



Nuclear Science  
Computing Center at CCNU



# QCD phase structure in strong magnetic fields

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based on *Phys. Rev. D* 104 (2021) 014505 & *Eur.Phys.J.A* 57 (2021) 6, 202,  
in collaboration with

Heng-Tong Ding, Jun-Hong Liu, Qi Shi, Akio Tomiya, Xiao-Dan Wang, Yu Zhang



# Outline

- Motivation and Introduction
- Lattice Setup
- Results
- Summary

# Motivation

## Heavy-Ion collisions

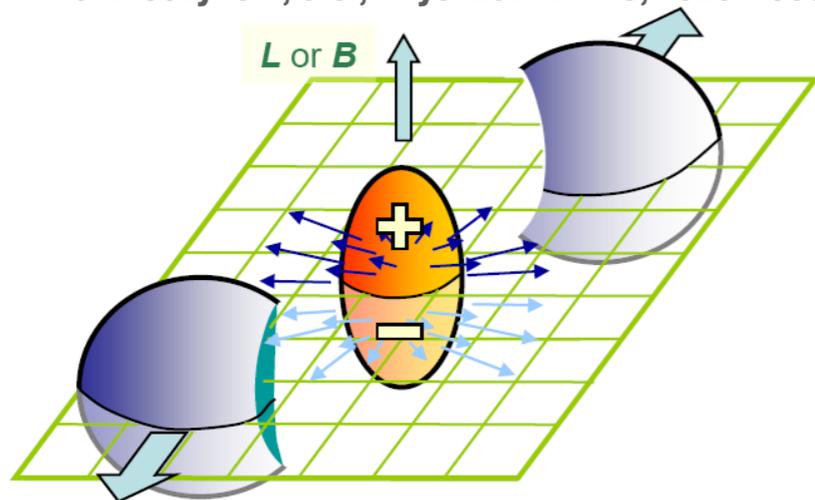
**RHIC:**  $eB \sim O(1)m_\pi^2$

**LHC:**  $eB \sim O(10)m_\pi^2$

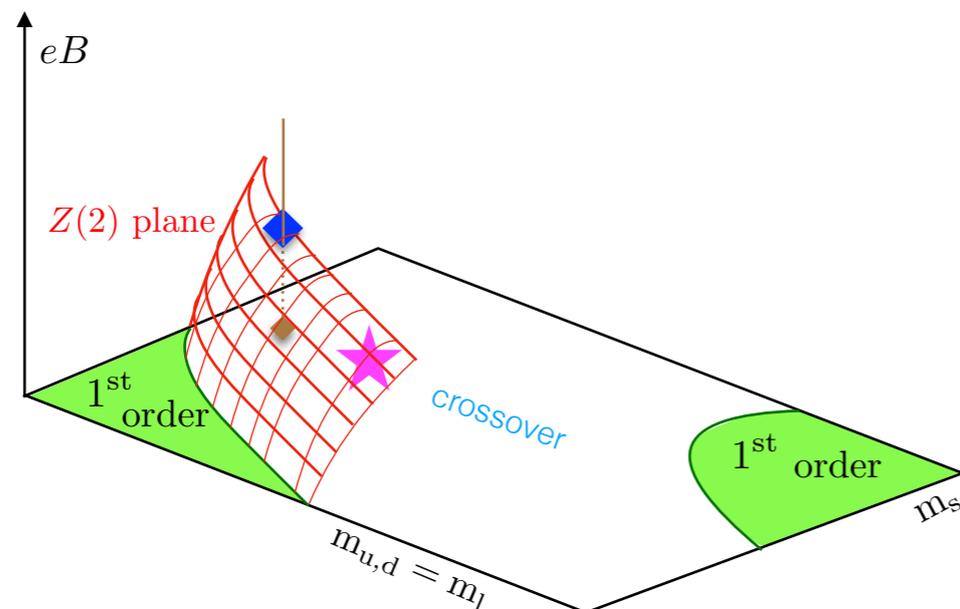
V. Skokov, et al., Int.J.Mod.Phys.A 24 (2009) 5925-5932

A. Bzdak, et al., Phys. Rev. Lett. 110, 192301 (2013).

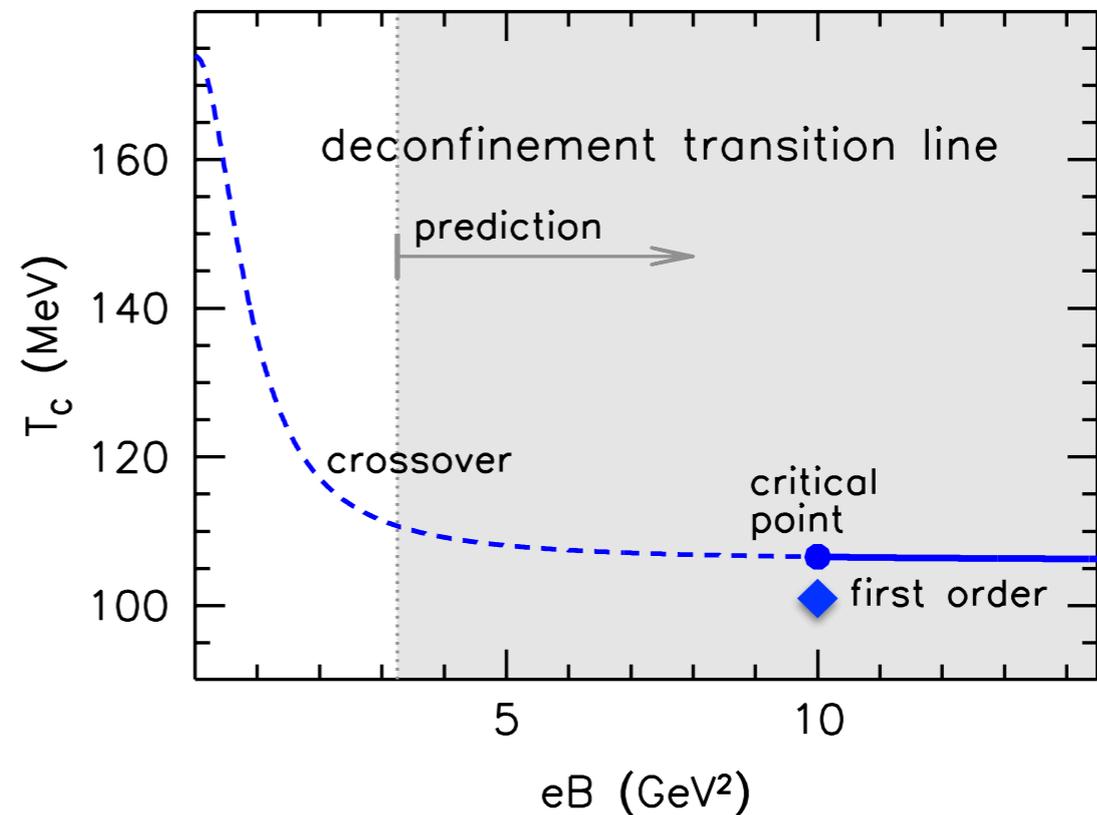
J. Błoczyński, et al., Phys. Lett. B 718, 1529-1535 (2013).



STAR Collaboration (B.I. Abelev (Illinois U., Chicago) et al.). Phys.Rev. C81 (2010) 054908



## T-eB plane



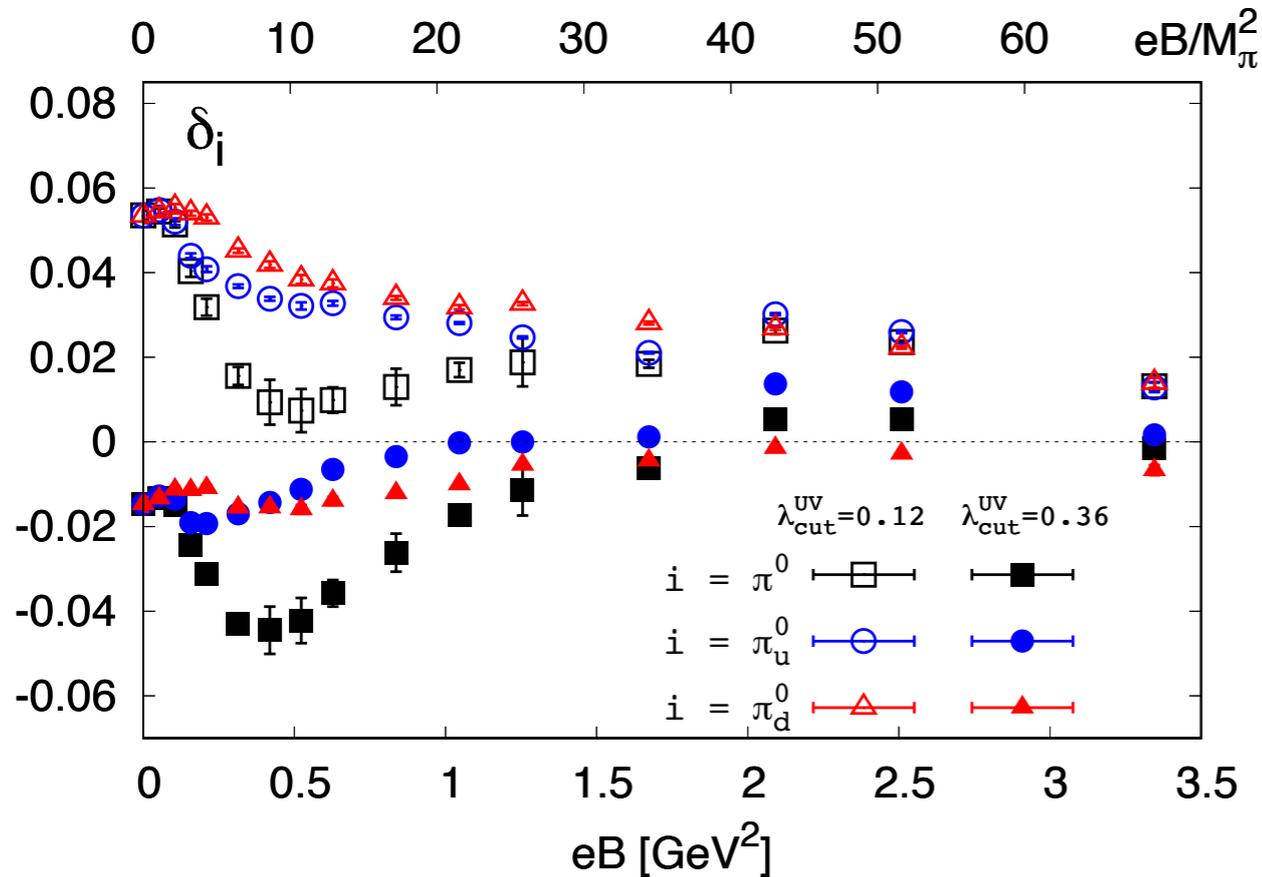
Gergely Endrodi(Regensburg U.) JHEP 07 (2015) 173 arxiv: 1504.08280

- A possible Z(2) critical end point in T-eB plane ?
- How to detect eB in HIC experiments ?

# First LQCD study on GMOR relation at $eB \neq 0$ and $T=0$

$N_f=2+1$  QCD,  $M_\pi(eB=0) \approx 220$  MeV,

$32^3 \times 96$  lattices with  $a^{-1} \approx 1.7$  GeV and HISQ action



H.T. Ding, STL, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

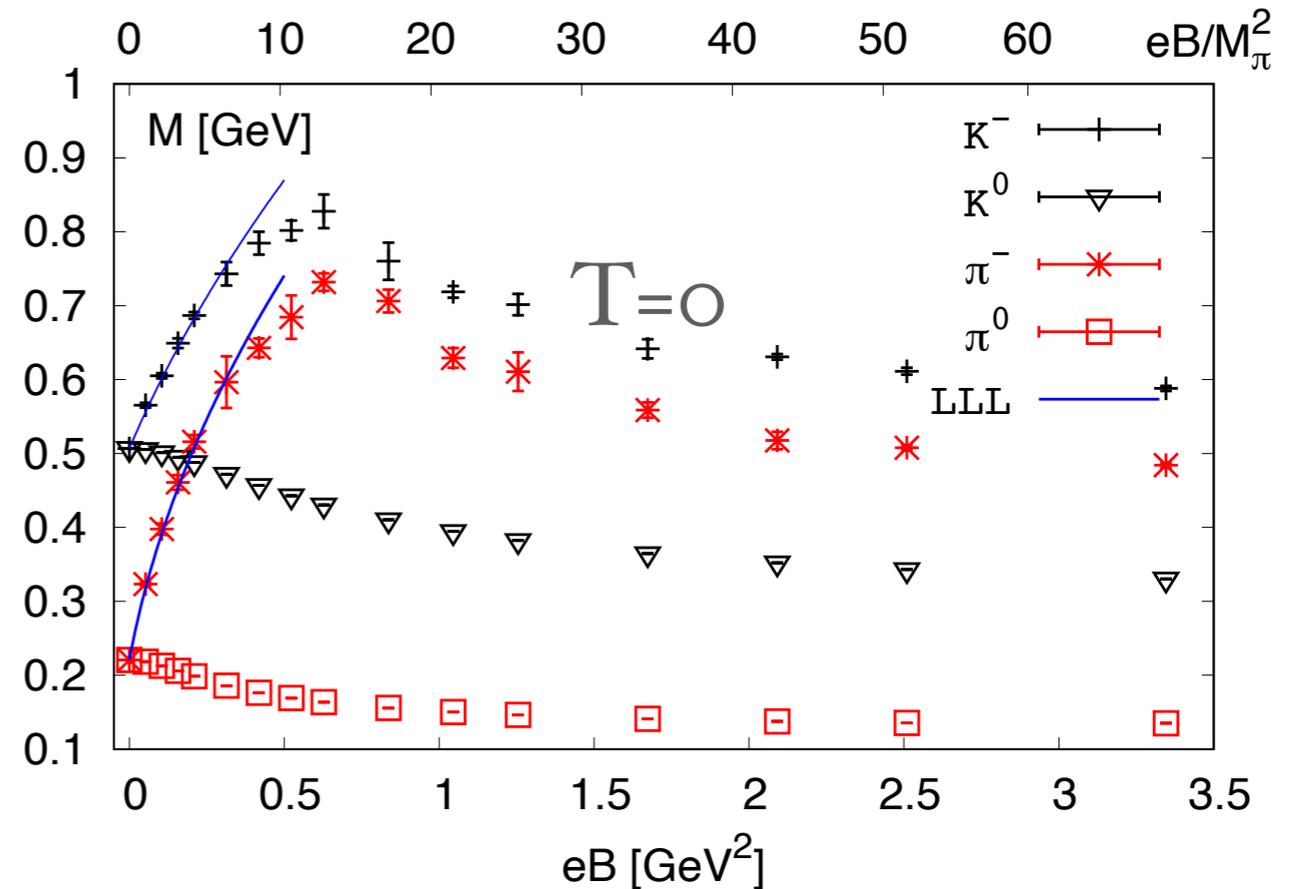
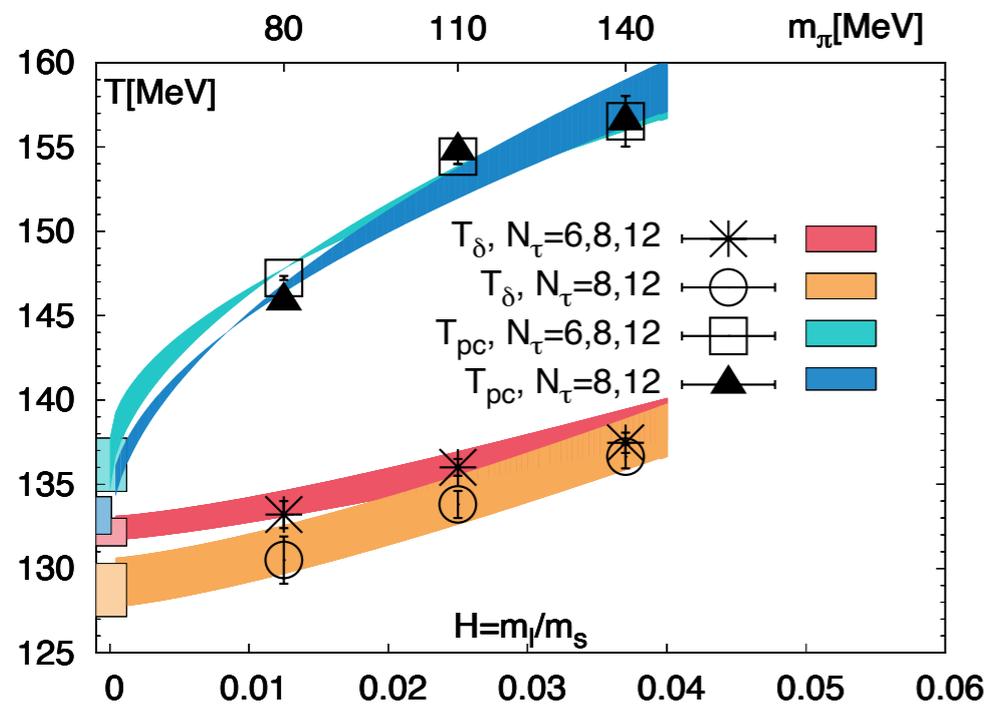
- $4m_u \langle \bar{\psi}\psi \rangle_u = 2f_{\pi_u^0}^2 M_{\pi_u^0}^2 (1 - \delta_{\pi_u^0})$
- $4m_d \langle \bar{\psi}\psi \rangle_d = 2f_{\pi_d^0}^2 M_{\pi_d^0}^2 (1 - \delta_{\pi_d^0})$
- $(m_u + m_d) (\langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d) = 2f_{\pi^0}^2 M_{\pi^0}^2 (1 - \delta_{\pi^0})$

**\* Neutral pion remains as a Goldstone boson with  $eB$  up to  $\sim 3.5$   $\text{GeV}^2$**

**★ GMOR relation holds in the presence of strong magnetic field**

# Masses of $\pi^{0,\pm}$ and $K^{0,\pm}$ and pressure

$N_f=2+1$  QCD,  $M_\pi(eB=0) \approx 220$  MeV,  
 $32^3 \times 96$  lattices with  $a^{-1} \approx 1.7$  GeV and HISQ action



H.-T. Ding, STL, P. Hegde O. Kaczmarek et al. [HotQCD], Phys. Rev. Lett. 123 062002

H.-T. Ding, STL, et al. Nuclear Physics A 1005 (2021):121940

H.-T. Ding, STL, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

See quenched LQCD results in Bali et al., PRD 97 (2018) 034505,  
 Lushevskaya et al., NPB 898 (2015) 627

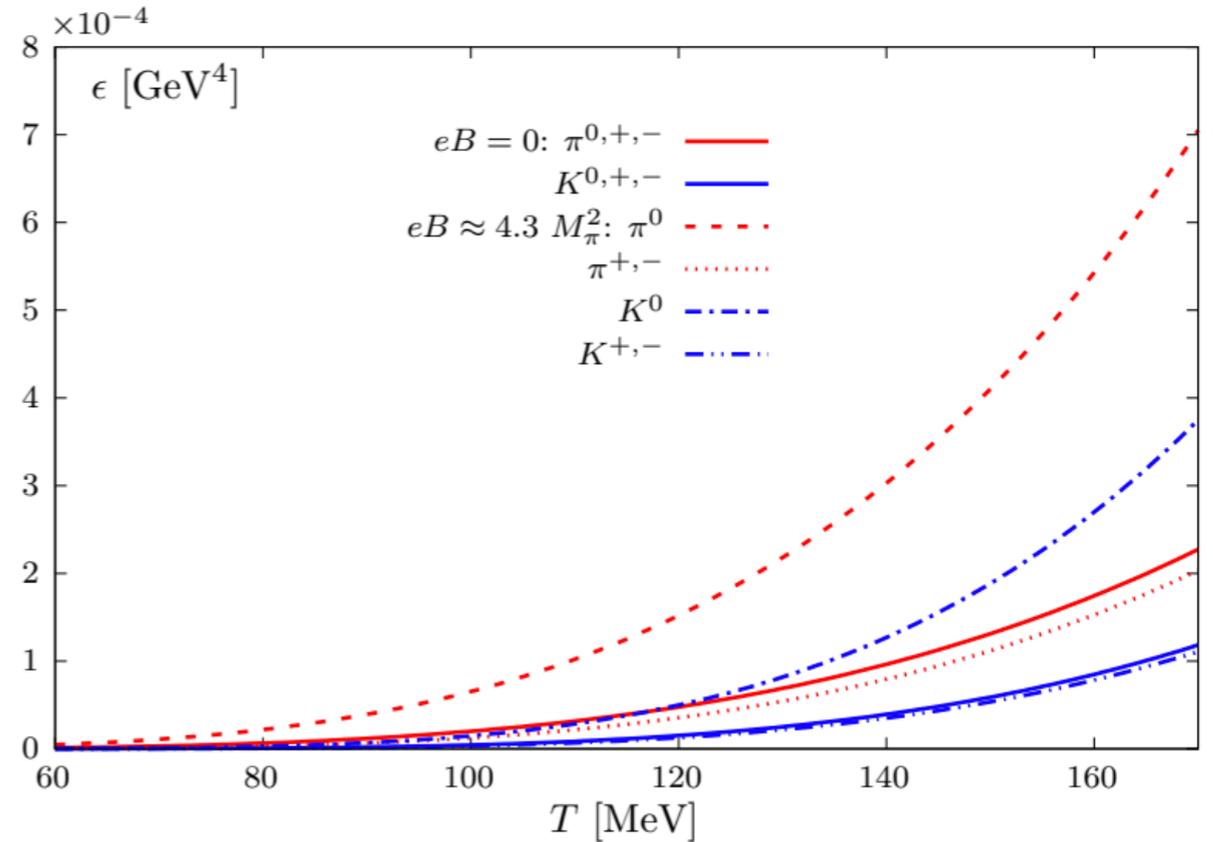
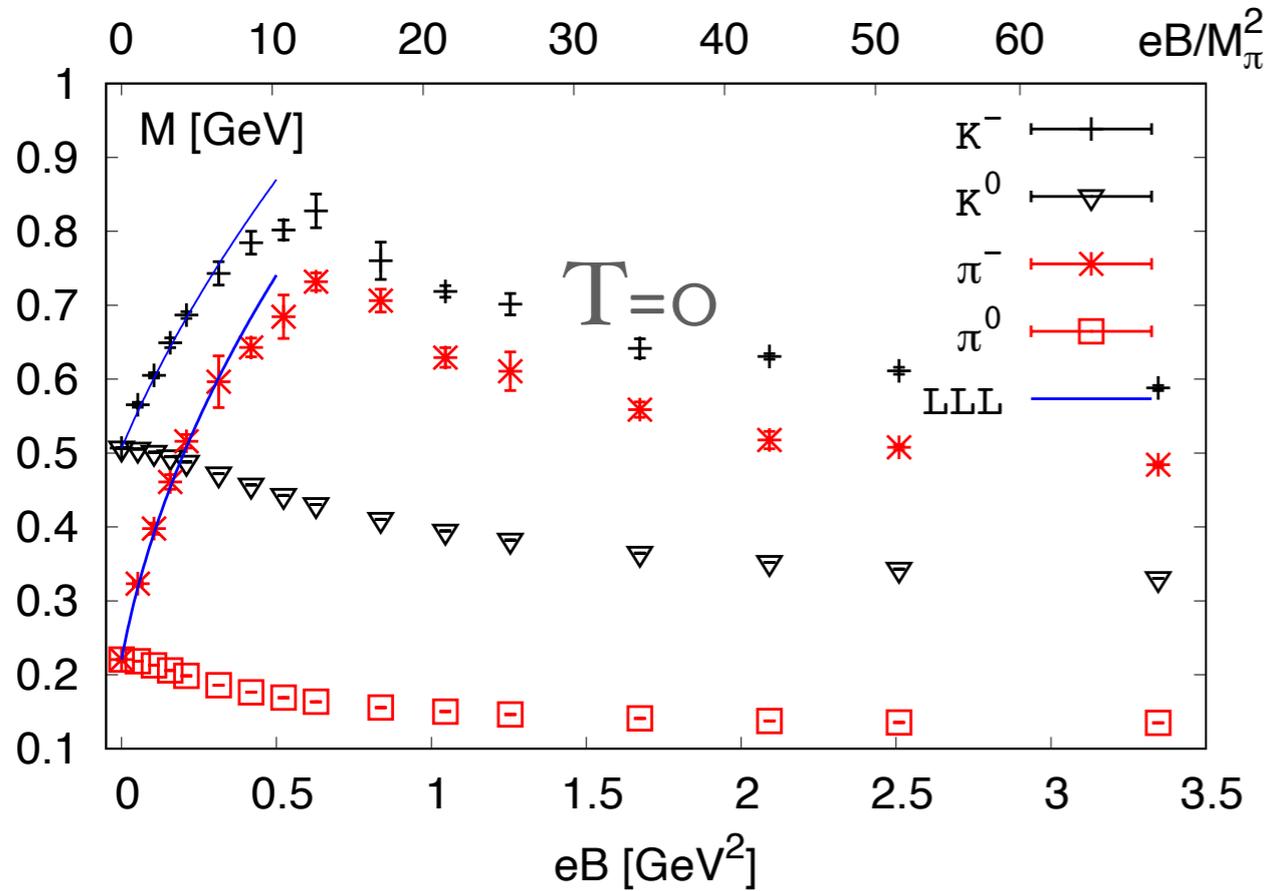
Pion is the lightest Goldstone boson

$T_{pc}$  decrease as decrease  $M_\pi$

# Masses of $\pi^{0,\pm}$ and $K^{0,\pm}$ and pressure

$N_f=2+1$  QCD,  $M_\pi(eB=0) \approx 220$  MeV,  
 $32^3 \times 96$  lattices with  $a^{-1} \approx 1.7$  GeV and HISQ action

Energy density in Hadron resonance gas model



H.T. Ding, STL, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

See quenched LQCD results in Bali et al., PRD 97 (2018) 034505,  
 Lushevskaya et al., NPB 898 (2015) 627

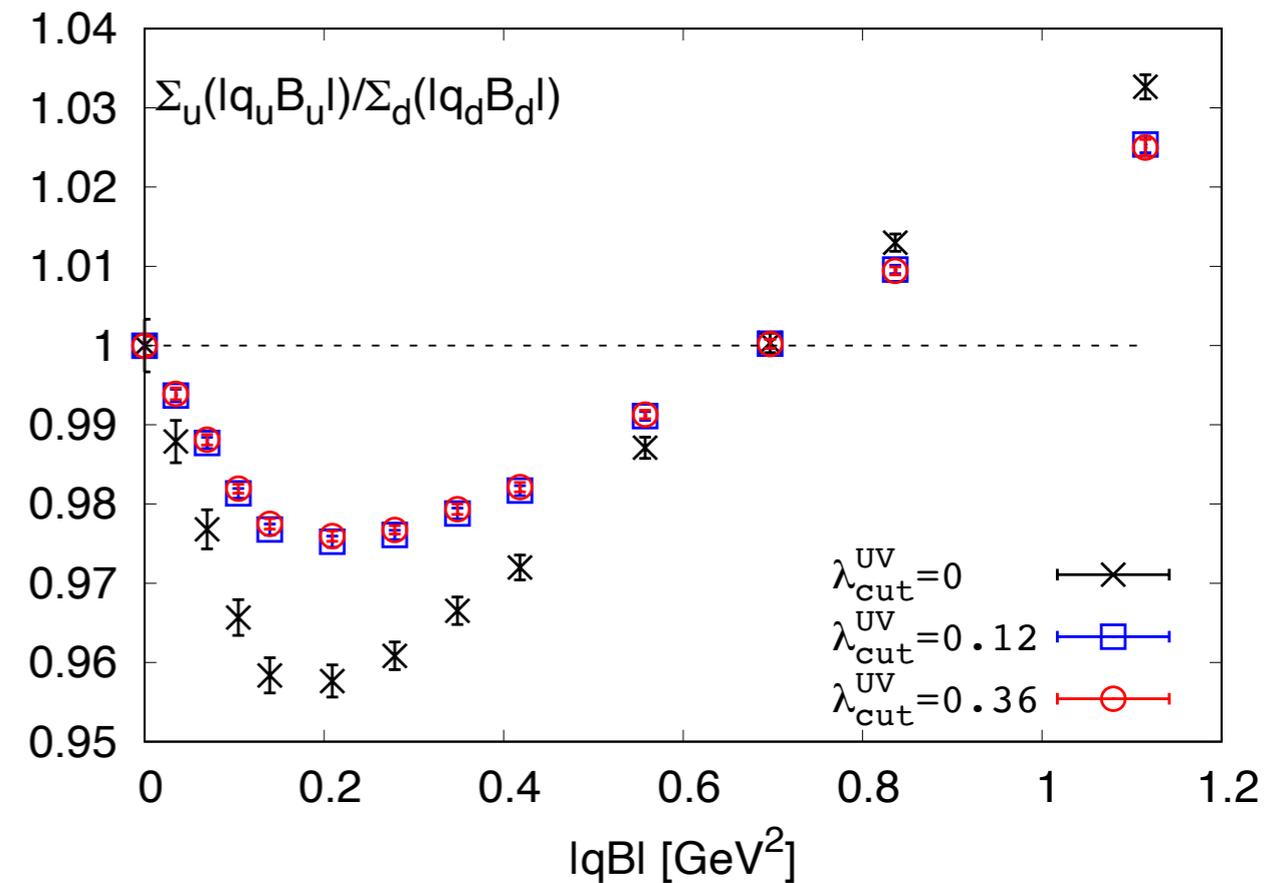
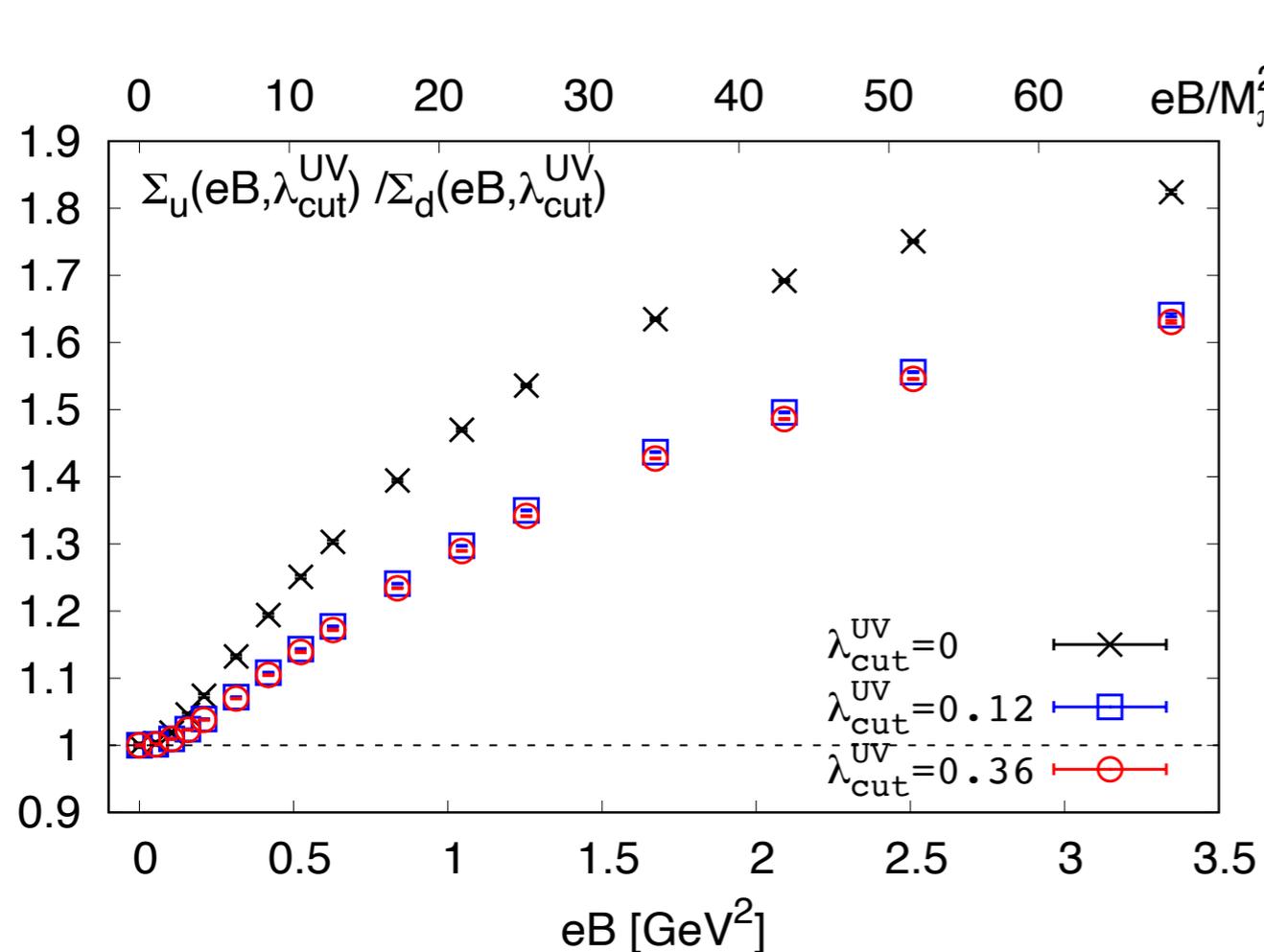
H.T. Ding, STL, Q. Shi, A. Tomiya, X.-D. Wang, Y. Zhang, arXiv: 2011.04870

# Chiral Condensates

complete Dirac eigenvalue spectrum

H.-T. Ding, S.-T. Li, Swagato Mukherjee, A. Tomiya, X.-D. Wang, and Y. Zhang  
Phys. Rev. Lett. 126, 082001(2021)

$$\Sigma_l(B, \lambda_{cut}^{UV}) = \frac{2m_l}{M_\pi^2 f_\pi^2} \left( \langle \bar{\psi} \psi \rangle_l(B) - \langle \bar{\psi} \psi \rangle_l^{UV}(B=0, \lambda_{cut}^{UV}) \right) + 1$$

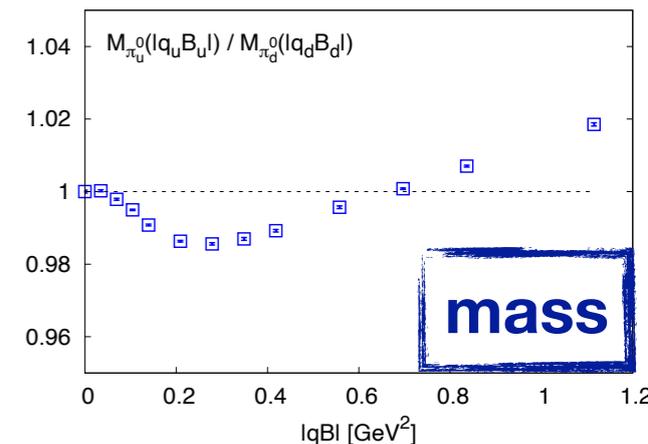
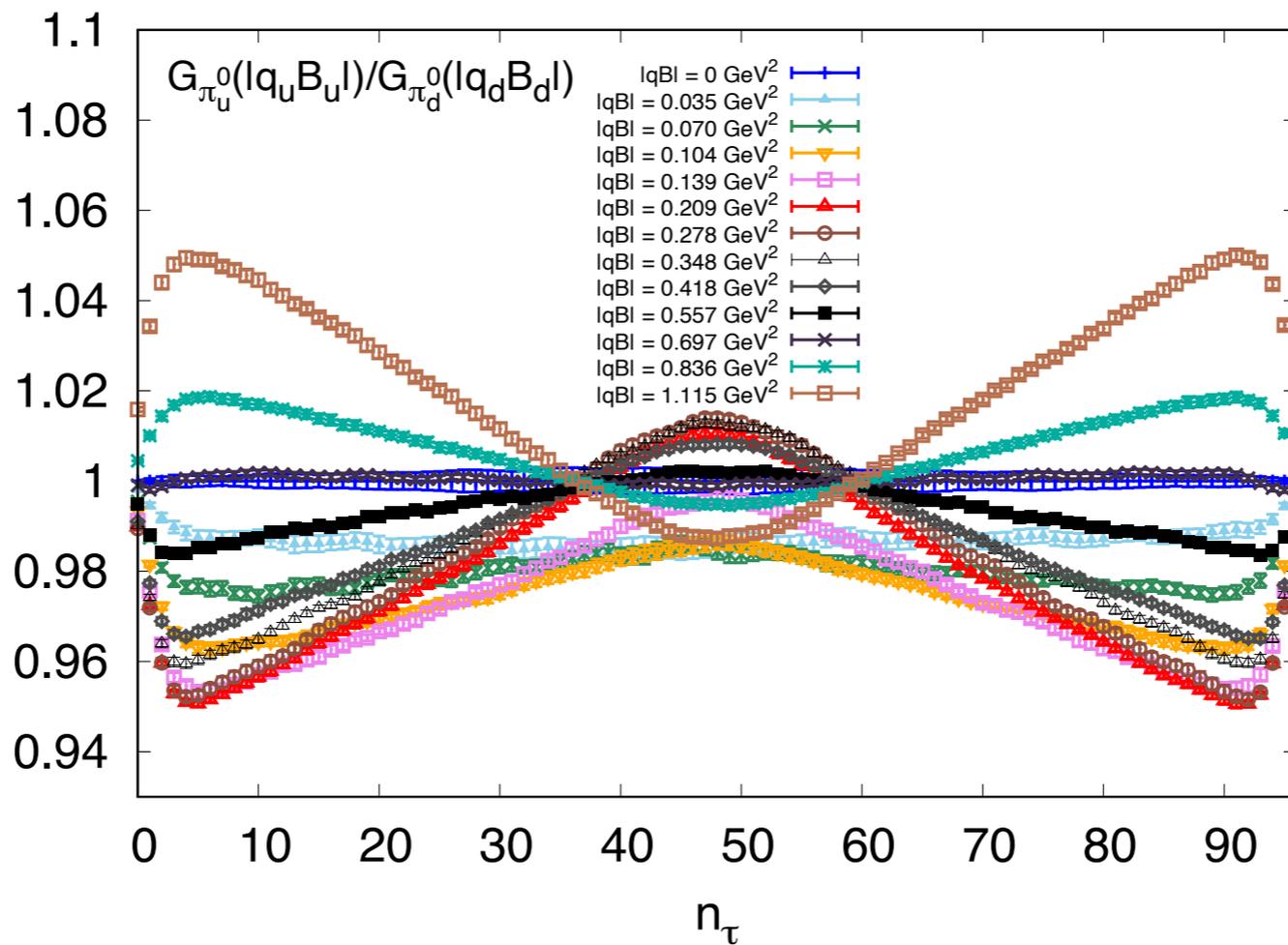
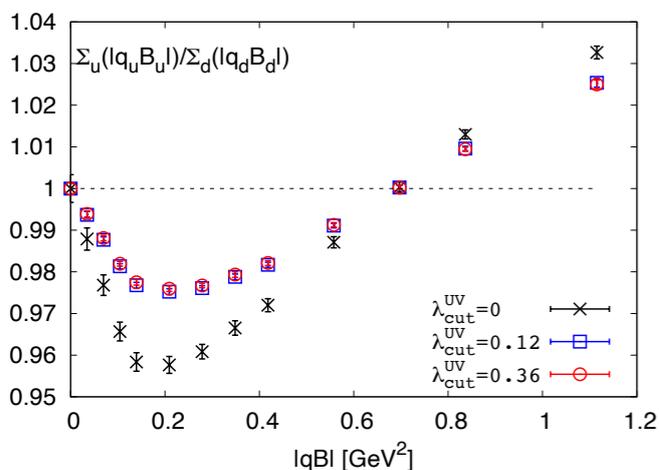


**qB scaling:**

**It is the qB that affects the behavior of quantities**

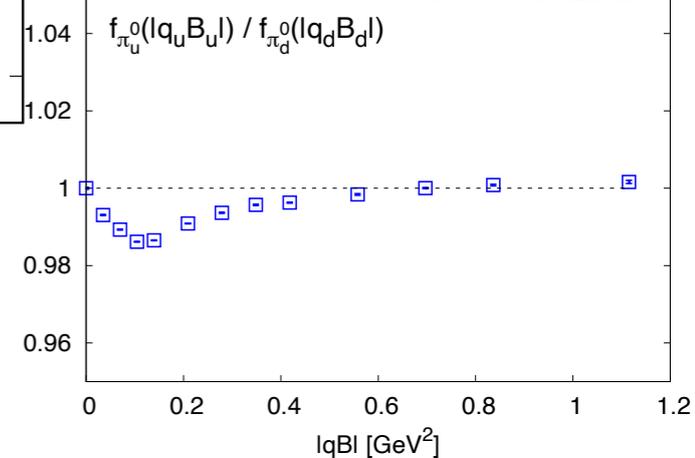
# qB scaling

chiral condensate



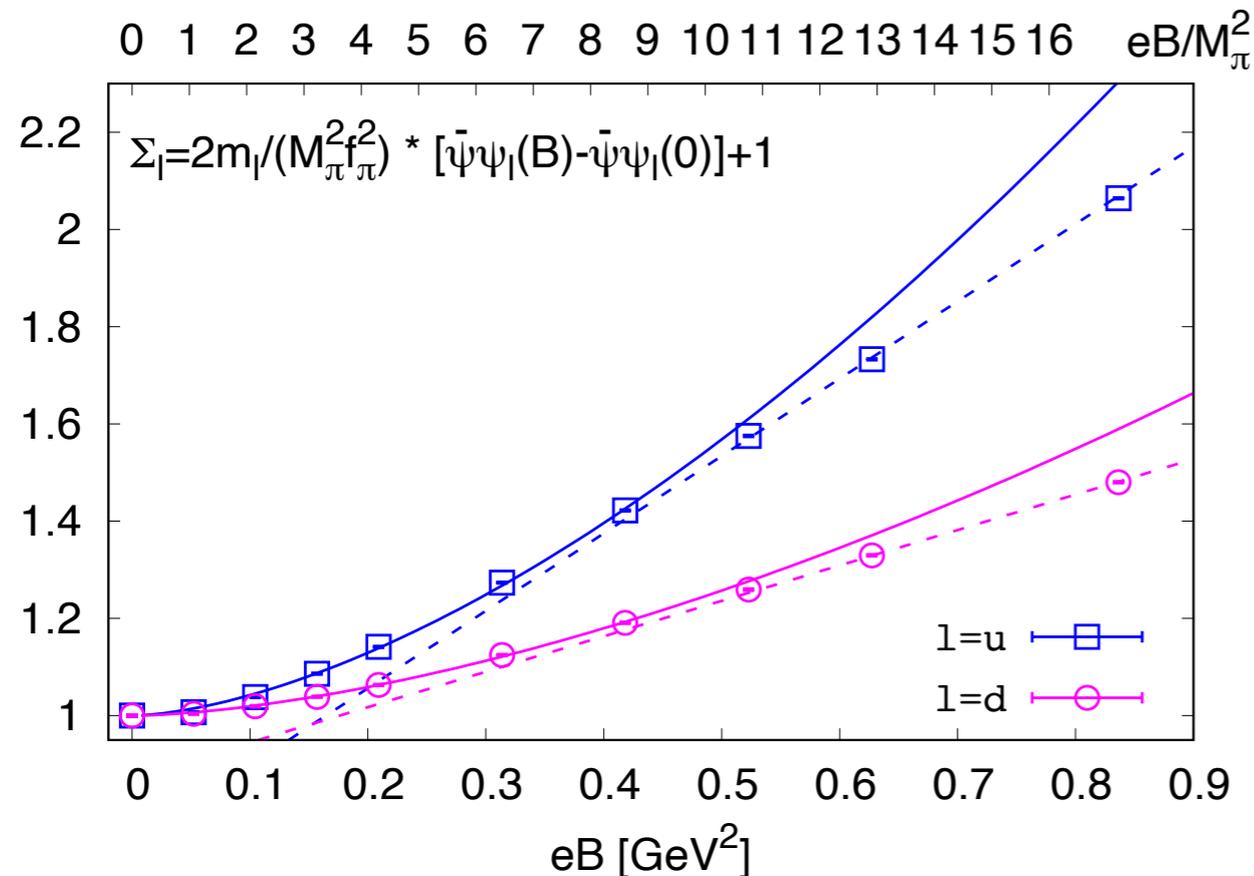
mass

decay constant



The origin of all is the correlator,  $G_{\pi_u^0}(\tau, q_u B_u) / G_{\pi_d^0}(\tau, q_d B_d)$  itself holds for qB scaling.

# Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates



**Not accessible in HIC experiments**

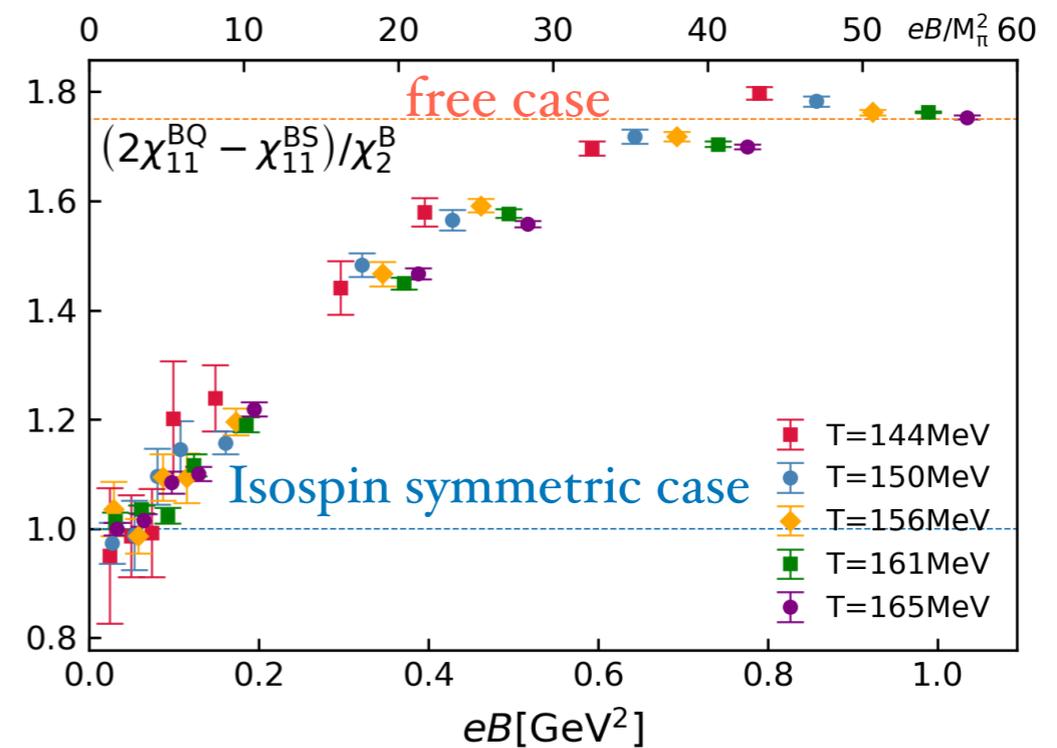
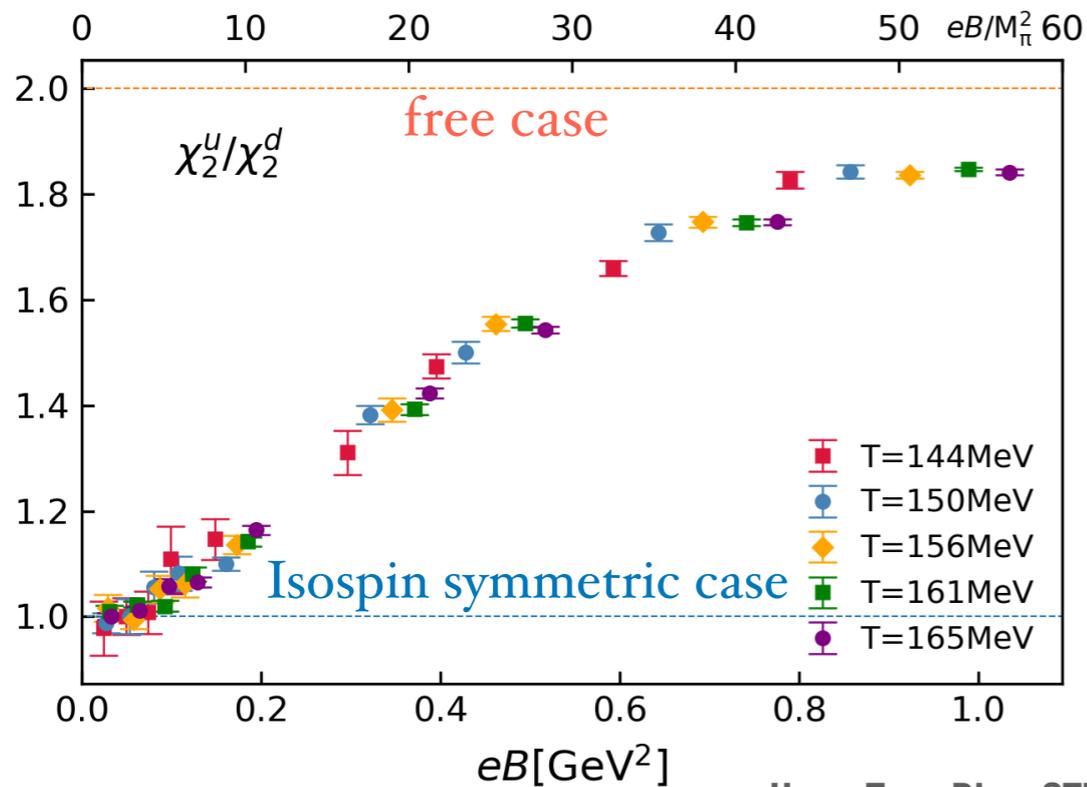


H.-T. Ding, STL, A. Tomiya, X.-D. Wang and Y. Zhang, PRD 126 (2021) 082001

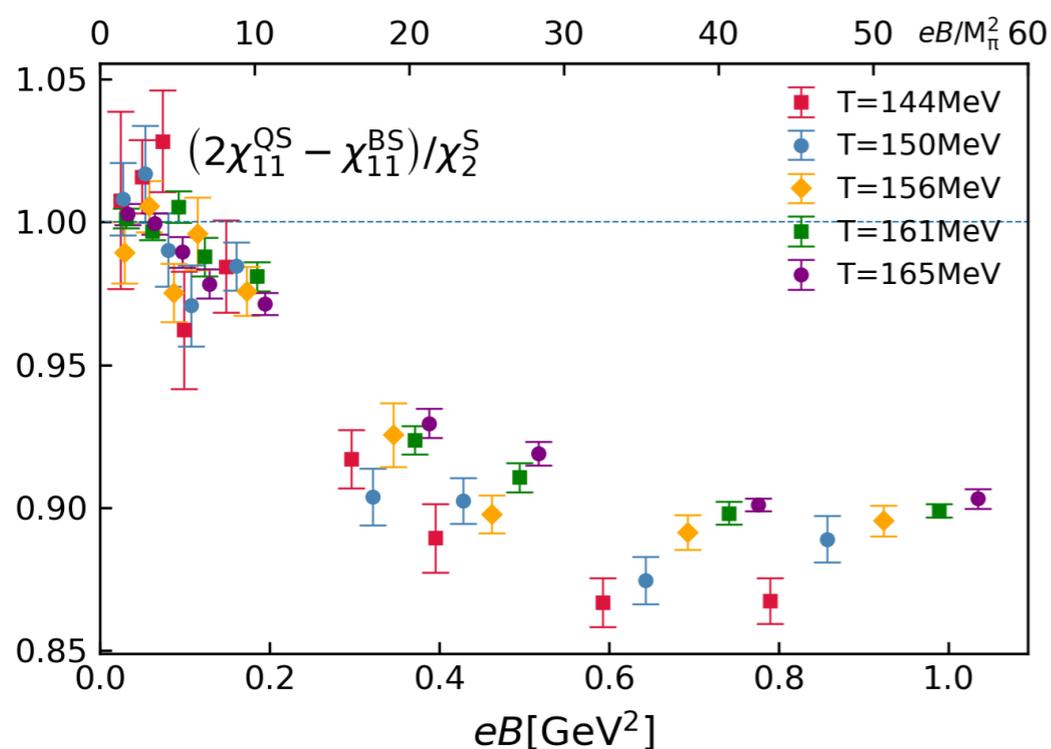
See also in reviews e.g. M. D'Elia, Lect.NotesPhys.871(2013)181

# Isospin symmetry breaking at $eB \neq 0$ with physical pion mass

$N_f=2+1$  QCD,  $M_\pi(eB=0) \approx 135$  MeV,  $T_{pc}(eB=0) \approx 157$  MeV,  $32^3 \times 8$  lattices with HISQ action



Heng-Tong Ding, STL, Jun-Hong Liu, Xiao-Dan Wang, work in progress

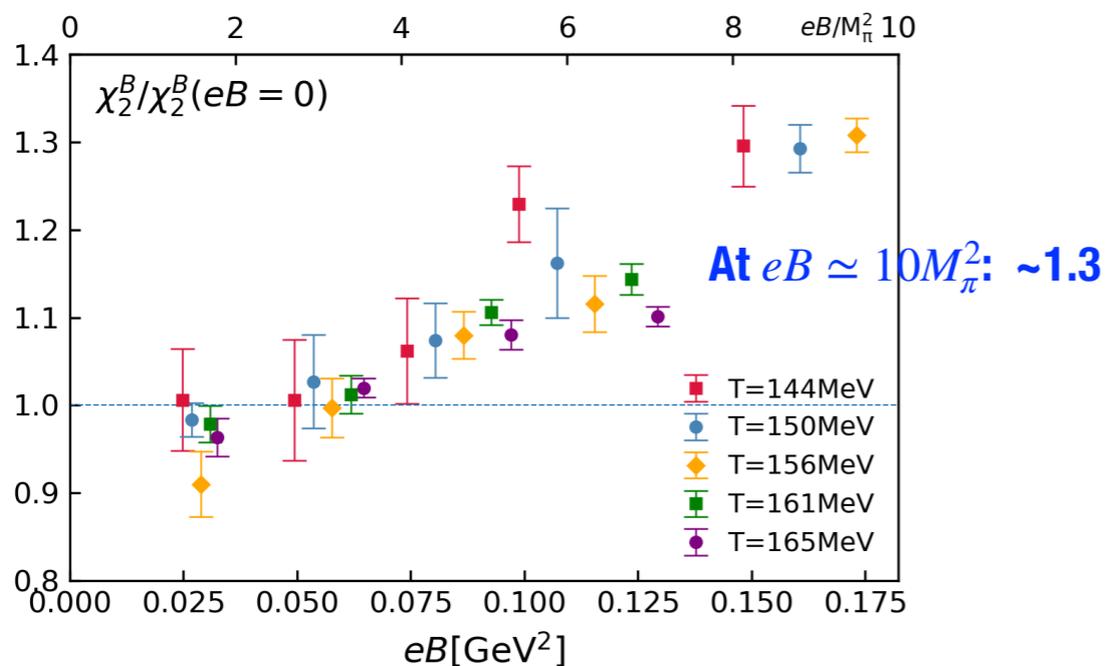


**At  $eB=0$ :**  $\chi_2^u / \chi_2^d = 1,$   
 $2\chi_{11}^{BQ} - \chi_{11}^{BS} = \chi_2^B,$   
 $2\chi_{11}^{QS} - \chi_{11}^{BS} = \chi_2^S$

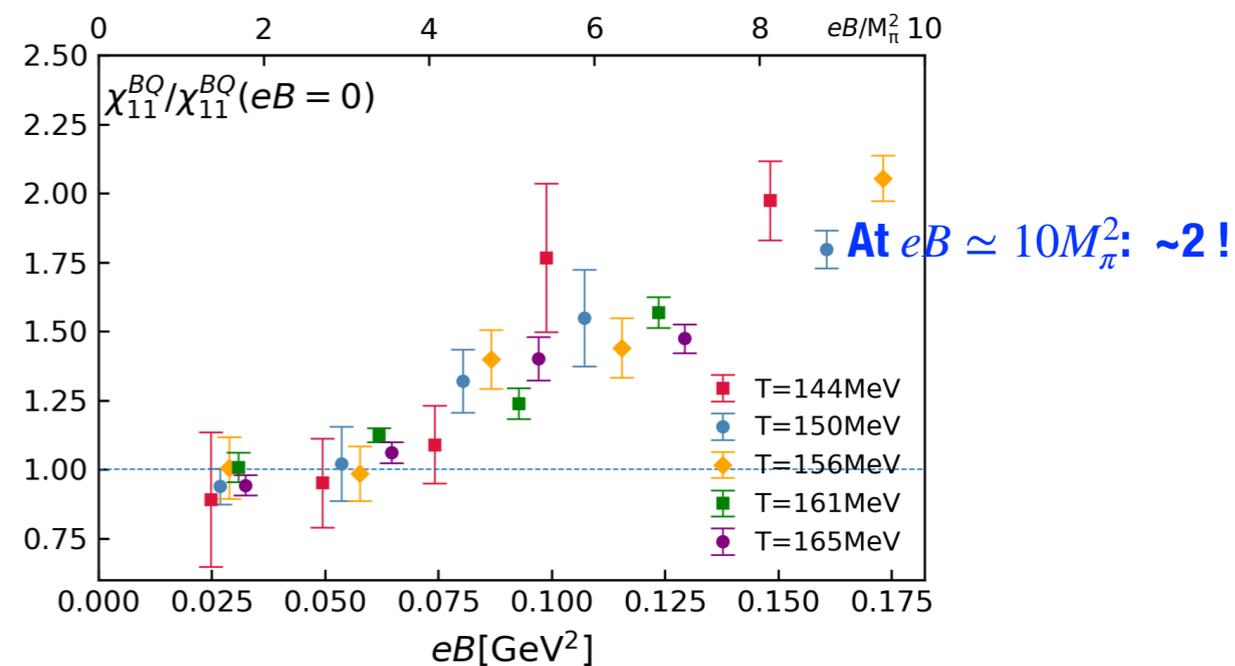
**Quantities to study isospin symmetry at  $eB \neq 0$**

# Conversed charge number fluctuation and their correlations at $T \neq 0, eB \neq 0$

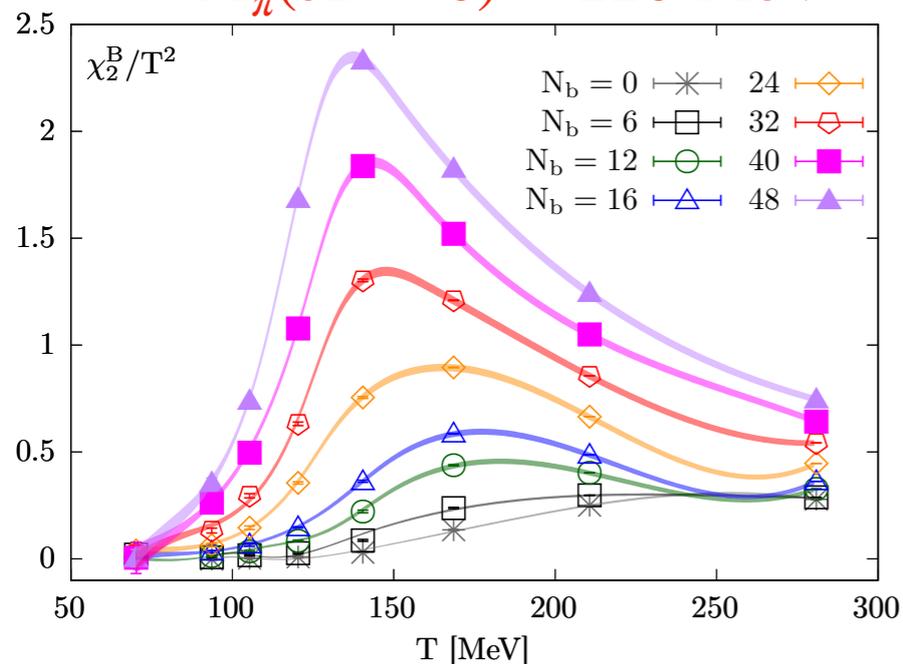
$M_\pi(eB = 0) \approx 135 \text{ MeV}$



$M_\pi(eB = 0) \approx 135 \text{ MeV}$



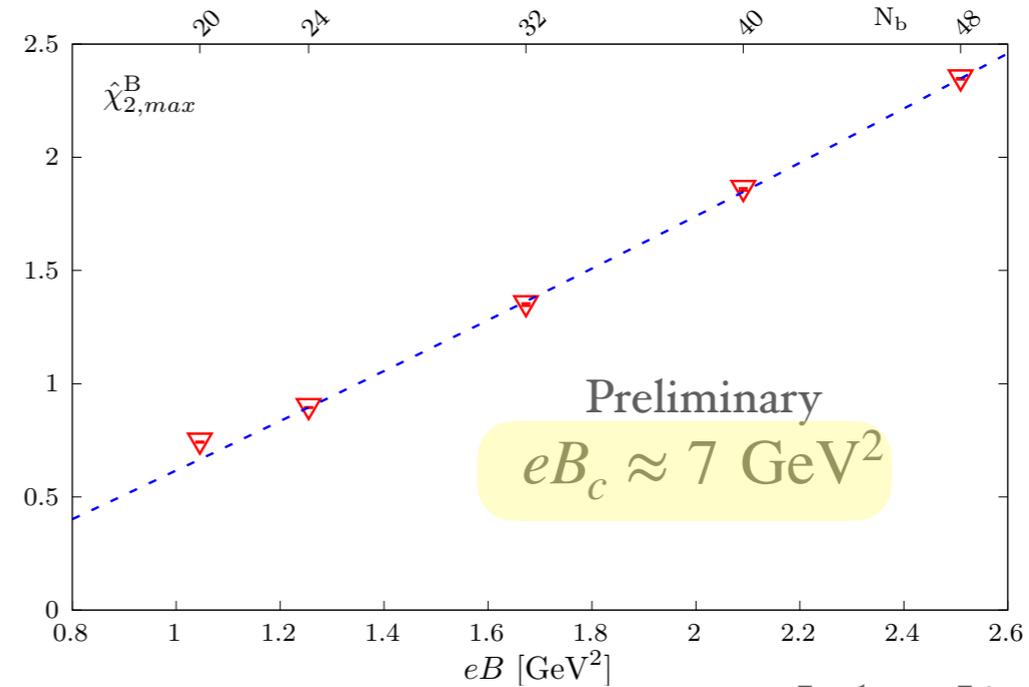
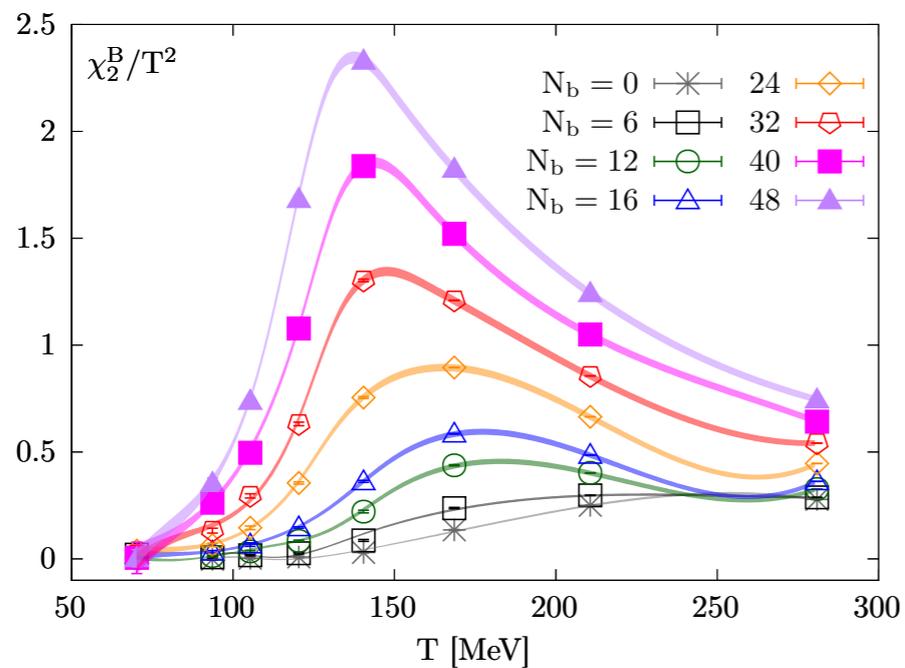
$M_\pi(eB = 0) \approx 220 \text{ MeV}$



- The quantities to detect  $eB$  in HIC experiments
- $eB \uparrow, T_{pc} \downarrow$ , stronger phase transition & closer to  $Z(2)$  critical point in  $T$ - $eB$  plane

# Conversed charge number fluctuation and their correlations at $T \neq 0, eB \neq 0$

$N_f=2+1$  QCD,  $M_\pi(eB=0) \approx 220$  MeV, with  $a^{-1} \approx 1.7$  GeV and HISQ action, fixed  $a$  approach ( $T = a^{-1}/N_\tau$ )



Junhong Liu, work in progress

$$\chi_{2,max}^B = b (eB_c - eB)^{(1-\alpha)/\beta\delta} + d$$

At  $eB=0$ :  $\chi_n^B \propto (-2\kappa_q)^{n/2} h^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z)$

Friman et al., Eur. Phys. J. C 71(2011)1694

Model	$(1-\alpha)/\beta\delta$
Z(2)	0.5693
O(4)	0.6643

★ The possible  $eB_c$  is around 7 GeV<sup>2</sup>

# Summary

- GMOR relations valid with small corrections at zero T**
- $M_{\pi^0}$  decreases with increasing  $eB$  at  $T=0$**
- The 2nd order fluctuations and correlations of B, Q & S are strongly affected by  $eB$** 
  - (1) Could be useful to detect the existence of a magnetic field in HIC**
  - (2) Analogy to study the QCD critical point in the  $T-\mu_B$  plane**

**Thank you for your attention!**