Why chemical freezeout is at the QCD cross over?

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Talk based on
Sourendu Gupta, JKN & Sushant K Singh, PRD 103, 054023 (2021)
When heavy nuclei like Au, Pb are collided at relativistic energies, a locally equilibrated fireball of quarks and gluons is formed. Hadrons emitted from the fireball are measured experimentally.

Large internal pressure makes the fireball expand and cool. Typical timescale $\tau_{exp} \sim 10 - 20 \text{ fm}$.

The hadrons may further interact changing the yields and momentum distribution of individual species. Let $\tau_R$ denotes the typical timescale over which yields change.

If $\tau_R < \tau_{exp}$, then chemical equilibrium is maintained.

If $\tau_R \gtrsim \tau_{exp}$, then system falls out-of-chemical equilibrium. Yields do not change significantly. Chemical Freeze Out occurs at temperature $T_{CFO}$ and chemical potential $\mu_{B,CFO}$. 
Hadron Resonance Gas model

- Number and identity of hadrons (yields) are described by HRG.

- Non-interacting gas of hadrons and resonances. The grand canonical partition function is

\[
\ln Z = \sum_{i=1}^{N} \ln Z_i \quad \text{and} \quad \ln Z_i = \pm \frac{V g_i}{2 \pi^2} \int dp \ p^2 \ln \left[ 1 \pm \exp \left( - \frac{E_i - \mu_i}{T} \right) \right]
\]

and

\[
N_i = -T \frac{\partial}{\partial \mu_i} \ln Z = \frac{g_i V}{2 \pi^2} \int_0^\infty \frac{p^2 dp}{\exp \left( (E_i - \mu_i)/T \right)} \pm 1 \quad \text{and} \quad \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q
\]

- The yields have been analyzed using the HRG model ([NPA 772 (2006) 167-199 and Nature 561, 321-330 (2018)]). Mean yields have been fitted fairly successfully. Fitting provides freezeout parameters \( T_{CFO} \) and \( \mu_{B,CFO} \).
$T_{CO} \approx T_{CFO}$: a coincidence?

Setting-up the Solution

- The hadron yields provide information (i.e. $T_{CFO}$ and $\mu_{B,CFO}$) only on the last scattering surface.
- To show that $T_{CO} \approx T_{CFO}$, we need to show
  - Chemical relaxation time in the chiral symmetric phase is small. This has been done in PRL 122, 142301 (2019). Typical timescale $\sim 1 – 2.5$ fm.
  - Chemical relaxation time in the chiral broken phase is large.
- In order to understand FO, we concentrate on broken phase and calculate $\tau_R$ of a hadron gas which is not very far from equilibrium.
- Set-up a transport theory for hadrons.
- For $\mu_B = 0$, $n_p^{eq}/n_{\pi}^{eq} \sim 0.01$ near $T_{CO}$. Neglect baryons and higher mass hadrons.
To calculate $\tau_R$, sufficient to consider a fluid at rest.

In order to understand freeze out, sufficient to calculate close to equilibrium. Hence, use of linear approximation i.e. for $i$th species

$$n_i = n_i^{eq} + \delta n_i$$

and keep terms linear in $\delta n_i$.


Equation determining the approach towards equilibrium

$$\frac{d\delta n_a}{dt} = - \sum_r \langle \sigma_r \nu_{ab} \rangle (n_a^{eq} \delta n_b + n_b^{eq} \delta n_a) + \sum_{\bar{r}} \langle \sigma_{\bar{r}} \nu_{cd} \rangle (n_c^{eq} \delta n_d + n_d^{eq} \delta n_c)$$

where $\langle \cdot \rangle \equiv$ averaging over the thermal distribution, $\sigma_r$ denotes cross-section of reactions where $a$ is in initial state, $\sigma_{\bar{r}}$ denotes cross-section of reactions where $a$ is in final state.
Isospin symmetry: since mass diff. b/w isospin partners $\Delta m \ll T_{CO}$, a reasonable approximation

$$\pi : (\pi^+, \pi^0, \pi^-), \quad K : (K^+, K^0), \quad \bar{K} : (\bar{K}^0, K^-)$$

Instead of 8, only 4 independent densities i.e. $\delta n_\pi, \delta n_K, \delta n_{\bar{K}}, \delta n_\eta$.

Strangeness: $S = n_{\bar{K}} - n_K$ conserved

Accidental conservation: $N = n_\pi + n_K + n_{\bar{K}} + n_\eta$ conserved.

$S$ and $N$ conservation can be taken into account through following parametrization of $\delta n_i$'s

$$\delta n_\pi = h_\pi, \quad \delta n_\eta = h_\eta, \quad \delta n_K = \delta n_{\bar{K}} = -(h_\pi + h_\eta)/2$$

so that

$$\frac{d}{dt} \begin{pmatrix} h_\pi \\ h_\eta \end{pmatrix} = C \begin{pmatrix} h_\pi \\ h_\eta \end{pmatrix}, \quad \text{elements of } C \text{ have dimension } [t]^{-1}$$
Octet of pseudoscalar mesons: \((\pi^+, \pi^-, \pi^0, K^+, K^0, \bar{K}^0, K^-, \eta)\) forms the Goldstone bosons of chiral symmetry breaking of QCD \((SU(3)_L \times SU(3)_R \to SU(3)_V)\).

Chiral symmetry constrains the interactions among the pseudo-scalar mesons. At leading order

\[
\mathcal{L} = \frac{f_0^2}{4} \text{Tr} \left[ \partial_\mu U^\dagger \partial^\mu U + M_0 (U + U^\dagger) \right]
\]

where \(U = \exp(i \sqrt{2} \Phi / f_0)\) and \(\Phi\) is a SU(3) matrix containing the 8 meson fields and \(M_0 = \text{diag}(M^2_{0\pi}, M^2_{0\pi}, 2M^2_{0K} - M^2_{0\pi})\).

NLO ChPT scattering-amplitudes have been computed in Ref. PRD65, 054009 (2002). Lagrangian contains 8 LECs.

Unitarity \((SS^\dagger = I)\) enforced in different \((I, J)\) channels through Inverse Amplitude Method. At every mass threshold, number of states increases by one. So dimension of S-matrix increases by one. So discontinuities in amplitudes at thresholds.
Some features of the Calculation

- Calculation involves 12 input parameters:
  \[ m_\pi, \ m_K, \ m_\eta, \ f_\pi, \ L_1, \ldots, L_8 \]
- Error uncertainties due to \( L_i \)'s only. Errors in others negligible.
- Dynamical generation of resonances in Unitarized ChPT. Scalar and vector resonances till 1.2 GeV are reproduced. Need not include resonances as explicit degrees of freedom.
- No UV cutoff needed as the amplitudes are unitarized.
The slow mode has a relaxation time of 100 fm at 150 MeV and 1000 fm at 100 MeV.

The fast mode has a relaxation time of 10 fm at 150 MeV and 100 fm at 100 MeV.
Figure: Angle between normal modes and pion direction shown as a function of the temperature. Ref: PRD 103, 054023 (2021)

- The slow mode is dominantly $\pi$’s.
- The fast mode is dominantly $\eta$’s.
Earlier studies in this direction

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PLB319 (1993) 401

PRC55, 3026 (1997)

- Relaxation time at high temperatures underestimated due to unphysical inputs.
Goldstone Physics

- Goldstone bosons are non-interacting. This means in the limit of exact chiral symmetry $\pi$, $K$, $\eta$ do not interact.

- Explicit breaking of symmetry gives rise to interaction proportional to symmetry breaking parameter. Here the symmetry breaking parameters are masses.

- Fast mode being $\eta$ and slow mode being $\pi$ suggests Goldstone behaviour.

- Best check should be to vary the masses and see whether $m \rightarrow 0$ gives $\tau_R \rightarrow \infty$. Unfortunately this cannot be done as LECs are fixed.

- Another check will be to remove $\eta$ from the system. The relaxation time must increase. This we have checked and results consistent with Goldstone physics.
Define the dimensionless ratio $\Pi = \frac{\tau(T) n_{\text{eq}}(m, T) m^2}{(4f_\pi^4)}$.

For slow mode ($m = m_\pi$ and $\tau = \tau_s$): $\Pi$ is of order unity (changes from 1 to 2 in the temperature range 100-150 MeV).

With our values of $\tau_s$, we find that $1/\tau_s \approx \sigma n_{\text{eq}}(m_\pi, T)$ with $\sigma = \frac{m_\pi^2}{4f_\pi^4} \approx 25 \text{ mb}$.

The result agrees with kinetic theory arguments !!!

This simple result is only obtained when the complexity of NLO ChPT and meson resonances are included.

For fast mode ($m = m_\eta$ and $\tau = \tau_f$): $\Pi$ is of order unity (changes from $2/3$ to $4/3$ in the temperature range 100-150 MeV).
Is $T_{CFO} \approx T_{CO}$?

- Slow mode relaxation time is 100 fm near $T_{CO}$ much larger than $\tau_{exp}$ typically 10-20 fm. Meson gas cannot remain in chemical equilibrium. This together with "short equilibration time in chiral symmetric phase" suggests $T_{CFO} \approx T_{CO}$.

- Inclusion of baryons will modify the slow mode relaxation time as

$$\frac{1}{\tau'_s} \approx \sigma_{\pi\pi} n_{\pi}^{eq} + \sigma_{\pi N} n_{N}^{eq}$$

At low energies, $\sigma_{\pi N}/\sigma_{\pi\pi} \approx 2$ and whereas $n_{N}^{eq}/n_{\pi}^{eq}$ varies between 0.001 and 0.01 when $T$ changes from 100 to 150 MeV. Therefore, the effect of adding baryons will be a few percent.

- Future calculation to include baryons are being undertaken but our results and arguments give a strong indication that we understand the basic physics of chemical freezeout.
Conclusions

- Constructed a kinetic theory of yields using pseudoscalar mesons and unitarized cross sections from NLO ChPT. This takes care of the fact that pseudoscalar mesons are pseudo-Goldstone bosons of chiral symmetry breaking in QCD.

- Neglected baryons. Although $\pi N$ cross sections are similar to $\pi\pi$ cross sections but densities are 2 to 3 orders of magnitude smaller.

- Interactions are completely fixed by chiral symmetry but due to low densities relaxation times are large.

- Clear physical reason why fireball cannot be in equilibrium after chiral symmetry breaking.

Thank you for your attention !!!