Berry curvature and topological aspects of color superconductor

Will nature fabric topological matter inside the neutron star?

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Noriyuki Sogabe, YY, to appear

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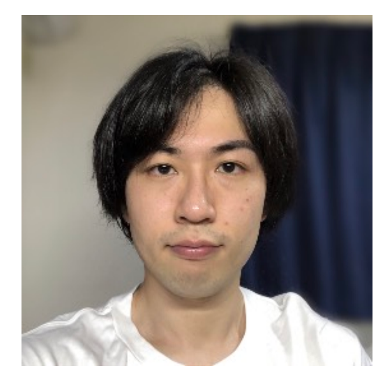

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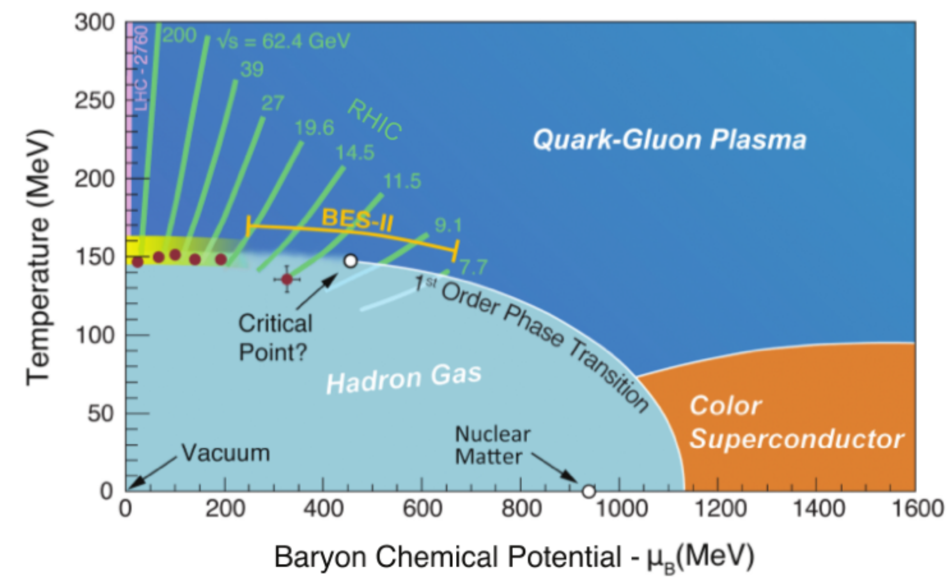
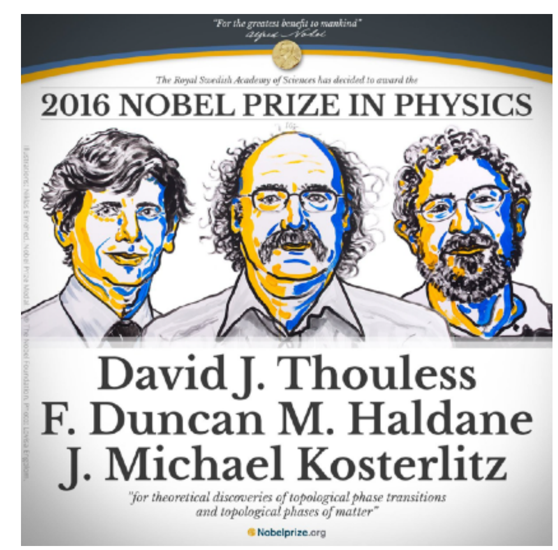
QCD phase diagram and topology

- Key question: identify possible phases of QCD.
- Traditional paradigm (Landau-Ginzburg): based on symmetry breaking pattern.
- Topological aspects: important in understanding and characterizing quantum phases.





Single flavor pairing in QCD

Baryonic matter at asymptotically high density is a color superconductor in color-flavor locked (CFL) phase. *Alford, Schmitt, Rajagopal, Schaffer, Rev. Mod. Phys. 2008*

Single flavor pairing might be favored at environments in stars. (mismatch between Fermi surface at different flavors induced by charge neutrality condition).

Gap function is spin-one triplet.

pairing between quarks with opposite chirality is energetically preferred.

Extensively studied but why we revisit?

e.g. works by Mei Huang, Defu Hou, T. Schaffer, A. Schmitt, Qun Wang, Pengfei Zhuang and many others

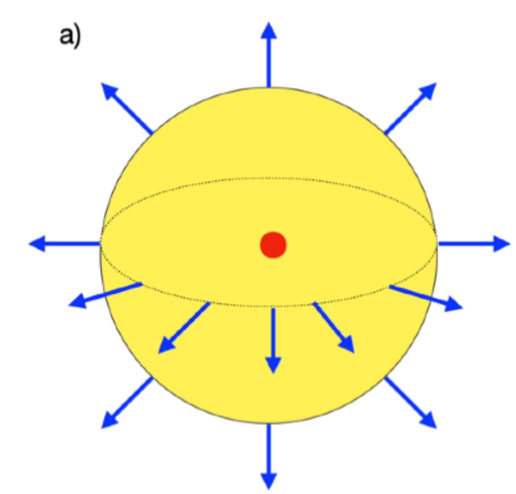
Topological aspects has been overlooked for years

Topological structure of the chiral Fermion surface (F.S.)

- Consider positive helicity spinor $\xi_R(\hat{k})$ and define Berry connection and curvature.

$$\vec{A}(\vec{k}) \equiv i\xi_R^\dagger(\hat{k})\,\vec{\nabla}_k \xi_R(\hat{k}) \qquad \vec{b} \equiv \vec{\nabla} \times \vec{A} = \frac{\vec{k}}{2k^2}$$

Similar for left helicity



Berry flux of F.S. = $4\pi q_{R/L}$

(with monopole charge $q_{R/L} = \pm 1/2$)

- The presence of monopole implies that helicity spinor can not be continuously on the whole F. S. .

- Berry monopole plays important role in anomaly-induced transport phenomena.

- What happens when F.S. collapses in super-conducting phase?

Berry structure in Cooper pair

Murakami-Nagaosa, PRL 03; Li-Haldane, PRL 18';

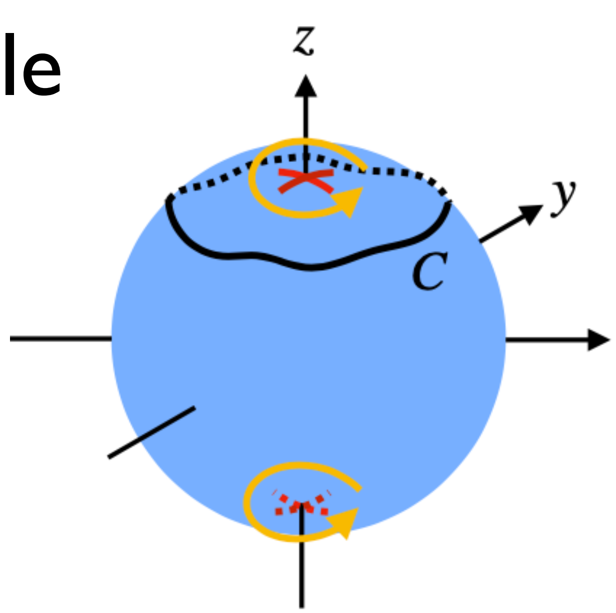
- Consider $\uparrow\uparrow$ component of gap matrix in spin space.

$$H_{int} \propto \Delta_{\uparrow\uparrow}(\vec{k}) \left[\xi_{R,\uparrow}(\hat{k})\xi_{L,\uparrow}(-\hat{k})c_R^\dagger(\vec{k})c_L^\dagger(-\vec{k})\right]$$

- Since spinor can not be continuous on the whole F.S., neither can the phase of $\Delta_{\uparrow\uparrow}$, $\phi_\Delta(\hat{k})$.

$\Rightarrow \Delta_{\uparrow\uparrow}$ must have nodes where $\phi_\Delta(\hat{k})$ is ambiguous.

- The winding number for the "velocity field" $\vec{v}^\Delta = \nabla_k \phi^\Delta$ around the nodes inherits the monopole charge of single particle F.S.



$$g_w = \oint d\vec{k} \cdot \vec{v}^\Delta = q_R - q_L$$

Single flavor pairing

- Accounting for color structure

$$\Delta_{\uparrow\uparrow}(\overrightarrow{k}) \rightarrow M = J_A\, O_i^A\, S^i(\overrightarrow{k})$$

matrix in color space

$$J_A = \left(\lambda_7, -\lambda_5, \lambda_2\right)$$

collection of Gellmann matrices

$$\overrightarrow{S} \propto \langle \xi_R^*(\hat{k})\xi_L^*(-\hat{k})\, c_R(\overrightarrow{k})c_L(-\overrightarrow{k})\overrightarrow{\sigma}\rangle$$

spin expectation value

O_i^A (3 by 3 matrix)

specifying entanglement between color and spin.

- Each components of $S^i(\hat{k})$ must have zeros where its phase becomes ambiguous.

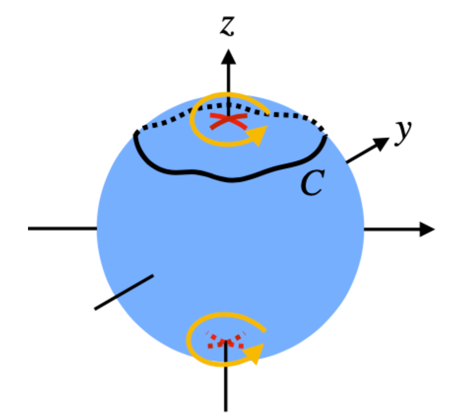
$$\overrightarrow{S} \propto \left(-\cos(\theta)\cos(\phi) + i\sin(\phi), -\cos(\theta)\sin(\phi) - i\cos(\phi), \sin(\theta)\right)$$

Polar phase (c.f. Haldane-Li)

- Polar phase (trivial in color structure, pair between red and blue quarks only):

$$O_i^A = \delta_3^A \, \delta_{i3} \qquad M_{\text{polar}} = \begin{pmatrix} 0 & S^3(\vec{k}) & 0 \\ -S^3(\vec{k}) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- The zeros of S^3 leads to topological nodes in M_{polar} .



Winding number = $q_R - q_L$

- The nodal structure of polar phase has been known since early 2000s, but the associated topological structure remains elusive before.

A. Schmitt, 2005

- Similar for A-phase: $O_i^A = \delta_3^A(\delta_{i1} + \delta_{i2})$.

Topological structure in excitations

Sogabe-YY, to appear

- For color-spin locking (CSL) phase (ground state), the gap matrix has no nodes, i.e. , $M \neq 0$ for any $\vec{k}$.

$$O_i^A = \delta_i^A \qquad M_{\rm CSL} = \begin{pmatrix} 0 & S^3 & S^2 \\ -S^3 & 0 & S^1 \\ S^2 & -S^1 & 0 \end{pmatrix}$$

- Where does the topology structure go?

- Among 12 excitation branches, 4 branches are gapless and carry monopoles charges (2 positive and 2 negative)

e.g. $$d^\dagger(\vec{k}) \propto S^1 c^\dagger_{L,red} + S^2 c^\dagger_{L,blue} + S^3 c^\dagger_{L,green}$$

Gapless excitation inherit the topology of Berry structure.

Energetic consideration

- Topology of Berry structure can be inherited by
  - Cooper pair ⇒ topologically protected nodes in gap matrix/function, (Polar and A phase). *Li-Haldane, PRL 18'*
  - Gappless excitations ⇒ no nodes in gap matrix/function, but excitations carries monopole charge, (CSL and planar phase) *Sogabe-YY, to appear*
- We may argue the second possibility is energetically favorable:

$$\Delta p \propto \int_{\hat{k}} |\phi(\hat{k})|^2 \qquad |\phi(\hat{k})|^2 \propto \mathrm{tr}(M M^\dagger)$$

Condensation energy

Gapless excitations play less topological inheritance tax.

Summary and outlook

- We find non-trivial topological structure in the same flavor pairing with opposite chirality: either in gap functions or in excitations.
- Hadron-quark continuity.
- The "topological" excitations in CSL and anomaly matching. (c.f. Chiral kinetic theory).
- Observational consequence in astrophysics.

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Back-up