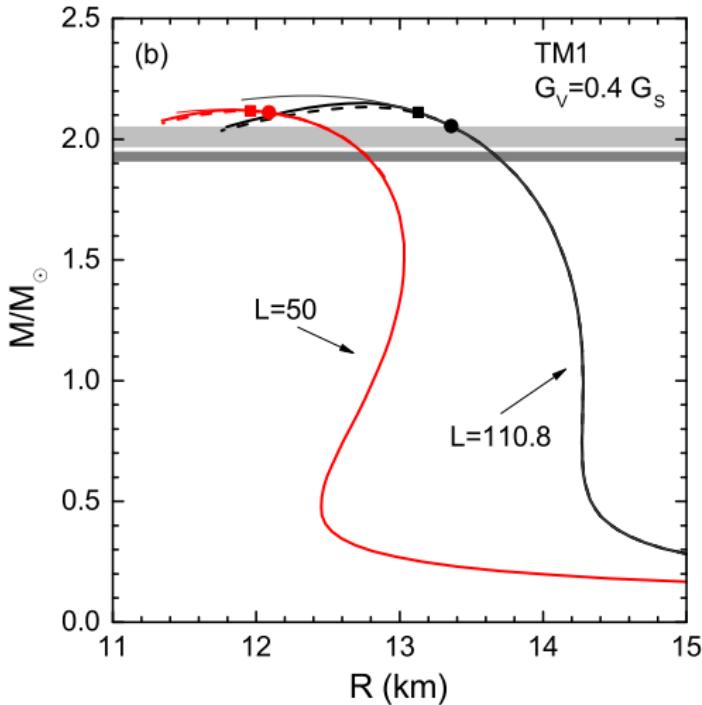


Fig. from X. H. Wu et al., PRC 99, 065802 (2019)

Lagrangian of the eLSM

based on: *P. Kovács et al. Phys. Rev. D* **93**, no. 11, 114014 (2016),
J. Takátsy et al., Universe **5**, 174 (2019)

$$\begin{aligned} \mathcal{L} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & + \textcolor{red}{c_1} (\det \Phi + \det \Phi^\dagger) + \textcolor{cyan}{\text{Tr}[H(\Phi + \Phi^\dagger)]} - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \text{Tr} \left[\left(\frac{m_1^2}{2} \mathbb{1} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\ & + \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi, \end{aligned}$$

$$\begin{aligned} D^\mu \Phi &= \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ie A_e^\mu [T_3, \Phi], \\ L^{\mu\nu} &= \partial^\mu L^\nu - ie A_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ie A_e^\nu [T_3, L^\mu]\}, \\ R^{\mu\nu} &= \partial^\mu R^\nu - ie A_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ie A_e^\nu [T_3, R^\mu]\}, \\ D^\mu \Psi &= \partial^\mu \Psi - i G^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a. \end{aligned}$$

+ Polyakov loop potential (for $T > 0$)

Determination of the parameters

14 unknown parameters ($m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F$) → determined by the min. of χ^2 :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[\frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

$(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1, \dots, x_N)$ → from the model, Q_i^{exp} → PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multipiparametric minimization → MINUIT

- ▶ PCAC → 2 physical quantities: f_π, f_K
- ▶ Curvature masses → 16 physical quantities:
 $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- ▶ Decay widths → 12 physical quantities:
 $\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^\star \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^\star}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi}, \Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$
- ▶ Pseudocritical temperature T_c at $\mu_B = 0$

Features of our approach

- ▶ D.O.F's:
 - scalar, pseudoscalar, vector, and axial-vector nonets
 - u, d, s constituent quarks ($m_u = m_d$)
 - Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\text{YM}}$ or $\mathcal{U}_{\log}^{\text{glue}}$
- ▶ no mesonic fluctuations, only fermionic ones

$$\mathcal{Z} = e^{-\beta V \Omega(T, \mu_q)} = \int_{\text{PBC}} \prod_a \mathcal{D}\xi_a \int_{\text{APBC}} \prod_f \mathcal{D}q_f \mathcal{D}q_f^\dagger \exp \left[-\int_0^\beta d\tau \int_V d^3x \left(\mathcal{L} + \mu_q \sum_f q_f^\dagger q_f \right) \right]$$

approximated as $\Omega(T, \mu_q) = U_{\text{meson}}^{\text{tree}}((M)) + \Omega_{\bar{q}q}^{(0)}(T, \mu_q) + \mathcal{U}_{\log}(\Phi, \bar{\Phi})$, $\tilde{\mu}_q = \mu_q - iG_4$

$$e^{-\beta V \Omega_{\bar{q}q}^{(0)}} = \int_{\text{APBC}} \prod_{f,g} \mathcal{D}q_g \mathcal{D}q_f^\dagger \exp \left\{ \int_0^\beta d\tau \int_x q_f^\dagger \left[\left(i\gamma_0 \vec{\gamma} \cdot \vec{\nabla} - \frac{\partial}{\partial \tau} + \tilde{\mu}_q \right) \delta_{fg} - \gamma_0 \mathcal{M}_{fg} \Big|_{\xi_a=0} \right] q_g \right\}$$

- ▶ tree-level (axial)vector masses
- ▶ fermionic vacuum and thermal fluctuations included in the (pseudo)scalar curvature masses used to parameterize the model
- ▶ 4 coupled T/μ_B -dependent field equations for the condensates $\phi_N, \phi_S, \Phi, \bar{\Phi}$
- ▶ thermal contribution of π, K, f_0^L included in the pressure, however their curvature mass contains no mesonic fluctuations

Inclusion of vector meson Yukawa term

$$\mathcal{L}_{\text{Yukawa-vec}} = -g_v \sqrt{6} \bar{\Psi} \gamma_\mu V_0^\mu \Psi$$

$$V_0^\mu = \frac{1}{\sqrt{6}} \text{diag}\left(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8\right)$$

mean-field treatment

$$\langle v_0^\mu \rangle = v_0 \delta^{0\mu}, \quad \langle v_8^\mu \rangle = 0^\mu$$

Modification of the grand canonical potential

$$\Omega(T=0, \mu_q) \rightarrow \Omega(T=0, \tilde{\mu}_q) - \frac{1}{2} m_v^2 v_0^2, \text{ with } \tilde{\mu}_q = \mu_q - g_v v_0$$

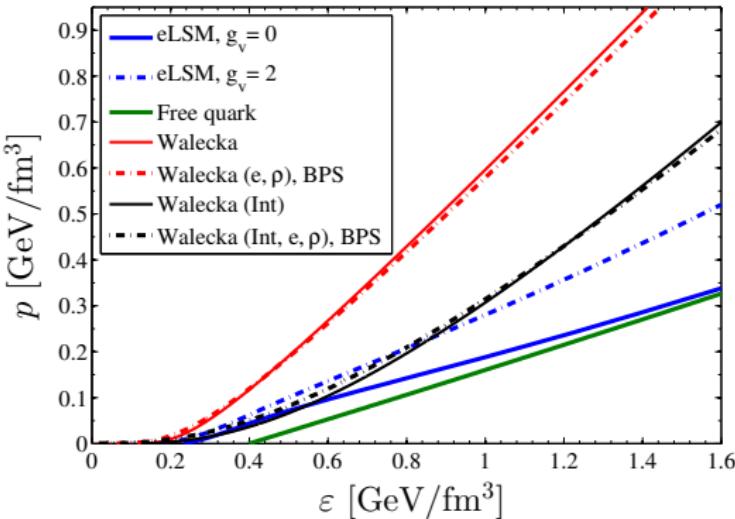
While the field equations

$$\frac{\partial \Omega}{\partial \phi_N} \Big|_{\phi_N=\bar{\phi}_N} = \frac{\partial \Omega}{\partial \phi_S} \Big|_{\phi_S=\bar{\phi}_S} = 0 \quad \text{and} \quad \frac{\partial \Omega}{\partial v_0} \Big|_{v_0=\bar{v}_0} = 0,$$

The EOS of eLSM

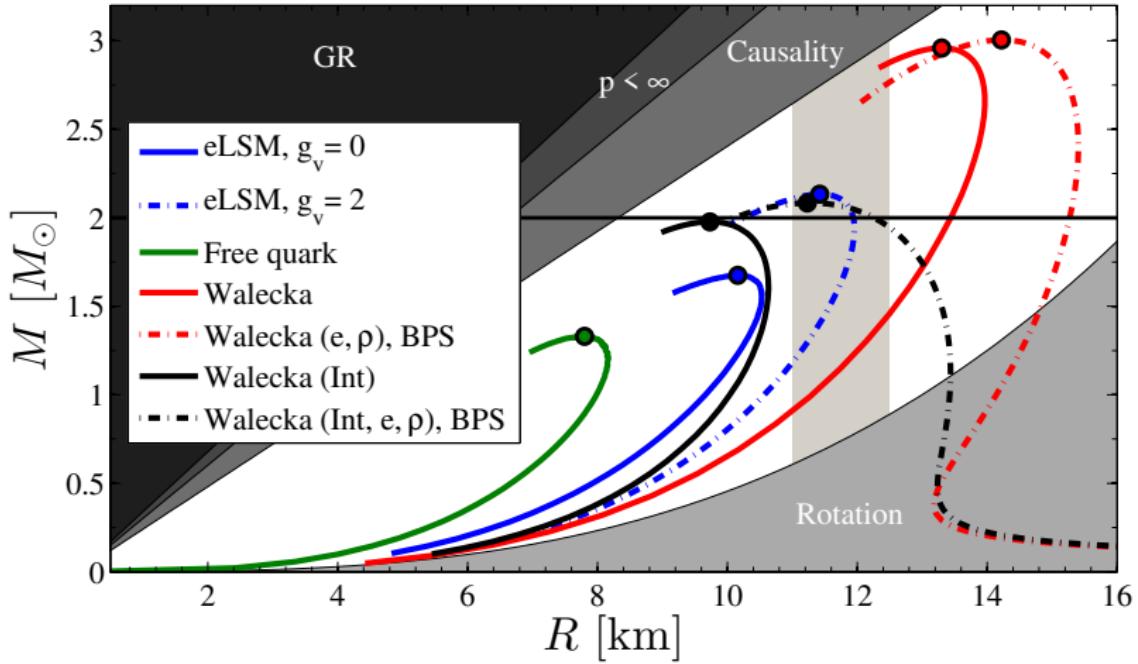
- ▶ Pure eLSM compared to Walecka and free quark models
- ▶ At low energies the EoS of the eLSM is close to the EoS of the Walecka - model
- ▶ At higher energies it tends to the EoS of the free quark model
- ▶ note: Walecka Int means

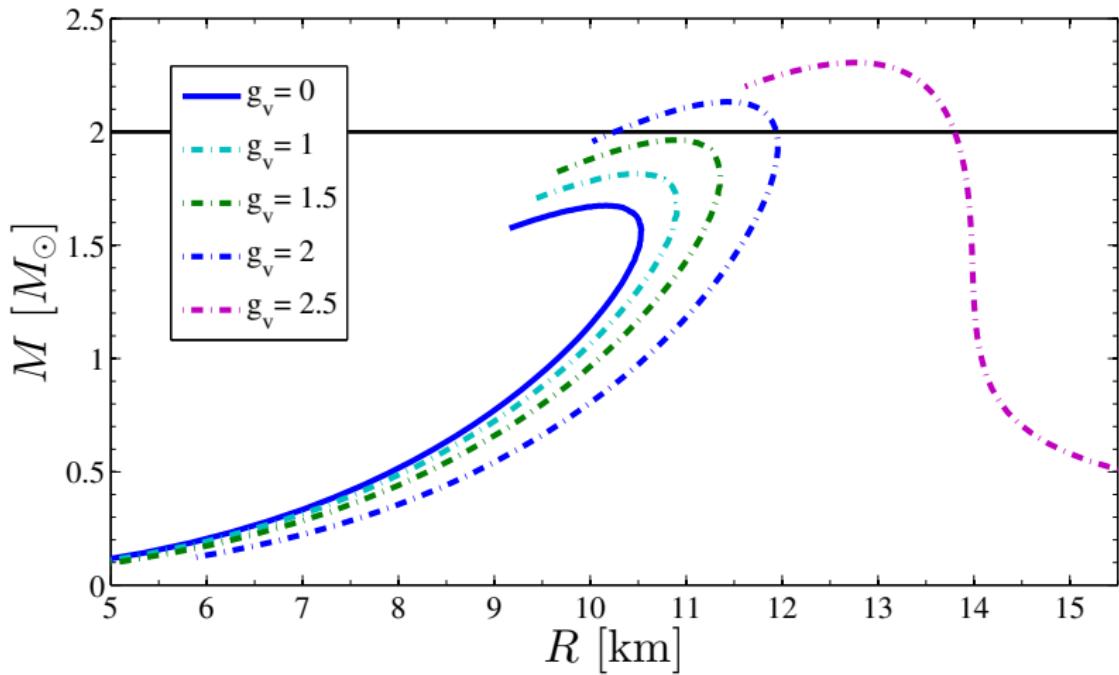
$$\mathcal{L}_{W,Int} = -\frac{b}{3}m_n(g_\sigma\sigma)^3 - \frac{c}{4}(g_\sigma\sigma)^4$$



- ▶ In the Walecka models electrons and ρ mesons are also included (dashed-dotted lines)

$M - R$ relations of eLSM



Changing the g_v vector coupling in eLSM

For $g_v \gtrsim 2.5$ $p(\varepsilon)$ becomes zero at $\varepsilon = 0 \Rightarrow$ small M , large R

Conclusion

Conclusion

- ▶ Different hybrid star constructions exist → current astrophys. restrictions (like $2M_\odot$ limit) can be fulfilled in many ways
- ▶ The quark–vector meson Yukawa interactions have a very important role in the EoS and consequently in the $M - R$ curves
- ▶ Even pure quark stars are not excluded by current observations

Plans

- ▶ Different hybrid star constructions with eLSM
- ▶ Better (more consistent) approximations in the eLSM part
- ▶ Inclusion of the total vector-quark Yukawa term, consistent treatment
- ▶ Beyond mean-field calculations

Thank you for your attention!

Particle content

- **Vector** and **Axial-vector** meson nonets

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$\rho \rightarrow \rho(770)$, $K^* \rightarrow K^*(894)$

$\omega_N \rightarrow \omega(782)$, $\omega_S \rightarrow \phi(1020)$

$a_1 \rightarrow a_1(1230)$, $K_1 \rightarrow K_1(1270)$

$f_{1N} \rightarrow f_1(1280)$, $f_{1S} \rightarrow f_1(1426)$

- **Scalar** ($\sim \bar{q}_i q_j$) and **pseudoscalar** ($\sim \bar{q}_i \gamma_5 q_j$) meson nonets

$$\Phi_S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{*+} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{*0} \\ K_0^{*-} & \bar{K}_0^{*0} & \sigma_S \end{pmatrix}$$

$$\Phi_{PS} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

unknown assignment

mixing in the $\sigma_N - \sigma_S$ sector

$\pi \rightarrow \pi(138)$, $K \rightarrow K(495)$

mixing: $\eta_N, \eta_S \rightarrow \eta(548)$, $\eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$
fields shifted by their expectation value: $\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S}$