Heavy Quarks in Quark-Gluon Plasma and Heavy Dark Matter Particles in Early Universe

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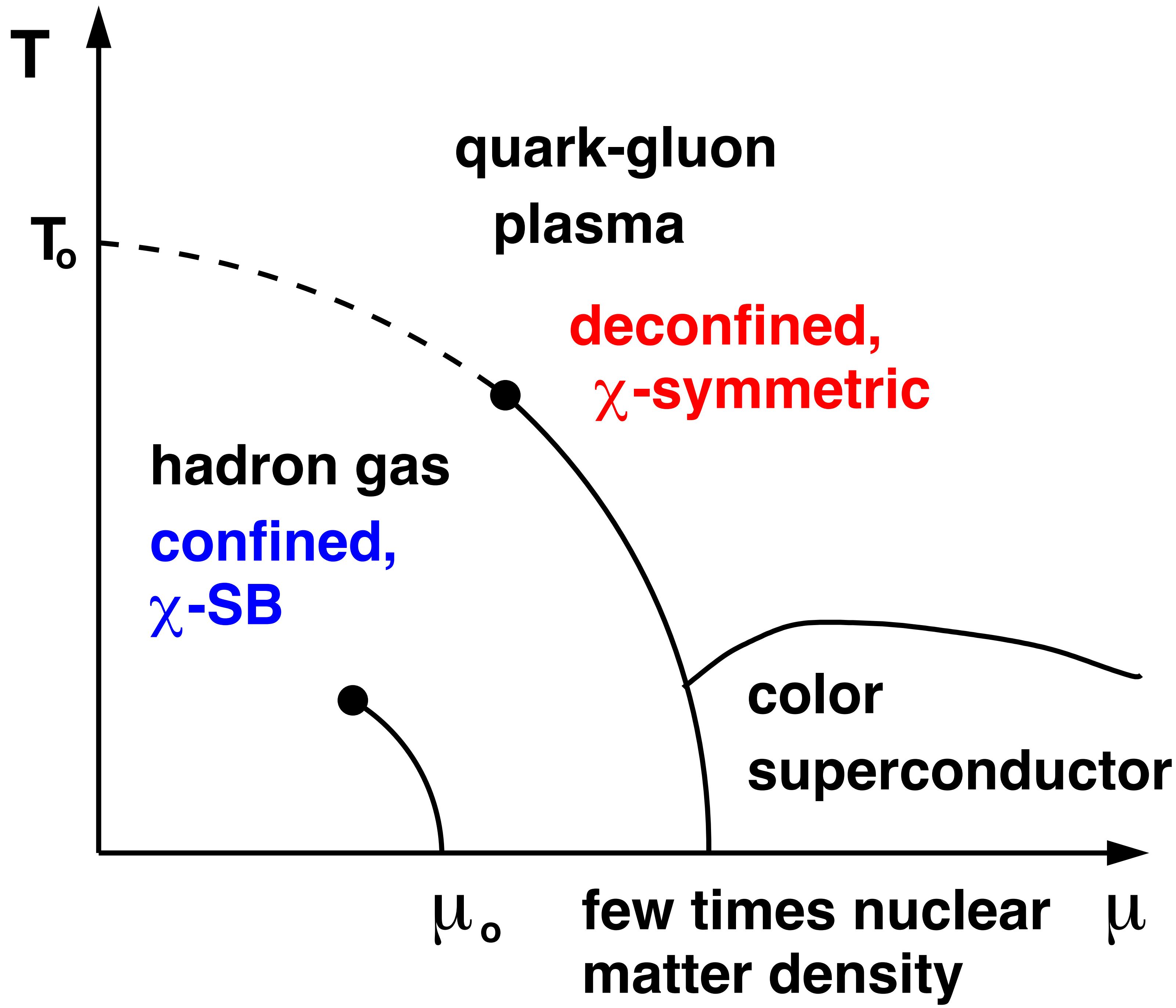
1. S. Kim and M. Laine, “Rapid thermal co-annihilation through bound states in QCD”, JHEP1607 (2016) 143

2. S. Kim and M. Laine, “On thermal corrections to near-threshold annihilation”, JCAP1701 (2017) 013

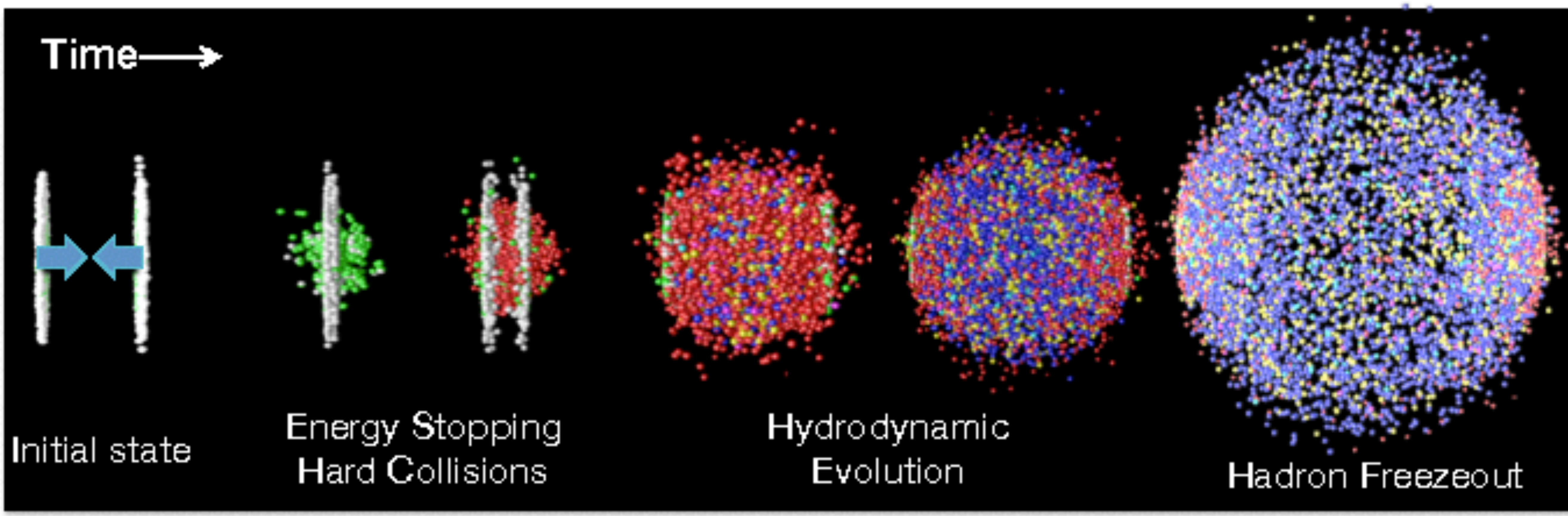
3. S. Kim and M. Laine, “Studies of a thermally averaged p-wave Sommerfeld factor”, Phys. Lett. B795 (2019) 469

4. S. Biondini, S. Kim and M. Laine, “Non-relativistic susceptibility and a dark matter application”, arXiv: 1908.07541 [hep-ph].

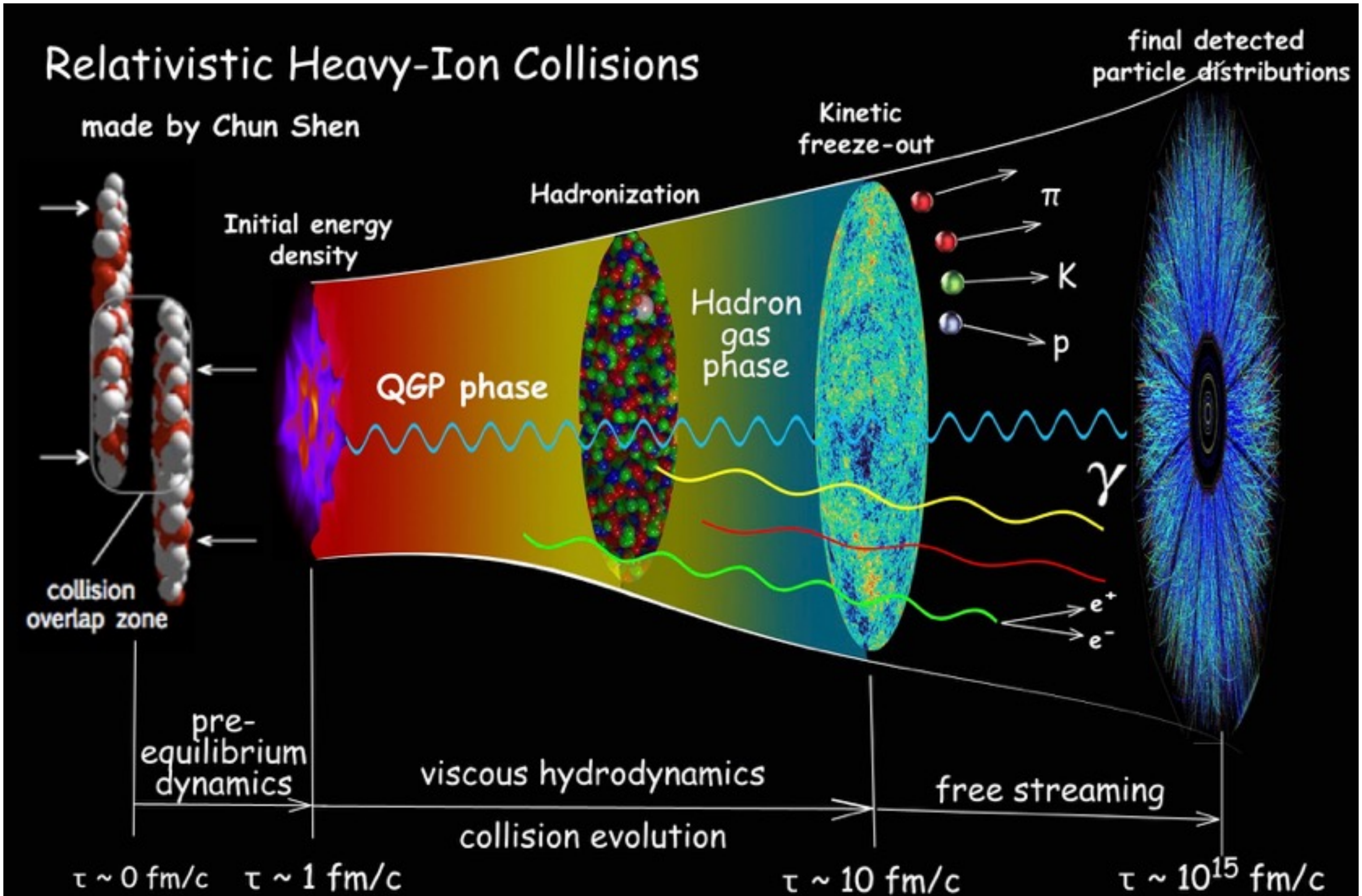
QCD phase diagram



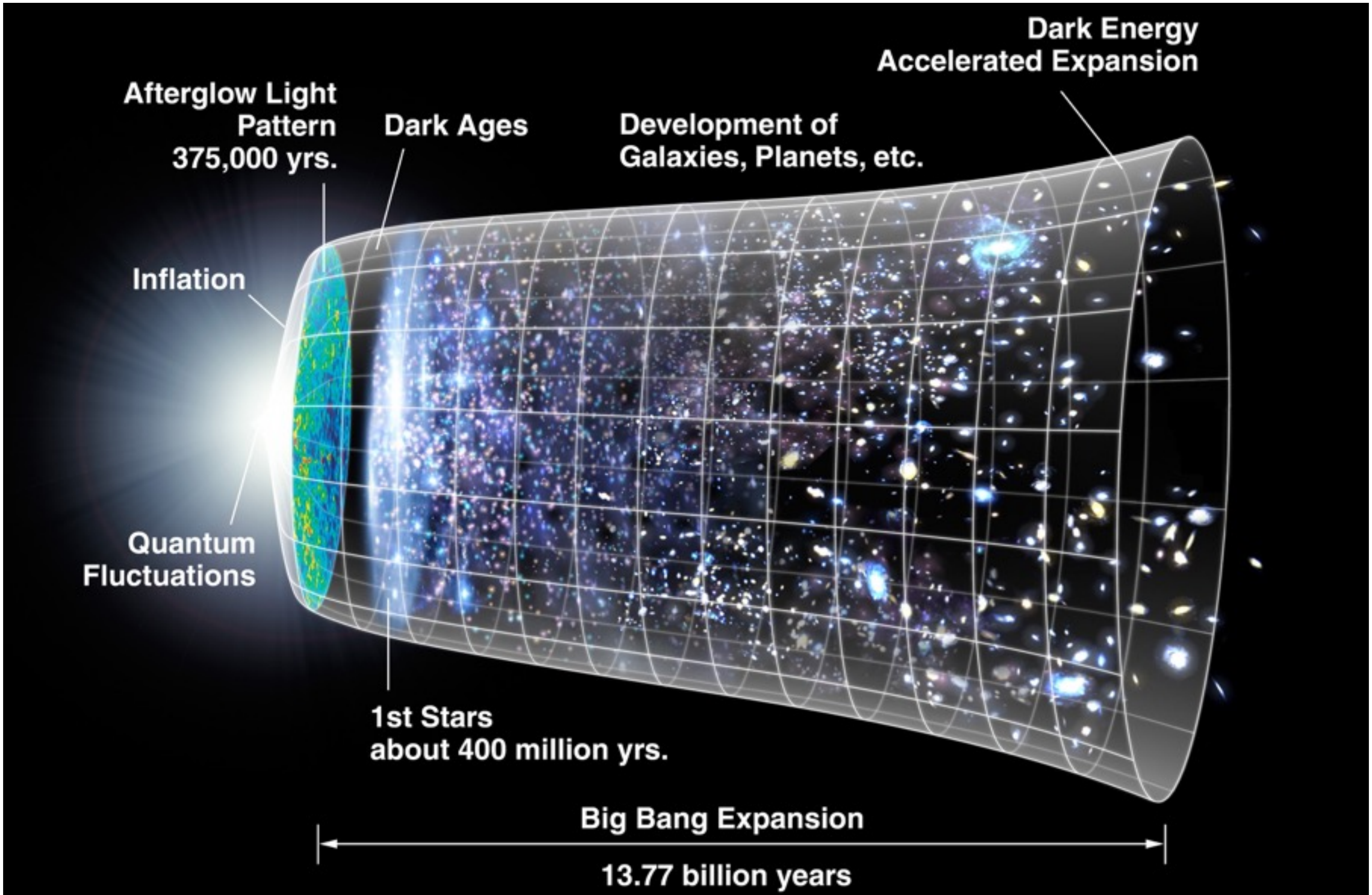
Relativistic heavy ion collisions



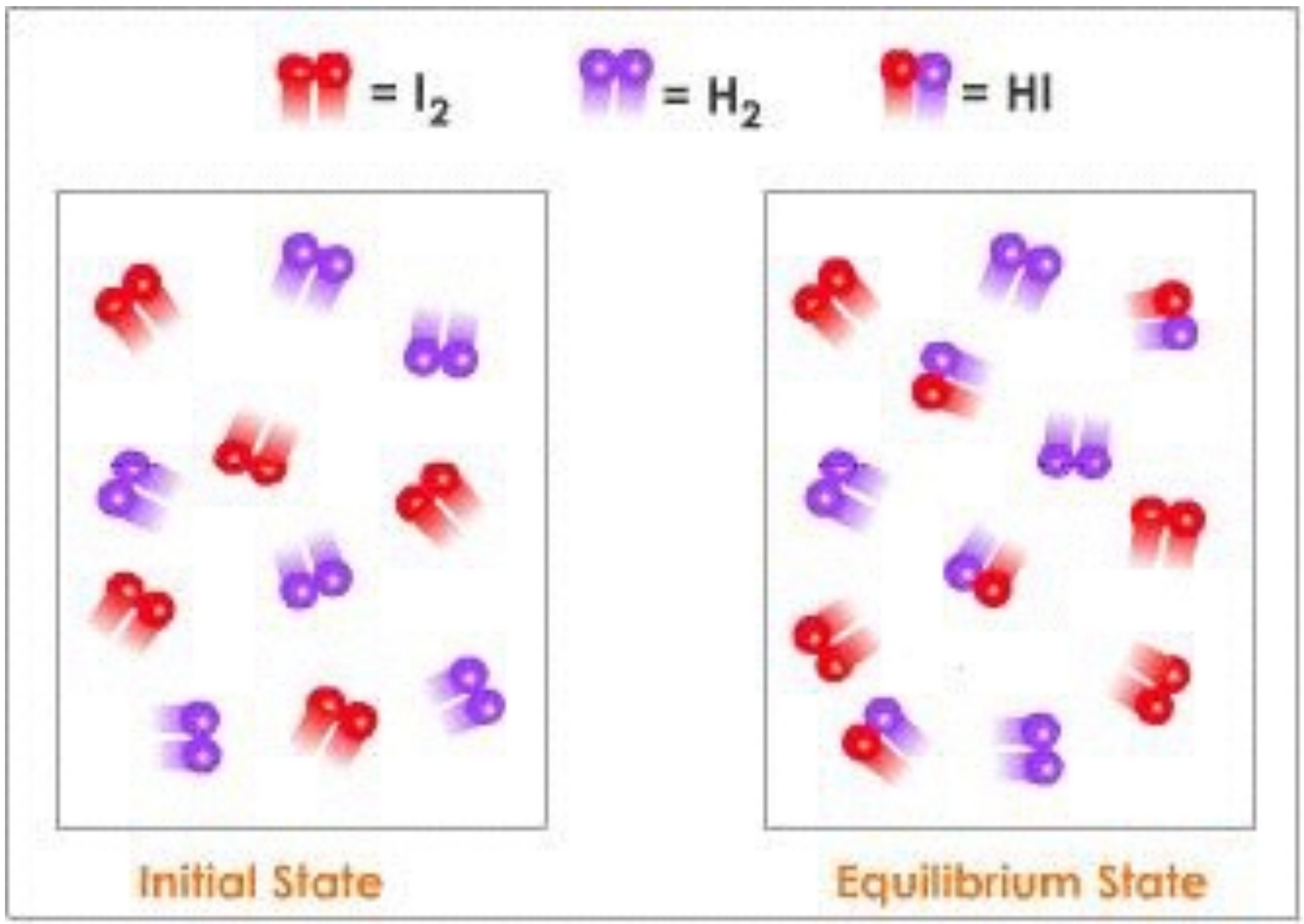
Evolution of Nuclear Collision



Evolution of Universe



chemical equilibrium



https://chem-guide.blogspot.com/2010/04/dynamic-nature-of-chemical-equilibrium.html

evolution of the number density in thermal environment: Lee-Weinberg equation (B.W. Lee and S. Weinberg, PRL39 (1977) 165)

- the number density (n) of heavy quarks or dark matter (Boltzmann equation)

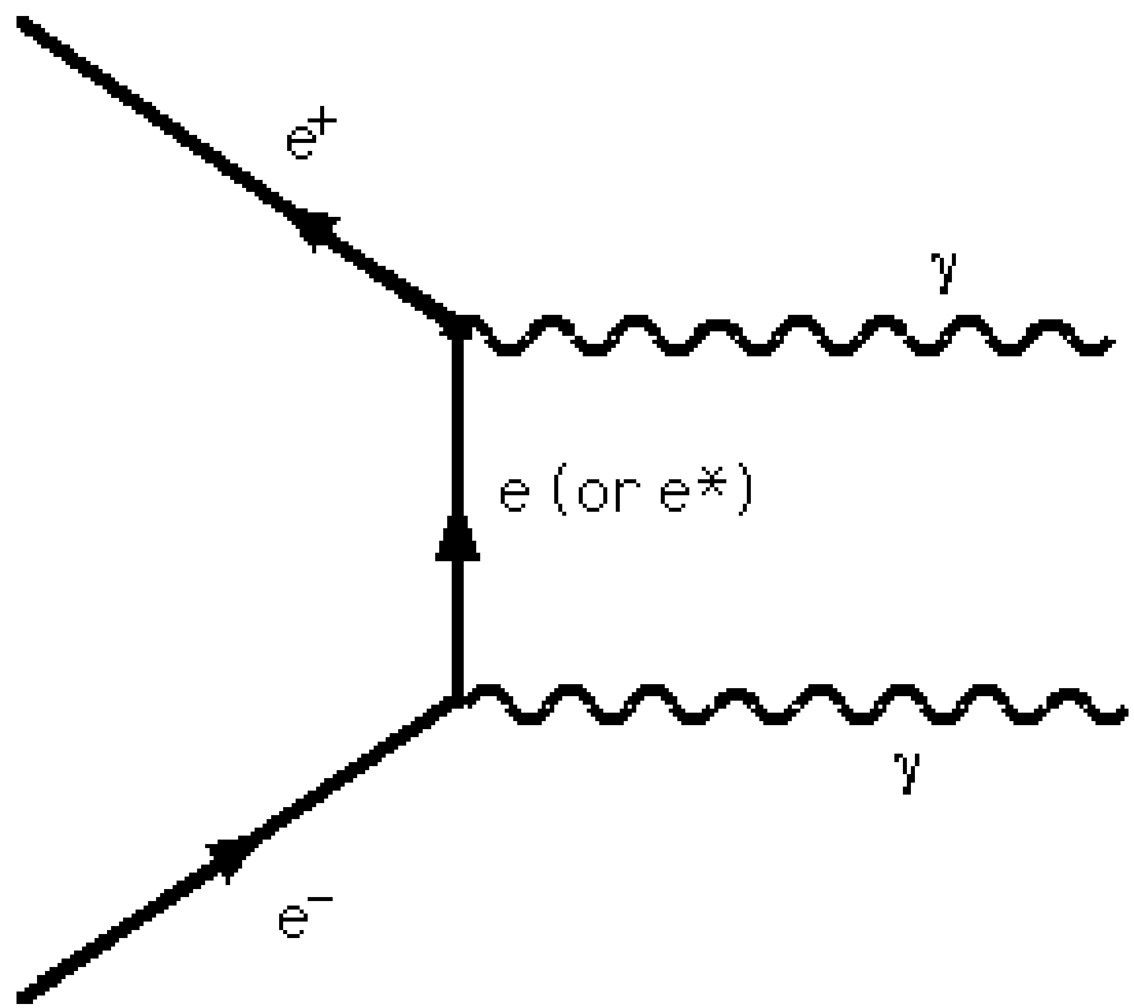
$$(\partial_t + 3H)n = - < \sigma v > (n^2 - n_{eq}^2)$$

- in linearized form

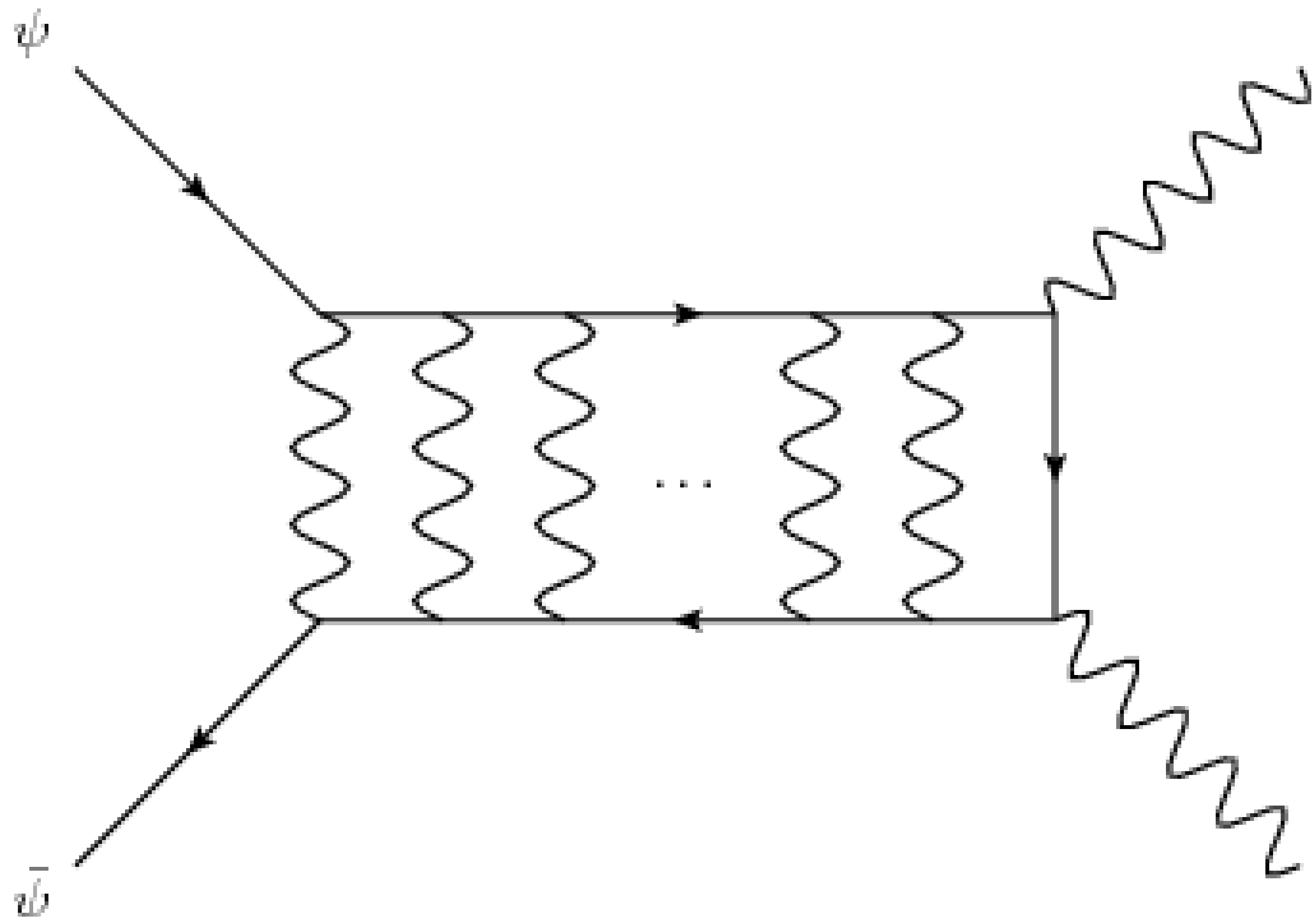
$$(\partial_t + 3H)n = -\Gamma_{\rm chem}(n - n_{eq}) + O(n - n_{eq})^2$$

where $\Gamma_{\rm chem} = 2 < \sigma v > n_{eq}$, chemical equilibriation rate

Pair annihilation/Pair creation



Pair annihilation/Pair creation



Sommerfeld factor

- Sommerfeld effect enhances the Born matrix elements

$$|\mathcal{M}_{\text{resummed}}|^2 = \mathcal{S}|\mathcal{M}_{\text{tree}}|^2$$

evolution of the number density
in thermal environment:
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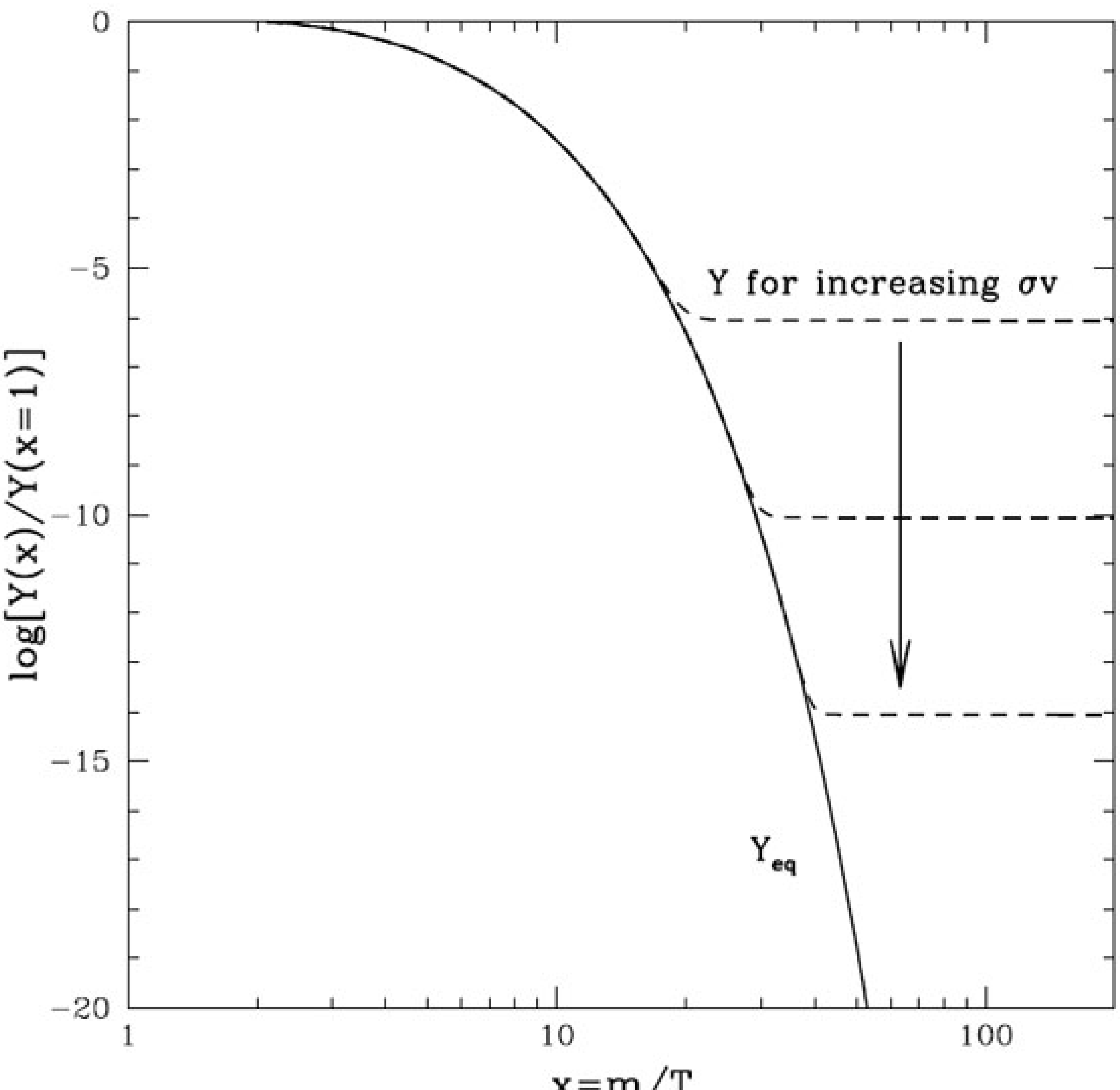
Evolution of dark matter number density



Heavy quark annihilations vs heavy dark matter particle annihilations in thermal environment

- heavy quark and anti heavy quark annihilates into light quarks and gluons
- heavy dark matter particles annihilate into light standard model particles
- we are interested in the case that the temperature (T) scale is far below the mass (M) scale, M >> T

Heavy quarks/quarkonium and NRQCD factorization (E. Braaten, G.T. Bodwin, G.P. Lepage, PRD51 (1995) 1125)

- inclusive decay rates (sum over all the possible final states) of heavy quarkonium can be separated into the short distance perturbative QCD effect and into the long distance structure associated with quarkonium structure using NRQCD (non-relativistic effective field theory QCD)

- M >> Mv >> Mv^2

- factorization theorem (the above structure can be "proven" analytically)

NRQCD

- expanding QCD Lagrangian in power of heavy quark velocity
- expanding operators in powers of heavy quark velocity

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L},$$

with

$$\mathcal{L}_0 = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi,$$

and

$$\begin{aligned}
\delta\mathcal{L} = \quad & -\frac{c_1}{8M^3} \left[\psi^\dagger (\mathbf{D}^2)^2 \psi - \chi^\dagger (\mathbf{D}^2)^2 \chi \right] \\
& + c_2 \frac{ig}{8M^2} \left[\psi^\dagger (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) \psi + \chi^\dagger (\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}) \chi \right] \\
& - c_3 \frac{g}{8M^2} \left[\psi^\dagger \sigma \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \psi + \chi^\dagger \sigma \cdot (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}) \chi \right] \\
& c_4 \frac{g}{2M} \left[\psi^\dagger \sigma \cdot \mathbf{B} \psi - \chi^\dagger \sigma \cdot \mathbf{B} \chi \right].
\end{aligned}$$

Lattice gauge theory

- Spacetime regularized Euclidean quantum field theory
- Fully non-perturbative definition of a QFT
- Quantum mechanics as a statistical mechanics problem
- Monte Carlo method
- Difficult to handle real time physics

Lattice gauge theory I

- In quantum mechanics ((0+1)-dimensional quantum field theory), solve

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \hat{H}\psi(\mathbf{x},t)$$

and calculate

$$\langle\psi_f|e^{-\frac{i}{\hbar}\hat{H}(t_f-t_i)}|\psi_i\rangle$$

- where

$$\hat{H} = \frac{\mathbf{p}^2}{2m} + V(\mathbf{x})$$

Lattice gauge theory II

• consider

$$
\begin{aligned}
&\langle\psi_f(t_f)|e^{-\frac{i}{\hbar}\hat{H}(t_f-t_i)}|\psi_i(t_i)\rangle \\
&= \langle\psi_f(t_f)|\int d^3x|\mathbf{x}\rangle\langle\mathbf{x}e^{-\frac{i}{\hbar}\hat{H}(t_f-t_i)}|\int d^3x'|\mathbf{x}'\rangle\langle\mathbf{x}'|\psi_i(t_i)\rangle \\
&= \int d^3x\int d^3x'\psi_f(\mathbf{x},t_f)^\dagger\psi_i(\mathbf{x}',t_i)\langle\mathbf{x}|e^{-\frac{i}{\hbar}\hat{H}(t_f-t_i)}|\mathbf{x}'\rangle
\end{aligned}
$$

• and Green function method

$$G(\mathbf{x},t_f|\mathbf{x}',t_i) = \langle\mathbf{x}|e^{-\frac{i}{\hbar}\hat{H}(t_f-t_i)}|\mathbf{x}'\rangle,$$

and

$$\psi_f(\mathbf{x},t_f) = \int d^3x' G(\mathbf{x},t_f|\mathbf{x}',t_i)\psi_i(\mathbf{x}',t_i)$$

$$G(x,t_f|x',t_i) = \langle x|e^{-i\hat{H}T}|x'\rangle = \int\prod_{i=1}^{N}dx_i e^{-i\varepsilon\sum_i L(t_i)} = \int\mathcal{D}x(t)e^{-iS},$$

and

$$S = \int_{t_i}^{t_f}dt\,L[x(t)] = \int_{t_i}^{t_f}dt\left[\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 - V(x(t))\right]$$

Lattice gauge theory III

- partition function

$$Z = \mathrm{Tr}\left(e^{-\frac{\hat{H}}{k_B T}}\right) = \sum_n e^{-\frac{E_n}{k_B T}} = \int d\,x \langle x | e^{-\frac{\hat{H}}{k_B T}} | x \rangle$$

With $k_B = 1$ and $\hbar = 1$ and $\tau_f - \tau_i = \frac{1}{T}$ and the periodic boundary condition,

$$Z = \int \mathcal{D}x(\tau) e^{-S},$$

and

$$S = \int_{\tau_i}^{\tau_f} d\,\tau \left[\frac{1}{2} m \left(\frac{dx}{d\tau}\right)^2 + V(x(\tau))\right],$$

- Euclidean space ($it \to \tau$) and statistical mechanics

Lattice gauge theory IV

- quantum mechanics is a statistical mechanics problem
- cf. M. Creutz and B Freedman, “A statistical approach to quantum mechanics”, Annals. Phys. 132 (1981) 427
- generalization to quantum field theory (thermodynamic limit of statistical mechanics)
- Green functions are correlation functions in statistical mechanics

Chemical equilibration rate as a transport phenomenon

- D. Boedeker and M. Laine, JHEP1207 (2012) 130 and JHEP1301 (2013) 037
- In a non-relativistic field theory, the number density is related to the energy density (Hamiltonian)
- change of the number density can be studied by insertion of four-quark operator
- In thermal system, “trace over states” is performed and the three point function becomes the two point function

What to calculate for S-wave Sommerfeld factor?

$$P_1 \equiv \frac{1}{2N_c}\mathrm{Re}\left\langle G^{\theta}_{\alpha\alpha;ii}(\beta,\vec{0};0,\vec{0})\right\rangle ,$$

$$P_2 \equiv \frac{1}{2N_c}\left\langle G^{\theta}_{\alpha\gamma;ij}(\beta,\vec{0};0,\vec{0})G^{\theta\dagger}_{\gamma\alpha;ji}(\beta,\vec{0};0,\vec{0})\right\rangle ,$$

$$P_3 \equiv \frac{1}{2N_c^2}\left\langle G^{\theta}_{\alpha\alpha;ij}(\beta,\vec{0};0,\vec{0})G^{\theta\dagger}_{\gamma\gamma;ji}(\beta,\vec{0};0,\vec{0})\right\rangle .$$

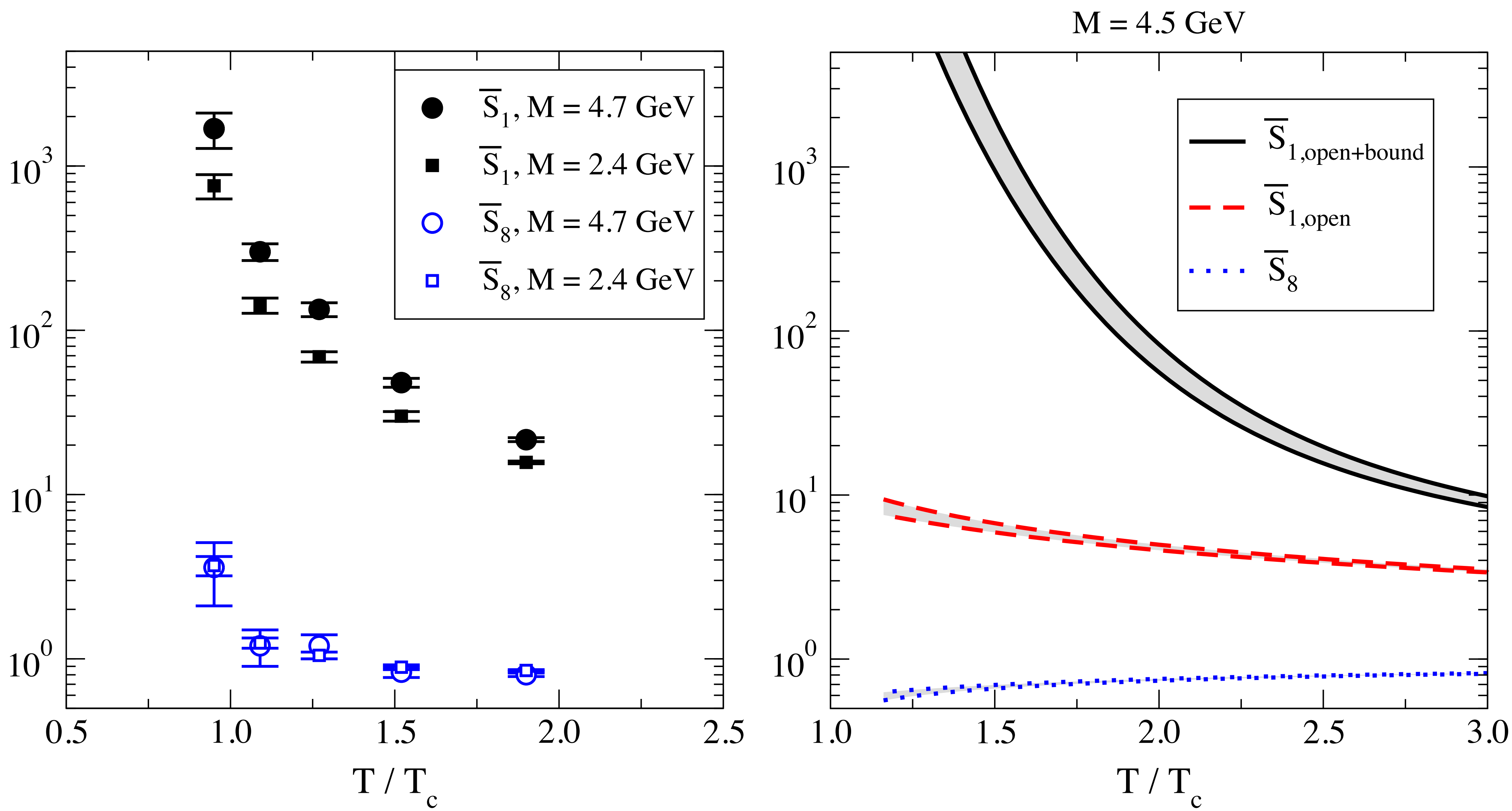
- singlet Sommerefeld factor

$$\bar{S}_1 = \frac{P_2}{P_1^2} .$$

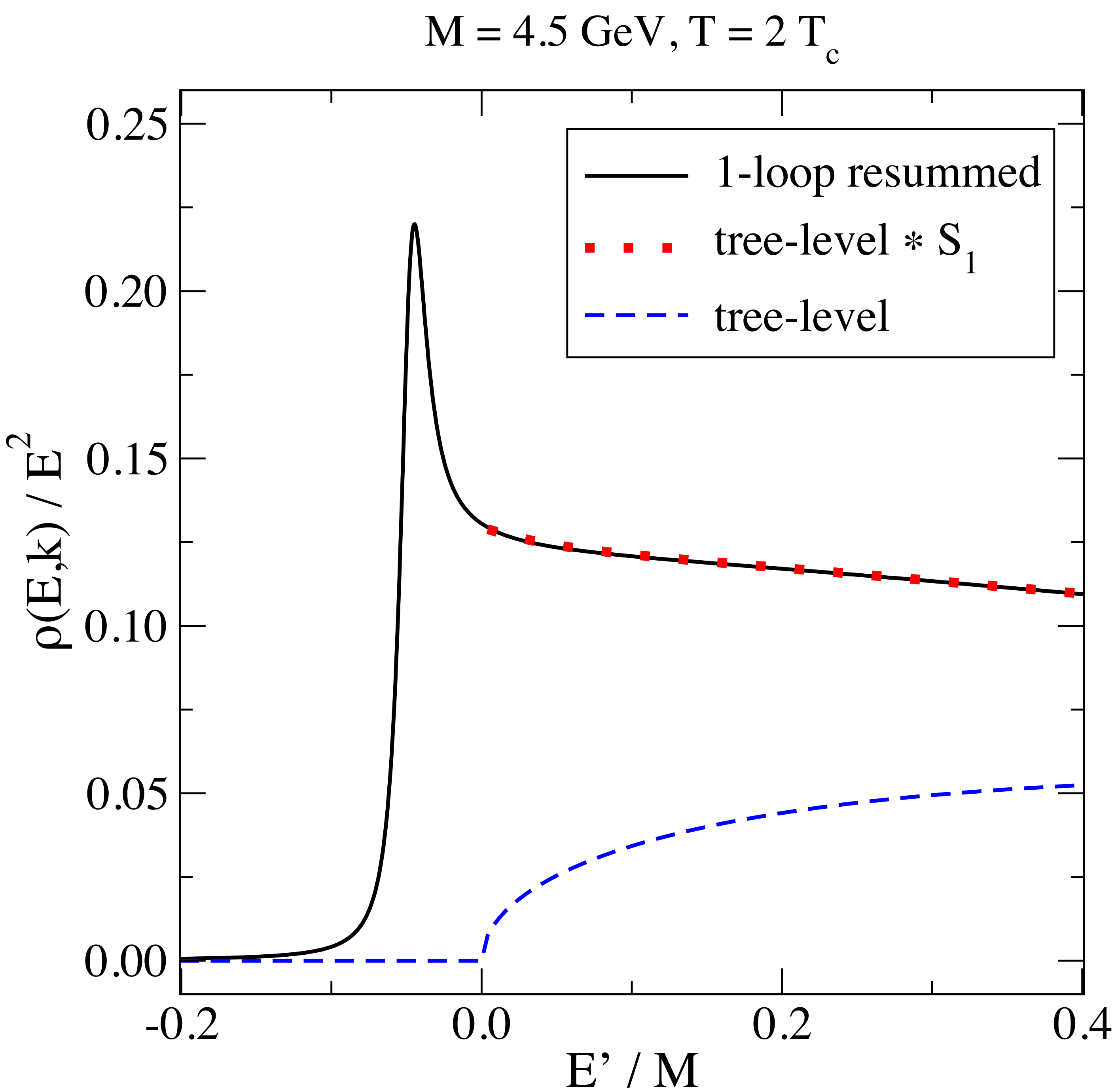
- octet Sommerefeld factor

$$\bar{S}_8 = \frac{N_c^2 P_3 - P_2}{(N_c^2-1)P_1^2} .$$

S-wave Sommerfeld factor (lattice and perturbative)



Bound state effect



Why larger Sommerfeld factor?

- When incoming heavy particle belongs to a bound state, the probability to meet the other particle is higher than free particle case
- strong interactions have larger coupling constants

Why P-wave?

- incoming particle system can have any relative angular momentum

- the total probability of an annihilation process is sum of all possible channels

What to calculate for P-wave Sommerfeld factor

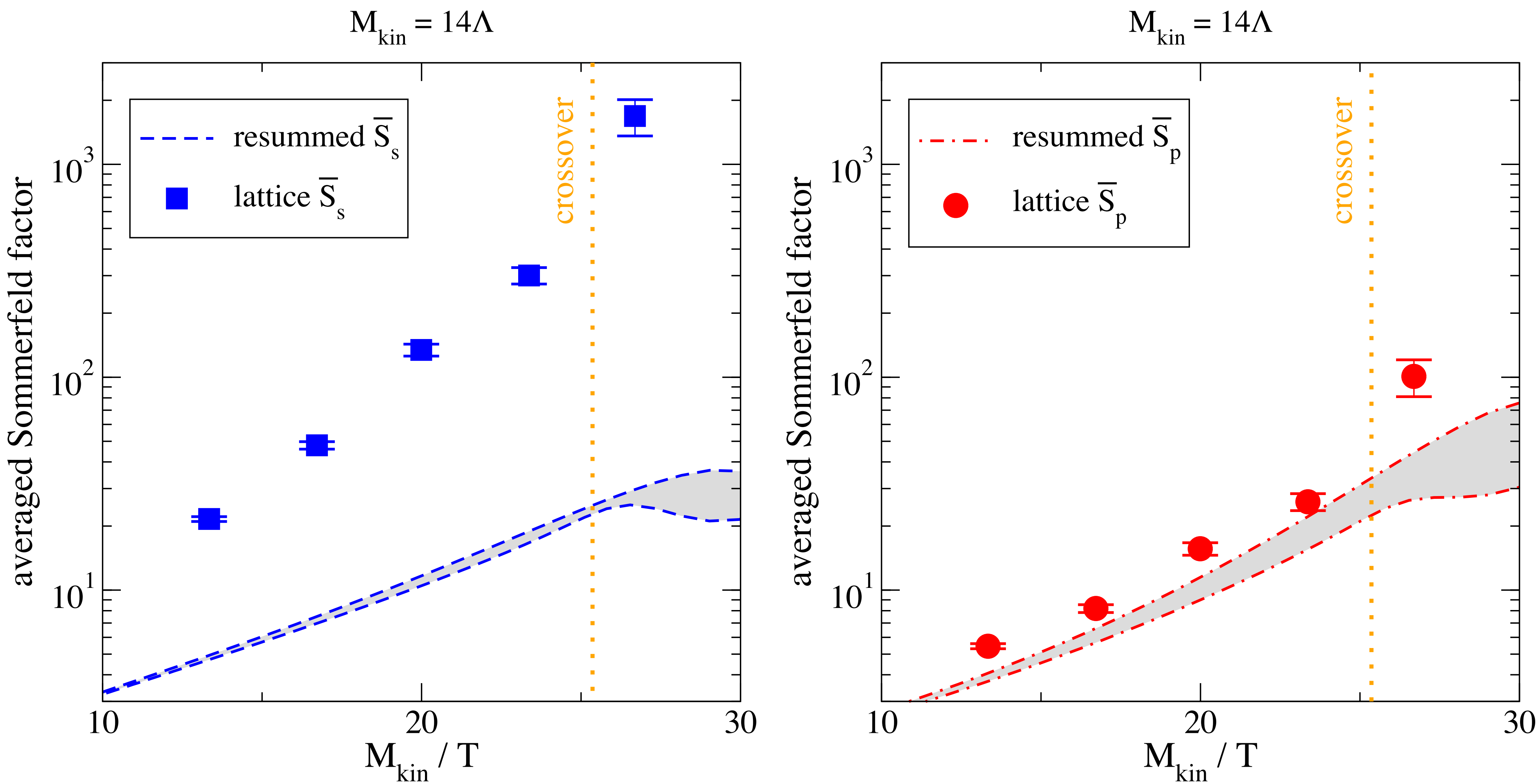
- P-wave Sommerfeld factor

$$\bar{S}_p = \frac{P_p}{M^2 P_1^2}$$

with

$$p_p = \mathrm{Tr}\langle \Delta_i G_V(\beta,\vec{0};0,\vec{0};i) G^\dagger(\beta,\vec{0};0,\vec{0})\rangle - \mathrm{Tr}\langle G_V(\beta,\vec{0};0,\vec{0};i)\Delta_i G^\dagger(\beta,\vec{0};0,\vec{0})\rangle$$

S- and P-wave Sommerfeld factor



Beyond linear response theory
(T. Binder et al, PRD98 (2018) 115023)

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- in linearized form

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where $\Gamma_{\mathrm{chem}} = 2 < \sigma v > n_{eq}$, chemical equilibriation rate

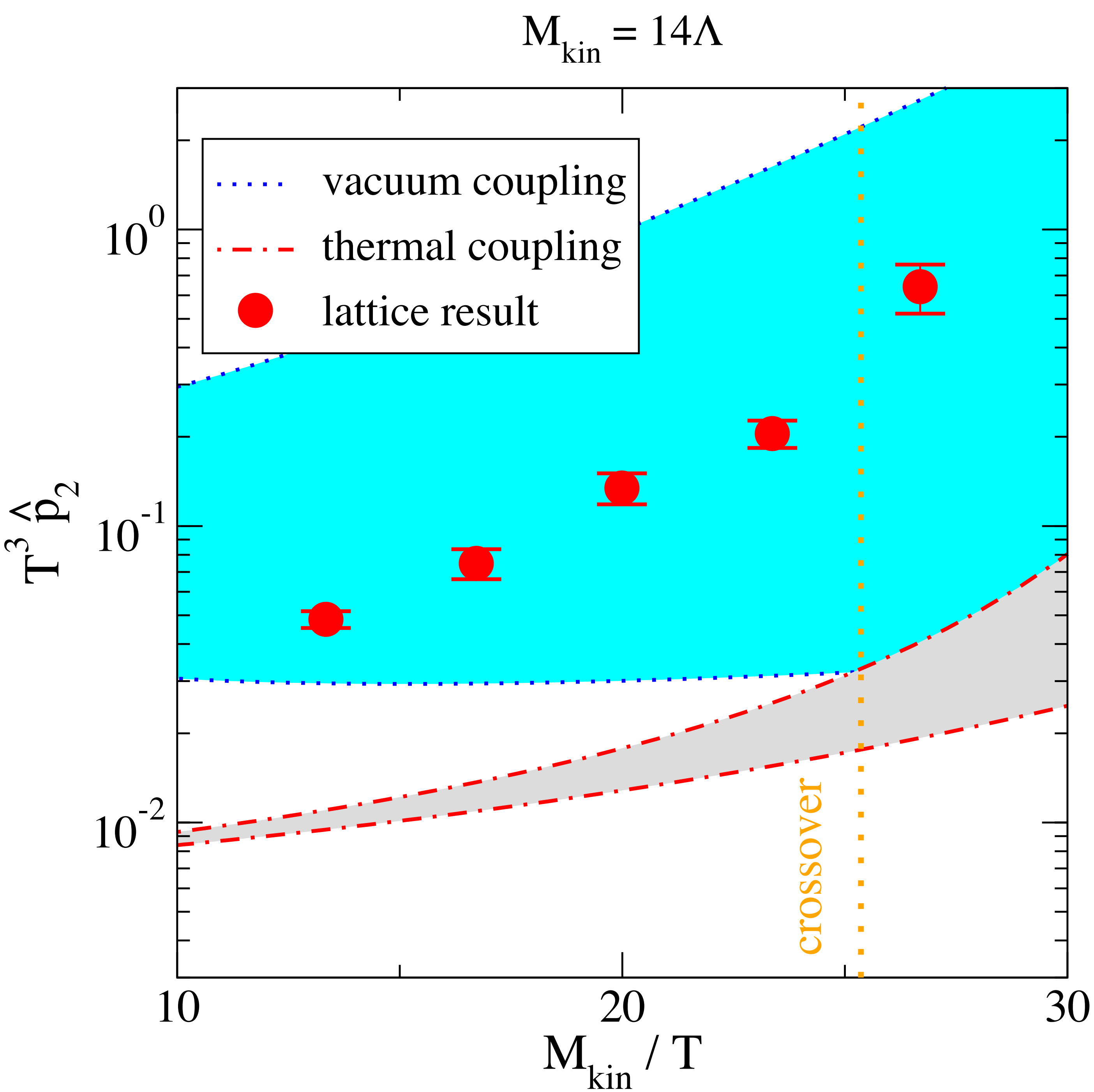
- beyond linear response regime

$$(\partial_t + 3H)n = - < \sigma v > (e^{2\frac{\mu(n)}{k_B T}} - 1)n_{eq}^2$$

What to calculate for non-relativistic particle number susceptibility?

$$\hat{p}_2 = \frac{\int_{\mathbf{x}} \left\{ \left\langle \mathrm{Re}\,\mathrm{Tr}\, G_{\mathbf{x}}\, \mathrm{Re}\,\mathrm{Tr}\, G_{\mathbf{0}} \right\rangle - \left\langle \mathrm{Re}\,\mathrm{Tr}\, G_{\mathbf{0}} \right\rangle^2 \right\}}{2 \left\langle \mathrm{Re}\,\mathrm{Tr}\, G_{\mathbf{0}} \right\rangle^2} \,.$$

Number susceptibility for non-relativistic particles



Conclusion I

- For the first time, non-perturbative computation of chemical equilibration rate becomes possible using Euclidean lattice field theory for heavy particles.

- Non-perturbative effect can be ~100 larger than a naive estimate

- A question, “the number density of heavy quarks in quark-gluon plasma” is similar to “the number density of heavy dark matter particles in early universe” in the sense that “non-relativistic particle in thermal environment”

Conclusion II

- Lattice gauge theory is necessary when perturbative estimate is not possible at all because non-perturbative effect can be an inherent property of the problem

- Physics of dark matter particles are not known and interaction can be non-perturbative interaction. In this case, thermal effect is quite subtle and lattice gauge theory method is essential