

# Suppressed flow harmonics: A signature of the QCD critical point ?

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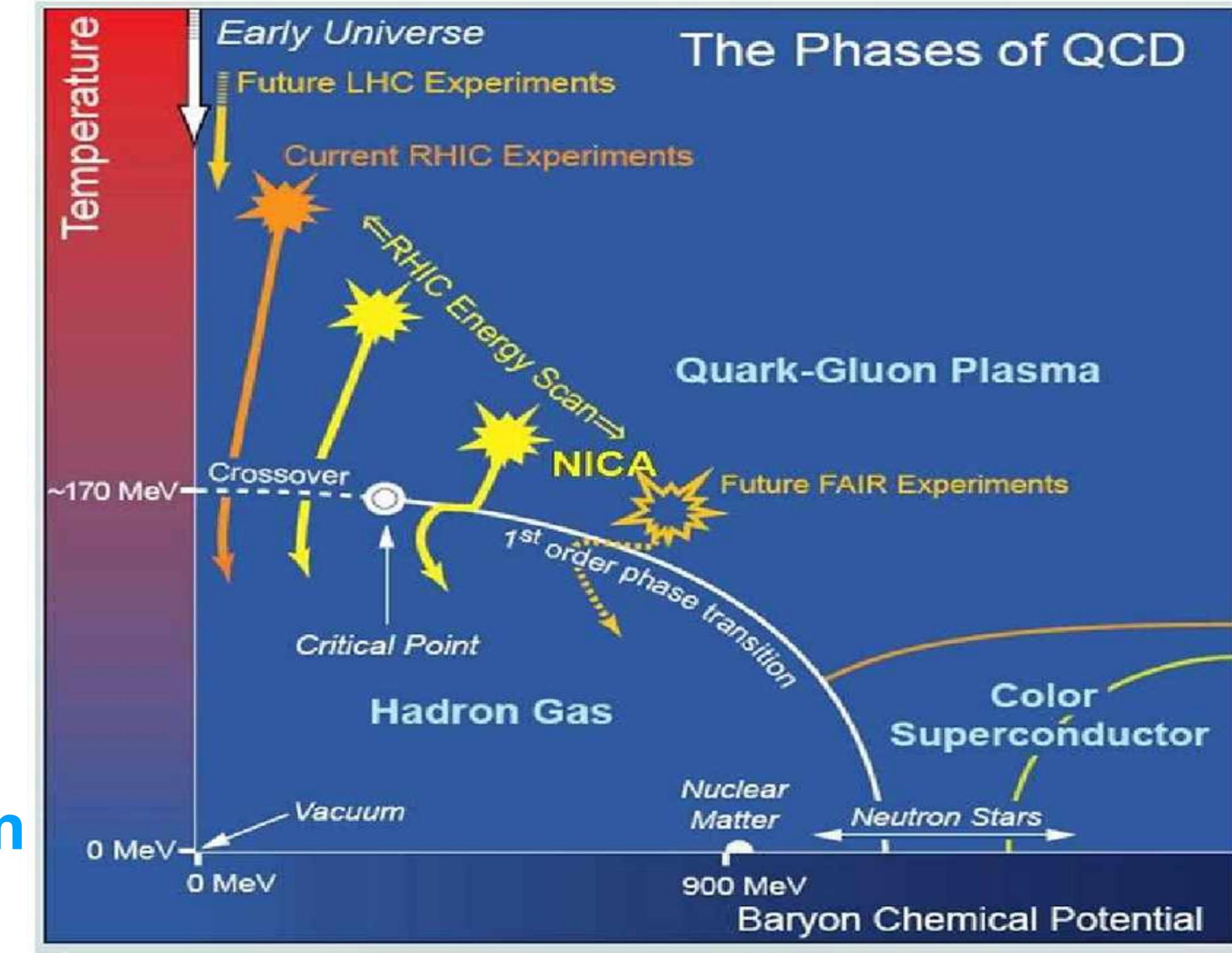
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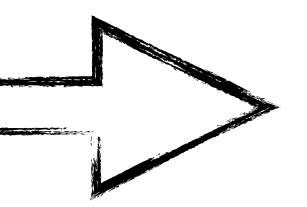
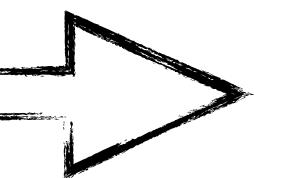
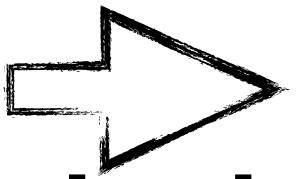
Based on  
**PRC 102, 034910 (2020), PLB 820 (2021) 136583**

# Motivation

- Thermodynamics  $\rightarrow$  Critical End-Point at the end of the first order phase transition line
- QCD based effective models  $\rightarrow$  The existence of the CEP based on the parameters of the models used
- Lattice calculations, not valid  $\mu_B \neq 0$ ,  $\rightarrow$  sign problem  
exact location of the CEP is still not conclusive.
- Lack of conclusiveness of the precise location of the CEP demands for more theoretical as well as experimental study
- Beam Energy Scan (BES) at BNL is particularly dedicated for finding the CEP by tuning the  $\sqrt{s}$ .

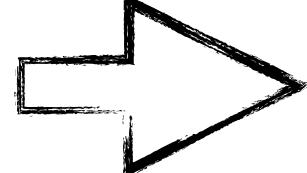


# Motivation

- The propagation of the perturbations  unveil the thermodynamic state of a fluid.
- The hydrodynamic response to the perturbation  imprinted on the particle spectra.
- Promising observables  **Flow harmonics**, is attributed to the hydrodynamic response of the QGP to the initial geometry.
- Consequences of the CEP on the hydrodynamic evolution if an isentropic trajectory passes through the critical region.

# Equation of State

Nonaka and Asakawa, PRC(2005), MH et al, PRC (2020)

- The effect of the CEP enters into the hydrodynamic evolution through the EoS which contains the critical point.
- Constructed on the basis of **universality hypothesis**  CEP of the QCD belongs to the same universality class of 3D Ising model.
- The critical entropy density  $S_c$  is analogous to the magnetization of the Ising model

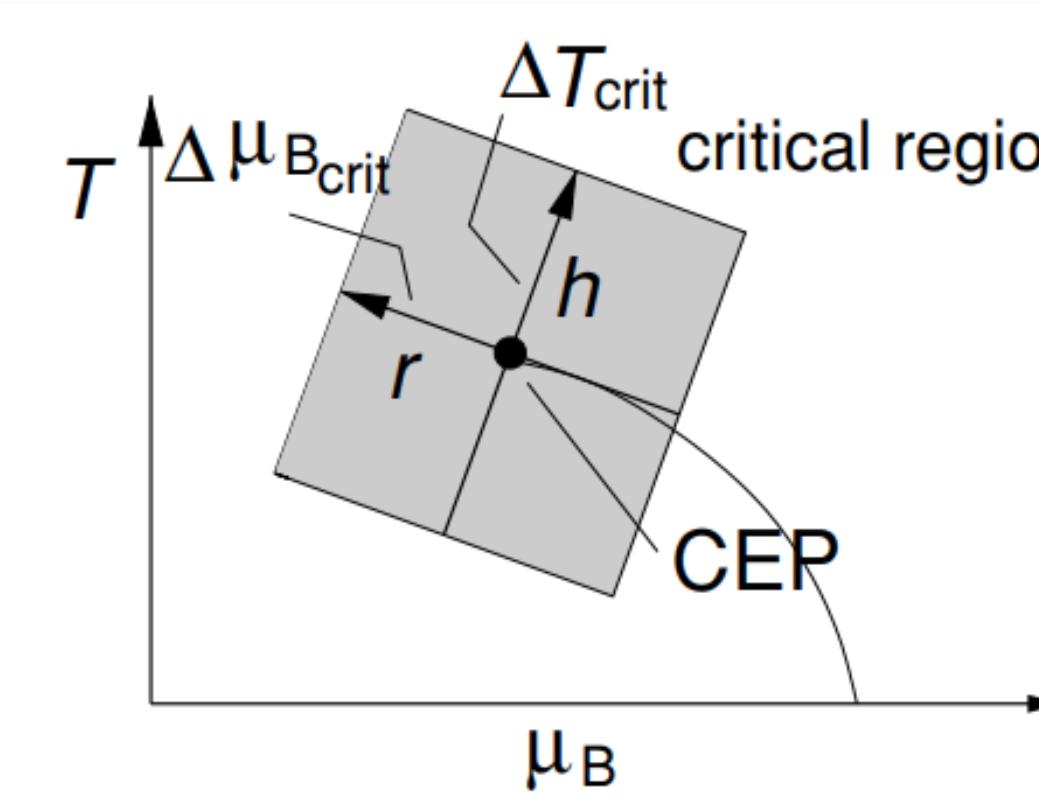
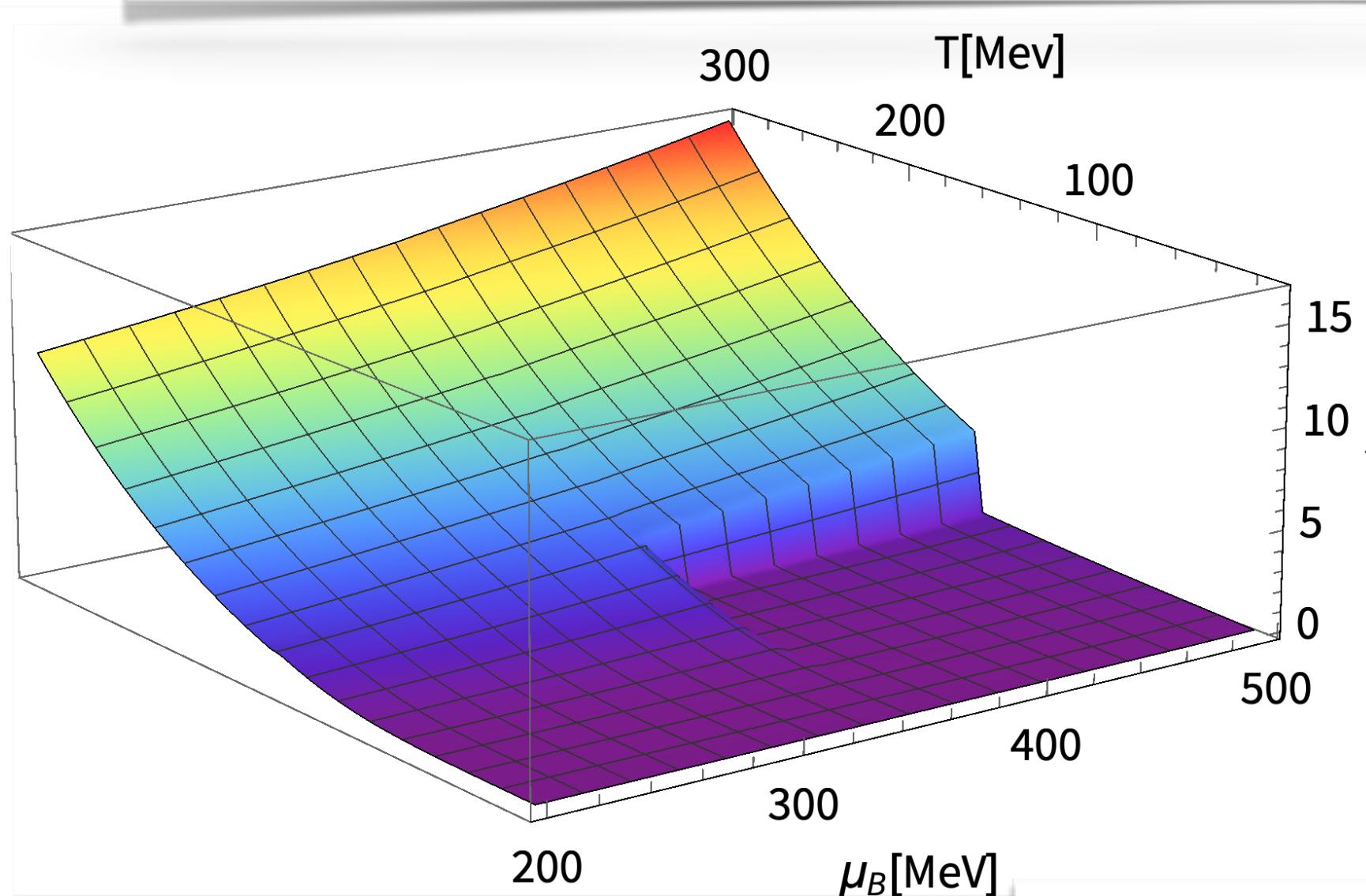
$$S_c = \frac{M(r, h)}{\Delta T_c} = M\left(\frac{\mu - \mu_c}{\Delta \mu_c}, \frac{T - T_c}{\Delta T_c}\right) \frac{1}{\Delta T_c}$$

- In our study,  $(T_c, \mu_c) = (154, 367)\text{MeV}$ .

# Equation of State

- The entropy density of  $s_Q$  and  $s_H$  is connected by using  $S_c$  as switching function as:

$$S(T, \mu) = \frac{1}{2}[1 - \tanh S_c(T, \mu)]s_Q(T, \mu) + \frac{1}{2}[1 + \tanh S_c(T, \mu)]s_H(T, \mu)$$



## Critical behavior of the bulk viscosity in QCD

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We study the behavior of the bulk viscosity  $\zeta$  in QCD near a possible critical end point. We verify the expectation that  $(\zeta/s) \sim a(\xi/\xi_0)^{x_\zeta}$ , where  $s$  is the entropy density,  $\xi$  is the correlation length,  $\xi_0$  is the noncritical correlation length,  $a$  is a constant, and  $x_\zeta \simeq 3$ . Using a recently developed equation of state that includes a critical point in the universality class of the Ising model we estimate the constant of proportionality  $a$ . We find that  $a$  is typically quite small,  $a \sim O(10^{-4})$ . We observe, however, that this result is sensitive to the commonly made assumption that the Ising temperature axis is approximately aligned with the QCD chemical potential axis. If this is not the case, then the critical  $\zeta/s$  can approach the noncritical value of  $\eta/s$ , where  $\eta$  is the shear viscosity, even if the enhancement of the correlation length is modest,  $\xi/\xi_0 \sim 2$ .

DOI: [10.1103/PhysRevD.100.074017](https://doi.org/10.1103/PhysRevD.100.074017)

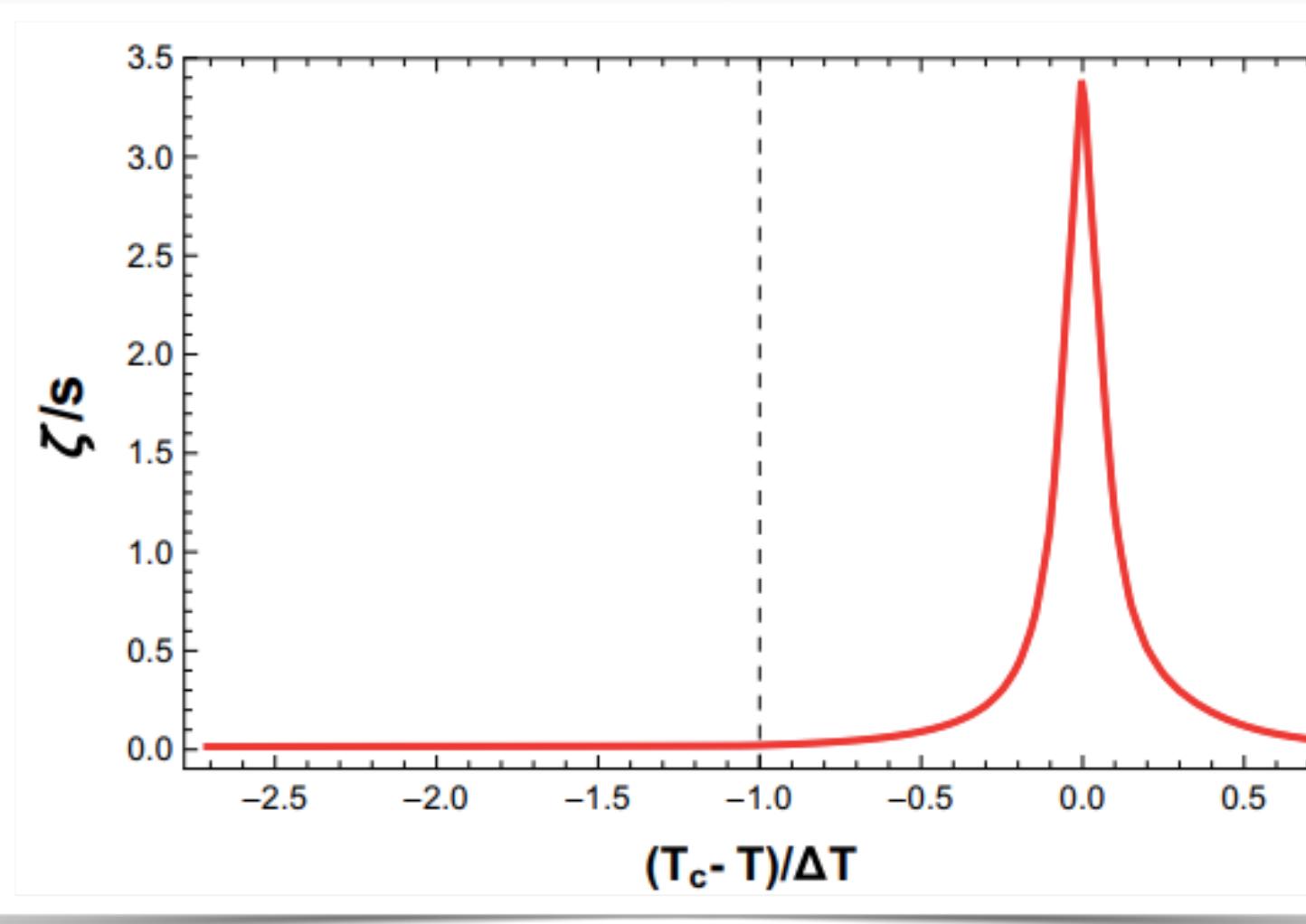
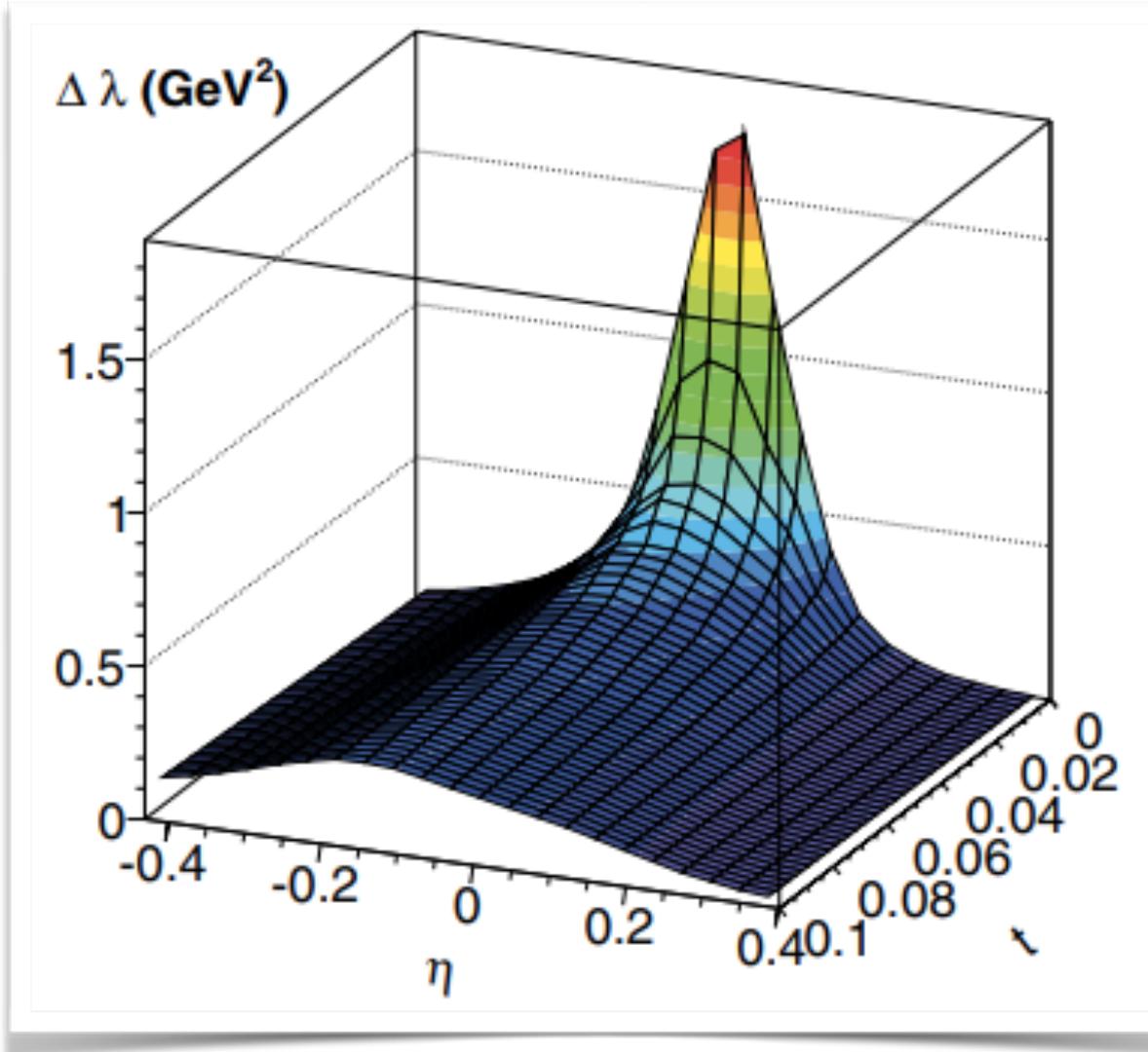
# Transport coefficients

- The bulk evolution of a system is controlled by the EoS and the transport coefficients.

PHYSICAL REVIEW C 86, 054911 (2012)

## Thermal conductivity and chiral critical point in heavy ion collisions

Joseph I. Kapusta<sup>1</sup> and Juan M. Torres-Rincon<sup>2</sup>



PHYSICAL REVIEW C 95, 034902 (2017)

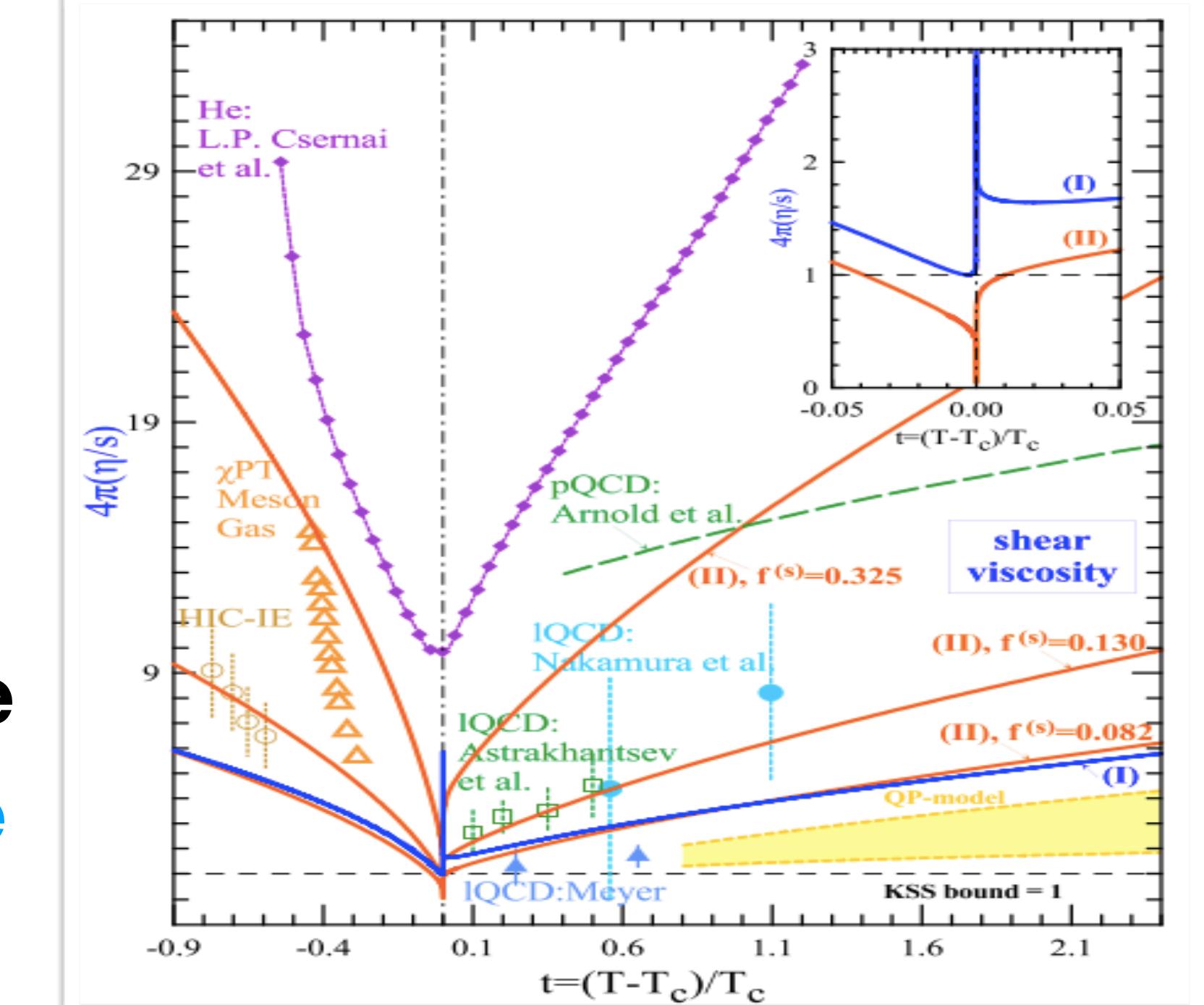
## Phenomenological consequences of enhanced bulk viscosity near the QCD critical point

Akihiko Monnai,<sup>1,2</sup> Swagato Mukherjee,<sup>3</sup> and Yi Yin<sup>3,4</sup>

PHYSICAL REVIEW C 96, 055207 (2017)

## Viscosity of a net-baryon fluid near the QCD critical point

N. G. Antoniou,<sup>\*</sup> F. K. Diakonos,<sup>†</sup> and A. S. Kapoyannis<sup>‡</sup>



- To observe the effects of EoS only in the evolution, we have kept the transport coefficients  $\eta, \zeta, \chi$  to be at the lower bound.

# Framework: Hydrodynamics

- Ideal Hydro is not suitable
- First order theory is known to violate causality and gives unstable solution.
- We use Second-order hydrodynamics to study the propagation of the perturbation.
- Causality is restored by introducing a time delay in the response of the dissipative currents from gradients of the fluid dynamical variables.

Bemfica et al, PRD(2021), arXiv:2009.11388

First-order theories can be causal and stable.

Landau-Lifshitz frame:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N^\mu = n u^\mu + \nu^\mu = n u^\mu - \frac{n q^\mu}{\epsilon + p}$$

$$\Pi = -\frac{1}{3}\zeta(\partial_\mu u^\mu + \beta_0 D\Pi - \alpha_0 \partial_\mu q^\mu)$$

$$\pi^{\lambda\mu} = -2\eta\Delta^{\mu\nu\alpha\beta}[\partial_\alpha u_\beta + \beta_2 D\pi_{\alpha\beta} - \alpha_1 \partial_\alpha q_\beta]$$

$$q^\mu = \kappa T \Delta^{\mu\nu} \left[ \frac{nT}{\epsilon + P} (\partial_\nu \alpha) - \beta_1 D q_\nu + \alpha_0 \partial_\nu \Pi + \alpha_1 \partial_\lambda \pi_\nu^\lambda \right]$$

# Strategy

- A space-time dependent perturbation,  $e^{-i(\omega t - kx)}$  is placed into the fluid via  $P, \epsilon, n, T, \mu$ , and  $u^\mu$ .

S. Weinberg, *Astrophys. J.* 168, 175 (1971)

- The EOM for different components of perturbation is obtained by the conservation equations:

$$\partial_\mu T^{\mu\nu} = 0; \partial_\mu N^\mu = 0$$

- Real and imaginary part dictates the propagation and dissipation of the perturbation into the fluid.

- A threshold wavelength ( $\lambda_{th} = 2\pi/k_{th}$ ) is evaluated by the condition

$$\left| \frac{\omega_{Re}}{\omega_{Im}} \right| = 1.$$

Liao and Koch, PRC (2010)

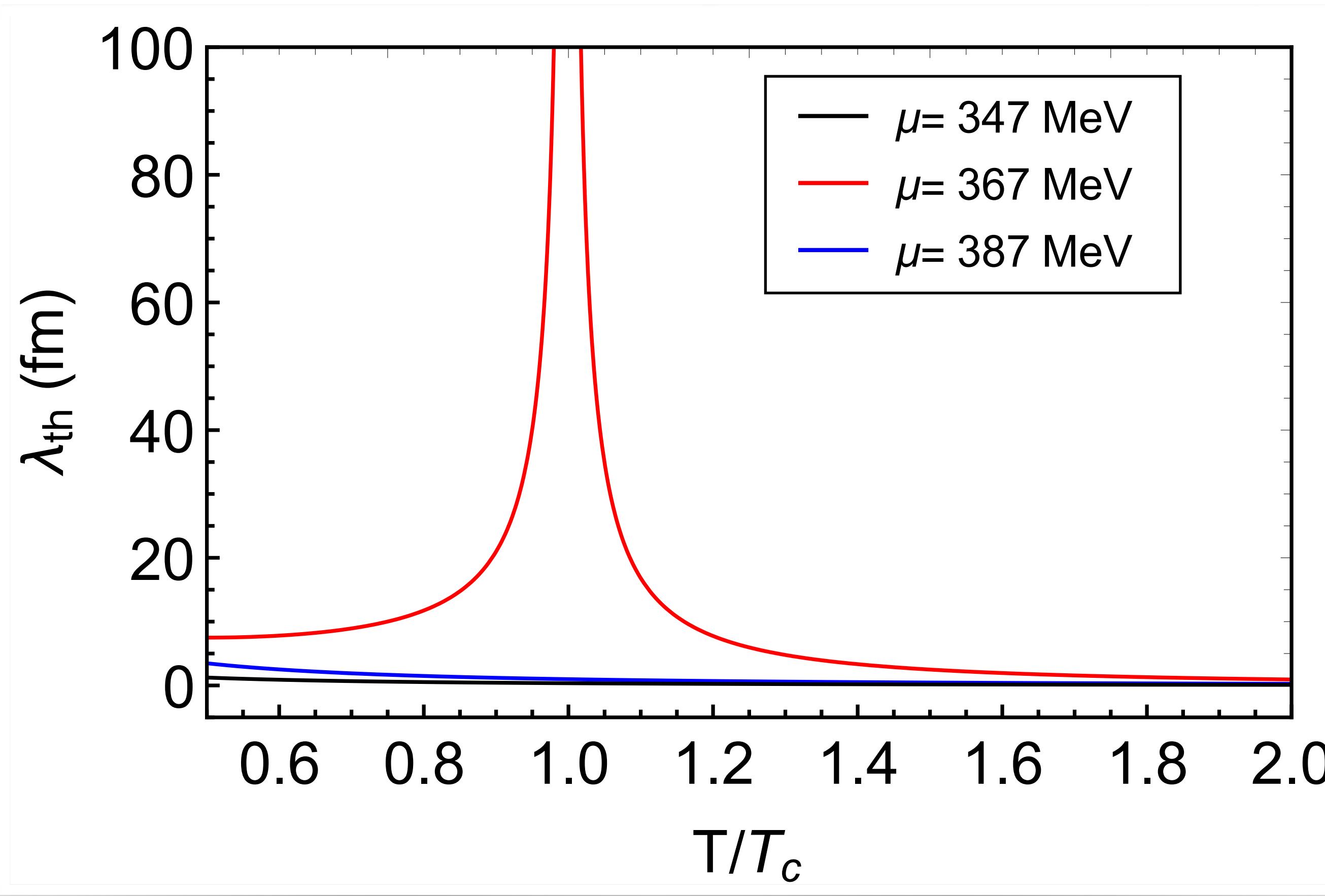
# Strategy

- A wave with wavelength  $\lambda \geq \lambda_{th}$ , can only propagate and rest will dissipate in the medium.
- Viscous horizon is a length scale  $(R_\nu) \sim \lambda_{th}$ : separates the wavelengths of the sound which are and are not dissipated by the viscosity effects.

**Staig and Shuryak, PRC(2011)**
- $R_\nu \rightarrow$  highest order of surviving flow harmonics  $(n_\nu) \sim 1/R_\nu$ 

**Roy A. Lacey et al, arXiv:1105.3782**

# Results

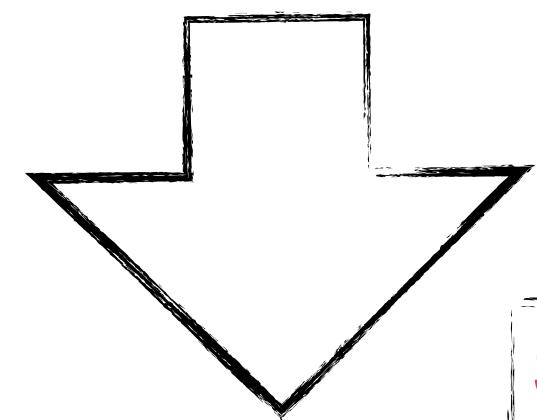


- At the CEP, we found  $\lambda_{th}$  to be very large, implies survival of no wave.
- As consequences, Mach cone formation is prevented.

MH, M. Rahaman, A. Bhattacharyya, J-e Alam, PRC(2020)

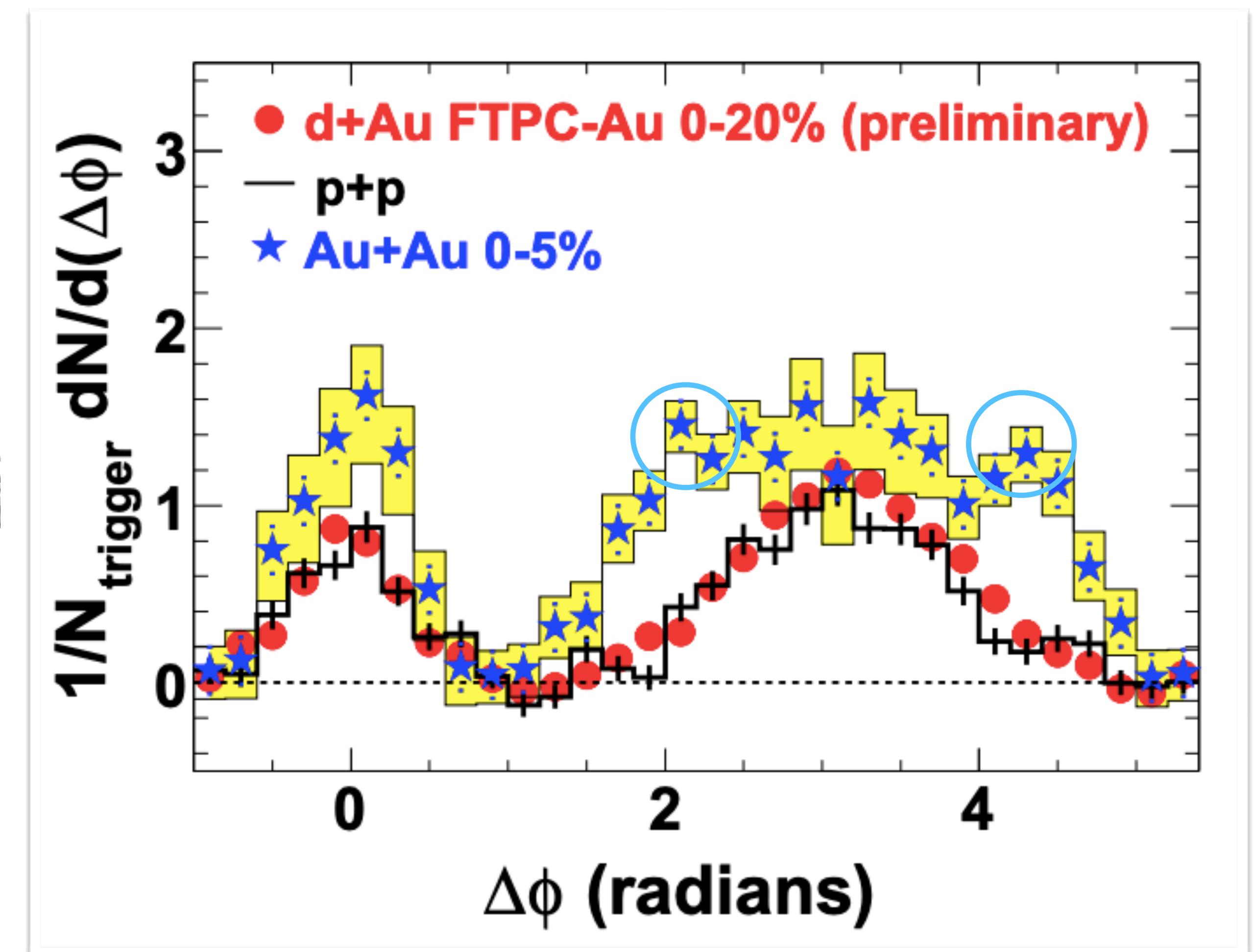
# Results

## Formation of Mach cone



STAR collaboration, PRL (2005)

- Double hump structure at away side jet in particle correlation
- Disappearance of local maxima's in data for hydro evolution through the CEP



STAR Collaboration, NPA(2006)

# Results



PROCEEDINGS  
OF SCIENCE

- $\lambda_{th}$  is divergent at the CEP, the viscous horizon scale is also divergent, means no wave can survive from viscous damping.
- All the possible flow harmonics, collapse at the CEP.
- Ideally fully collapsed harmonics may be the signature of the CEP, but in experiments from the formation of the QGP to hadron is superposition of all the  $T, \mu$  states, one expects suppression.

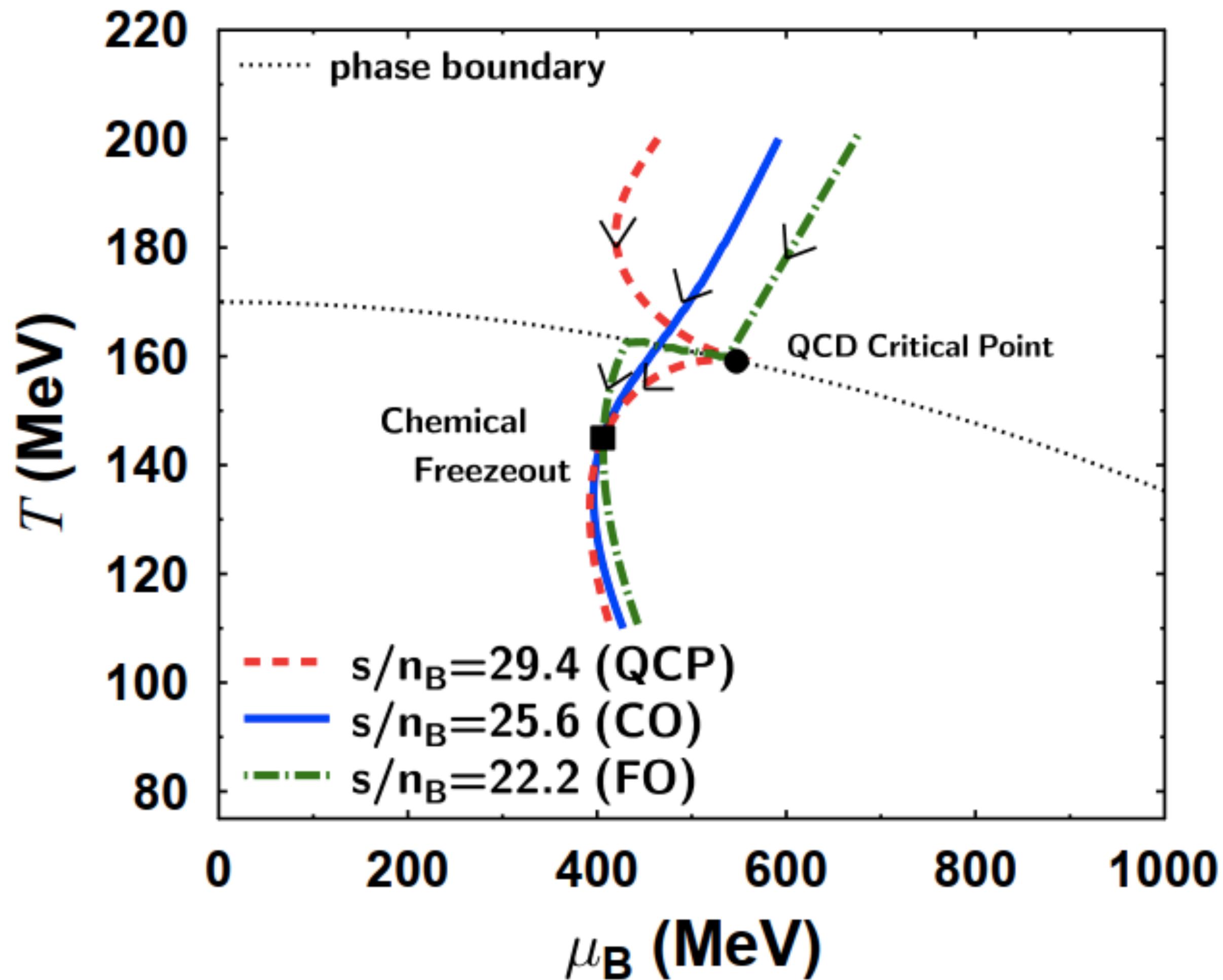
Collapse of Flow: Probing the Order of the Phase Transition

Horst Stöcker\*

H. Stoecker, arXiv:0710.5089

# Results

Nonaka et al, NPA (2009)



- Focussing effect around the CEP.

Stephanov et al, PRL (1998)

- In event-by-event analysis, isentropic trajectories form a few events may go through the CEP and the others away from the CEP, cause **large event-by-event fluctuation of flow harmonics**.

# Summary

- Second-order hydro + EoS with the critical point to study the propagation of a perturbation.
- $\lambda_{th}$  diverges at the CEP, implying that all the modes of the perturbation are dissipated at this point.
- Formation of Mach cone is prevented, the double hump structure at away side jet in particle correlation, disappear.
- All the possible flow harmonics will be suppressed.
- The vanishing Mach cone effects, enhancement of fluctuation of flow harmonics in event-by-event analysis may be considered as the possible signature of detecting the CEP.

**Thank You for your kind attention**