The anomalous enhancement of dilepton production from diquark correlations in dense quark matter

Toru Nishimura
Osaka University
Collaborators: Masakiyo Kitazawa, Teiji Kunihiro
ATHIC2021 (9 November 2021)
Color superconductivity (CSC)

QCD phase diagram


* Induced by diquark condensation.

* Low-temperature and high-density.

* It is difficult to observe CSC in experiment.

* We focus on 2SC.

Realize in the relatively low-density region.
Recently, heavy ion collision (HIC) experiments allow us to search for high-dense matter with high statistics.
Recently, heavy ion collision (HIC) experiments allow us to search for high-density matter with high statistics. Ongoing projects include:

- **BES II at RHIC**
- **NA61/SHINE at LHC**
- **HADES at GSI**
- **FAIR at GSI**
- **HIAF**

Future projects include:

- **FAIR at GSI**
- **NICA at JINR**
- **J-PARC-HI (planned)**
How to observe CSC at HIC?

**Problem I**
Matter produced by HIC is high temperature.
→ Is CSC realized?

**Solution I**
Focus on diquark fluctuation.
...This develops at $T > T_c$

**Problem II**
CSC can exist only for short time in early stage.
→ Hadrons are bad as probes of CSC.

**Solution II**
Focus on dilepton.
...This doesn’t interact strongly.

Kitazawa, Koide, Kunihiro, Nemoto, PTP(2005), Kunihiro, Kitazawa, Nemoto, 0711.4429
The purpose of this study

Through “Diquark fluctuations” “Dilepton”, We research the observability of CSC (2SC) at HIC.

Calculate the effect of the fluctuations on the dilepton production rate.

Need the “photon self-energy”

\[
\frac{d^4 \Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^\beta \omega - 1} g_{\mu \nu} \text{Im} \Pi^{R \mu \nu}(k)
\]
The purpose of this study

Through "Diquark fluctuations" "Dilepton",
We research the observability of CSC (2SC) at HIC.

Calculate the effect of the fluctuations on the dilepton production rate.

Virtual photon \( l^- + l^+ \) dilepton

medium (CFL)

medium (2SC)

diquark

virtual photon

dilepton

\[
\frac{d^4 \Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^\beta \omega - 1} g_{\mu\nu} \text{Im} \Pi^{R\mu\nu}(k)
\]

Dilepton from CFL

Jaikumar, Rapp, Zahed (2002)
2-flavor NJL model

\[ \mathcal{L} = \bar{\psi} i \partial \psi + \mathcal{L}_S + \mathcal{L}_C \]

\[ \mathcal{L}_S = G_S [(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \tau \psi)^2] \]

\[ \mathcal{L}_C = G_C (\bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi^c)(\bar{\psi}^c i \gamma_5 \tau_2 \lambda_A \psi) \]

parameter

\[ G_S = 5.01 \text{MeV}, \quad G_C = 0.6 G_S, \quad \Lambda = 650 \text{MeV} \]

Chiral Symmetry \((m_q = 0)\)

\(SU_c(3)\) Symmetry

Lorentz Symmetry

Because we consider 2SC.

Kitazawa, Koide, Kunihiro, Nemoto (2002)
Diquark fluctuations

The propagator of the fluctuations

$$\Xi^R(\mathbf{k}, \omega) =$$

Random phase approx.

$$= G_C +$$

$$= \frac{G_C}{1 + G_C Q^R(\mathbf{k}, \omega)}$$

diquark correlation

$$Q^R(\mathbf{k}, \omega) =$$

Kitazawa, Koide, Kunihiro, Nemoto (2005)

- The fluctuations form **soft mode**.
- The peak in **space-like** region at low energy-momentum.
Photon self-energy

\[
\frac{d^4 \Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^{\beta \omega} - 1} g_{\mu\nu} \text{Im} \Pi^{\mu\nu}(k)
\]
Photon self-energy

\[ d^4 \Gamma = \alpha \frac{1}{\omega^2} \]

Thermodynamic potential:

\[ \text{One loop of diquark fluctuations} \]

\[ = \bigoplus \text{patterns of photon interaction} \]

Interact at two points.

→ Consider every interaction.

→ Satisfy Ward Identity.

4 patterns of photon interaction
Aslamasov-Larkin term • Maki-Thompson term • Density of states term

AL • MT • DOS terms are well known in condensed matter theory.

Electric conductivity in metallic SCs

Skopal, Tinkham (1975)
Aslamasov-Larkin term • Maki-Thompson term • Density of states term

Review: Larkin (2008)

AL term has contribution to pair annihilation of fluctuations.

$$\text{Im} \Pi_{AL} =$$

Optical theorem

These terms include three-loop momentum integral.

Approximate these so that soft property can be properly evaluated.
Time-dependent Ginzburg-Landau (TDGL) approx.

\[ \Xi^R(k, \omega) = \frac{G_C}{1 + G_C Q^R(k, \omega)} = \frac{1}{a + b\omega + ck^2} \]

\[ 1 + G_C Q^R(0, 0) = 0 \text{ at } T = T_C \]

Coefficients are determined by NJL.

\[ a = [\Xi(0, 0)]^{-1}, \quad b = \frac{\partial [\Xi(0, 0)]^{-1}}{\partial \omega}, \quad c = \frac{\partial [\Xi(0, 0)]^{-1}}{\partial k^2} \]

This approximation is valid around \( T_C \) in the low energy-momentum region.
Approximation of vertex by Ward Identity

**AL**: \( \Pi_{\text{AL}}^{\mu\nu}(k) = \Gamma^\mu \Gamma^\nu \)
\[
= \int \frac{d^4q}{(2\pi)^4} \Gamma^\mu(q, q + k) \Xi(q + k) \Gamma^\mu(q + k, q) \Xi(q)
\]

**W-I of photon self-energy**

- **time component**
  \[ \Pi^{R00}(k) = \frac{k^2 \Pi^{R11}(k)}{k_0^2} \]
  at \( k = (k_0, |k|, 0, 0) \)

  Need only space components of vertices \( \Gamma^i \).

- **longitudinal component**

**W-I of vertex of AL**

\[ k_\mu \Gamma^\mu(q, q + k) = \Xi^{-1}(q + k) - \Xi^{-1}(q) \]

Compare the lowest order terms of \( k \) and \( \omega \).

\[ \Gamma^i(q, q + k) = \frac{\partial \Xi^{-1}(q + k)}{\partial k_i} = c_1(2q + k)^i \]
Approximation of vertex by Ward Identity

Aproximated vertices are all real.

Need $g_{\mu\nu}\text{Im} \Pi^{\mu\nu}(k)$ for production rate:

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^\beta \omega - 1} g_{\mu\nu}\text{Im} \Pi^{R\mu\nu}(k)$$

Imaginary part of MT and DOS term cancel.
Consistent with the metallic SC !!!

$$\text{Im}(\text{MT}) + \text{DOS} = 0$$

Only AL term is necessary for dilepton production rate.
Production rate at $k=0$

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} e^{\beta \omega} \frac{1}{1 - g_{\mu\nu} \text{Im} \Pi^{R\mu\nu}(k)}$$

Red: fluctuations
Green: free quark

Enhance at low $\omega$ as $T \rightarrow T_c$

Expected from soft property of fluctuations

$\mu = 350\text{MeV}$
Production rate at $k=0$

\[
\frac{d^4 \Gamma}{dk^4} = \frac{\alpha}{12\pi^4} \frac{1}{k^2} \frac{1}{e^\beta \omega - 1} g_{\mu \nu} \text{Im} \Pi^{R\mu\nu}(k)
\]

★ Diquark fluctuation has the peak in space-like region.

★ Dilepton production rates enhance in time-like region.

This process is dominant!!
Invariant mass spectrum

$$\frac{d\Gamma}{dM^2} = \frac{1}{2\omega} \int d^3k \frac{d^4\Gamma}{d^4k}$$

Red: fluctuations
Green: free quark
Comparable with experiments

Enhancement in low-invariant mass region

Signal for observation of CSC!?
Summary

★ Calculate the effects of “diquark fluctuations” to the “dilepton production rate” in order to observe CSC at HIC.
* Consider AL, MT, DOS terms.
* Satisfy Ward Identity.
* Simplify momentum integration by TDGL approximation.
* The production rate per invariant mass.
=> Enhancement in low-invariant mass!!

Outlook

★ Are the enhancement at low-M observable??
* Apply our result to dynamical model.
* Consider the competition with other dilepton production process. (Dalitz decay etc..)