Baryon fluctuations in Extended linear sigma model

Győző Kovács
PhD student
ELTE University

Collaborators:
Peter Kovacs, Wigner
Gyorgy Wolf, Wigner
Zsolt Szep, ELTE

Zimányi School, Budapest
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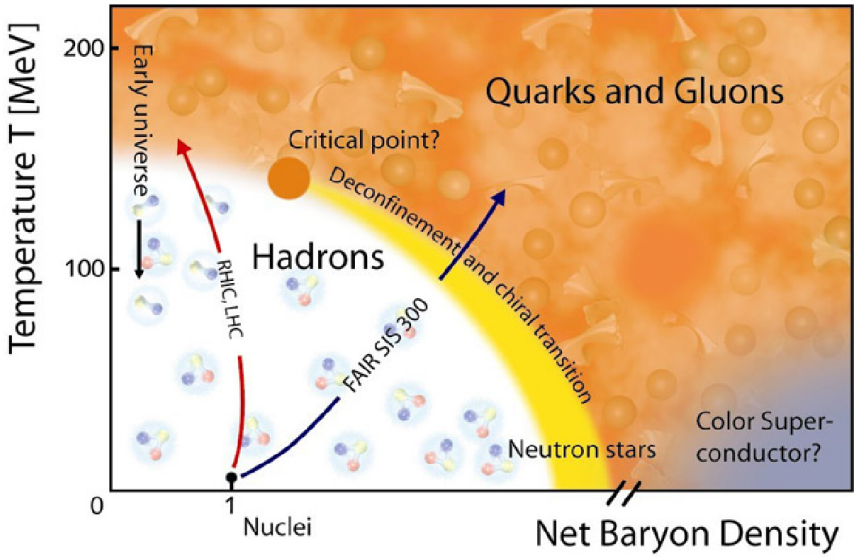
Outline

- Motivations.
- Extended linear sigma model and its thermodynamics.
- Baryonfluctuations in the ELσM.
- Outlook.

Critical end point

The phase structure of the strongly interacting matter, especially the region of chiral phase transition, is a heavily studied field of particle physics.



- There might be a first order phase transition.
- At $\mu_q = 0$ there is a crossover according to lattice calculations.
- There is a Critical end point (CEP) between the two regions.
- Effective models trying to prove the existence and location the CEP.
- The upcomming FAIR experiment will hopefully give insight into the energy region, where the CEP might be. PS **94**, no. 3, 033001 (2019)

Uncertainty of CEP

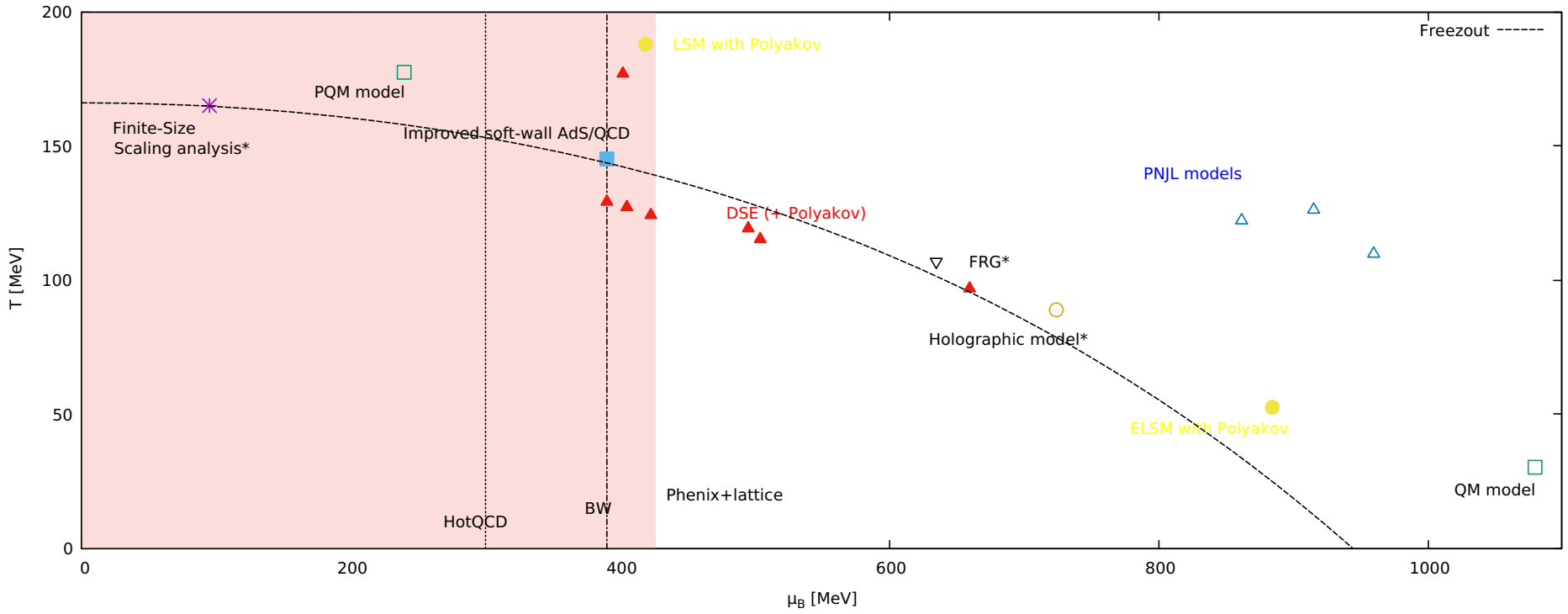


Figure: Location of CEP according to different models and the freeze-out line for comparison. The shadowed region and the vertical lines corresponds to lower limits of the CEP.

Budapest-Wupperal	Phys. Lett. B **751**, 559 (2015)	Phenix	Phys. Rev. C **93**, no. 1, 011901 (2016)
HotQCD	Phys. Lett. B **795**, 15 (2019)	Freezout	J. Phys. G **32**, S165 (2006)

Extended linear sigma model

Already existing model, published in: Phys. Rev. D **93**, no. 11, 114014 (2016) .

- Linear sigma model with full Scalar, Pseudoscalar, Vector and Axialvector nonets.
- Isospin symmetric case: 16 mesonic degrees of freedom.
- The Lagrangian build up from the fields

$$L^\mu = \sum_a (V_a^\mu + A_a^\mu)\, T_a, \quad R^\mu = \sum_a (V_a^\mu - A_a^\mu)\, T_a, \quad M = \sum_a (S_a + iP_a)\, T_a, \tag{1}$$

  with terms up to fourth order, taking care of the symmetry properties.
- Constituent quarks (2+1 flavors) are included in a Yukawa type term,

$$\mathcal{L}_{Yukawa} = \bar{\psi}\, [i\gamma_\mu D^\mu - \mathcal{M}]\, \psi, \tag{2}$$

  where $\mathcal{M} = g_F\, (\mathbb{1} M_S + i\gamma_5 M_P)$ is the quark mass matrix and $M_S = \sum_a S_a T^a, M_P = \sum_a P_a T^a$.

Extended linear sigma model

- The parameters of the Lagrangian are fitted with χ^2 test. The used physical parameters are the (pseudo)scalar meson masses, decay widths and the $T_C(\mu = 0)$ critical temperature, getting from lattice simulations.
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- In the 0-8 sector N/S bases is used

$$\xi_N = \frac{1}{\sqrt{3}}\left(\sqrt{2}\xi_0 + \xi_8\right) \qquad \xi_S = \frac{1}{\sqrt{3}}\left(\xi_0 - \sqrt{2}\xi_8\right). \tag{3}$$

- All fields can be written as $\varphi(x) := \phi + \varphi'(x)$, where $\varphi'(x)$ has no vacuum expectation value (vev).
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  Only the scalar-isoscalar sector has nonzero vev denoted as: ϕ_N, ϕ_S.
- Including Polyakov loop variables $\langle\Phi\rangle, \langle\bar{\Phi}\rangle$ as further order parameters, which mimic the deconfinment phase transition.

Grand potential

Assumptions

- Symmetric quark matter: $\mu_u = \mu_d = \mu_s = \mu_q = \mu_B/3$
- Fermionic determinant calculated with neglecting mesonic fluctuations, ie. (pseudo)scalar mesons treated at tree-level only.
- Effect of mesonic thermal fluctuations (of the three lightest: π, K, f_0^L can be) included in the level of pressure.

The Grand potential reads

$$\Omega(T, \mu_q) = U(\langle M \rangle) + U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle) + \Omega^{(0)}_{\bar{q}q}(T, \mu_q) \tag{4}$$

where the terms are the tree-level mesonic potential, the Polyakov loop potential, and the contribution of fermionic fluctuations respectively.

Grand potential

$U(\langle M \rangle)$ depends on the (up to fourth order of) scalar-isoscalar background and the parameters of the Lagrangian. It also contains the counterterms for the renormalization of the fermionic contribution.

$U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle)$ depends only on the Polyakov loop parameters.

$\Omega^{(0)}_{\bar{q}q}(T, \mu_q)$ coming from the fermion determinant. It can be written as

$$\Omega^{(0)}_{\bar{q}q}(T, \mu_q) = \Omega^{(0)v}_{\bar{q}q} + \Omega^{(0)T}_{\bar{q}q}(T, \mu_q) \tag{5}$$

where v denotes the zero temperature vacuum part, while T refers to the thermal part.

Grand potential

Vacuum and thermal part of $\Omega^{(0)}_{\bar{q}q}(T,\mu_q)$:

$$\begin{aligned}
\Omega^{(0)v}_{\bar{q}q} &= -2N_C \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_f(p) \\
&\xrightarrow{\text{After ren}} -\frac{3}{8\pi^2} \sum_{f=u,d,s} m_f^4 \frac{m_f}{M_0} \\
\Omega^{(0)T}_{\bar{q}q}(T,\mu_q) &= -2T \sum_f \int \frac{d^3p}{(2\pi)^3} \left[\ln g_f^+(p) + \ln g_f^-(p)\right]
\end{aligned} \tag{6}$$

where $\quad g_f^{\pm} = 1 + 3\left(\phi^{\pm} + \phi^{\pm} e^{-\beta E_f^{\pm}(p)}\right) e^{-\beta E_f^{\pm}(p)} + e^{-3\beta E_f^{\pm}(p)}$

whith $\phi^- = \phi$, $\phi^+ = \bar{\phi}$ and $E_f^{\pm}(p) = E_f \mp \mu_f$.

Curvature masses

Curvature meson masses

$$m^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} = \mathfrak{m}^2_{i,ab} + \Delta m^2_{i,ab} + \delta m^2_{i,ab} \tag{7}$$

The three terms are the tree-level mass and the corrections coming from the fermionic vacuum and thermal fluctuations, respectively. $a, b \in \{0, \ldots, 8\}$

- $i = S, P$ firstly calculated by Schaefer and Wagner in PRD **79**, 014018 (2009)
- $i = V, A$ should be also included. This is a work in progress.

Field equations

$$\frac{\partial \Omega}{\partial \bar{\Phi}} = \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S} = 0 \tag{8}$$

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In ELσM

$$\begin{aligned}
-\frac{d}{d\Phi}\left(\frac{U\left(\Phi,\bar{\Phi}\right)}{T^4}\right) + \frac{6}{T^3}\sum_f \int \frac{d^3p}{(2\pi)^3}\left(\frac{e^{-\beta E_f^-(p)}}{g_f^-(p)} + \frac{e^{-2\beta E_f^+(p)}}{g_f^+(p)}\right) &= 0 \\
-\frac{d}{d\bar{\Phi}}\left(\frac{U\left(\Phi,\bar{\Phi}\right)}{T^4}\right) + \frac{6}{T^3}\sum_f \int \frac{d^3p}{(2\pi)^3}\left(\frac{e^{-\beta E_f^+(p)}}{g_f^+(p)} + \frac{e^{-2\beta E_f^-(p)}}{g_f^-(p)}\right) &= 0 \\
m_0^2\phi_N + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - \frac{c}{\sqrt{2}}\phi_N\phi_S - h_{0N} + \frac{3}{2}g_F\sum_{l=u,d}\langle\bar{q}_l q_l\rangle_T &= 0 \\
m_0^2\phi_S + \left(\lambda_1 + \lambda_2\right)\phi_S^3 + \lambda_1\phi_N^2\phi_S - \frac{\sqrt{2}c}{4}\phi_N^2 - h_{0S} + \frac{3}{\sqrt{2}}g_F\langle\bar{q}_s q_s\rangle_T &= 0
\end{aligned} \tag{9}$$

Solutions in T and μ_B

- We need initial values for solving the field equations. It is possible at $T = 0$ (or $\mu_B = 0$, but now we are not interested in that case).
- We can start to move into finite T with small steps, using the last solutions as initial values.

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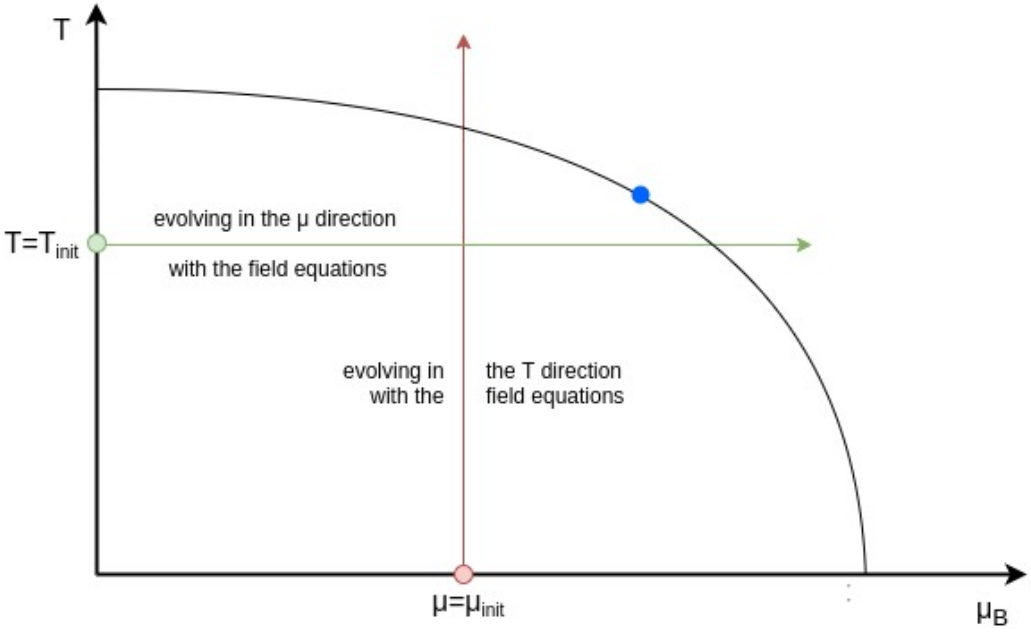

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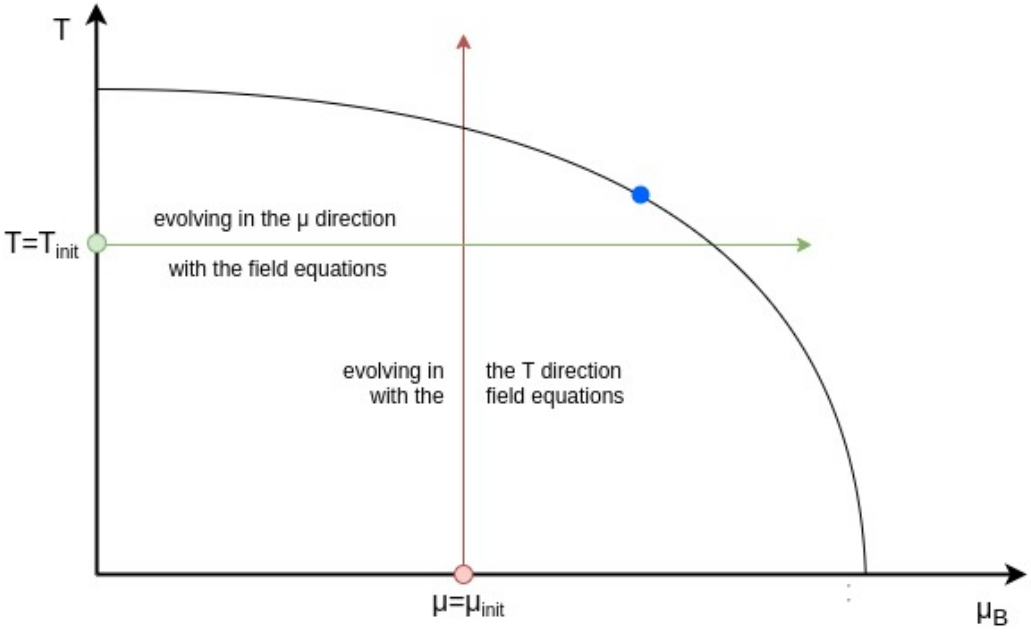

- One can calculate the pressure as $p(T, \mu_B) = \Omega(T = 0, \mu_B) - \Omega(T, \mu_B)$ for fixed μ_B as a function of temperature.

Baryon fluctuations

- One need quantities accessible both from measurements and theoretical calculations and are sensitive to the critical behavior.
- Baryon number fluctuations (and quantities resulting therefrom, such as kurtosis).

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One can characterize the baryon fluctuations with the susceptibilities.

Baryon number susceptibilities

$$\chi_n^B = \frac{\partial^n p/T^4}{\partial \left(\mu_B/T\right)^n} = T^{n-4}\frac{\partial^n p}{\partial \mu_B^n} \tag{10}$$

Baryon fluctuations

The (excess) kurtosis can be written as the ratio of the fourth and the square of the second order cumulants (variance),

$$\kappa = \frac{m_4}{m_2^2} = \frac{m_4}{m_2\sigma_2} = \frac{\chi_4}{\chi_2\sigma_2} \tag{11}$$

κ can be rewritten with the number susceptibilities (in the grand canonical ensemble). Thus often the ratio of susceptibilities defined as kurtosis.

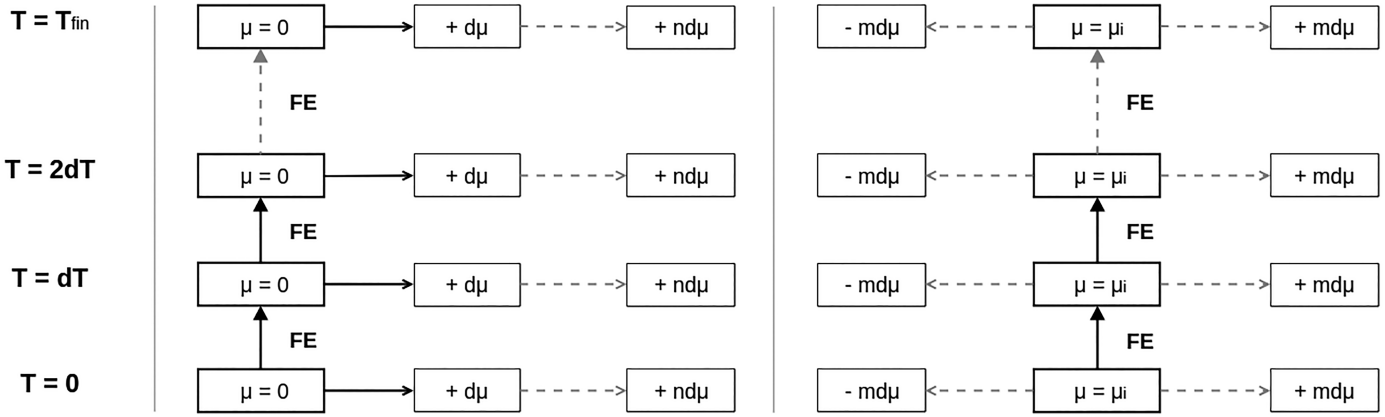
Kurtosis

$$\sigma_2\kappa = \frac{\chi_4}{\chi_2} \tag{12}$$

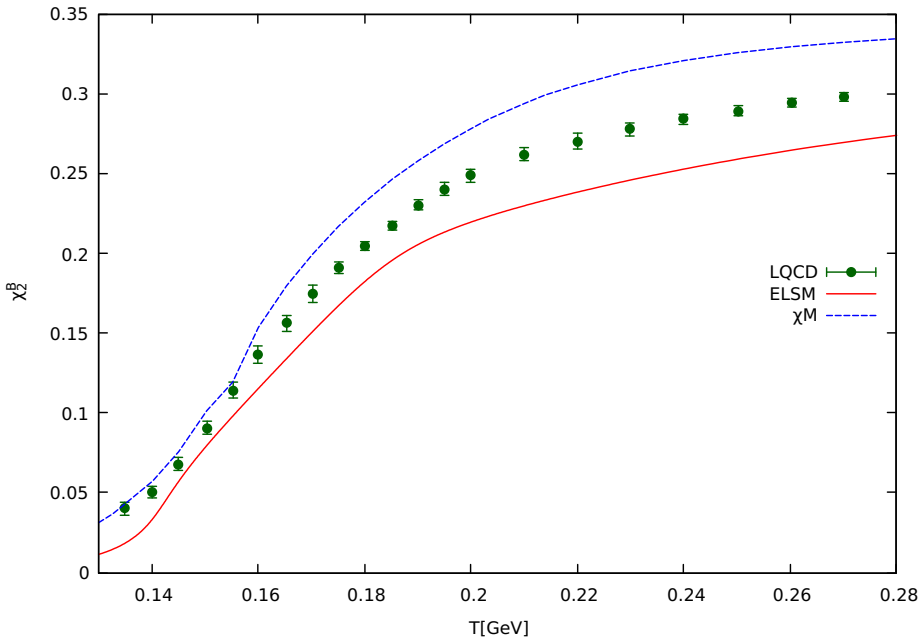
Further ratios can be defined. χ_3/χ_1 (skewness), χ_4/χ_2 (kurtosis), χ_6/χ_2, χ_8/χ_2 has higher importance in lattice calculations by the continuum extrapolation. Thus, these are good quantities to compare effective models with lattice results.

Calculation of μ_B derivatives

- We need the derivative of the pressure on the phase space in the μ_B direction while moving along T.
- By using finite difference method we just need the values of pressure in some neighbouring points (in μ_B) at a given T to construct the derivative, which can be calculated with the field equations.
- One can move to the next T with the field equations, too.



Results for cumulants



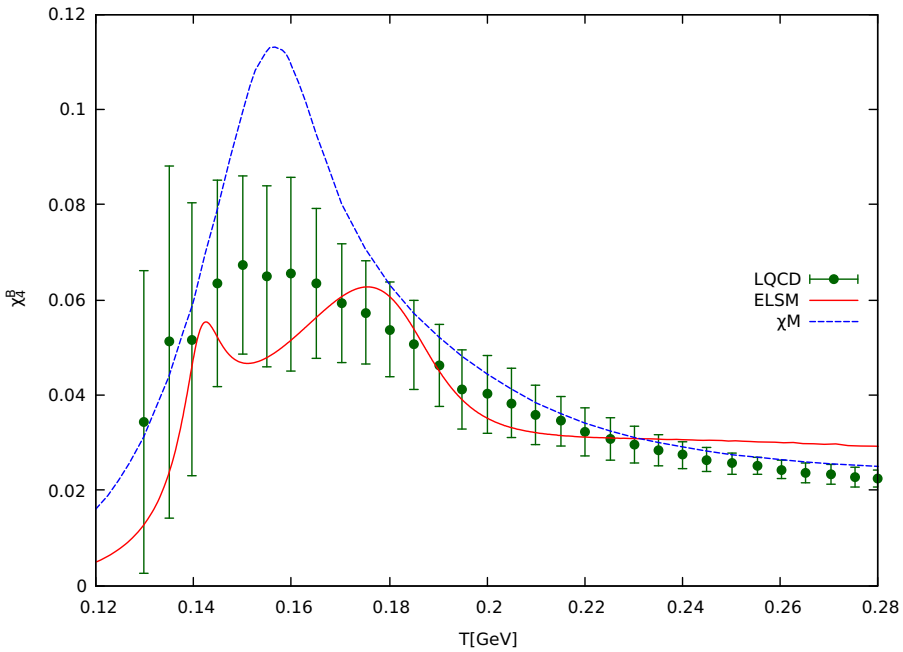


Figure: χ_2^B as the function of temperature in the extended linear sigma model, in the chiral matrix model, and in lattice calculations (left). The temperature dependence of χ_4^B compared again with the chiral matrix model and lattice data (right).

Chiral matrix model: Phys. Rev. D **94**, no. 3, 034015 (2016)

Results for cumulants

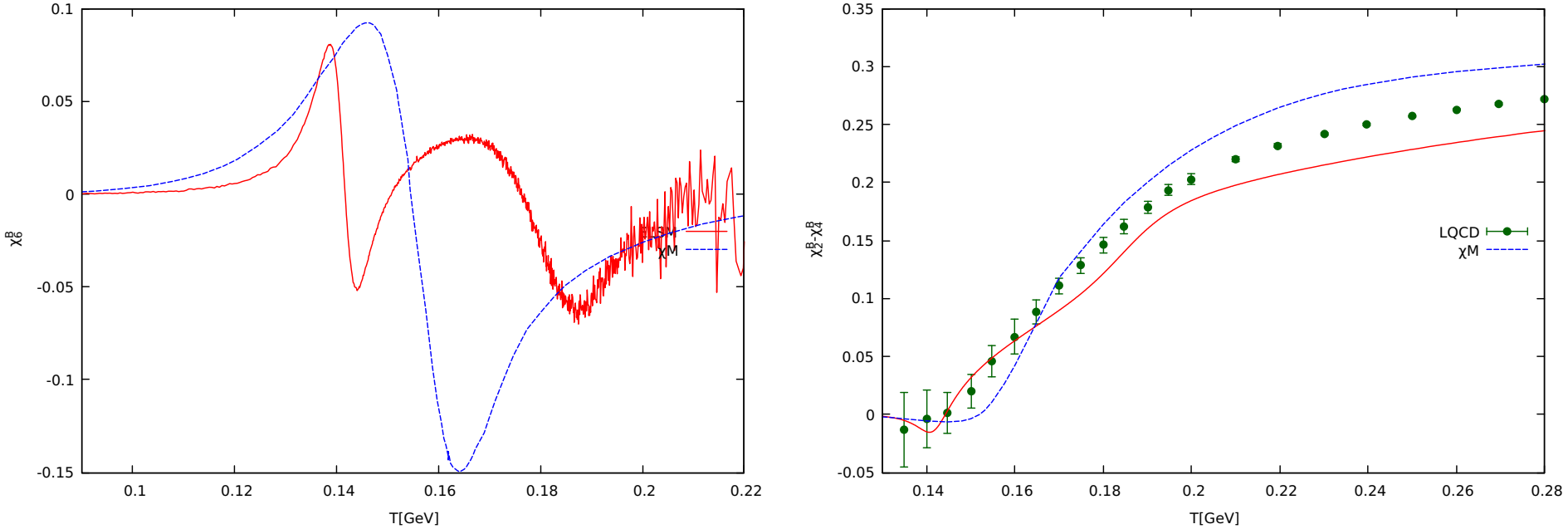
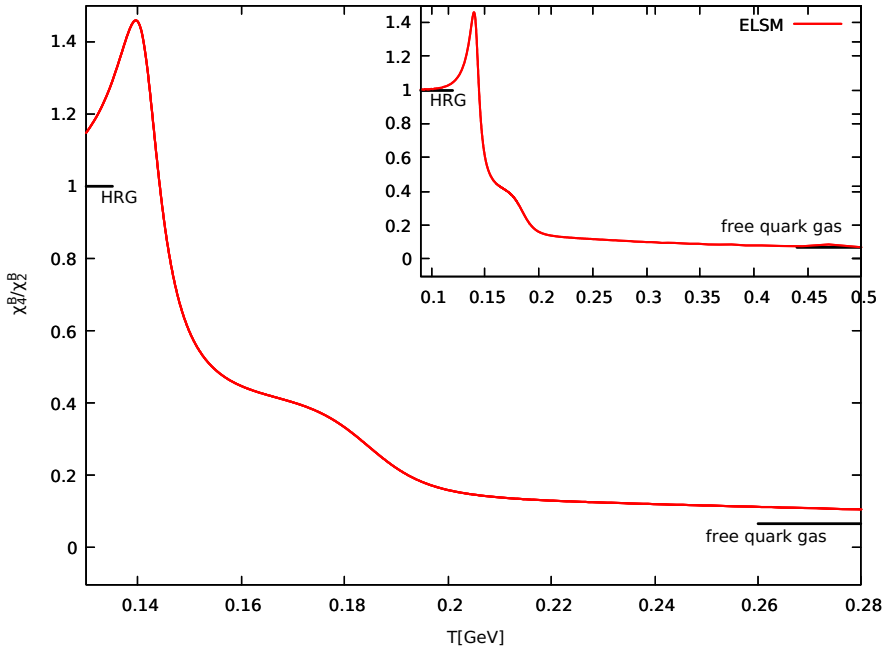


Figure: χ_6^B as the function of temperature (left) and $\chi_2^B - \chi_4^B$ (right) in the extended linear sigma model, in the chiral matrix model, and in lattice calculations.

Results: kurtosis



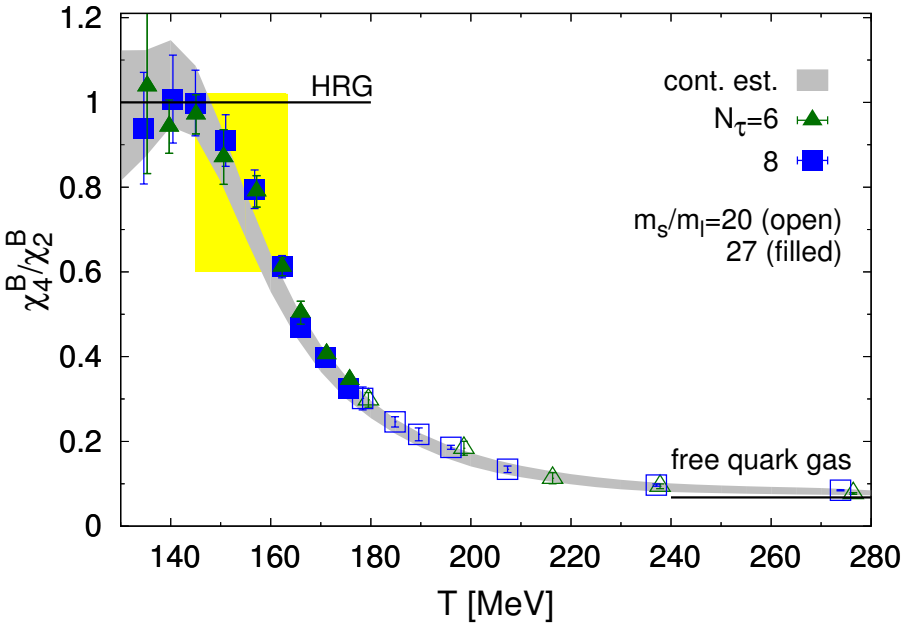


Figure: Kurtosis in the extended linear sigma model (left) compared with lattice data (right) as a function of temperature. The insert shows, how the kurtosis reaches the HRG and free quark gas limits in ELσM.

Lattice: Phys. Rev. D **95**, no. 5, 054504 (2017)

In finite baryon chemical potential

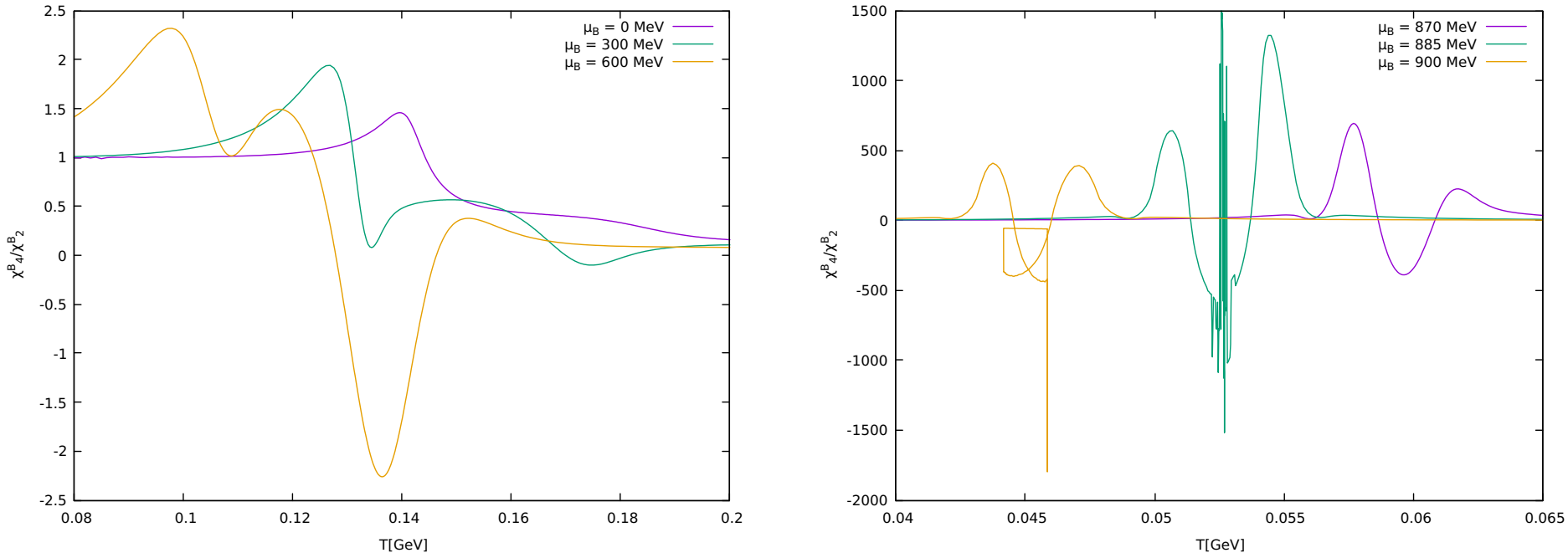


Figure: The kurtosis in finite μ_B. The phase transition moves to lower temperature as the μ_B grows (left). The transition is smooth, even close but below the critical $\mu_B = 885MeV$, while the spinodal behavior shows up above it. At the critical baryon chemical potential we get the expected singular result.

In finite μ_B

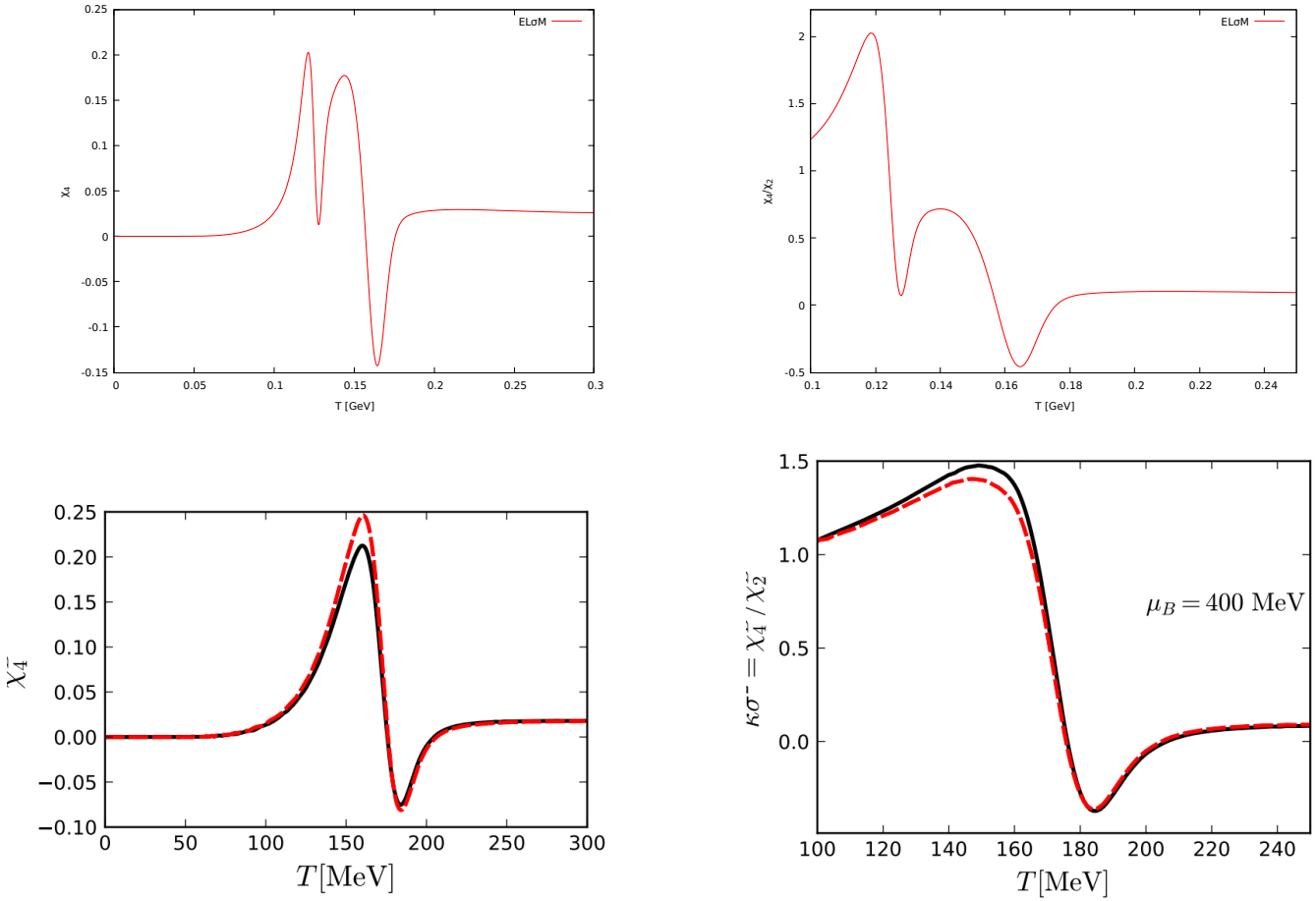


Figure: χ_4 (left) and the kurtosis (right) in finite μ_B, in the ELσM (top) and in FRG calulation (bottom). Phys. Rev. D **100**, no. 9, 094029 (2019)

Summary and outlook

- ELσM with Polyakow loop. Gives CEP at $(885, 52.7)\ MeV$.
- Baryon fluctuations and Kurtosis results are close to lattice data in $\mu_B = 0$ and in finite μ_B behaves similarly as expected in case of a CEP.

Improving the model

- Choose better way for the calculation of derivatives with respect to μ_B.
- Including also (axial)vector curvature masses.
- Extend the approximation for the field equation and the curvature masses selfconsisntently.