Heavy quarkonia in a bulk viscous QGP

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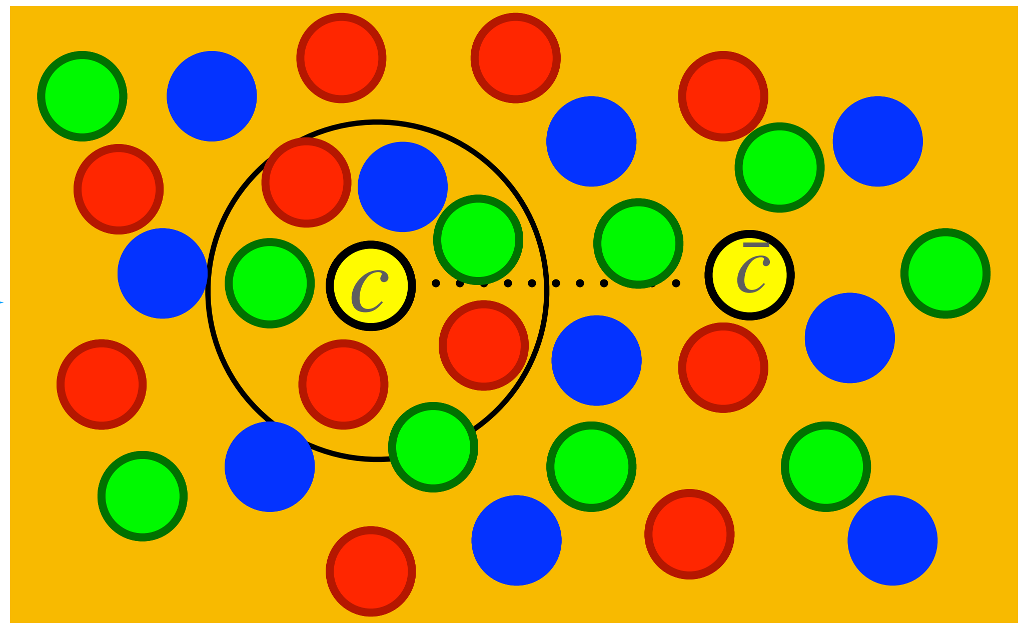
Introduction

- **Quarkonia** are the bound state of heavy quarks and its own antiquark

Charmonium Bottomonium

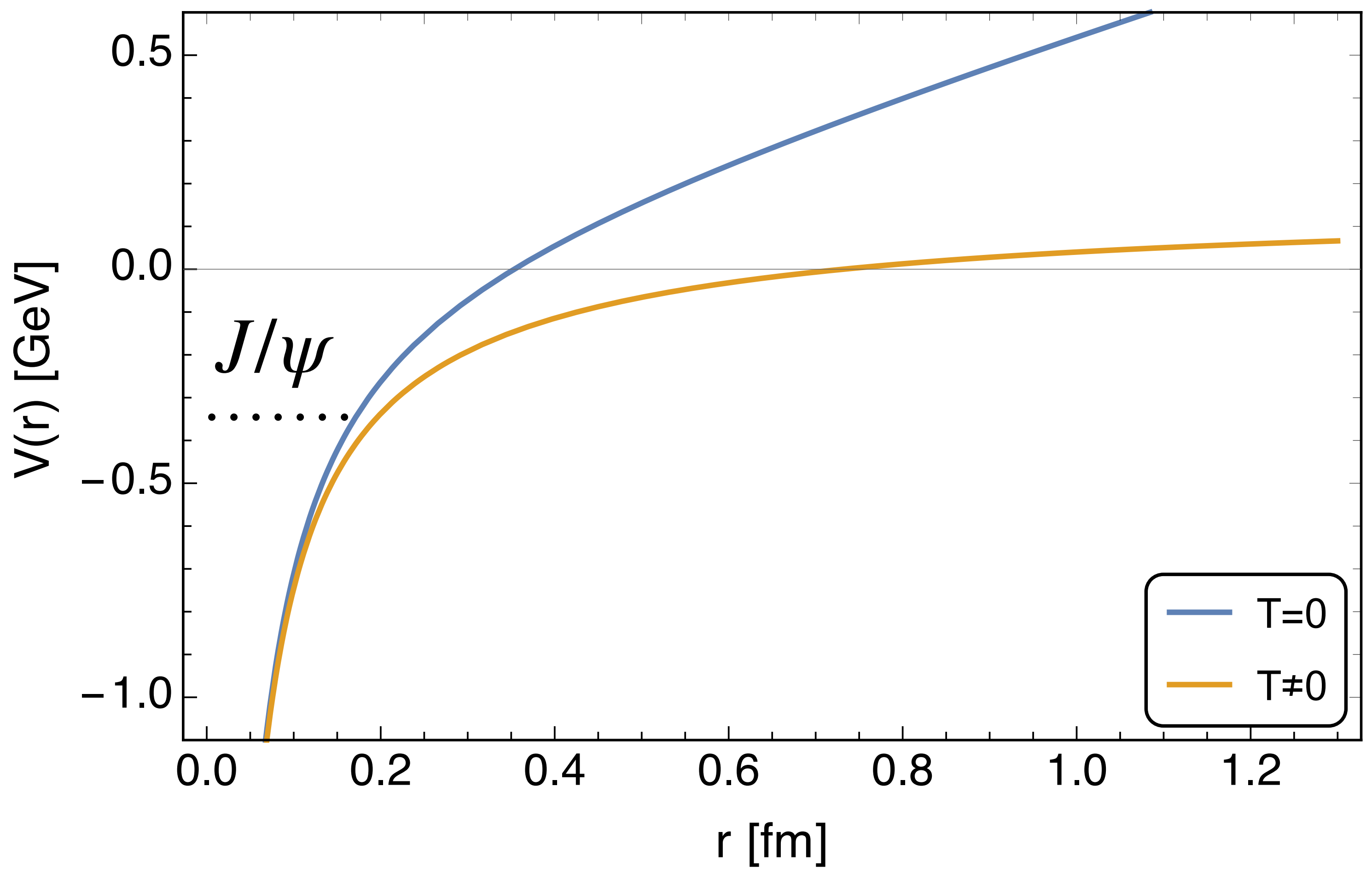
- **Quarkonium in QGP** → as a probe of the quark gluon plasma formed in heavy ion collision

Color screening

J/ψ → 

→ **Dissociated into c and $\bar{c}$**

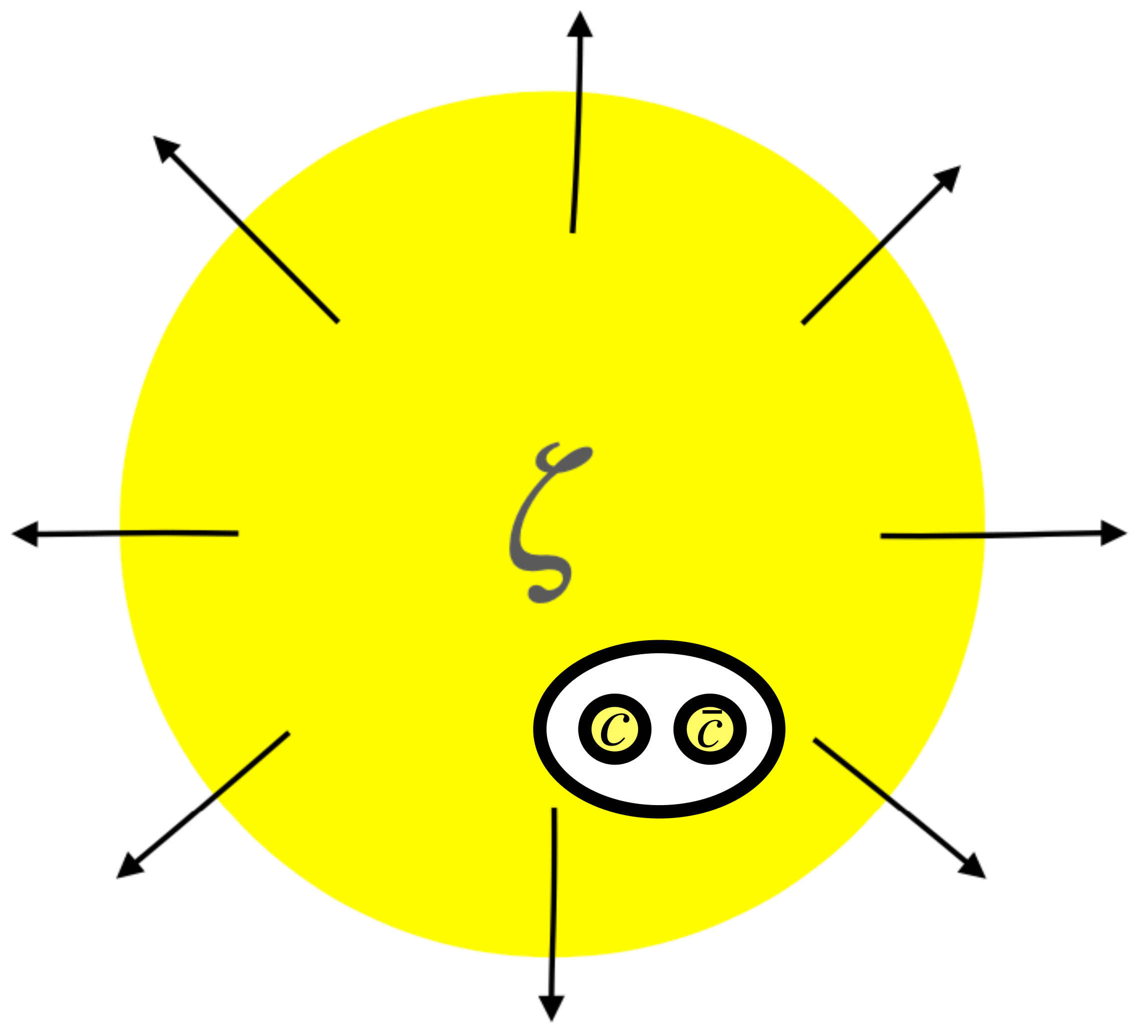
QGP medium



Matsui and Satz, PLB 178 (1986) 416

Question: heavy quarkonia as a probe of non-eq. QGP?

- QGP has many different **non-equilibrium properties:**
  - Dissipative effects
    - Shear viscosity
    - **Bulk viscosity**
  - Magnetic field
  - …



- QCD matter has non-zero bulk viscosity, which affects the evolution of the medium Ryu et. al. PRL 115 132301 (2015)
- Do **heavy quarkonia** work as an alternative probe for non-equilibrium nature of QGP?

Need to know

✓ **How sensitive are heavy quarkonia to the bulk viscous nature of the fluid ?**
✓ **How sensitive are physical observables?**

How to incorporate bulk viscous correction

In-medium spectral functions

Burnier et. al., JHEP 01 (2008) 043

In-medium heavy quark potential

$$V_{\text{in-medium}}(p) = \varepsilon^{-1}(p)\, V_{\text{Cornell}}(p)$$

Dielectric permittivity $\varepsilon(p)$

$$\varepsilon^{-1}(p) = \lim_{p^0 \to 0} p^2 D^{00}(P)$$

Propagators $D(p)$

$$D^{00}(p) = \frac{1}{2}(D_R + D_A + D_S)$$

$$\text{Re}\, D^{00} = \frac{1}{2}(D_R + D_A)$$

$$\text{Im}\, D^{00} = \frac{1}{2} D_S$$

How to incorporate bulk viscous correction

Deformed distribution function in the presence of bulk viscous correction

$$f(k) \approx f_{\rm id}(\tilde{k}) + \frac{m^2 \Phi}{2T\sqrt{k^2+m^2}} f_{\rm id}(\tilde{k})(1 \pm f_{\rm id}(\tilde{k}))$$

→ Non-equilibrium corrections from bulk viscosity

$$\Phi = -\zeta\, \partial_\mu u^\mu$$

Du, Dumitru, Guo, Strickland, JHEP 01 (2017) 123

Retarded self energy

$$\Pi_R(P) = \widetilde{m}^2_{D,R}\left(\frac{p^0}{2p}\ln\frac{p^0+p+i\epsilon}{p^0-p+i\epsilon} - 1\right)$$

Modified retarded Debye mass

$$\widetilde{m}^2_{D,R} = m^2_{D,R} + \delta m^2_{D,R}$$

Symmetric self energy

$$\Pi_S(P) = -2\pi i\, \widetilde{m}^2_{D,S}\, \frac{T}{p}\, \Theta(p^2 - p_0^2)$$

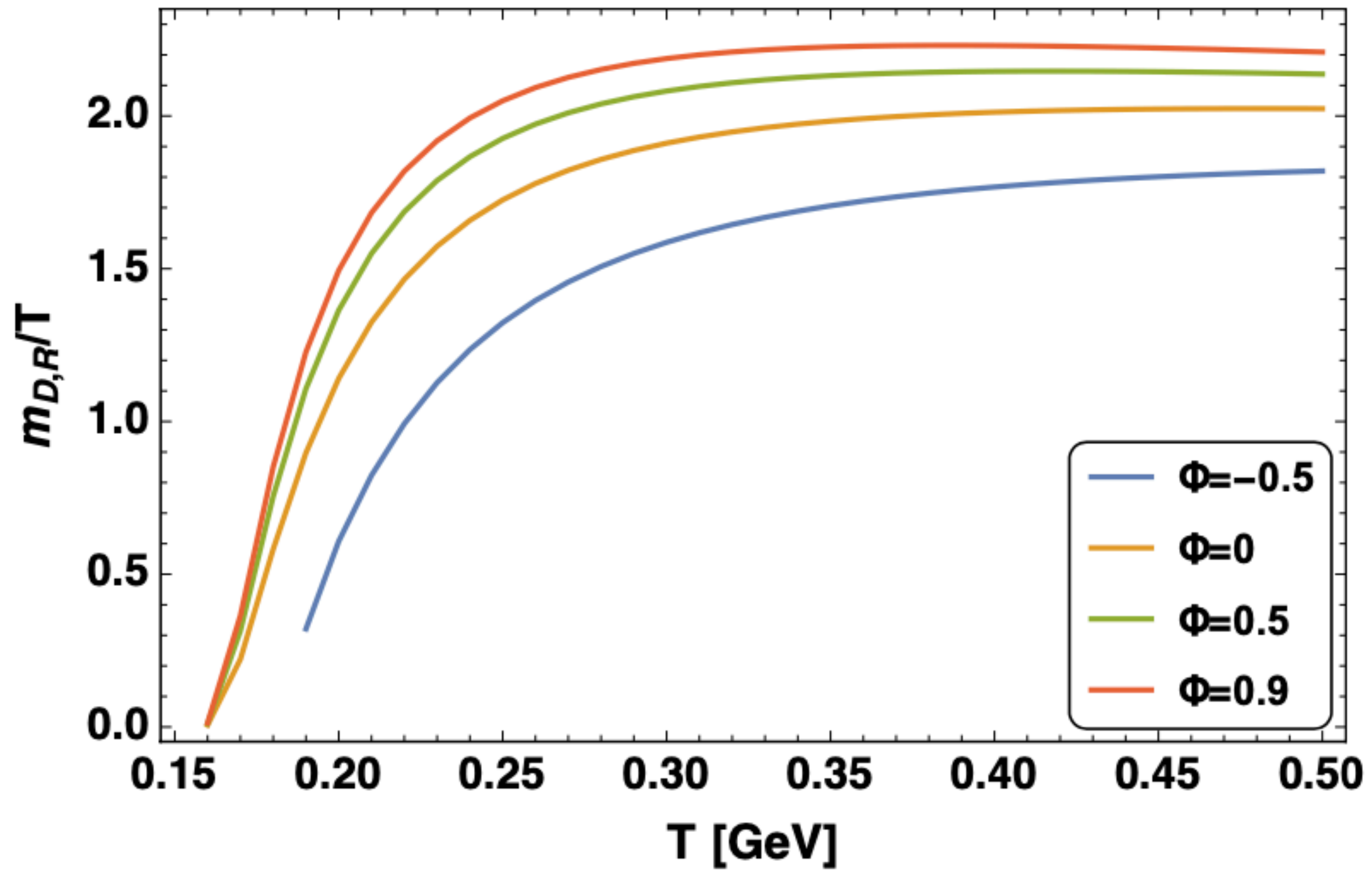
Modified symmetric Debye mass

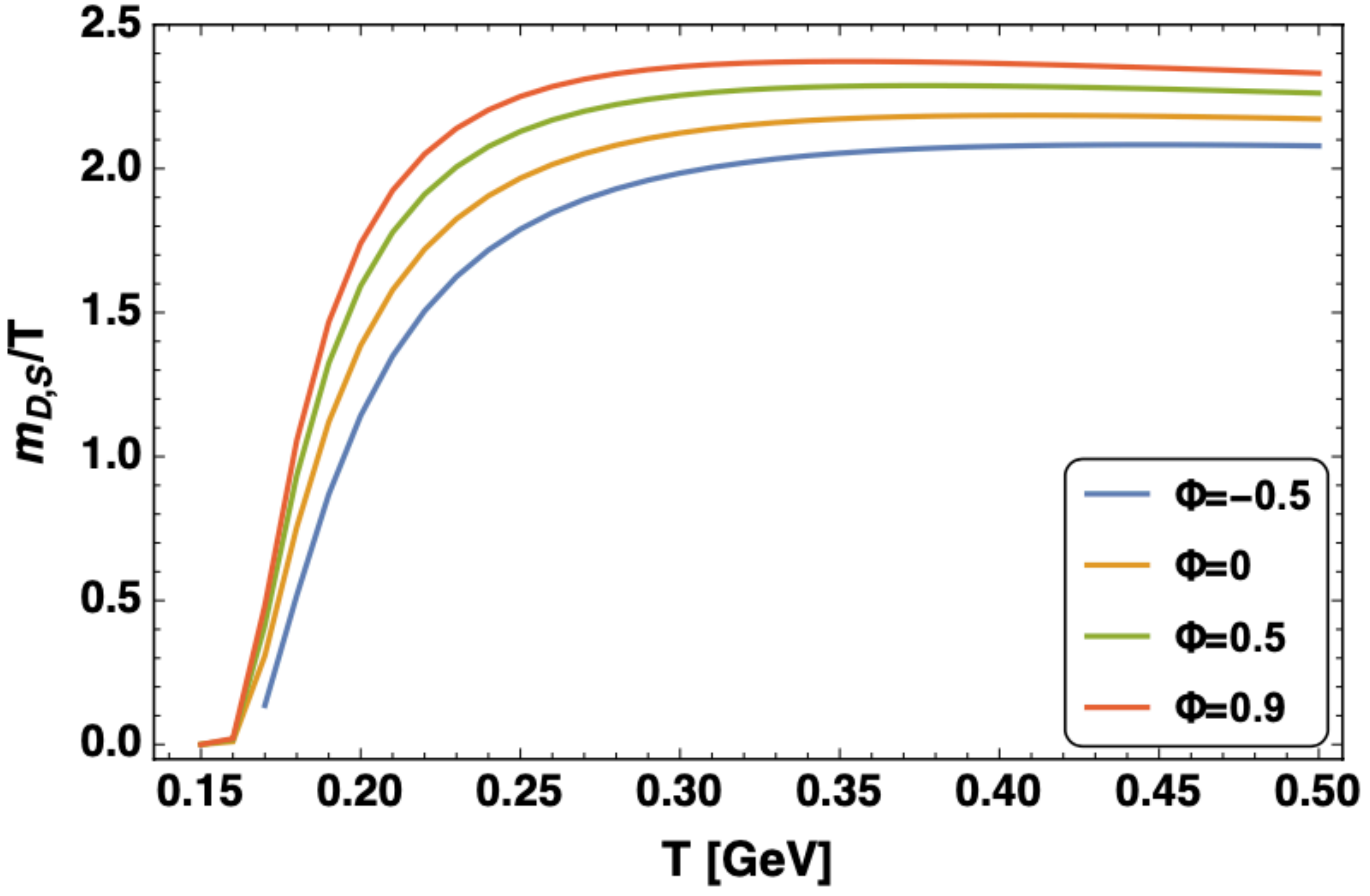
$$\widetilde{m}^2_{D,S} = m^2_{D,S} + \delta m^2_{D,S}$$

Dielectric permittivity

$$\varepsilon^{-1}(p) = \frac{p^2}{p^2+\widetilde{m}^2_{D,R}} - i\frac{\pi T p\, \widetilde{m}^2_{D,S}}{(p^2+\widetilde{m}^2_{D,R})^2}$$

Debye masses

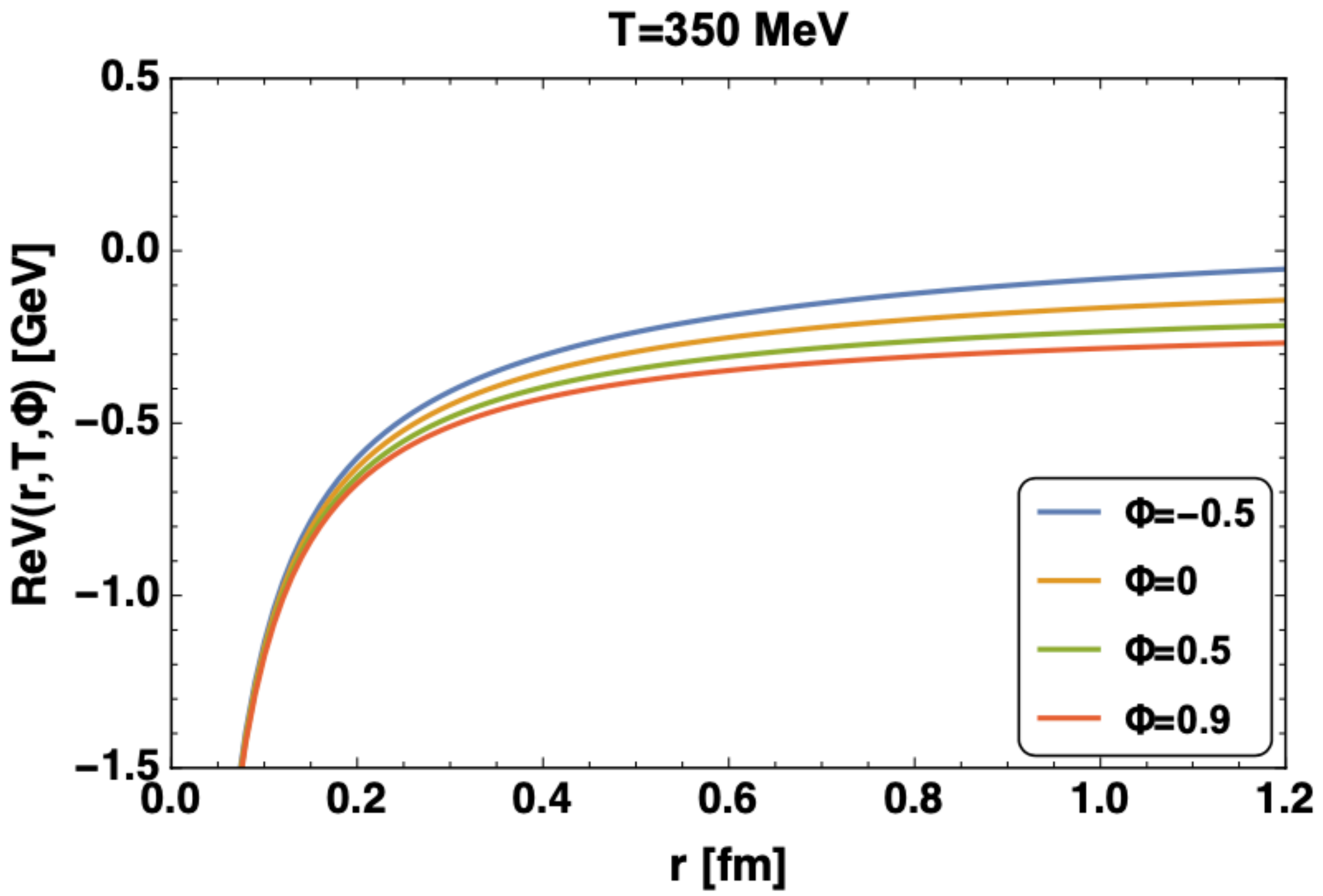


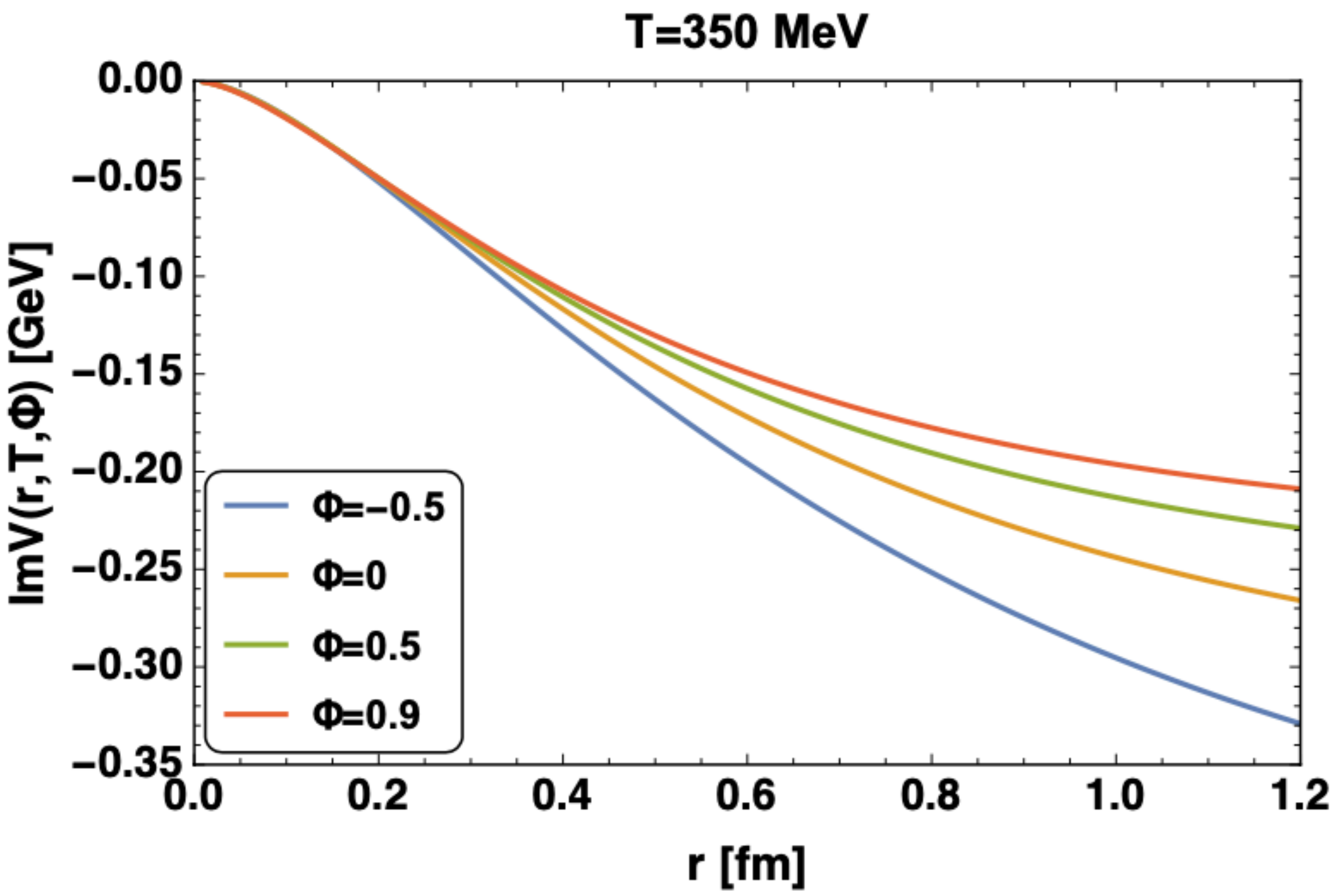


- Debye screening increases as a function bulk viscous correction

In-medium heavy quark complex potential

$$V_{\text{in-medium}}(p) = \varepsilon^{-1}(p)\, V_{\text{Cornell}}(p)$$

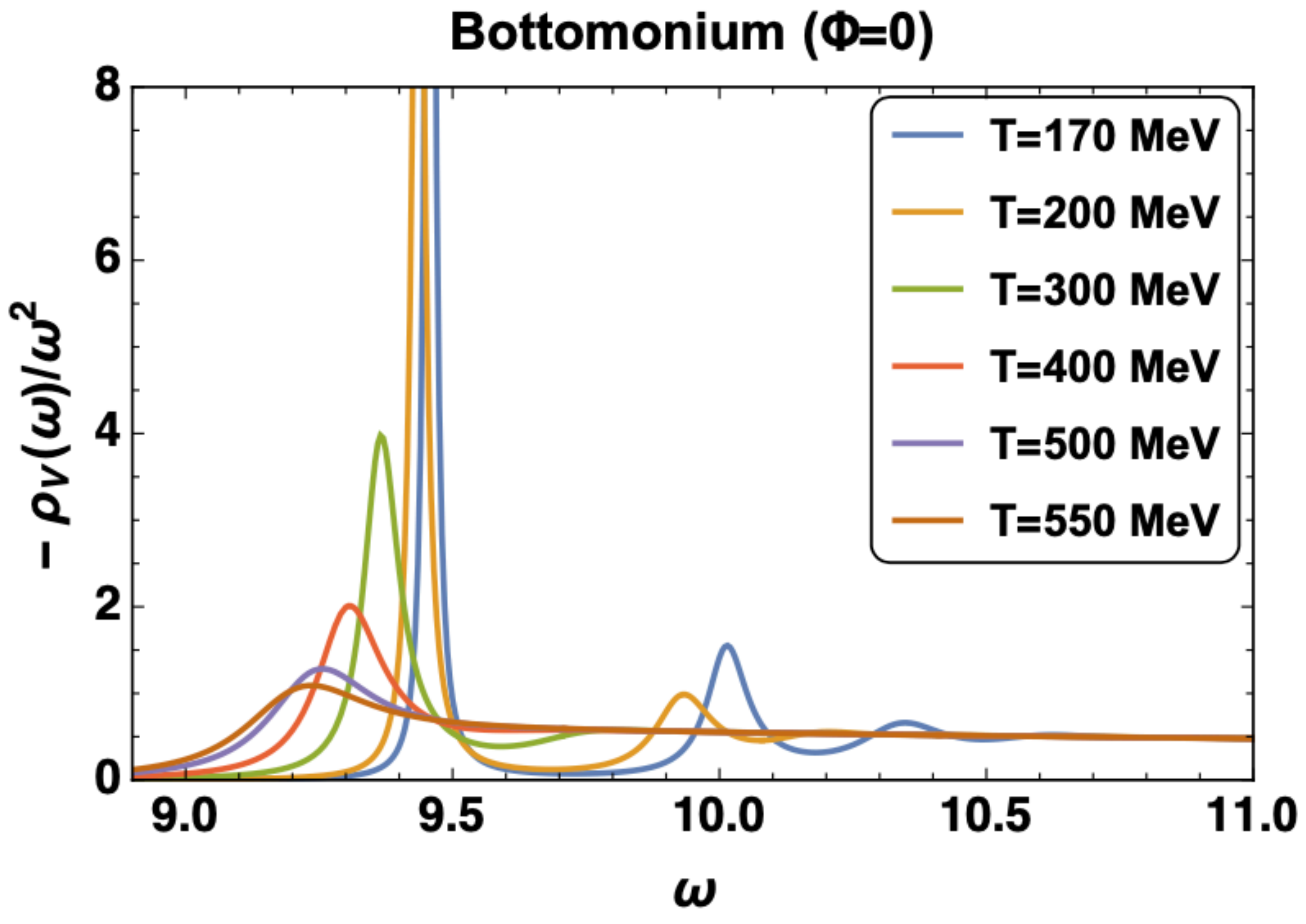


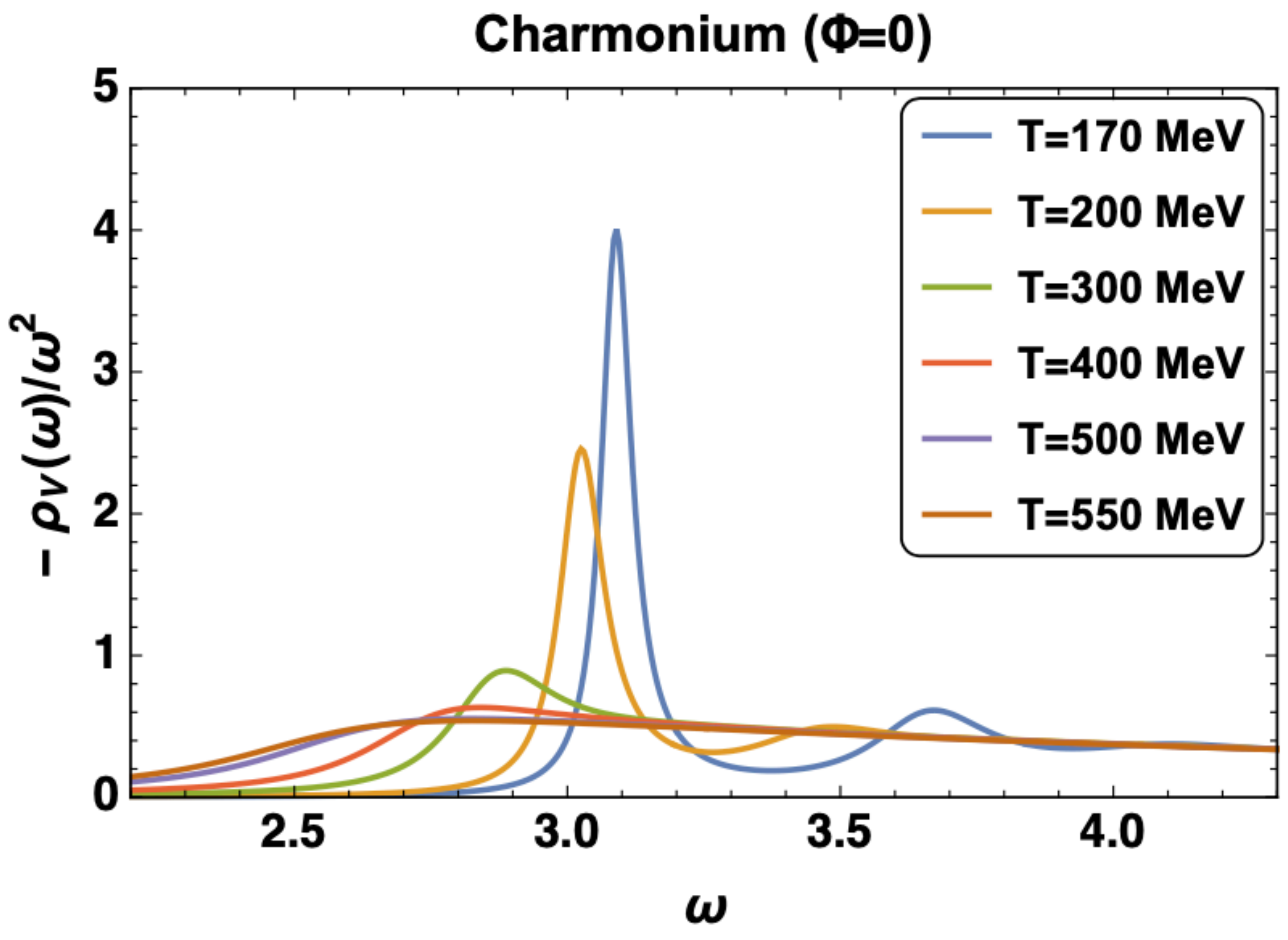


- Effect of bulk viscous corrections:
  - Larger screening
  - Suppression of $|\text{Im}V|$ at large r

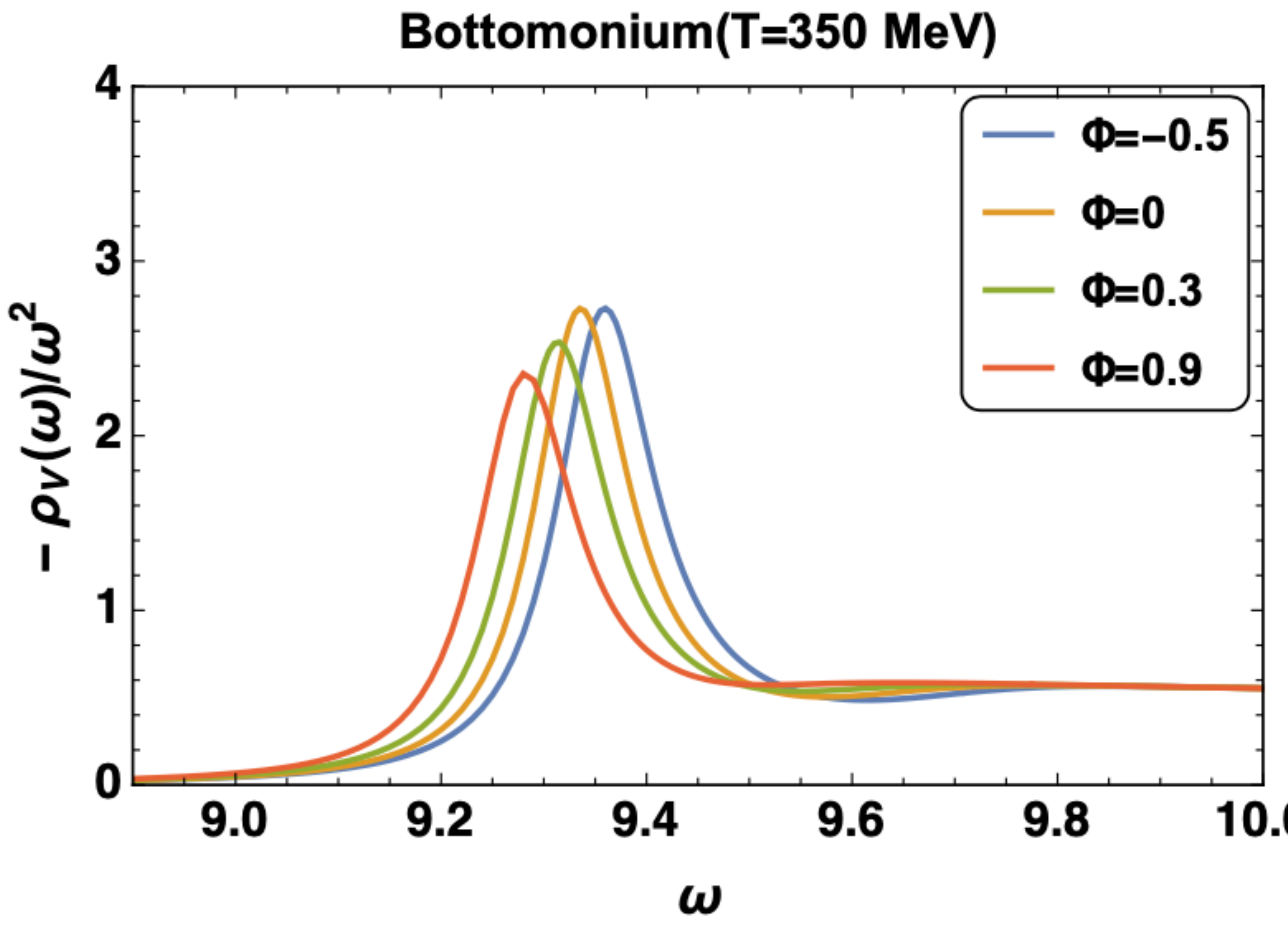
Quarkonium spectral functions

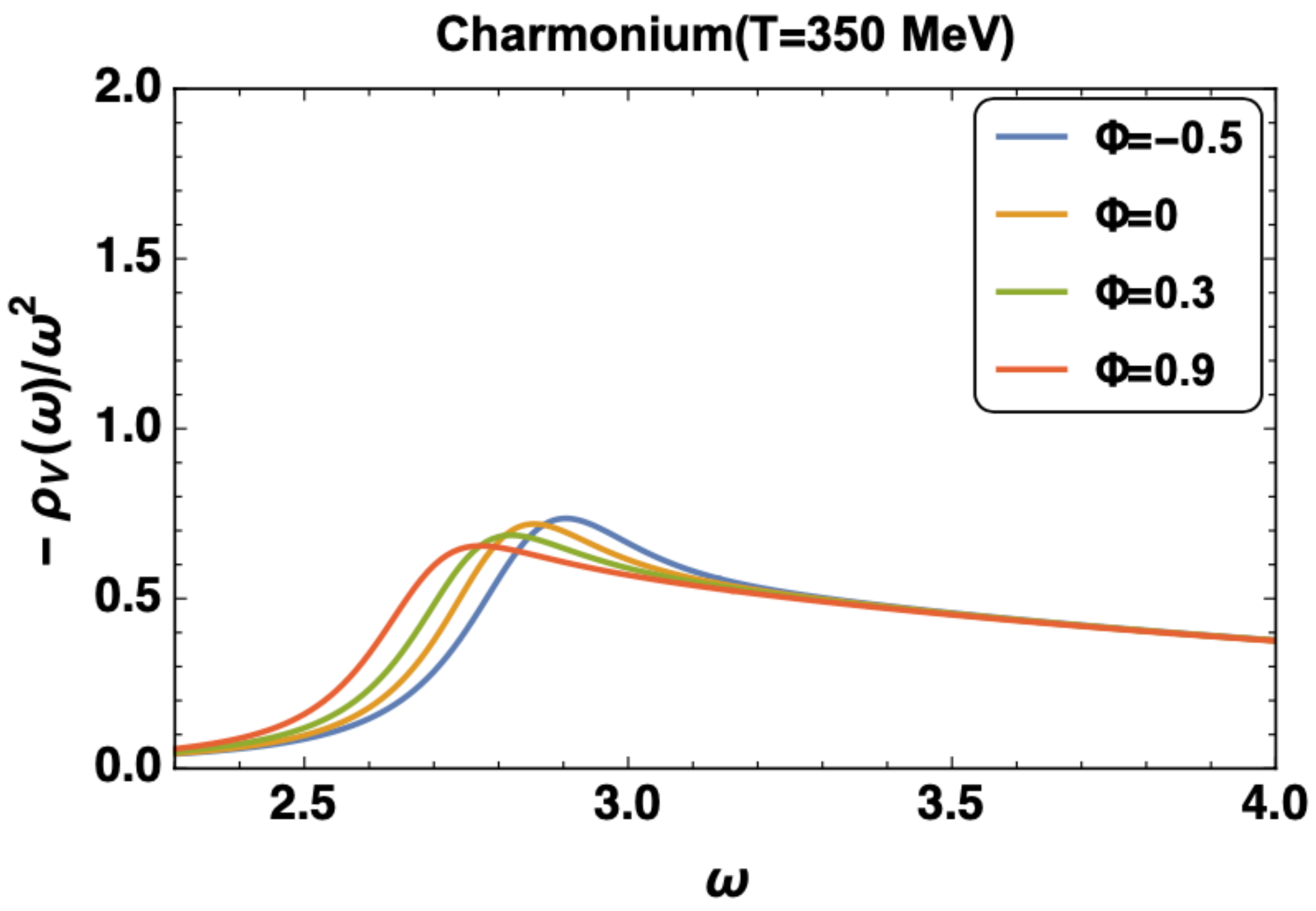
For $\Phi = 0$





For $\Phi \neq 0$



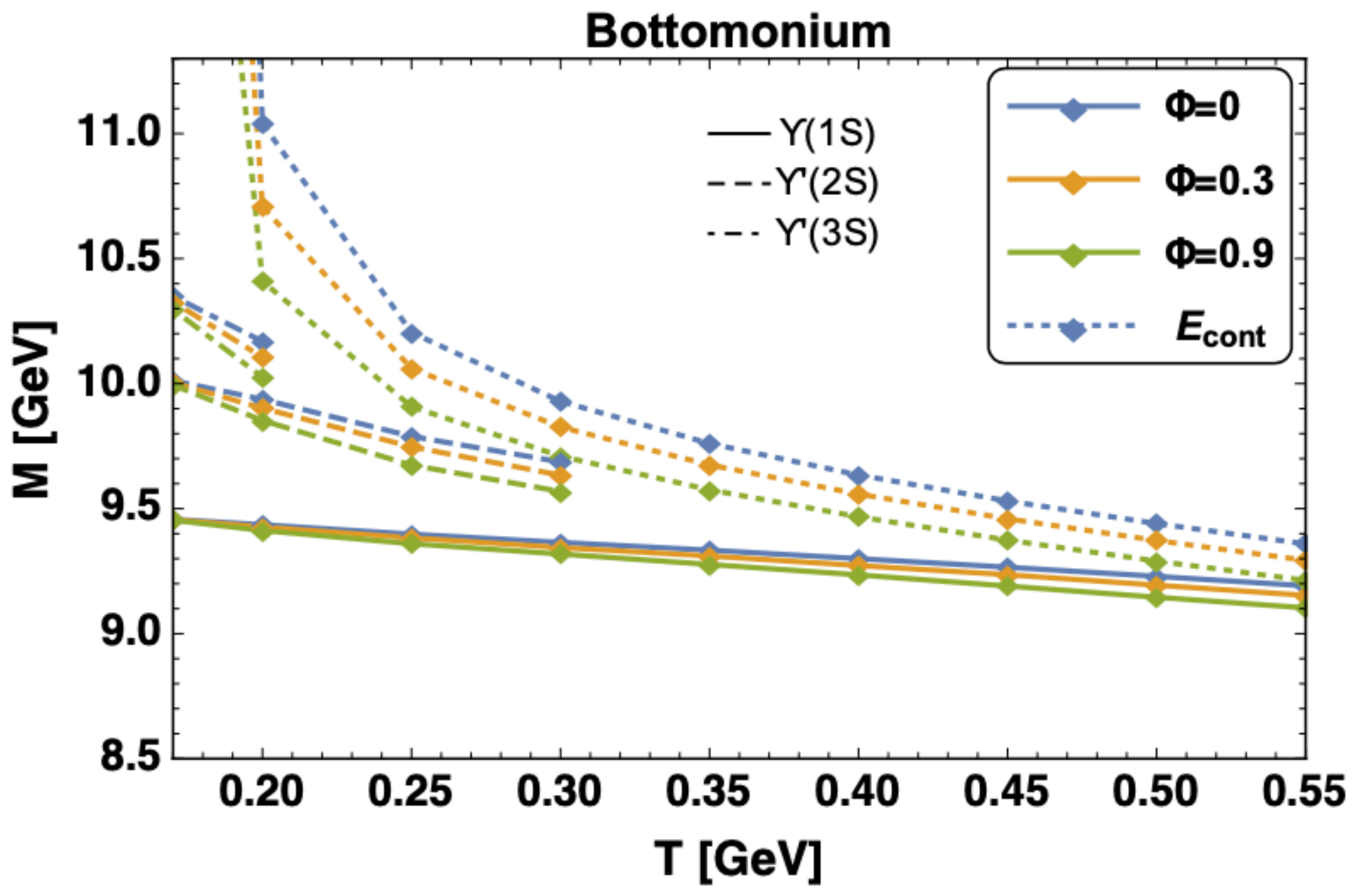


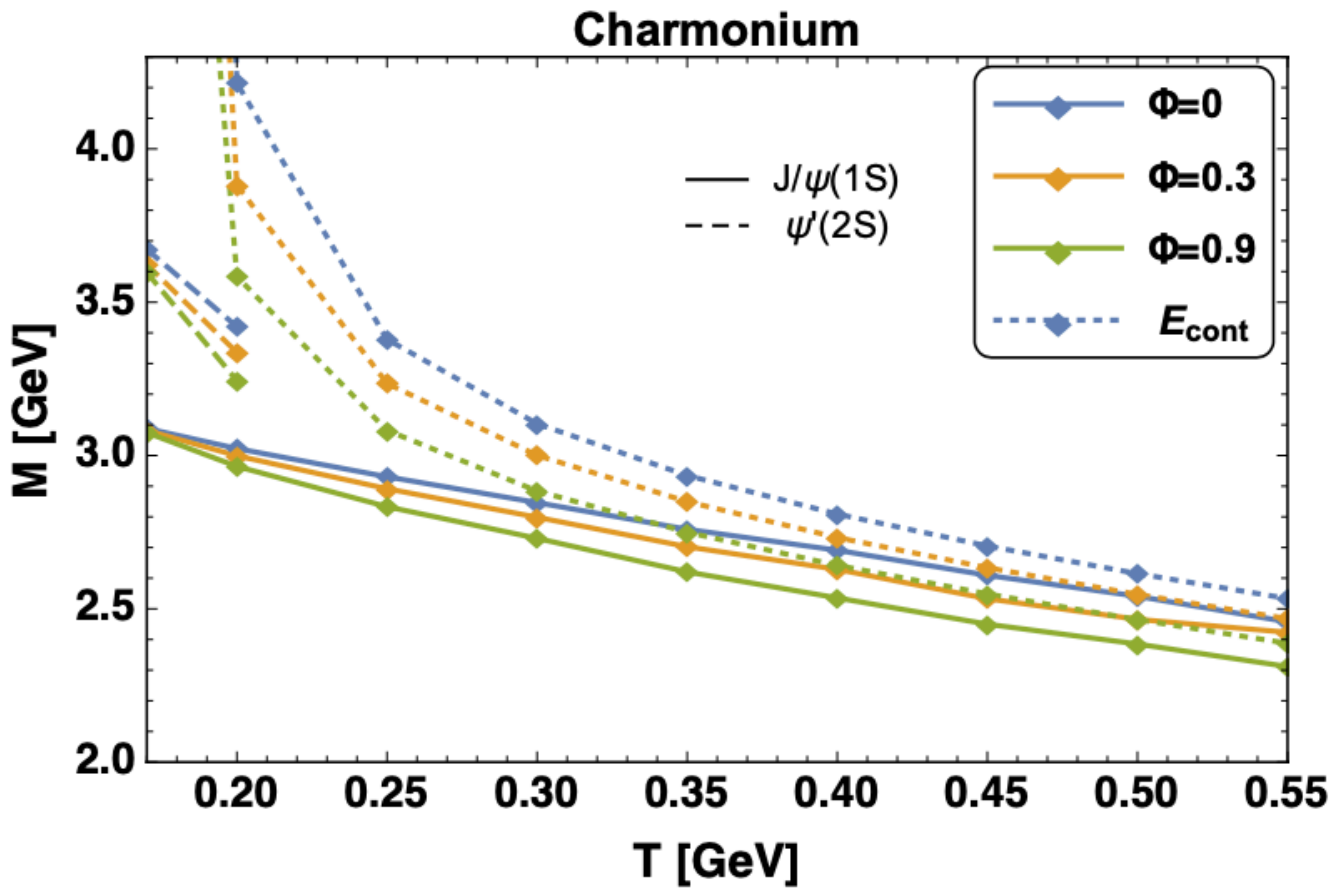
In-medium masses of quakonium states

Fitting of the spectral function with the skewed Breit-Wigner form

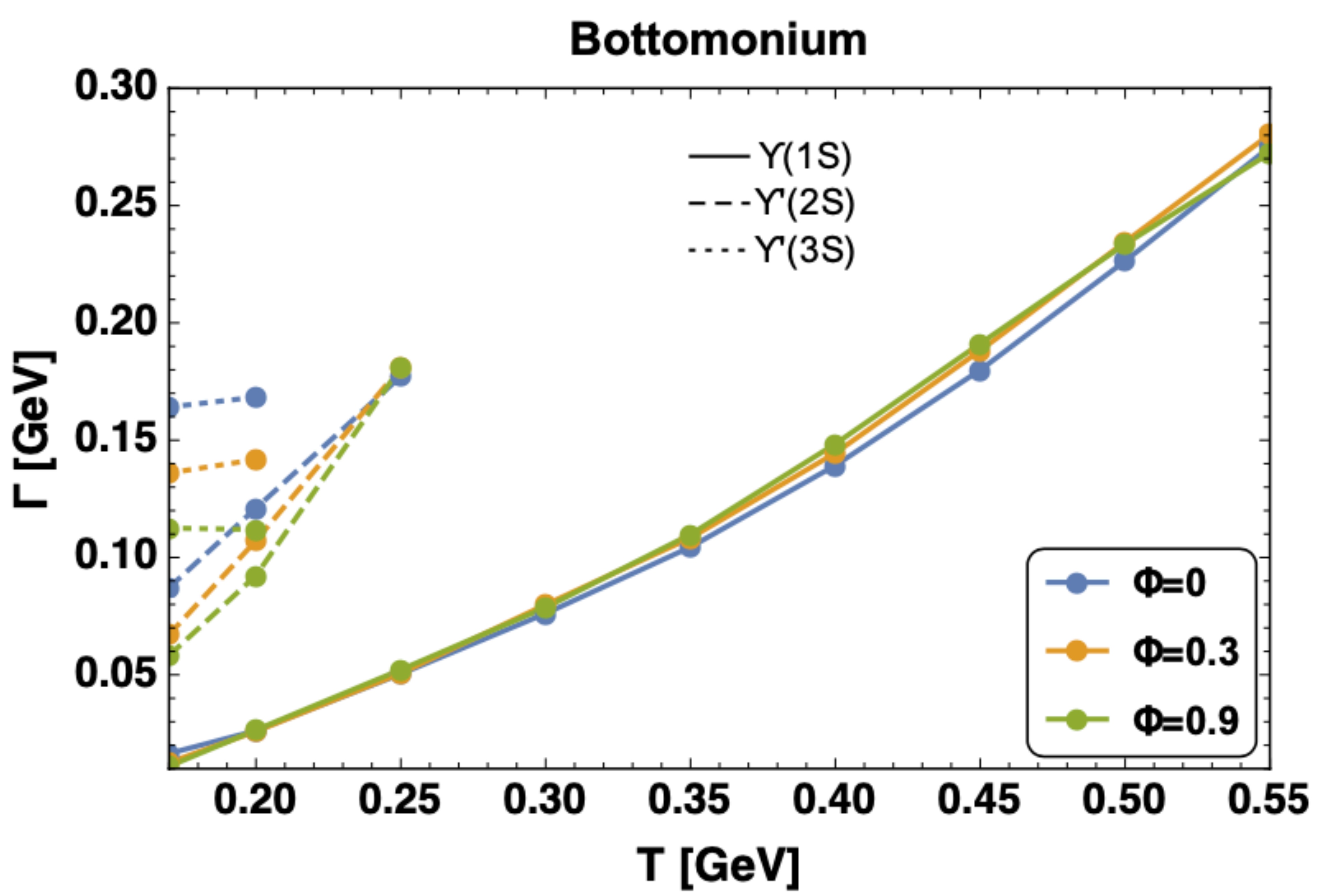
$$\rho(\omega \approx E) = C\frac{(\Gamma/2)^2}{(\Gamma/2)^2+(\omega-E)^2} + 2\delta\frac{(\omega-E)\Gamma/2}{(\Gamma/2)^2+(\omega-E)^2} + A_1 + A_2(\omega-E) + O(\delta^2)$$

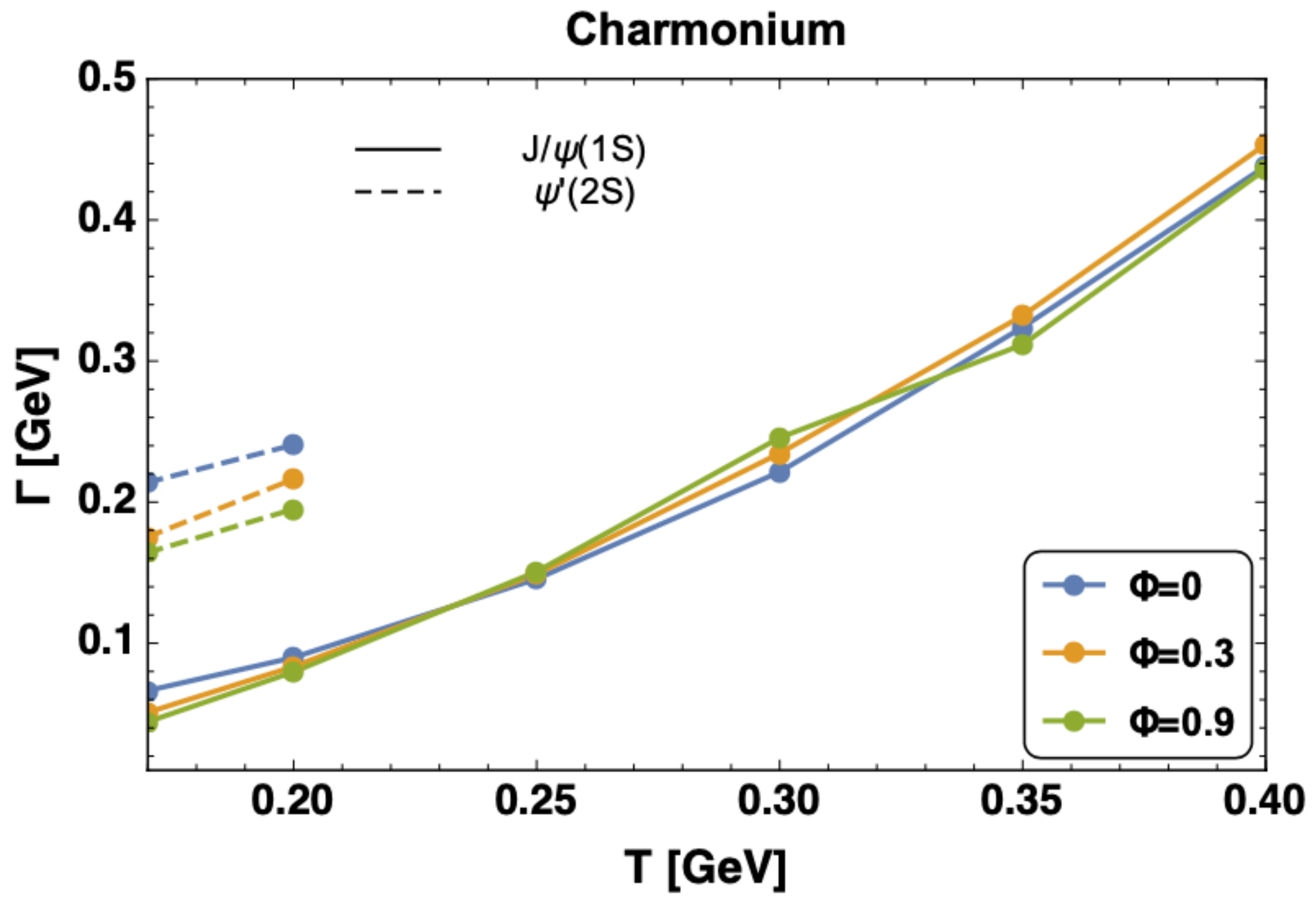
Lafferty and Rothkopf, PRD 101 (2020) 056010





Decay widths of quarkonium states





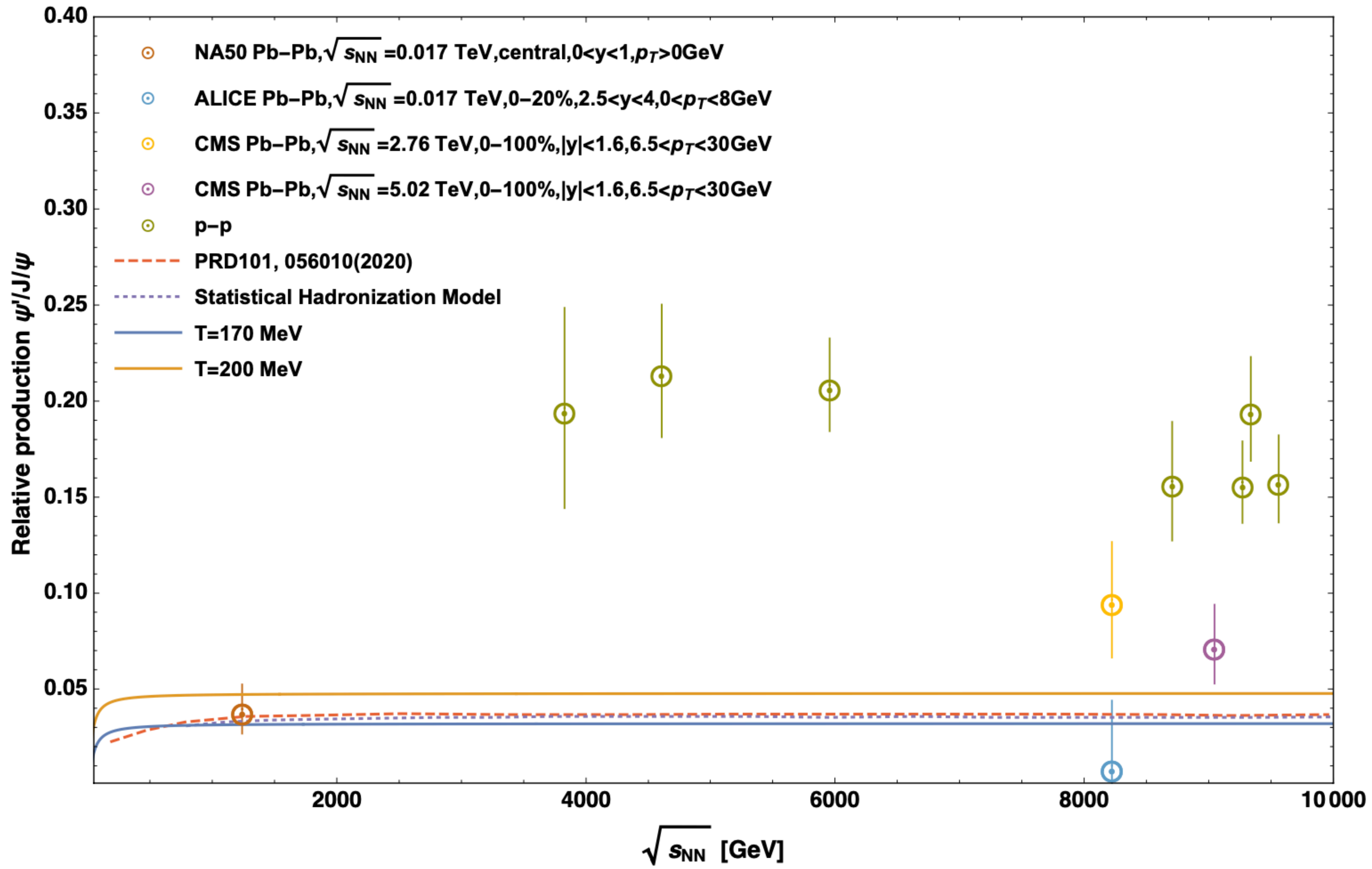
- Two competing effects
  - Spreading of wave functions $\longrightarrow$ increase decay widths
  - Suppression of $|\mathrm{Im}V|$ at large r $\longrightarrow$ decrease decay widths

$$\Gamma \approx \int \psi^* |\mathrm{Im}V| \psi$$

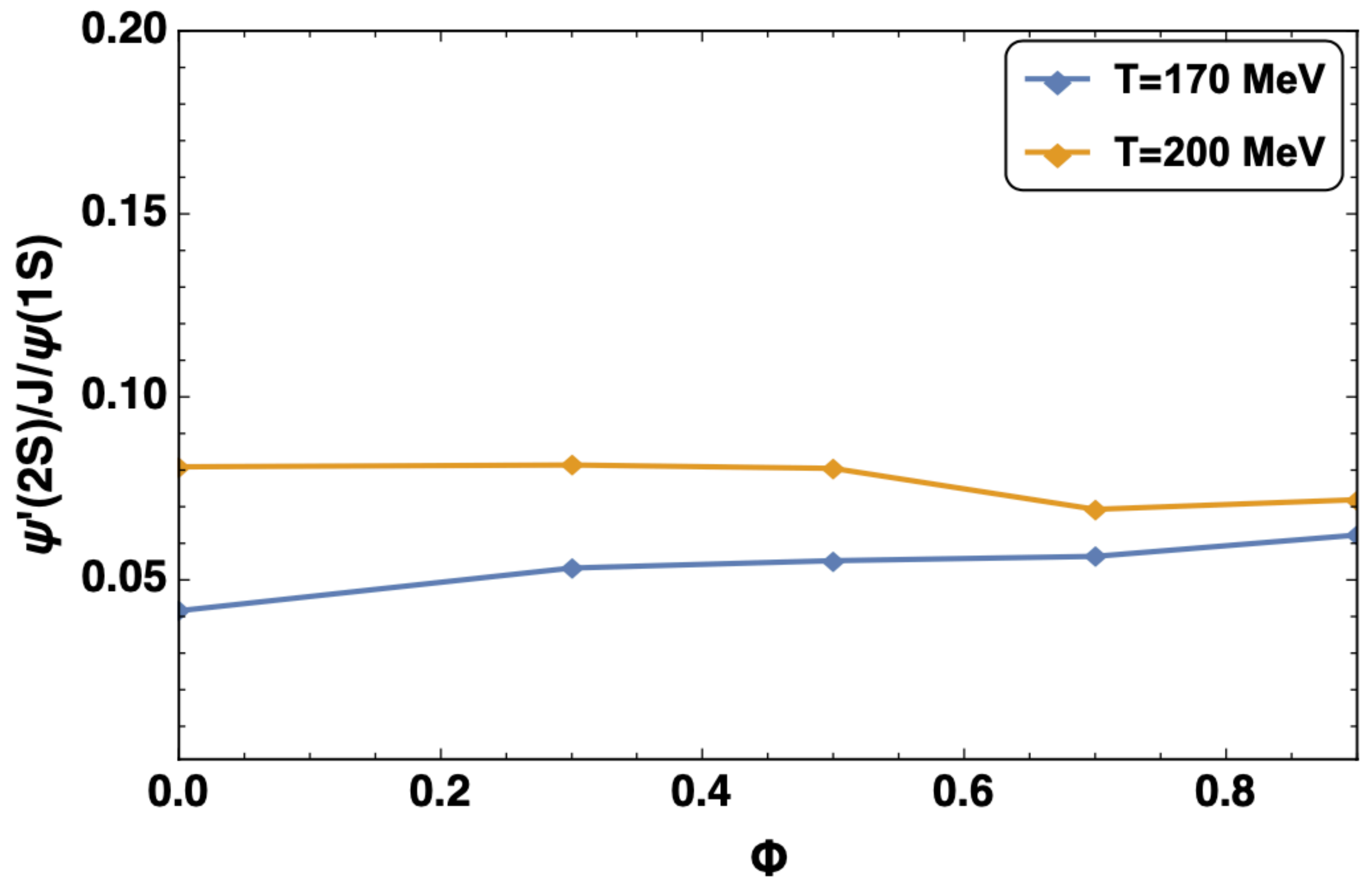
R_{AA} of excited states are more sensitive to bulk viscous corrections than R_{AA} of ground states

$\psi' / J/\psi$ ratio

$$\frac{N_{\psi'}}{N_{J/\psi}}$$

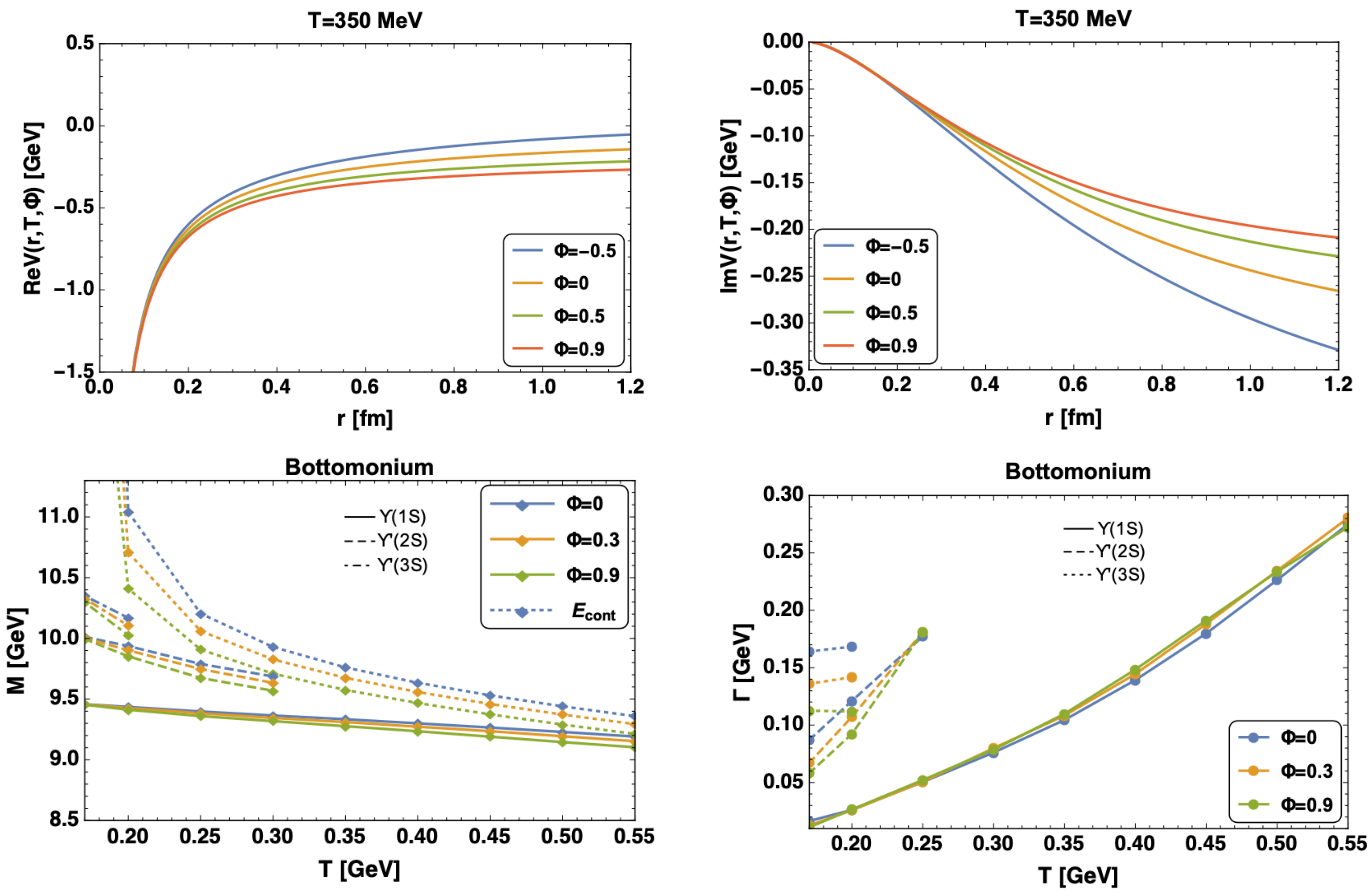


$\psi'/J/\psi$ ratio as a function of Φ



Summary

- **Heavy quarkonia properties in a bulk viscous plasma**
- **R_{AA} of excited states are more sensitive to bulk viscous corrections than R_{AA} of ground states**
  - **Potentially useful for critical point search**



Backup slides

Relative Production yield ψ' to J/ψ ratio

$$\frac{N_{\psi'}}{N_{J/\psi}} = \frac{R_{l\bar{l}}^{\psi'}}{R_{l\bar{l}}^{J/\psi}} \cdot \frac{M_{\psi'}^2 |\psi_{J/\psi}(0)|^2}{M_{J/\psi}^2 |\psi_{\psi'}(0)|^2}$$

G. T. Bodwin, E. Braaten, and G. P. Lepage, PRD 51 (1995) 1125–1171

$$\rho^V(\omega)/\omega^2 \longrightarrow \sum_n A_n \delta(\omega - M_n)$$

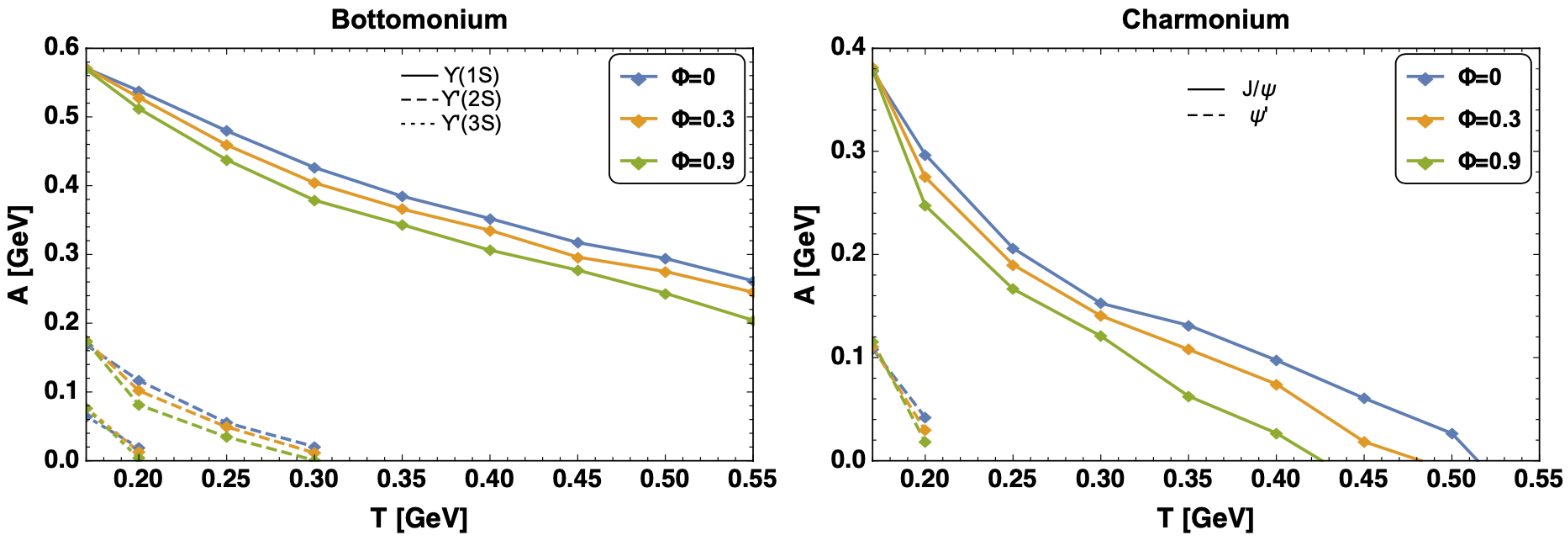
Where

$$R_{l\bar{l}} \propto A_n \int d^3\mathbf{k}\; n_B\left(\sqrt{M_n^2 + \mathbf{k}^2}\right) \frac{M_n}{\sqrt{M_n^2 + \mathbf{k}^2}}$$

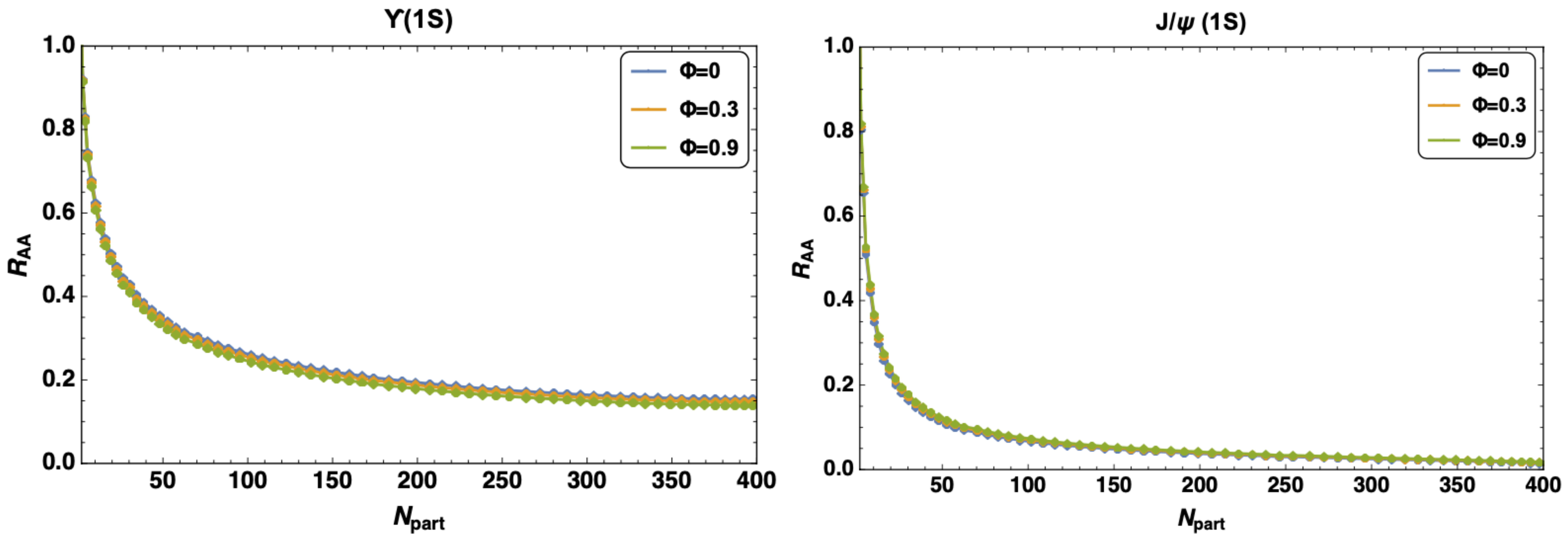
A_n: Area under the bound states peak

M_n: In-medium masses

Area under the bound states peak

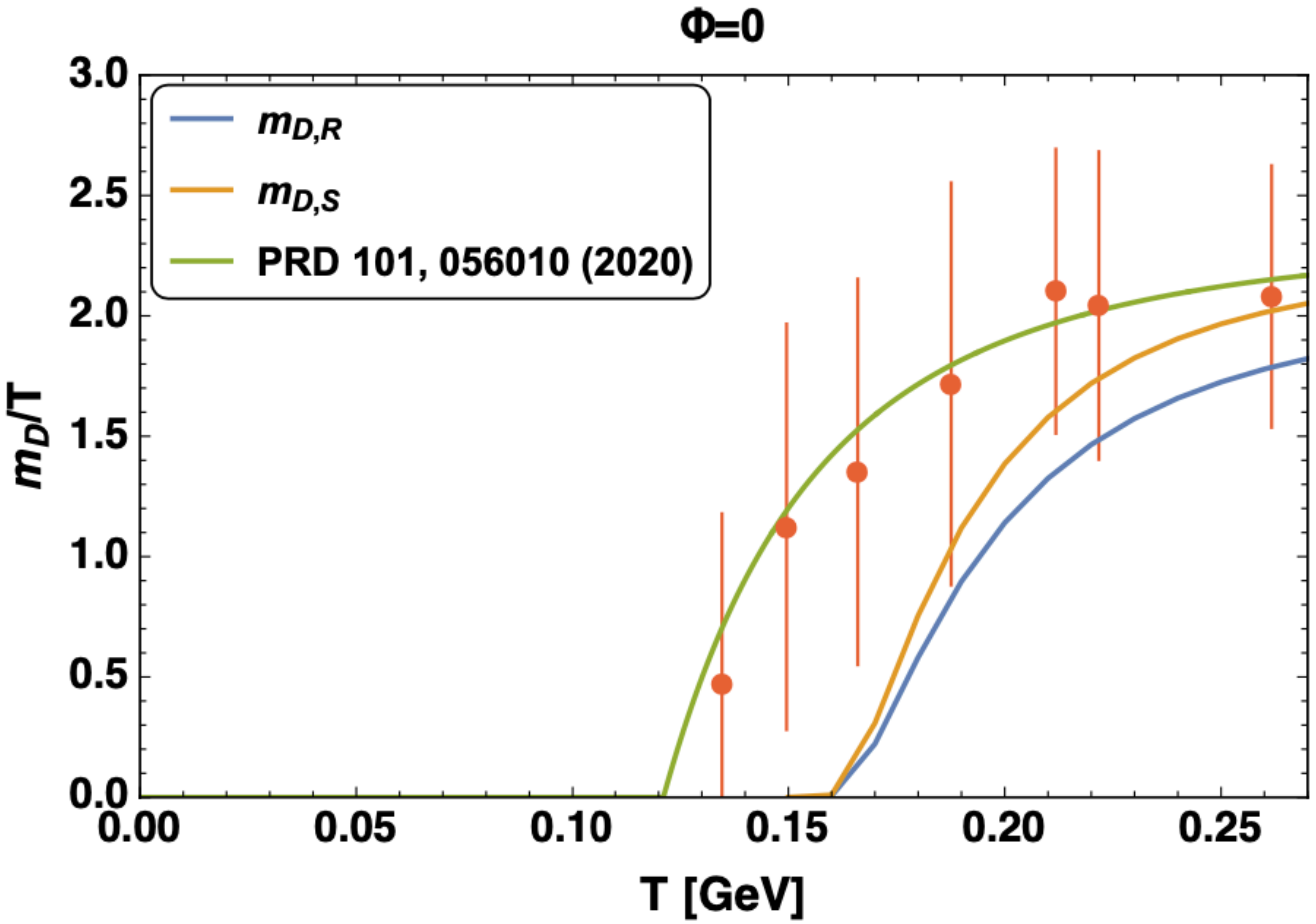


R_{AA}



Υ(1S)
1.0
0.8
0.6
0.4
0.2
0.0
R_{AA}
50
100
150
200
250
300
350
400
N_{part}
Φ=0
Φ=0.3
Φ=0.9
J/ψ (1S)
1.0
0.8
0.6
0.4
0.2
0.0
R_{AA}
50
100
150
200
250
300
350
400
N_{part}
Φ=0
Φ=0.3
Φ=0.9

Debye mass



Φ=0
$m_{D,R}$
$m_{D,S}$
PRD 101, 056010 (2020)
m_D/T
T [GeV]
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.05
0.10
0.15
0.20
0.25

Parameters

$$c = -0.161 \text{ GeV}$$

$$\alpha = 0.513 \text{ GeV}$$

$$\sigma = (0.412 \text{ GeV})^2$$

$$m_b = 4.88 \text{ GeV}$$

$$m_c = 1.4692 \text{ GeV}$$

$$\Lambda = 176 \text{ MeV}$$

$$M_{\psi'} = 3.684 \text{ GeV and } M_{J/\psi} = 3.0969 \text{ GeV}$$

$$\psi_{J/\psi}(0) = 1.454 \text{ GeV}^3 \text{ and } \psi_{\psi'}(0) = 0.927 \text{ GeV}^3$$

$$T(\sqrt{s_{NN}}) = \frac{T_c}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}})/0.45)}$$

$$\begin{aligned}\tilde{m}^2_{D,R} &= m^2_{D,R} + \delta m^2_{D,R} \\ &= \frac{g^2T^2}{6}\left[N_f\left(\left(1+\frac{3\tilde{\mu}^2}{\pi^2}\right)f_q(\tilde{m},\tilde{\mu}) + \frac{\tilde{m}^2\Phi}{\pi^2}\left(\frac{3}{1+e^{\tilde{m}-\tilde{\mu}}}+\frac{3}{1+e^{\tilde{m}+\tilde{\mu}}}\right)\right)\right. \\ &\left. +2N_c\left(f_g(\tilde{m}) + \frac{3\tilde{m}^2\Phi}{\pi^2}\left(\frac{1}{e^{\tilde{m}}-1}\right)\right)\right].\end{aligned}$$

$$f_q(\tilde{m},\tilde{\mu}) = 2\left[1 - \frac{3\tilde{m}\tilde{\mu} - 3\tilde{m}\ln[(1+e^{\tilde{m}+\tilde{\mu}})(1+e^{\tilde{\mu}-\tilde{m}})] - 3[\mathrm{Li}_2(-e^{\tilde{m}-\tilde{\mu}})] + \mathrm{Li}_2(-e^{\tilde{m}+\tilde{\mu}})}{\pi^2+3\tilde{\mu}^2}\right]$$

$$\begin{aligned}\tilde{m}^2_{D,S} &= m^2_{D,S} + \delta m^2_{D,S} \\ &= \frac{g^2T^2}{6}\left[N_f\frac{6\tilde{m}^2}{\pi^2}\sum_{n=1}^{\infty}(-1)^{n+1}\cosh(\tilde{\mu}n)\left(K_2(\tilde{m}n)+\Phi\frac{\tilde{m}}{2}nK_1(\tilde{m}n)\right)\right. \\ &\left. +2N_c\frac{3\tilde{m}^2}{2\pi^2}\sum_{n=1}^{\infty}\left(K_2(\tilde{m}n)+\Phi\frac{\tilde{m}}{2}nK_1(\tilde{m}n)\right)\right],\end{aligned}$$

$$f_g(\tilde{m}) = \frac{\left(3\tilde{m}^2 + 2\pi^2 - 6\tilde{m}\ln[e^{\tilde{m}}-1] - 6\mathrm{Li}_2(e^{\tilde{m}})\right)}{\pi^2}$$

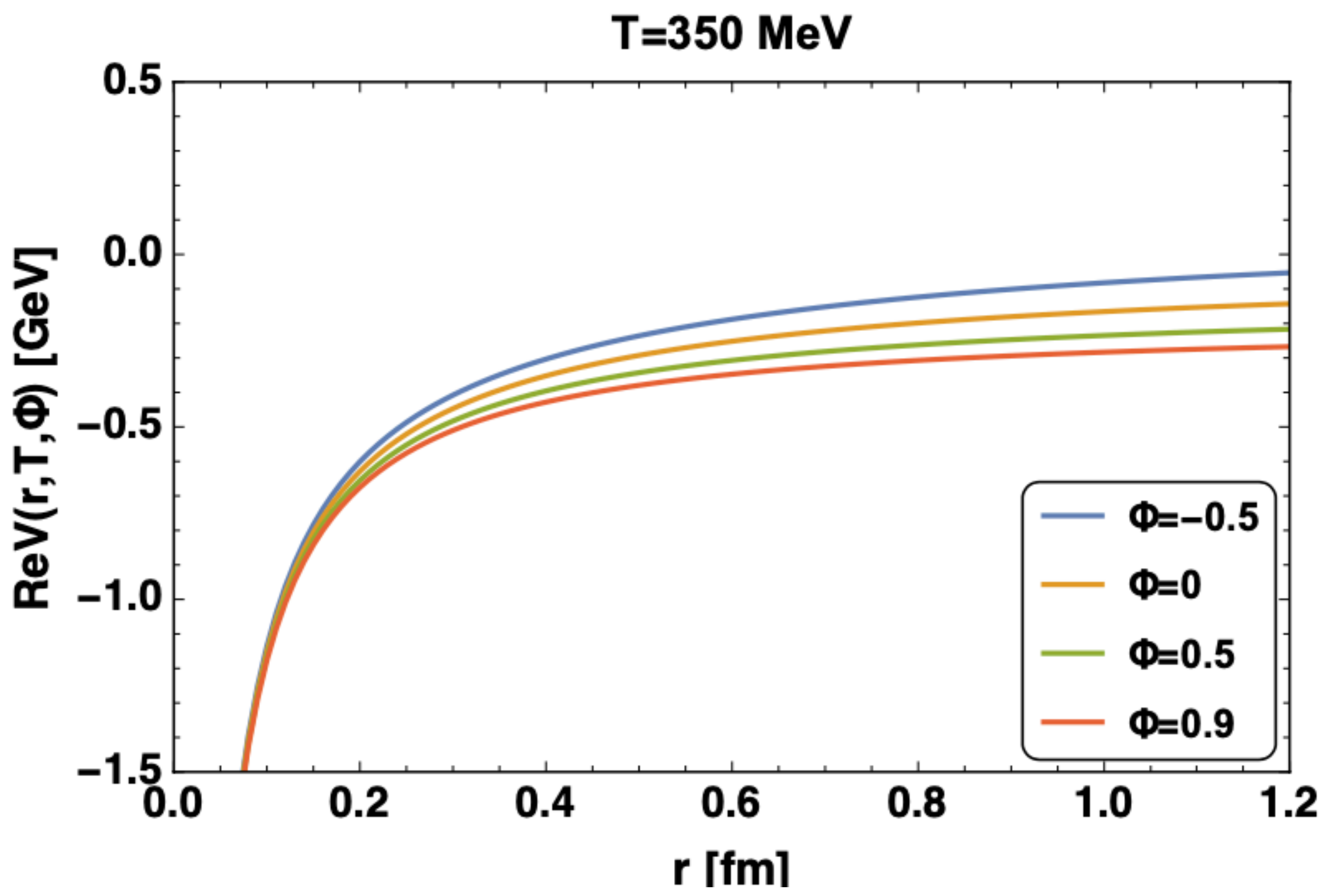
$$m^2(T,\mu) = \frac{G^2(T)T^2N_c}{9} + \frac{G^2(T)T^2N_f}{18}\left(1+\frac{3\mu^2}{\pi^2T^2}\right)$$

Peshier et.al. , PRD 54 (1996) 2399–2402

In-medium heavy quark complex potential

Real part of the potential

$$\mathrm{Re}\,V(r,T,\Phi) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}}(e^{i\mathbf{p}\cdot\mathbf{r}}-1)V_{\mathrm{Cornell}}(p)\mathrm{Re}\,\varepsilon^{-1}(p)$$
$$= -\alpha\,\widetilde{m}_{D,R}\left(\frac{e^{-\widetilde{m}_{D,R}\,r}}{\widetilde{m}_{D,R}\,r}+1\right)+\frac{2\sigma}{\widetilde{m}_{D,R}}\left(\frac{e^{-\widetilde{m}_{D,R}\,r}-1}{\widetilde{m}_{D,R}\,r}+1\right)+c$$

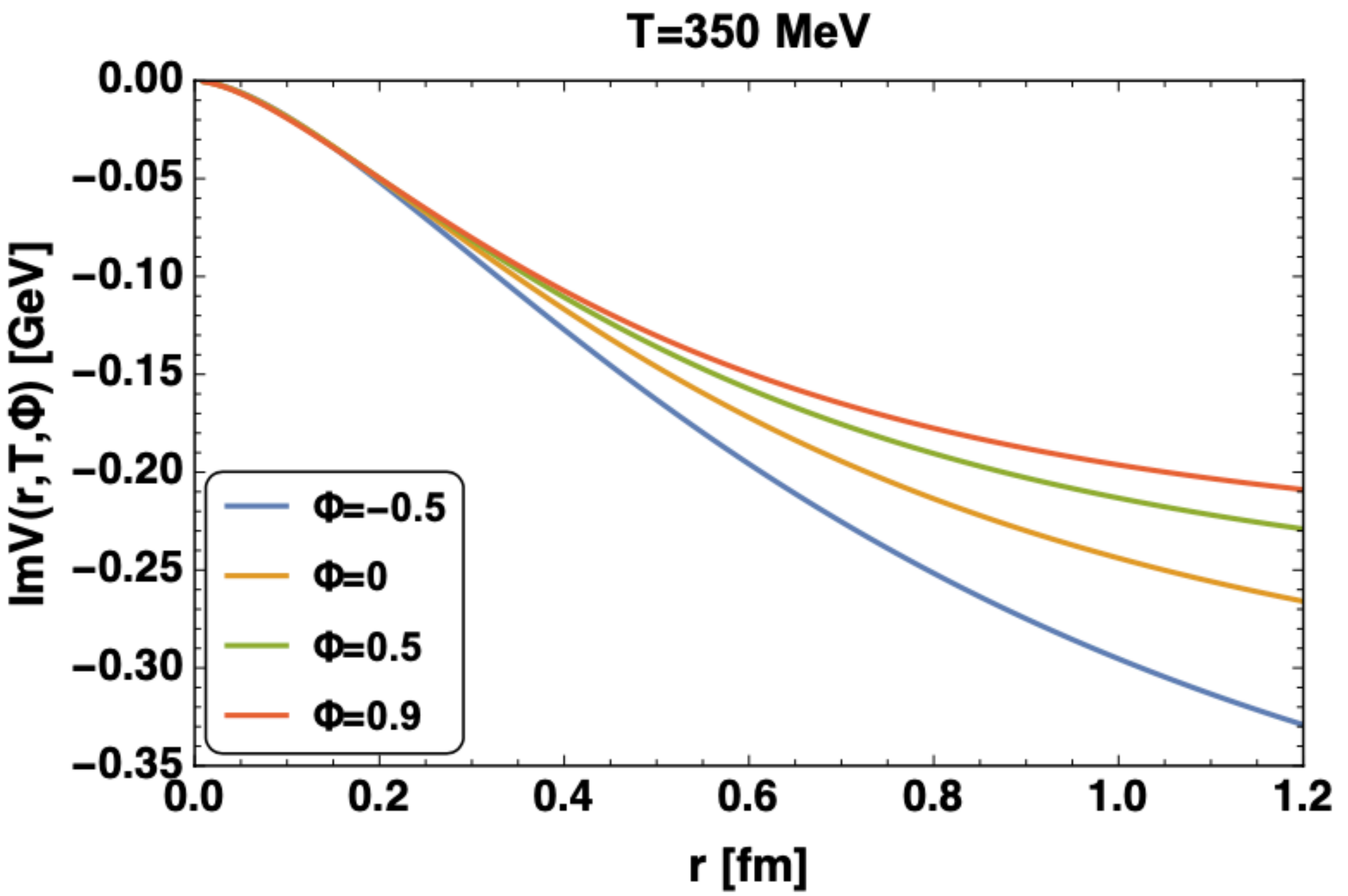


Imaginary part of the potential

$$\mathrm{Im}\,V(r) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}}(e^{i\mathbf{p}\cdot\mathbf{r}}-1)V_{\mathrm{Cornell}}(p)\,\mathrm{Im}\,\varepsilon^{-1}(p)$$
$$= -\alpha\lambda T\,\phi_2(\widetilde{m}_{D,R}\,r) - \frac{2\sigma T\lambda}{\widetilde{m}^2_{D,R}}\chi(\widetilde{m}_{D,R}\,r)$$

$$\phi_n(x) \equiv 2\int_0^\infty dz\,\frac{z}{(z^2+1)^n}\left[1-\frac{\sin(xz)}{xz}\right]$$
$$\chi(x) \equiv 2\int_0^\infty \frac{dz}{z(z^2+1)^2}\left[1-\frac{\sin(xz)}{xz}\right].$$

$$\lambda \equiv \frac{\widetilde{m}^2_{D,S}}{\widetilde{m}^2_{D,R}}$$



Quarkonium spectral functions

Schrödinger equation

$$\left[\hat{H} \mp i|\mathrm{Im}V(r,T,\Phi)|\right] G^{>}(t;\mathbf{r},\mathbf{r}') = i\partial_t G^{>}(t;\mathbf{r},\mathbf{r}')$$

Burnier et. al., JHEP 01 (2008) 043

Where

$$\hat{H} = 2m_Q - \frac{\nabla_r^2}{m_Q} + \frac{l(l+1)}{m_Q r^2} + \mathrm{Re}\,V(r,T,\Phi)$$

S-wave vector channel spectral function

$$\rho^V(\omega) = \lim_{\mathbf{r},\mathbf{r}'\to 0} \frac{1}{2}\tilde{G}(\omega;\mathbf{r},\mathbf{r}')$$

$$\tilde{G}(\omega;\mathbf{r},\mathbf{r}') = \int_{-\infty}^{\infty} dt e^{i\omega t} G^{>}(t;\mathbf{r},\mathbf{r}')$$

Binding energies of quarkonium states

$$E_{\text{bin}} = 2m_{c,b} + V(r \to \infty) - M$$