Instabilities

JAI lectures - Hilary Term 2020

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References

- H. Wiedemann, Particle Accelerator Physics, Springer.
- G. Romulo, Beam Instabilities, CAS lectures.
- M. Miggliorati, Instabilities and Wake fields, JUAS lectures.
- S. Sheehy, Instabilities, JAI lectures.

Many different approaches depending on the source you take.

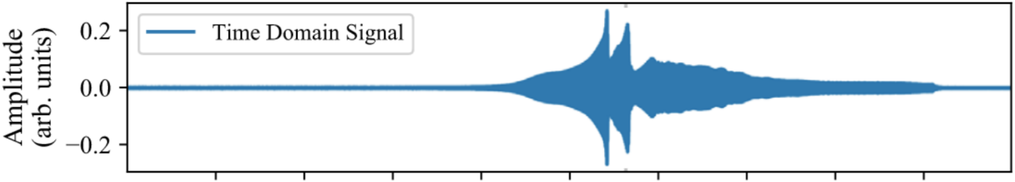
Goals of this course

- To give an overview of the mechanisms that drive to instabilities.
- Understand that the beam couples with the accelerator environment throughout the concepts of wake fields and impedance.
- Identify different types of instabilities in the transverse and longitudinal planes.
- Identify single-bunch and multi-bunch instabilities.

Approach

The deep treatment of wakefields/impedances/instabilities requires a detailed mathematical treatment. In this course a qualitative approach is given. Details can be found in the references.

How does an instability look like?



Observables

- Increase of oscillation amplitude.
- Beam losses.
- Intensity loss.

Impedance

- We have studied the single particle dynamics (transverse dynamics lectures).
- We have seen what happens when the bunches are treated as distribution of self-interacting charges (beam-beam and space charge lectures).
- A circulating beam resembles an electric circuit.
- Impedance plays an important role in determining the induced voltage on circulating current.

Impedance

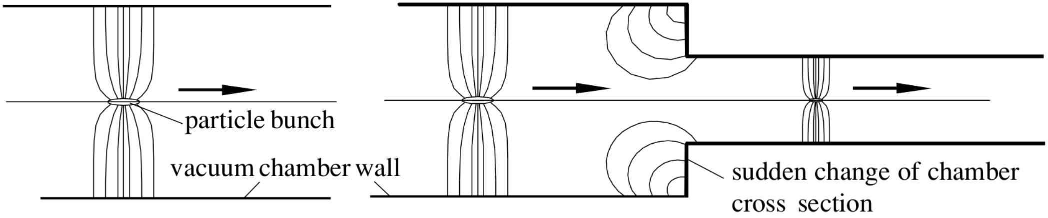
Fourier transform of the electromagnetic waves induced by the passing charged particle beam (wakefield).

Impedance

Impedance

Origins

Accelerator components such as resistive wall vacuum cahmber, space charge, image charge on vacuum chamber, broad-band impedance due to bellows, vacuum ports, and BPMs, and narrow-band impedance due to high-Q resonance modes in rf cavities, septum and kicker tanks.



Impedance

- The particle beam covers a wide frequency spectrum from the kHz regime to the order of the revolution frequency up to many GHz limited only by bunch length.
- The vacuum chamber environment consitutes an impedance which can become significant in the same frequency regime and efficient coupling can occur leading to collective effects.
- Strong coupling between RF-cavity and beam.
- The ultimate electron beam intensity is limited by instabilities caused by electromagnetic interaction of the beam current with the environment of the vacuum chamber.

Impedance and wake fields

- Wake fields are created after the interaction of the beam charge with the environment.
- They have the ability to pull or push the charges of the distribution.
- Energy losses and gains of a single or collection of particles can cause modifications in the beam dynamics, eventually driving to a beam instability.
- Transverse wakefields: $W_{||}$.
- Longitudinal wakefields: $W_{\perp}$.

Impedance

- In time domain, the interaction is described by wake fields that act on charges.
- In frequency domain, vacuum chamber components can be represented as a frequency dependent impedance $Z(\omega)$.

$$V(\omega) = -Z(\omega)I(\omega) \tag{1}$$

- The $-$ sign indicates that the induced voltage leads to an energy loss. $Z(\omega) \in \mathbb{C}$.
- All elements can be seen as accidental cavities. Z depends on their shape, material and frequency under consideration.
- Narrow band: $Q \ll 1$ (cavity like).
- Broad band: $Q \approx 1$ (sudden change in chambre cross section).

Interlude: RLC circuit

A cavity can be modeled as an AC resonant circuit.

$$\omega_r = \frac{1}{\sqrt{LC}} \tag{2}$$

Quality factor:

$$Q = R\sqrt{C/L} = R/L\omega_r = RC\omega_r \tag{3}$$

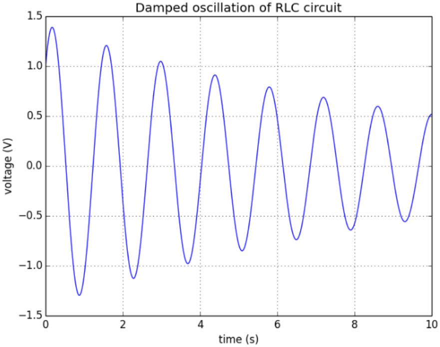
$$\ddot{V} + \frac{\omega_r}{Q}\dot{V} + \omega_r^2 V = \omega_r \frac{R}{Q}\dot{I} \tag{4}$$

Interlude: RLC circuit

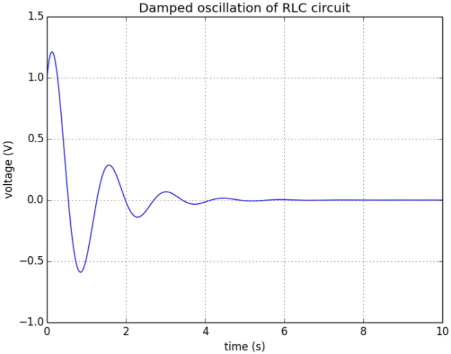
Damped oscillator:

$$V = V_0 e^{-\omega_r/2Q} \sin \left[\omega_r \sqrt{1 - \frac{1}{4Q^2}}\, t \right] + \phi \tag{5}$$

Low Q



High Q



Impedance

Narrow band: $Q \ll 1$

- Persist for a long time.
- Multibunch instabilities (rarely affect single bunch limits).
- Accelerating cavities (fundamental and higher order modes).

Broad band: $Q \approx 1$

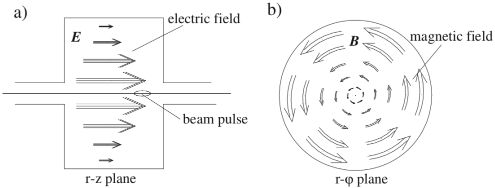
- Discontinuities in cross section of vacuum chamber (accelerating cavities, flanges, kicker magnets, BPMs,...)
- Many higher modes can be xcited by a passing short particle bunch.
- All modes decohere very fast.
- At the time of arrival of the next particle bunch or the same bunch after one or more revolutions these fields have vanished.
- Single bunch instabilities.

Impedance (Longitudinal vs. Transverse)

A bunch passing through a structure on axis excites a longitudinal electrical field and a transverse magnetic field.

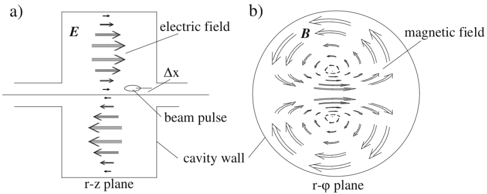
Longitudinal impedance

$$Z_{\|}(\omega) = -\frac{\int \vec{E}_{\|}(\omega) d\vec{z}}{I(\omega)} \tag{6}$$



Transverse impedance

$$Z_{\perp}(\omega) = i\frac{\int (\vec{E}(\omega) + [\vec{v} \times \vec{B}(\omega)])_{\perp} d\vec{z}}{I(\omega)\Delta x} \tag{7}$$



Impedance

In general:

$$Z(\omega) = Z_{\text{Re}}(\omega) + iZ_{\text{Im}}(\omega) = R\left[\frac{1 - iQ\frac{\omega^2-\omega_r^2}{\omega_r\omega}}{1 + iQ\frac{(\omega^2-\omega_r^2)^2}{\omega_r^2\omega^2}}\right] \tag{8}$$

- Resistive part (Z_{Re}): Tune shift.
- Reactive part (Z_{Im}): Damping or antidamping.
    - $\omega < \omega_r$. Inductive: $Z_{\text{Im}} > 0$.
    - $\omega > \omega_r$. Capacitive: $Z_{\text{Im}} < 0$.

Impedance in the accelerator environment

- The vacuum chamber of an accelerator is too complicated in geometry to allow an analytical expression for its impedance.
- Each section must treated independently.

Examples of impedances

- Resistive wall impedance.
- Space-charge impedance.
- Cavity-like impedance.

Resistive wall impedance

The particle beam induces an image current in the vacuum chamber wall in a thin layer.

- Since conductivity is not perfect, resistive losses apply a pull or decelerating field on the particles.
- The pull is proportional to the beam current,

$$\frac{Z_{||}(\omega_n)}{n} = \frac{1-i}{n}\frac{\bar{R}}{c r_w}\sqrt{\frac{\mu_r \omega_n}{2\epsilon_0 \sigma}} = \frac{1-i}{n}\frac{\bar{R}}{r_w \sigma \delta_{\mathrm{skin}}} \tag{9}$$

- Important role only for low frequencies.

The transverse resistive wall impedance for a round beam pipe is:

$$Z_{\perp}(\omega_n) = \frac{2\bar{R}}{r_w^2}\frac{Z_{||}(\omega_n)}{n} \tag{10}$$

Space-charge impedance

We know that there is an induced voltage leading to an energy gain or loss due to a collection of charged particles.

$$Z_{||,sc}(\omega) = -\frac{i}{\epsilon_0 c}\frac{n}{2\beta\gamma^2}\left(1 + 2\ln\frac{r_w}{r_0}\right) \tag{11}$$

Correct for long wavelength. Purely reactive.

$$Z_{\perp,sc}(\omega) = -\frac{i}{\epsilon_0 c}\frac{\bar{R}}{\beta^2\gamma^2}\left(\frac{1}{r_0^2} - \frac{1}{r_w^2}\right) \tag{12}$$

Specially strong for low energy particle beams.

Cavity-like structure impedance

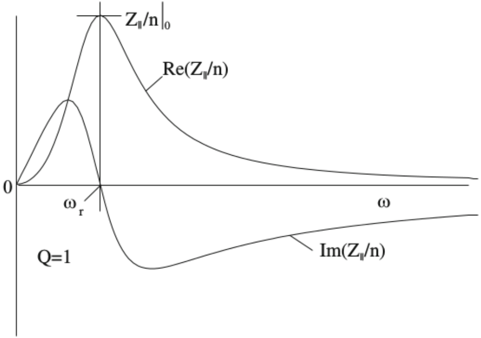
- Vacuum chamber impedances occur due to sudden changes of cross section.
- Collectively described by a cavity-like impedance with quality factor $Q \approx 1$.
- Broad band impedance.

$$\frac{Z_{||}}{n}_{\text{bb}}(\omega) = \left|\frac{Z_{||}}{n}\right|_0 \frac{1 - i\frac{\omega^2-\omega_r^2}{\omega_r\omega}}{1 + i\frac{(\omega^2-\omega_r^2)^2}{\omega_r^2\omega^2}} \tag{13}$$

- At low frequencies it is almost purely reactive and inductive.
- At high frequencies the impedance becomes capacitive.
- At resonance, purely resistive.
- Transverse broad-band impedance:

$$Z_\perp(\omega_n) \approx \frac{2\bar{R}}{r_w^2} Z_{||}(\omega_n) \tag{14}$$

Cavity-like structure impedance



- At low frequencies it is almost purely reactive and inductive.
- At high frequencies the impedance becomes capacitive.
- At resonance, purely resistive.

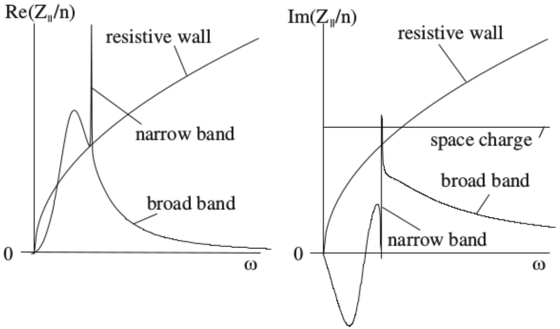
Cavity-like structure impedance

- Transverse broad-band impedance:

$$Z_{\perp}(\omega_n) \approx \frac{2\bar{R}}{r_w^2} Z_{||}(\omega_n) \tag{15}$$

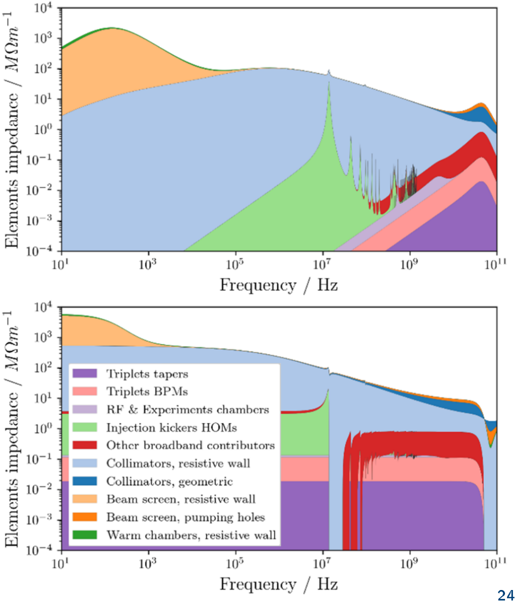
Overall accelerator impedance

- At low frequencies the reactive and the resistive component of the resistive wall impedance dominates.
- Space-charge impedance is independent of frequency.
- The narrow-band cavity spectrum includes the high impedances at the fundamental and higher mode frequencies.



LHC impedance model

- The impedance model summarizes the main impedance contributions from the different elements of the machine.
- In the LHC, the main contribution (for high frequencies) is coming from the collimators.
- New low-impedance collimators under investigation/implementation.



Instabilities

Instability: general concept

Assume some perturbation (i.e. impedance) in the dynamics of the beam motion. If:

- The perturbation is increased: Instability.
- The perturbation is decreased: Stability.

Many types of instabilities:

- Negative-mass instability.
- Robinson instability.
- Head-tail instability.
- Potential well distortion.
- Synchrotron oscillation tune shift.
- Bunch lengthening.
- Multi-bunch instability.
- ...

Negative-mass instability

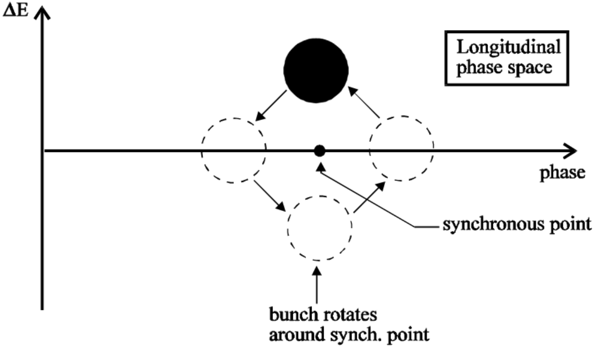
Observed in coasting beams with two possible scenarios:

- Below transition energy:
    - The repulsive electrostatic field from a lump in the distribution causes particles ahead of the lump to be accelerated and particles behind to be decelerated.
    - Stabilizing situation and the lumps smoothes out.
- Above transition energy:
    - Particles ahead gets a slower revolution frequency and it will move closer to teh lump.
    - A particle behind will be decelerated and then it will circulate faster.
    - Instability due to a "negative mass".
    - If attractive, then it is stable (Saturn's rings).

It should drive any unbunched beam mode unstable due to the resistivity of the vacuum chamber.

Robinson instability

- Most basic longitudinal instability mechanism that occurs in ciruclar accelerators.
- Induced by longitudinal impedance due to RF accelerating cavities (ω_{R}).
- Dipole mode oscillation. The whole bunch is moving back and forth around the synchronous position.
- Above transition, the beam will be unstable if ω_R is slightly above $h\omega_0$ and stable if slightly below. Below transition, it is the other way around.



Head-tail instability

In a circular proton accelerator:

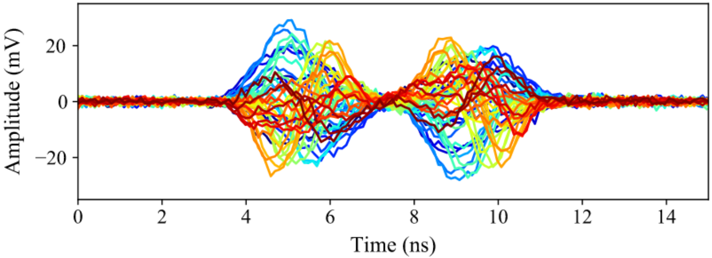
- No radiation damping.
- Weak transverse wake fields can lead to transverse bunch blow up and beam loss.
- Particles in the head of a bunch oscillate due to synchrotron oscillations.
- There is an intensity I_b from which the perturbation from wake fields is strong enough to produce instabilities.

$$I_b \leq \frac{4\pi q \gamma \omega_0 \nu_\beta \nu_s}{r_c \beta c \operatorname{Im}(Z_\perp / n)} \tag{16}$$

- Exceeding this limit leads to an immediate loss of the excess current.
- Most severe instability.

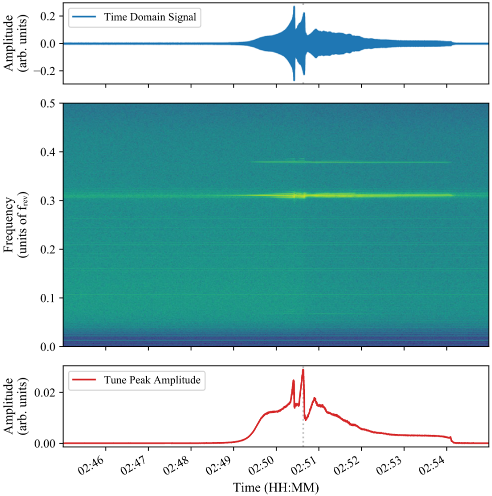
Head-tail instability

- Head and tail start to oscillate while the center of the bunch remains unperturbed.



(a) 8-bit oscillscope

Typical instability in the LHC



Landau damping

Landau damping

General concept

- Pretty complex topic without a clear/unified description in literature.
- Introduced by Lev Landau (greatest physicist ever) in 1946.
- First introduced in the context of plasmas. Many applications nowadays.
- Quite often, the physical interpretation is unclear.
- Not even for me...

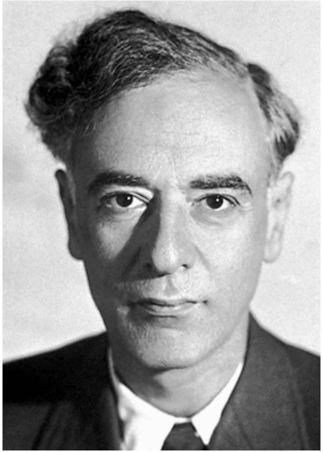
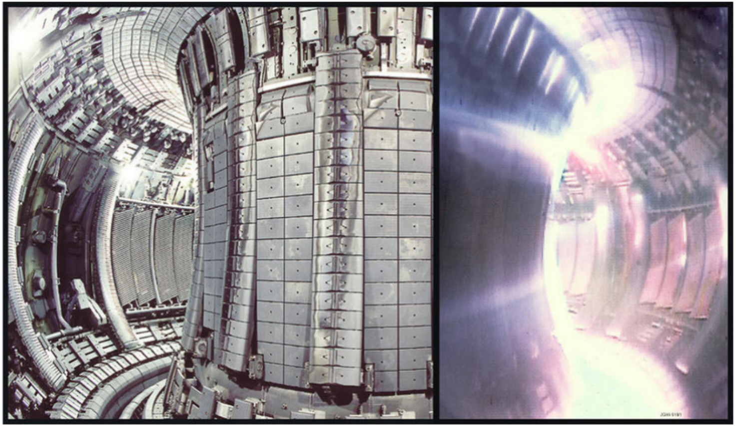


Figure: Lev D. Landau

Landau damping in plasmas

- Landau damping dumps collective oscillations.
- Leads to exponentially decaying oscillations.



Landau damping in accelerators

Not all particles in the beam have the same frequency. The coherent motion from an instability therefore de-coheres over time, potentially damping the instability.

- Landau damping does not damp anything.
- Exponentially decaying oscillations are not desired.
- It acts in the longitudinal and transverse planes.
- Along with active feedback systems, it is a powerful way to overcome coherent beam instabilities.

Landau damping is the absence of oscillations

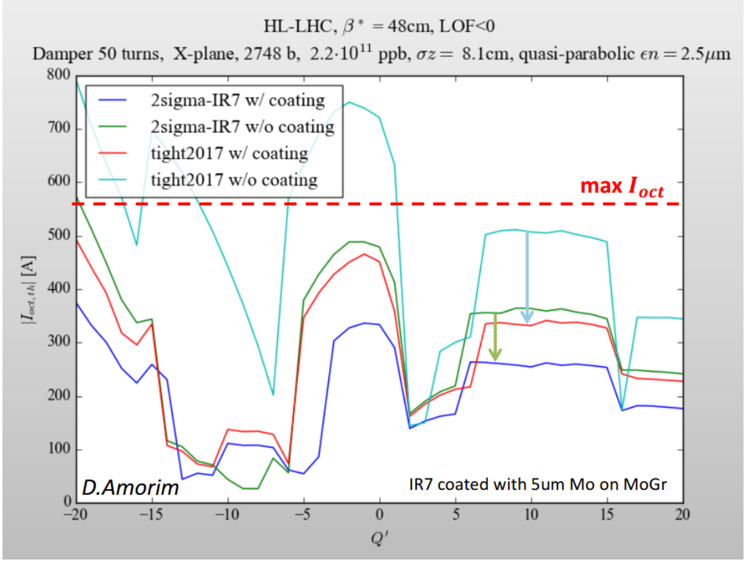
Recipe to generate Landau damping

- Compute detuning with amplitude $Q(J_x, J_y)$.
- Stability diagram may show that we are in the unstable area.
- Octupoles can be used to increase amplitude detuning.
- Check if now we are in a stable region.

Downside

- Octupoles introduce strong non-linearities at large amplitudes.
- Not many particules at large amplitudes $\Rightarrow$ requires large strengths.
- Reduction of dynamic aperture.
- Chromaticity change.

HL-LHC: Stability example



Summary

Summary

- Impedance is one of the main sources of beam instabilities.
- They are generated by the interaction/coupling between the beam and the environment.
- Impedances generate wake fields that perturb the beam motion.
- Eventually, this perturbation might lead to unstabilities.
- Different ways to mitigate instabilities (radiation damping, octupoles,...).

Thank you!