QCD phase structure in strong magnetic fields

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Outline

- Motivation and Introduction
- Lattice Setup
- Results
- Summary
Motivation

Heavy-Ion collisions

**RHIC:** \( eB \sim O(1) m_\pi^2 \)

**LHC:** \( eB \sim O(10) m_\pi^2 \)


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**T-eB plane**

- deconfinement transition line
- prediction
- crossover
- critical point
- first order


- A possible Z(2) critical end point in T-eB plane ?
- How to detect eB in HIC experiments ?
First LQCD study on GMOR relation at eB ≠ 0 and T=0

\[ 4m_u \langle \bar{\psi} \psi \rangle_u = 2f_{\pi^0_u}^2 M_{\pi^0_u}^2 (1 - \delta_{\pi^0_u}) \]

\[ 4m_d \langle \bar{\psi} \psi \rangle_d = 2f_{\pi^0_d}^2 M_{\pi^0_d}^2 (1 - \delta_{\pi^0_d}) \]

\[ (m_u + m_d) \left( \langle \bar{\psi} \psi \rangle_u + \langle \bar{\psi} \psi \rangle_d \right) = 2f_{\pi^0}^2 M_{\pi^0}^2 (1 - \delta_{\pi^0}) \]

Neutral pion remains as a Goldstone boson with eB up to ~3.5 GeV^2

GMOR relation holds in the presence of strong magnetic field
Masses of $\pi^0,\pm$ and $K^0,\pm$ and pressure

$N_f=2+1$ QCD, $M_\pi(eB = 0) \approx 220$ MeV, $32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action

Pion is the lightest Goldstone boson

$T_{pc}$ decrease as decrease $M_\pi$
Masses of $\pi^{0,\pm}$ and $K^{0,\pm}$ and pressure

$N_t=2+1$ QCD, $M_\pi(eB = 0) \approx 220$ MeV, $32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action

Energy density in Hadron resonance gas model

H.T. Ding, STL, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001


Chiral Condensates

\[ \Sigma_l(B, \lambda_{UV}^\text{cut}) = \frac{2m_l}{M^2_{\pi} f^2_{\pi}} \left( \langle \bar{\psi} \psi \rangle_l(B) - \langle \bar{\psi} \psi \rangle_l^\text{UV}(B = 0, \lambda_{UV}^\text{cut}) \right) + 1 \]

qB scaling:
It is the qB that affects the behavior of quantities
The origin of all is the correlator, $G_{\pi u}(\tau, q_u B_u)/G_{\pi d}(\tau, q_d B_d)$ itself holds for qB scaling.
Isospin symmetry breaking at $eB \neq 0$ manifested in chiral condensates

\[
\Sigma_l = \frac{2m_l}{(M_{\pi}^2/M_{\pi}^2)} \times [\bar{\psi}\psi_l(B) - \bar{\psi}\psi_l(0)] + 1
\]

Not accessible in HIC experiments


See also in reviews e.g. M. D’Elia, Lect.NotesPhys.871(2013)181
Isospin symmetry breaking at $e B \neq 0$ with physical pion mass

$N_f=2+1$ QCD, $M_\pi(eB = 0) \approx 135$ MeV, $T_{pc}(eB = 0) \approx 157$ MeV, $32^3 \times 8$ lattices with HISQ action

At $eB=0$: $\frac{\chi_2^u}{\chi_2^d} = 1$, 
$2\chi_{11}^{BQ} - \chi_{11}^{BS} = \chi_2^B$,  
$2\chi_{11}^{QS} - \chi_{11}^{BS} = \chi_2^S$

Quantities to study isospin symmetry at $eB \neq 0$
Conversed charge number fluctuation and their correlations at $T \neq 0$, $eB \neq 0$

$M_\pi(eB = 0) \approx 135$ MeV

- At $eB \simeq 10M_\pi^2$: ~1.3

$M_\pi(eB = 0) \approx 220$ MeV

The quantities to detect $eB$ in HIC experiments
- $eB \uparrow$, $T_{pc} \downarrow$, stronger phase transition & closer to $Z(2)$ critical point in $T$-$eB$ plane
Conversed charge number fluctuation and their correlations at \( T \neq 0, eB \neq 0 \)

At eB=0: \( \chi^B_2 \propto (-2 \kappa_q)^{n/2} h(2 - \alpha - n/2) / \beta \delta f_f^{(n/2)}(z) \)

\( N_f=2+1 \) QCD, \( M_\pi(eB=0) \approx 220 \) MeV, with \( a^{-1} \approx 1.7 \) GeV and HISQ action, fixed \( \alpha \) approach \( (T = a^{-1}/N_c) \)

\[ \chi^B_{2,\text{max}} = b \left( eB_c - eB \right)^{(1-\alpha)/\beta \delta} + d \]


\( \approx 7 \) GeV²

Junhong Liu, work in progress

\( eB_c \approx 7 \) GeV²

\( \star \) The possible \( eB_c \) is around 7 GeV²

<table>
<thead>
<tr>
<th>Model</th>
<th>((1-\alpha)/\beta \delta)</th>
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<tr>
<td>Z(2)</td>
<td>0.5693</td>
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<td>0(4)</td>
<td>0.6643</td>
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Summary

- GMOR relations valid with small corrections at zero $T$
- $M_{\pi^0}$ decreases with increasing $eB$ at $T=0$
- The 2nd order fluctuations and correlations of $B$, $Q$ & $S$ are strongly affected by $eB$

(1) Could be useful to detect the existence of a magnetic field in HIC
(2) Analogy to study the QCD critical point in the $T$-$\mu_B$ plane
Thank you for your attention!