Baryon fluctuations in Extended linear sigma model

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Outline

- Motivations.
- Extended linear sigma model and its thermodynamics.
- Baryon fluctuations in the EL$\sigma$M.
- Outlook.
The phase structure of the strongly interacting matter, especially the region of chiral phase transition, is a heavily studied field of particle physics.

- There might be a first order phase transition.
- At $\mu_q = 0$ there is a crossover according to lattice calculations.
- There is a Critical end point (CEP) between the two regions.
- Effective models trying to prove the existence and location the CEP.
- The upcoming FAIR experiment will hopefully give insight into the energy region, where the CEP might be.  \textit{PS 94}, no. 3, 033001 (2019)
Uncertainty of CEP

**Figure:** Location of CEP according to different models and the freeze-out line for comparison. The shadowed region and the vertical lines correspond to lower limits of the CEP.


Extended linear sigma model


- Linear sigma model with full Scalar, Pseudoscalar, Vector and Axialvector nonets.
- Isospin symmetric case: 16 mesonic degrees of freedom.
- The Lagrangian build up from the fields

\[ L^\mu = \sum_a (V^\mu_a + A^\mu_a) T_a, \quad R^\mu = \sum_a (V^\mu_a - A^\mu_a) T_a, \quad M = \sum_a (S_a + iP_a) T_a, \]

(1)

with terms up to fourth order, taking care of the symmetry properties.
- Constituent quarks (2+1 flavors) are included in a Yukawa type term,

\[ \mathcal{L}_{Yukawa} = \bar{\psi} [i\gamma_\mu D^\mu - \mathcal{M}] \psi, \]

(2)

where \( \mathcal{M} = g_F (\mathbb{1} M_S + i\gamma_5 M_P) \) is the quark mass matrix and \( M_S = \sum_a S_a T^a, M_P = \sum_a P_a T^a. \)
The parameters of the Lagrangian are fitted with $\chi^2$ test. The used physical parameters are the (pseudo)scalar meson masses, decay widths and the $T_C(\mu = 0)$ critical temperature, getting from lattice simulations.

All parameters and the fitting procedure can be found in PRD 93, no. 11, 114014 (2016).
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- In the 0-8 sector N/S bases is used
  \[
  \xi_N = \frac{1}{\sqrt{3}} \left( \sqrt{2} \xi_0 + \xi_8 \right) \quad \xi_S = \frac{1}{\sqrt{3}} \left( \xi_0 - \sqrt{2} \xi_8 \right). \tag{3}
  \]
- All fields can be written as $\varphi(x) := \phi + \varphi'(x)$, where $\varphi'(x)$ has no vacuum expectation value (vev).
  Only the scalar-isoscalar sector has nonzero vev denoted as: $\phi_N, \phi_S$. 
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Including Polyakov loop variables $\langle \Phi \rangle, \langle \bar{\Phi} \rangle$ as further order parameters, which mimic the deconfinement phase transition.
Grand potential

Assumptions

- Symmetric quark matter: $\mu_u = \mu_d = \mu_s = \mu_q = \mu_B/3$
- Fermionic determinant calculated with neglecting mesonic fluctuations, ie. (pseudo)scalar mesons treated at tree-level only.
- Effect of mesonic thermal fluctuations (of the three lightest: $\pi$, $K$, $f_0^L$ can be) included in the level of pressure.

The Grand potential reads

$$\Omega(T,\mu_q) = U(\langle M \rangle) + U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle) + \Omega^{(0)}_{qq}(T,\mu_q)$$  \hspace{1cm} (4)$$

where the terms are the tree-level mesonic potential, the Polyakov loop potential, and the contribution of fermionic fluctuations respectively.
Grand potential

\[ U(\langle M \rangle) \] depends on the (up to fourth order of) scalar-isoscalar background and the parameters of the Lagrangian. It also contains the counterterms for the renormalization of the fermionic contribution.

\[ U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle) \] depends only on the Polyakov loop parameters.

\[ \Omega_{qq}^{(0)}(T, \mu_q) \] coming from the fermion determinant. It can be written as

\[
\Omega_{qq}^{(0)}(T, \mu_q) = \Omega_{qq}^{(0)v} + \Omega_{qq}^{(0)T}(T, \mu_q) \quad (5)
\]

where \( v \) denotes the zero temperature vacuum part, while \( T \) refers to the thermal part.
Grand potential

 Vacuum and thermal part of $\Omega_{\bar{q}q}^{(0)}(T, \mu_q)$:

$$\Omega_{\bar{q}q}^{(0)v} = -2N_C \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_f(p)$$

After renormalization:

$$-\frac{3}{8\pi^2} \sum_{f=u,d,s} m_f^4 \frac{m_f}{M_0}$$

$$\Omega_{\bar{q}q}^{(0)T}(T, \mu_q) = -2T \sum_f \int \frac{d^3p}{(2\pi)^3} \left[ \ln g_f^+(p) + \ln g_f^-(p) \right]$$

where

$$g_f^\pm = 1 + 3 \left( \phi^\pm + \phi^\pm e^{-\beta E_f^\pm(p)} \right) e^{-\beta E_f^\pm(p)} + e^{-3\beta E_f^\pm(p)}$$

with $\phi^- = \phi$, $\phi^+ = \bar{\phi}$ and $E_f^\pm(p) = E_f \mp \mu_f$. 
Curvature masses

Curvature meson masses

\[ m_{i,ab}^2 = \frac{\partial^2 \Omega(T, \mu)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} = m_{i,ab}^2 + \Delta m_{i,ab}^2 + \delta m_{i,ab}^2 \]  \hspace{1cm} (7)

The three terms are the tree-level mass and the corrections coming from the fermionic vacuum and thermal fluctuations, respectively. \( a, b \in \{0, \ldots, 8\} \)

- \( i = S, P \) firstly calculated by Schaefer and Wagner in PRD 79, 014018 (2009)
- \( i = V, A \) should be also included. This is a work in progress.
Field equations

\[ \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S} = 0 \]
Grand potential and thermodynamics

Field equations

\[
\frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} = \frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S} = 0
\] (8)

In ELσM

\[
- \frac{d}{d\Phi} \left( \frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{6}{T^3} \sum_f \int \frac{d^3 p}{(2\pi)^3} \left( \frac{e^{-\beta E_f^{-}(p)}}{g_f^{-}(p)} + \frac{e^{-2\beta E_f^{+}(p)}}{g_f^{+}(p)} \right) = 0
\]

\[
- \frac{d}{d\bar{\Phi}} \left( \frac{U(\Phi, \bar{\Phi})}{T^4} \right) + \frac{6}{T^3} \sum_f \int \frac{d^3 p}{(2\pi)^3} \left( \frac{e^{-\beta E_f^{+}(p)}}{g_f^{+}(p)} + \frac{e^{-2\beta E_f^{-}(p)}}{g_f^{-}(p)} \right) = 0
\] (9)

\[
m_0^2 \phi_N + \left( \lambda_1 + \frac{\lambda_2}{2} \right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - \frac{c}{\sqrt{2}} \phi_N \phi_S - h_{0N} + \frac{3}{2} g_F \sum_{l=u,d} \langle \bar{q}_l q_l \rangle_T = 0
\]

\[
m_0^2 \phi_S + (\lambda_1 + \lambda_2) \phi_S^3 + \lambda_1 \phi_N \phi_S^2 - \frac{\sqrt{2}c}{4} \phi_N^2 - h_{0S} + \frac{3}{\sqrt{2}} g_F \langle \bar{q}_s q_s \rangle_T = 0
\]
Solutions in $T$ and $\mu_B$

- We need initial values for solving the field equations. It is possible at $T = 0$ (or $\mu_B = 0$, but now we are not interested in that case).
- We can start to move into finite $T$ with small steps, using the last solutions as initial values.
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One can calculate the pressure as $p(T, \mu_B) = \Omega(T = 0, \mu_B) - \Omega(T, \mu_B)$ for fixed $\mu_B$ as a function of temperature.
One need quantities accessible both from measurements and theoretical calculations and are sensitive to the critical behavior.

Baryon number fluctuations (and quantities resulting therefrom, such as kurtosis).
Baryon fluctuations

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Baryon number fluctuations (and quantities resulting therefrom, such as kurtosis).

One can characterize the baryon fluctuations with the susceptibilities.

**Baryon number susceptibilities**

\[
\chi^B_n = \frac{\partial^n p / T^4}{\partial (\mu_B / T)^n} = T^{n-4} \frac{\partial^n p}{\partial \mu_B^n} \tag{10}
\]
The (excess) kurtosis can be written as the ratio of the fourth and the square of the second order cumulants (variance),

$$\kappa = \frac{m_4}{m_2^2} = \frac{m_4}{m_2 \sigma_2} = \frac{\chi_4}{\chi_2 \sigma_2}$$  \hspace{1cm} (11)

\(\kappa\) can be rewritten with the number susceptibilities (in the grand canonical ensemble). Thus often the ratio of susceptibilities defined as kurtosis.

Kurtosis

$$\sigma_2 \kappa = \frac{\chi_4}{\chi_2}$$  \hspace{1cm} (12)

Further ratios can be defined. \(\chi_3/\chi_1\) (skewness), \(\chi_4/\chi_2\) (kurtosis), \(\chi_6/\chi_2\), \(\chi_8/\chi_2\) has higher importance in lattice calculations by the continuum extrapolation. Thus, these are good quantities to compare effective models with lattice results.
Calculation of $\mu_B$ derivatives

- We need the derivative of the pressure on the phase space in the $\mu_B$ direction while moving along $T$.
- By using finite difference method we just need the values of pressure in some neighbouring points (in $\mu_B$) at a given $T$ to construct the derivative, which can be calculated with the field equations.
- One can move to the next $T$ with the field equations, too.
Results for cumulants

Figure: $\chi_2^B$ as the function of temperature in the extended linear sigma model, in the chiral matrix model, and in lattice calculations (left). The temperature dependence of $\chi_4^B$ compared again with the chiral matrix model and lattice data (right).

Results for cumulants

Figure: $\chi_6^B$ as the function of temperature (left) and $\chi_2^B - \chi_4^B$ (right) in the extended linear sigma model, in the chiral matrix model, and in lattice calculations.
Results: kurtosis

Figure: Kurtosis in the extended linear sigma model (left) compared with lattice data (right) as a function of temperature. The insert shows, how the kurtosis reaches the HRG and free quark gas limits in ELσM.

In finite baryon chemical potential

Figure: The kurtosis in finite $\mu_B$. The phase transition moves to lower temperature as the $\mu_B$ grows (left). The transition is smooth, even close but below the critical $\mu_B = 885\,MeV$, while the spinodal behavior shows up above it. At the critical baryon chemical potential we get the expected singular result.
In finite $\mu_B$

Figure: $\chi_4$ (left) and the kurtosis (right) in finite $\mu_B$, in the EL$\sigma$M (top) and in FRG calculation (bottom).
Summary and outlook

- EL$\sigma$M with Polyakov loop. Gives CEP at (885, 52.7) MeV.
- Baryon fluctuations and Kurtosis results are close to lattice data in $\mu_B = 0$ and in finite $\mu_B$ behaves similarly as expected in case of a CEP.

Improving the model

- Choose better way for the calculation of derivatives with respect to $\mu_B$.
- Including also (axial)vector curvature masses.
- Extend the approximation for the field equation and the curvature masses selfconsisntently.

The diagram shows the flow of calculations from the Effective potential to the Curvature masses, with corrections to the field equations and curvature masses.