Two-particle correlation via Bremsstrahlung

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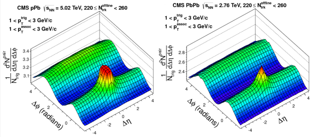
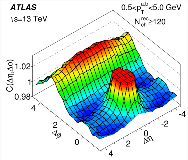
Motivation

Ridge structure

- η - independent shape in two-particle angular correlations
- Explained via elliptic and higher-order flows for AA collisions



However, flows in small systems?



Purpose

- Describe the Ridge through kinematics between jets and medium

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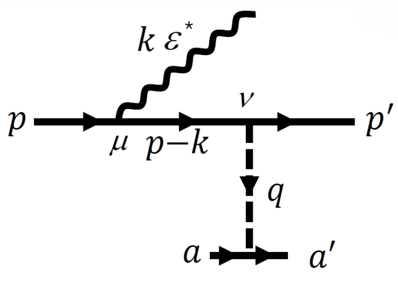
CMS collaboration, *Physical Letters B* **724**, 213240 (2013)

ATLAS collaboration, *Physical Review Letters* **116**, 172301 (2016)

Kinematic interaction between jet & medium

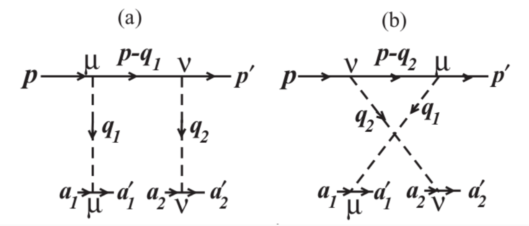
Jet particles lose their energy while passing through medium via...



1st order diagram for Bremsstrahlung

- Collision
- Radiation
    - Gluon radiation
    - **Photon radiation (Bremsstrahlung)**

Previous study in scattering



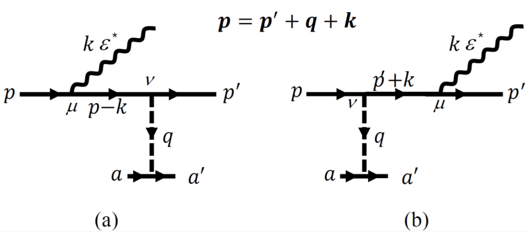
Two symmetric diagrams for parton scattering inside medium → **constructive interference**

$$d\sigma \sim |M_{(a)} + M_{(b)}|^2 = |M_{(a)}|^2 + |M_{(b)}|^2 + (\textbf{interference})$$

- Medium parton aligned along the jet particle
- Collective motion

C. Y. Wong, *Physical Review C* **85**, 064909 (2012)

Bremsstrahlung processes

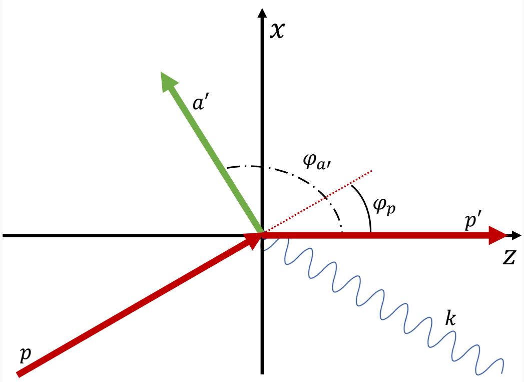


Two diagrams of γ emission and medium parton scattering might **interfere constructively**.

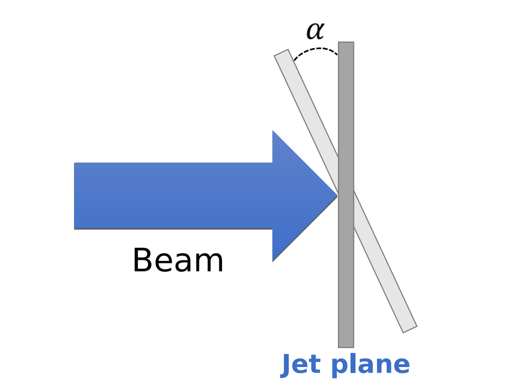
- **Expect to explain the Ridge behavior**

Coordinates

Use (E, y, φ) coordinates system



Jet plane



Jet plane + Beam direction

- Jet plane is independent of the beam direction
- Calculate cross section at $\alpha = 0^\circ$

Description for initial medium partons

Consider all possible initial medium partons' momentum

$$\int d^3\vec{a} \rightarrow \int f(y_a, a_T) \times dy_a da_T d\varphi_a \tag{1}$$

Distribution function $f(y_a, a_T)$ for describing momentums of initial medium partons

- Maxwell-Boltzmann distribution (**MB**)

$$f_{MB}(y_a, a_T) = \frac{E_a a_T}{2\pi m k_B T}\sqrt{\frac{1}{2\pi m k_B T}}\exp\left[-\frac{E_a^2 - m^2}{2mk_B T}\right] \tag{2}$$

- Jüttner-Synge distribution (**JS**)

$$f_{JS}(y_a, a_T) = \frac{E_a a_T}{4\pi m^2 k_B T K_2(m/k_B T)}\exp\left[-\frac{E_a}{k_B T}\right] \tag{3}$$

- Phenomenological Parton Distribution from Soft scattering model (**phPDs**)

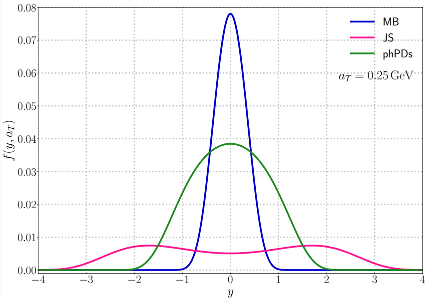
$$f_{phPDs}(y_a, a_T) = A_{Ridge}\left(1 - \frac{\sqrt{m_\pi^2 + p_T^2}\exp[|y_a| - y_B]}{m_\pi(m_d^2 + p_T^2)}\right)^a \exp\left[-\frac{\mathbf{E_a}}{k_B T}\right] \tag{4}$$

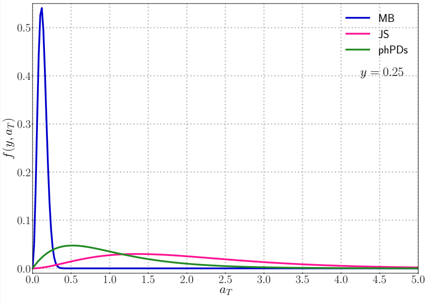
C. Y. Wong, *Physical Review C* **76**, 054908 (2007)

Description for initial medium partons

Set $T = 0.7\ \mathrm{GeV}$ From results of momentum kick model calculation

vs. Rapidity **vs. Transverse momentum**



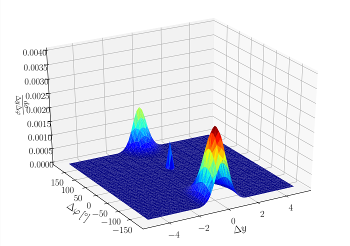


- JS/phPDs has larger range in rapidity than MB
- JS/phPDs is spread out in higher transverse momentum than MB

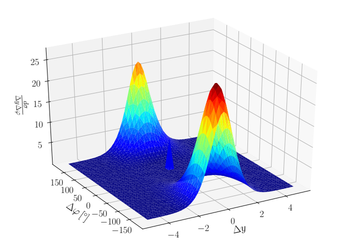
C. Y. Wong, *Physical Review C* **84**, 024901 (2011)

Previous results : Correlation between p' and a'

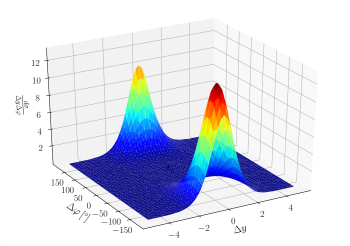
$$E_p = 10\,\mathrm{GeV} \to E_{p'} = 7\,\mathrm{GeV} \qquad \varphi_p = 10^\circ \to \varphi_{p'} = 0^\circ \qquad T = 0.7\,\mathrm{GeV}$$



MB



JS

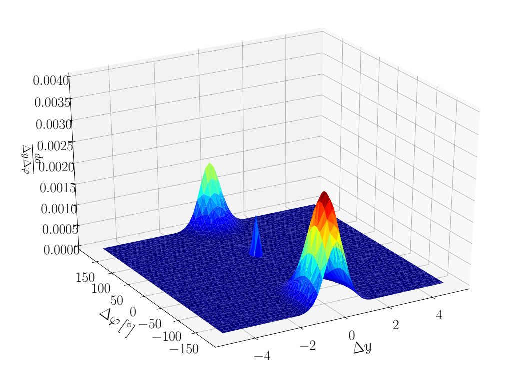


phPDs

- Dependency of distribution function, $f(y_a, a_T)$
  - $FWHM_{MB} = 0.4, \quad FWHM_{JS} = 1.0, \quad FWHM_{phPDs} = 0.6$
    - MB gives the most narrow peak
    - JS gives the most wide peak
  - Scales are different each other

Investigate abnormal peaks at $d\sigma$ distribution

$$E_p = 10\,\mathrm{GeV} \to E_{p'} = 7\,\mathrm{GeV} \qquad \varphi_p = 10^\circ \to \varphi_{p'} = 0^\circ \qquad T = 0.7\,\mathrm{GeV} \qquad \textbf{MB}$$

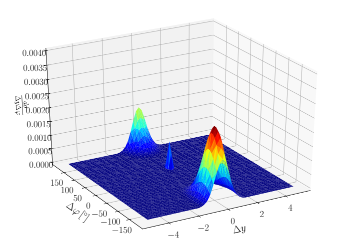


- Elastic scattering
  - Apply the cutoff parameter, Λ_q
    - $\frac{1}{\mathbf{q}^4} \to \frac{1}{(\mathbf{q}^2+\Lambda_q)^2}$
- Infrared
  - Cut E_k
    - For low E_k
  - Use the cutoff parameter, ε
    - For $\vec{p} \parallel \vec{k}$ and $\vec{p'} \parallel \vec{k}$
    - $\frac{1}{|(-2\mathbf{p}^\mu \mathbf{k}_\mu)+i\varepsilon|^2}$ and $\frac{1}{|(-2\mathbf{p'}^\mu \mathbf{k}_\mu)+i\varepsilon|^2}$

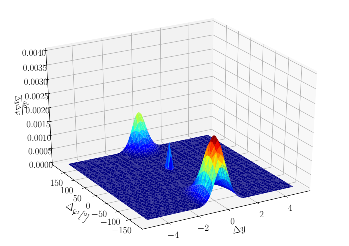
Check Λ_q dependency

Increase Λ_q to reduce effects from elastic scattering, $\mathbf{q}^2 \approx 0$

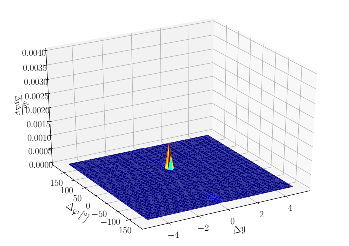
$E_p = 10\,\mathrm{GeV} \to E_{p'} = 7\,\mathrm{GeV}$ $\quad \varphi_p = 10^\circ \to \varphi_{p'} = 0^\circ$ $\quad T = 0.7\,\mathrm{GeV}$ $\quad$ **MB**



$\Lambda_q = 0.00000001\ (m_a^8)$



$\Lambda_q = 0.0001\ (m_a^4)$



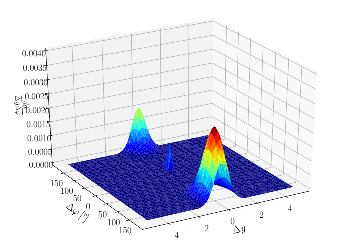
$\Lambda_q = 0.01\ (m_a^2)$

- The abnormal peak is reduced as Λ_q increased but not clearly
- Side peaks are more suppressed than the abnormal peak at $\Lambda_q = 0.01$
- Λ_q affects side peaks more than the abnormal peak

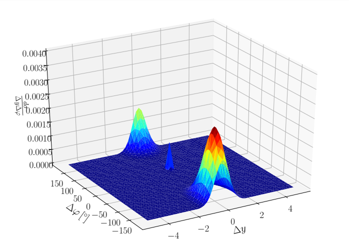
Check E_k dependency

Change the cut value for E_k to reduce effects from infrared processes, $E_k \approx 0$

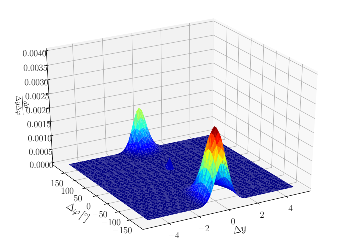
$E_p = 10\,\mathrm{GeV} \rightarrow E_{p'} = 7\,\mathrm{GeV}$ $\quad \varphi_p = 10^\circ \rightarrow \varphi_{p'} = 0^\circ$ $\quad T = 0.7\,\mathrm{GeV}$ $\quad$ **MB**



$E_k \leq 0.05\,\mathrm{GeV}\ (5m_a^2)$



$E_k \leq 0.1\,\mathrm{GeV}\ (10m_a^2)$



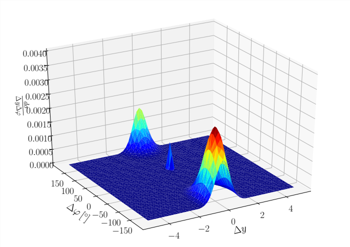
$E_k \leq 0.5\,\mathrm{GeV}\ (50m_a^2)$

- The abnormal peak is reduced as E_k increased
- Can $E_k = 0.5\,\mathrm{GeV}$ be regarded as low energy at other energy-loss cases?
  - ex) $E_p = 10\,\mathrm{GeV} \rightarrow E_{p'} = 9\,\mathrm{GeV}$

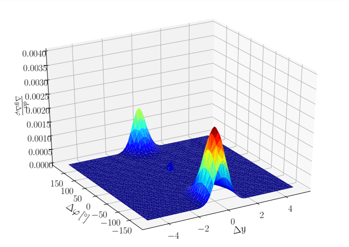
Check ε dependency

Increase ε to reduce effects from infrared processes, $\mathbf{p}^{\mu}\mathbf{k}_{\mu} \approx 0$ or $\mathbf{p}'^{\mu}\mathbf{k}_{\mu} \approx 0$

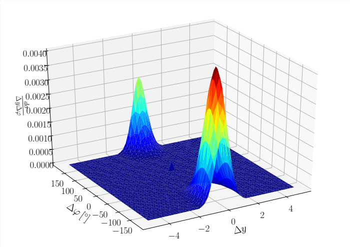
$E_p = 10\,\mathrm{GeV} \to E_{p'} = 7\,\mathrm{GeV}$ $\quad \varphi_p = 10^\circ \to \varphi_{p'} = 0^\circ$ $\quad T = 0.7\,\mathrm{GeV}$ $\quad$ **MB**



$\varepsilon = 0.0001$ $\left(\frac{1}{100} m_a^2\right)$



$\varepsilon = 0.01$ $\left(m_a^2\right)$

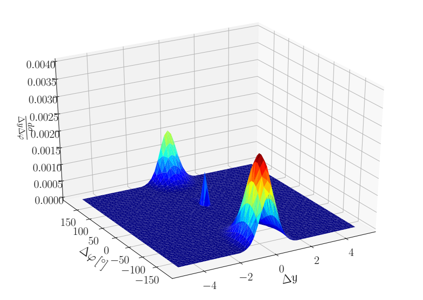


$\varepsilon = 1$ $\left(100 m_a^2\right)$

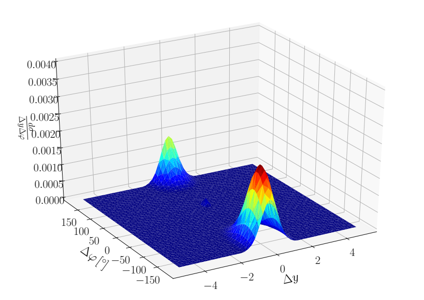
- The abnormal peak is obviously reduced at $\varepsilon = 0.01$ but not disappear
- Side peaks are enhanced at $\varepsilon = 1$
- $\vec{p} \parallel \vec{k}$ or $\vec{p'} \parallel \vec{k}$ cases largely contribute at this abnormal peak

Investigate abnormal peaks at $d\sigma$ distribution

$E_p = 10\,\mathrm{GeV} \rightarrow E_{p'} = 7\,\mathrm{GeV}$ $\quad \varphi_p = 10^\circ \rightarrow \varphi_{p'} = 0^\circ$ $\quad T = 0.7\,\mathrm{GeV}$ $\quad$ **MB**



Before reducing divergence



After reducing divergence

- To remove the abnormal peak...
  - $\Lambda_q = 0.0001\,(m_a^4)$
  - Cut $E_k \leq 0.1\,\mathrm{GeV}\,(m_a)$
  - $\varepsilon = 0.01\,(m_a^2)$

$\Rightarrow$ The peak is reduced, but not disappear

Correlation function

Correlation function in experiment :

$$C\left(\Delta\eta,\,\Delta\varphi\right) = \frac{S\left(\Delta\eta,\,\Delta\varphi\right)}{B\left(\Delta\eta,\,\Delta\varphi\right)}$$

To compare to experimental results,
We use correlation function between arbitary two particles :

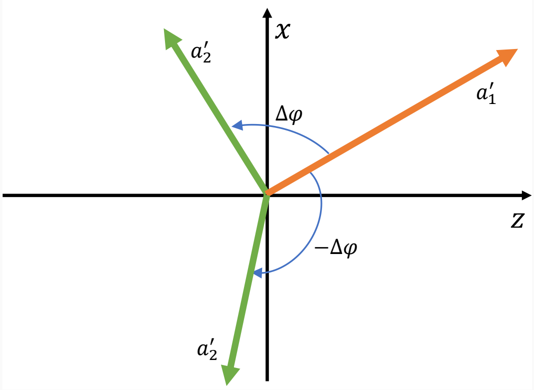
- $P_i\left(\mathbf{a}'\right)$: Probability of final medium partons **before interaction with jet particle**
- $P_f\left(\mathbf{a}'\right)$: Probability of final medium partons **after interaction with jet particle**

$$C\left(\mathbf{a}'_1,\,\mathbf{a}'_2\right) = \frac{P_f\left(\mathbf{a}'_1,\,\mathbf{a}'_2\right)}{P_i\left(\mathbf{a}'_1\right)\cdot P_i\left(\mathbf{a}'_2\right)}$$

In our calculation,

- $P_i\left(\mathbf{a}'\right) = \int f(\mathbf{a}')d^3\vec{a'}$
- $P_f\left(\mathbf{a}'\right) = \int f(\mathbf{a})\times d\sigma\left(\mathbf{a}_i\to\mathbf{a}'\right)d^3\vec{a}, \qquad P_f\left(\mathbf{a}'_1,\,\mathbf{a}'_2\right) = P_f\left(\mathbf{a}'_1\right)\cdot P_f\left(\mathbf{a}'_2\right)$

Determine Δy and $\Delta\varphi$



On the Jet plane

- $\Delta y = y_1 - y_2$
- $|\Delta\varphi| = |\varphi_1 - \varphi_2|$
  - $|\Delta\varphi| < \pi$
    - $\varphi_1 < \varphi_2 : \Delta\varphi = |\Delta\varphi|$
    - $\varphi_1 > \varphi_2 : \Delta\varphi = -|\Delta\varphi|$
  - $|\Delta\varphi| > \pi$
    - $\varphi_1 < \varphi_2 : \Delta\varphi = -(2\pi - |\Delta\varphi|)$
    - $\varphi_1 > \varphi_2 : \Delta\varphi = (2\pi - |\Delta\varphi|)$

Description for medium partons

Consider all possible medium partons' momentum

$$\int d^3\vec{a} \to \int f(y_a, a_T) \times dy_a da_T d\varphi_a \tag{5}$$

Distribution function $f(y_a, a_T)$ for describing momentums of initial medium partons

- Maxwell-Boltzmann distribution (**MB**)

$$f_{MB}(y_a, a_T) = \frac{E_a a_T}{2\pi m k_B T}\sqrt{\frac{1}{2\pi m k_B T}}\exp\left[-\frac{E_a^2 - m^2}{2mk_B T}\right]$$

- Jüttner-Synge distribution (**JS**)

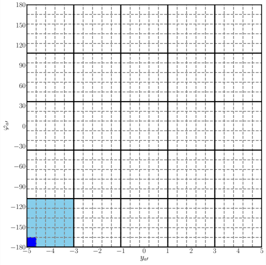
$$f_{JS}(y_a, a_T) = \frac{E_a a_T}{4\pi m^2 k_B T K_2(m/k_B T)}\exp\left[-\frac{E_a}{k_B T}\right]$$

- Phenomenological Parton Distribution from Soft scattering model (**phPDs**)

$$f_{phPDs}(y_a, a_T) = A_{Ridge}\left(1 - \frac{\sqrt{m_\pi^2 + a_T^2}\exp[|y_a| - y_B]}{m_\pi(m_d^2 + a_T^2)}\right)^a \exp\left[-\frac{\mathbf{E_a}}{k_B T}\right]$$

C. Y. Wong, *Physical Review C* **76**, 054908 (2007)

Set bins and meshpoints

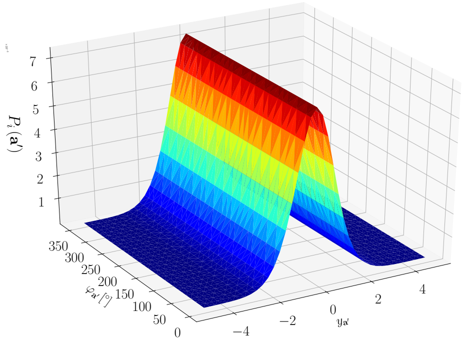


Calculate $\frac{d\sigma}{\Delta y \Delta \varphi}$ with enough meshpoints / bins [in KIAF]

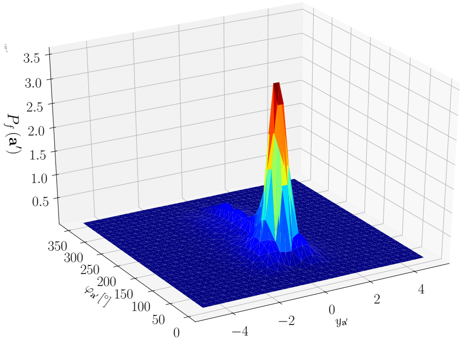
1. Divide the area in $|y_{a'}| < 5.0$, $|\varphi_{a'}| < 180.0^\circ$ into $(M \times N)$ sections
2. Within sections,
   - Divide the section into $(m \times n)$ $\mathbf{a}'$ bins
   - Generate $\mathbf{a}'$ using $(i \times j)$ random # for each **bin**
     - Generate for $y_{a'}$ **and** for $\varphi_{a'}$, **simultaneously**
     - **To aviod elastic scattering cases, $\vec{a} = \vec{a'}$**
   - Calculate $\int d^3\vec{a}$ for a given $\mathbf{a}'$
   - Combine $\frac{d\sigma}{\Delta y \Delta \varphi}$ on each bin
   - Repeat same processes in other sections
3. Finally, $(M \times N) \times (m \times n)$ bins for $\mathbf{a}'$
4. In this calculation,
   - $(40 \times 30)\ \times (5 \times 5) = 30000$ bins of $\mathbf{a}'$
   - $(250 \times 250 \times 250)$ meshpoints of $\mathbf{a}$

Probability of final partons for a single particle

$$E_p = 10\,\mathrm{GeV} \rightarrow E_{p'} = 9\,\mathrm{GeV} \qquad \varphi_p = 10° \rightarrow \varphi_{p'} = 0°$$



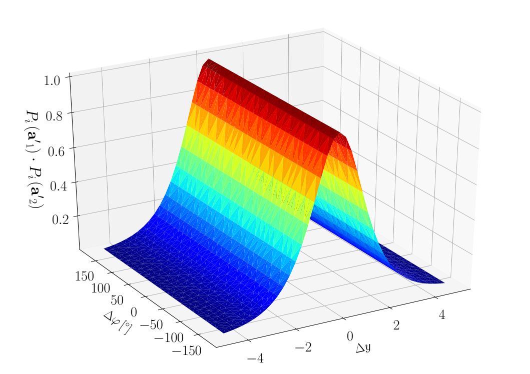
$P_i\left(\mathbf{a}'\right)$



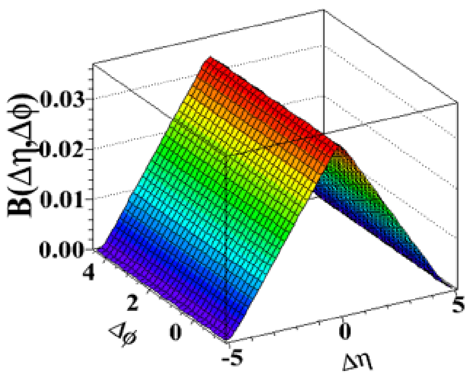
$P_f\left(\mathbf{a}'\right)$

Before interaction and Background distribution

$$E_p = 10\,\mathrm{GeV} \to E_{p'} = 9\,\mathrm{GeV} \qquad \varphi_p = 10^\circ \to \varphi_{p'} = 0^\circ$$



$P_i(\mathbf{a}'_1) \cdot P_i(\mathbf{a}'_2)$

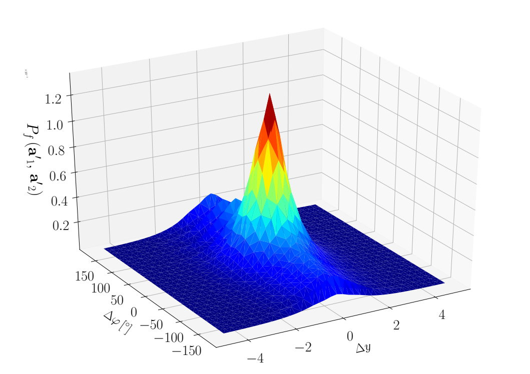


Background distribution from experiments

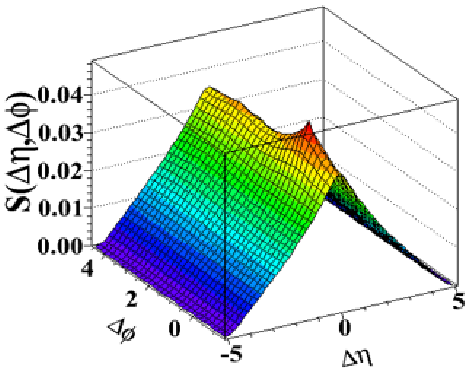
CMS collaboration, *Journal of High Energy Physics* **09**, 091 (2010)

After interaction and Signal distribution

$$E_p = 10\,\mathrm{GeV} \rightarrow E_{p'} = 9\,\mathrm{GeV} \qquad \varphi_p = 10^\circ \rightarrow \varphi_{p'} = 0^\circ$$



$P_f\,(\mathbf{a}'_1, \mathbf{a}'_2)$

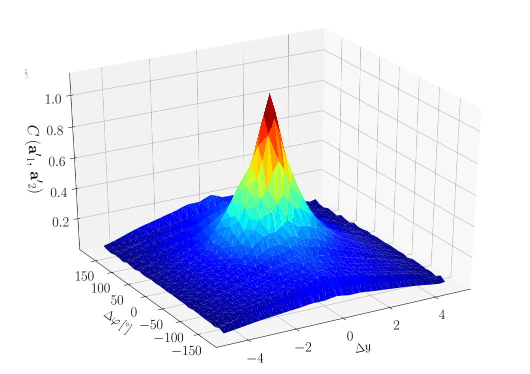


Signal distribution from experiments

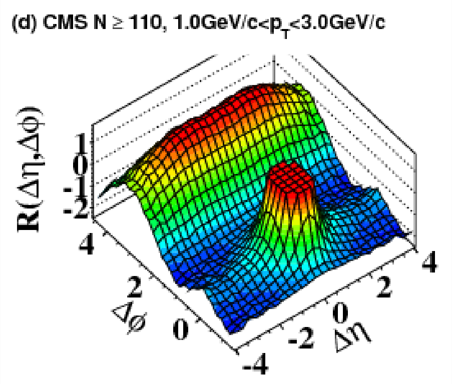
CMS collaboration, *Journal of High Energy Physics* **09**, 091 (2010)

Correlation function between final medium partons

$$E_p = 10\,\mathrm{GeV} \to E_{p'} = 9\,\mathrm{GeV} \qquad \varphi_p = 10^\circ \to \varphi_{p'} = 0^\circ$$



$C\,(\mathbf{a}'_1, \mathbf{a}'_2)$



Correlation from experiments

CMS collaboration, *Journal of High Energy Physics* **09**, 091 (2010)

Summary & Outlooks

Summary

- Inverstigate strange peak from divergences
  - Elastic scattering
    - Apply the cutoff parameter, $\Lambda_q = 0.0001 \left(m_a^4\right)$
  - Infrared
    - Cut $E_k \leq 0.1\,\mathrm{GeV}\,(m_a)$
    - Use the cutoff parameter, $\varepsilon = 0.01 \left(m_a^2\right)$
  - Not enough to remove divergent effects
- Adopt correlation function

$$C\left(\mathbf{a'}_1,\, \mathbf{a'}_2\right) = \frac{P_f\left(\mathbf{a'}_1,\, \mathbf{a'}_2\right)}{P_i\left(\mathbf{a'}_1\right) \cdot P_i\left(\mathbf{a'}_2\right)}$$

  - Calculate for final medium partons, $\mathbf{a'}_1,\, \mathbf{a'}_2$
  - Use Phenomenological Parton Distribution from Soft scattering model (phPDs)
  - Compare calculation to experimental results

Outlooks

- Calculate with various conditions
  - For various E_p, $E_{p'}$ and φ_p, $\varphi_{p'}$
  - Rotate jet plane on beam axis
- Modify $P_f(\mathbf{a}')$ calculation to include probability not to participate in collision

$$\int f(\mathbf{a}) \times d\sigma(\mathbf{a} \to \mathbf{a}')\, d^3\vec{a}$$

$$\Rightarrow \int f(\mathbf{a}) \times d\sigma(\mathbf{a} \to \mathbf{a}')\, d^3\vec{a}$$

$$+ \int f(\mathbf{a}') \times (1 - d\sigma(\mathbf{a}' \to \mathbf{a}''))\, d^3\vec{a''}$$

Thanks for your attention!

Differential cross section for correlation between $\mathbf{p}'$ and $\mathbf{a}'$

$$\frac{d\sigma}{d\Delta y d\Delta\varphi d^3\vec{a}} = \frac{1}{2\,(2\pi)^5}\frac{m_p^2 m_a^2}{pa_0 k_0}|M|^2\delta^{(4)}(\mathbf{p}' + \mathbf{a}' + \mathbf{k} - \mathbf{p} - \mathbf{a})$$
$$\times \frac{p_0' p_T' a_0' a_T'}{p_0' a_0'} dy_{p'}\, dp_T'\, d\varphi_{p'}\, dy_{a'}\, da_T'\, d\varphi_{a'}$$
$$= \frac{1}{2\,(2\pi)^5}\frac{m_p^2 m_a^2}{pa_0 k_0}|M|^2\delta^{(4)}(\mathbf{p}' + \mathbf{a}' + \mathbf{k} - \mathbf{p} - \mathbf{a})$$
$$\times\, p_T' a_T' dy_{p'}\, dp_T'\, d\varphi_{p'}\, dy_{a'}\, da_T'\, d\varphi_{a'}$$

Amplitude

According to diagrams in the previous slide, $M_{(a)}$ and $M_{(b)}$ are written as

$$M_{(a)} = -i\bar{u}(\mathbf{p}')\left(-ig\gamma^{\nu}\right)\frac{1}{q^2}\bar{u}(\mathbf{a}')\left(-ig\gamma_{\nu}\right)u(\mathbf{a}) \\ \times \frac{i\left(\not{\mathbf{p}} - \not{\mathbf{k}} + m\right)}{\left(\mathbf{p}-\mathbf{k}\right)^2 - m^2 + i\epsilon}\left(-ig\gamma^{\mu}\right)\epsilon_{\mu}^{*}u(\mathbf{p})$$

$$M_{(b)} = -i\bar{u}(\mathbf{p}')\epsilon_{\mu}^{*}\left(-ig\gamma^{\mu}\right)\frac{i\left(\not{\mathbf{p}}' + \not{\mathbf{k}} + m\right)}{\left(\mathbf{p}'+\mathbf{k}\right)^2 - m^2 + i\epsilon} \\ \times \left(-ig\gamma^{\nu}\right)\frac{1}{q^2}\bar{u}(\mathbf{a}')\left(-ig\gamma_{\nu}\right)u(\mathbf{a})u(\mathbf{p})$$

Amplitude

Total amplitude is obtained by $M_{(a)} + M_{(b)}$

$$
\begin{aligned}
M = ig^3 \frac{1}{q^2} \bar{u}(\mathbf{p}') \Bigg[\gamma^\nu \bar{u}(\mathbf{a}') \gamma_\nu u(\mathbf{a}) \frac{\left(\not{\mathbf{p}}' + \not{\mathbf{k}} + m\right) \not{\epsilon}^*}{(\mathbf{p}' + \mathbf{k})^2 - m^2 + i\epsilon} \\
+ \frac{\left(\not{\mathbf{p}}' + \not{\mathbf{k}} + m\right) \not{\epsilon}^*}{(\mathbf{p}' + \mathbf{k})^2 - m^2 + i\epsilon} \bar{u}(\mathbf{a}') \gamma_\nu u(\mathbf{a}) \gamma^\nu \Bigg] u(\mathbf{p}) \\
= ig^3 \frac{1}{q^2} \bar{u}(\mathbf{p}') \gamma^\nu u(\mathbf{p}) \bar{u}(\mathbf{a}') \gamma_\nu u(\mathbf{a}) \left[\frac{2\mathbf{p} \cdot \epsilon}{-2\mathbf{p} \cdot \mathbf{k} + i\epsilon} + \frac{2\mathbf{p}' \cdot \epsilon}{2\mathbf{p}' \cdot \mathbf{k} + i\epsilon} \right]
\end{aligned}
$$

Amplitude

Sum over whole possible polarizations of the photon to complete $|M|^2$

$$
\begin{aligned}
|M|^2 &= \frac{g^6}{q^4}\left|\bar{u}(\mathbf{p}')\gamma^\nu u(\mathbf{p})\right|^2\left|\bar{u}(\mathbf{a}')\gamma_\nu u(\mathbf{a})\right|^2 \Sigma_\epsilon \left|\frac{2\mathbf{p}\cdot\epsilon}{-2\mathbf{p}\cdot\mathbf{k}+i\epsilon}+\frac{2\mathbf{p}'\cdot\epsilon}{2\mathbf{p}'\cdot\mathbf{k}+i\epsilon}\right|^2 \\
&= \frac{g^6}{q^4}\left|\bar{u}(\mathbf{p}')\gamma^\nu u(\mathbf{p})\right|^2\left|\bar{u}(\mathbf{a}')\gamma_\nu u(\mathbf{a})\right|^2 \\
&\quad\times\left[\frac{4p^2\sin^2\theta_{kp}}{|(-2\mathbf{p}\cdot\mathbf{k})+i\epsilon|^2}+\frac{4p'^2\sin^2\theta_{kp'}}{|(2\mathbf{p}'\cdot\mathbf{k})+i\epsilon|^2}+4pp'\sin\theta_{kp}\sin\theta_{kp'}\cos\phi_{pp'}\right. \\
&\qquad\left.\times\left(\frac{1}{(-2\mathbf{p}\cdot\mathbf{k}+i\epsilon)(2\mathbf{p}'\cdot\mathbf{k}-i\epsilon)}+\frac{1}{(-2\mathbf{p}\cdot\mathbf{k}-i\epsilon)(2\mathbf{p}'\cdot\mathbf{k}+i\epsilon)}\right)\right]
\end{aligned}
$$

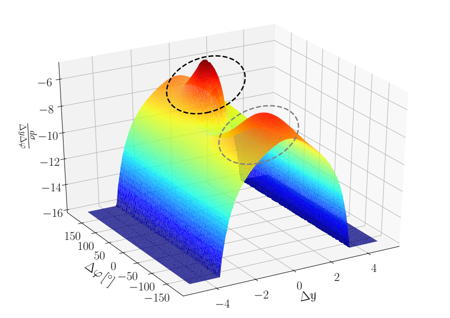
Inverstigate M_{fac}

- The peak comes from small $\mathbf{p}^\mu \mathbf{k}_\mu$ or $\mathbf{p}'^\mu \mathbf{k}_\mu$
  - Denominator produces large values
  - It can be regarded as divergence
    - Is this also from elastic or infrared processes?

$$|M|^2 \propto \underbrace{\sum_\epsilon \left| \frac{2\mathbf{p}\cdot\epsilon}{-2\mathbf{p}^\mu \mathbf{k}_\mu + i\varepsilon} + \frac{2\mathbf{p}'\cdot\epsilon}{2\mathbf{p}'^\mu \mathbf{k}_\mu + i\varepsilon} \right|^2}_{M_{fac}}$$

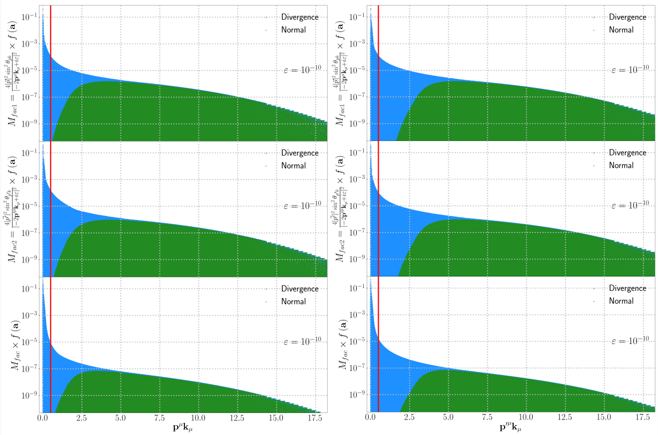
$$\propto \left[\underbrace{\frac{4p^2 \sin^2\theta_{kp}}{|(-2\mathbf{p}^\mu \mathbf{k}_\mu) + i\varepsilon|^2}}_{M_{fac1}} + \underbrace{\frac{4p'^2 \sin^2\theta_{kp'}}{|(2\mathbf{p}'^\mu \mathbf{k}_\mu) + i\varepsilon|^2}}_{M_{fac2}} + (\cdots) \right]$$

Log scale, $\varepsilon = 0.01$



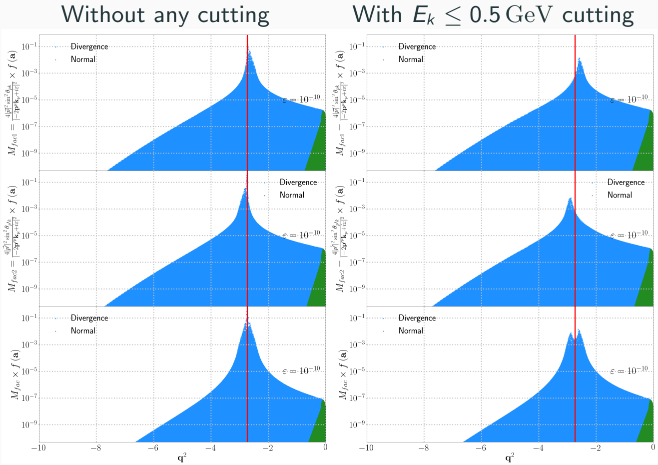
- Divergent peak $\sim 3.0 \times 10^{-5}$
  - $|\Delta y| \leq 0.125,\ 75^\circ < \Delta\varphi < 90^\circ$
- Normal peak $\sim 2.5 \times 10^{-6}$
  - $|\Delta y| \leq 0.25,\ -150^\circ < \Delta\varphi < -110^\circ$

Dependence on $\mathbf{p}^{\mu}\mathbf{k}_{\mu}$ and $\mathbf{p}'^{\mu}\mathbf{k}_{\mu}$



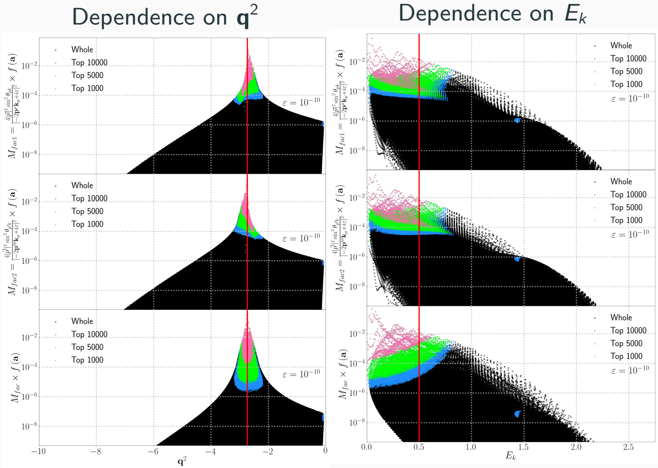
- Choose specific bin for each peak
  - Included divergence
    - $-0.15 \leq \Delta y \leq -0.01$
    - $81.6^{\circ} \leq \Delta\varphi \leq 84.0^{\circ}$
  - Normal
    - $0.00 \leq \Delta y \leq 0.05$
    - $-120.0^{\circ} \leq \Delta\varphi \leq -117.6^{\circ}$
- Figure obtained
  - Large amplitude values appears at $\mathbf{p}^{\mu}\mathbf{k}_{\mu} \to 0$ and $\mathbf{p}'^{\mu}\mathbf{k}_{\mu} \to 0$ simultaneously
    - $E_k \to 0$
    - $(E_p - |\vec{p}|\cos\theta_{pk}) \to 0$ and $\left(E_{p'} - |\vec{p'}|\cos\theta_{p'k}\right) \to 0$

Dependence on q^2



- Choose specific bin for each peak
  - Included divergence
    - $-0.15 \leq \Delta y \leq -0.01$
    - $81.6^\circ \leq \Delta\varphi \leq 84.0^\circ$
  - Normal
    - $0.00 \leq \Delta y \leq 0.05$
    - $-120.0^\circ \leq \Delta\varphi \leq -117.6^\circ$
- Figure obtained
  - $\Delta\mathbf{p}^2 \approx \mathbf{q}^2$ major contribution
    - $E_k \leq 0.5\,\mathrm{GeV}$
    - $\cos\theta_{\Delta pq} \approx 1.0$
    - $\cos\theta_{\Delta pk} \approx 0.5$
  - Cutting E_k is not enough to reduce divergent effects
    - Which one needs more?

Check top n-th entries sorted by $|M|^2$



- Choose specific bin for each peak
  - Included divergence
    - $-0.15 \leq \Delta y \leq -0.01$
    - $81.6^\circ \leq \Delta\varphi \leq 84.0^\circ$
- Figure obtained
  - Hard to set constraint for removing divergence